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SIMPLE CORRELATION TO EVALUATE EFFICIENCY OF ANNULAR ELLIPTICAL FIN CIRCUMSCRIBING CIRCULAR TUBE

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ABSTRACT

Efficiency of annular elliptical fin has been studied numerically. Results show that constant temperature lines do not remain circular in high aspect ratio fins. So, it is expected that some common methods such as equivalent fin area or sector method do not have enough accuracy. For this reason, at first, a new simple correlation has been proposed to approximate annular circular fin efficiency based on longitudinal fin efficiency and then comparing to the numerical results, it has been shown that the proposed formula is still applicable if one uses arithmetic mean of elliptical fin lengths and geometric mean of elliptical fin radiuses.

1. INTRODUCTION

Fins or extended surfaces are used vastly in industry to increase the heat transfer area. The usefulness of fin employing is judged by fin efficiency and many investigations are dedicated to improve fin efficiency [1-3]. Fin efficiency is defined as the ratio of the actual heat transfer to the ideal heat transfer in which, all fin area is in its base temperature.

One of the most common fins is annular fin. Although circular type of annular fin has been studied widely, the elliptical one has not been studied yet. Pressure drop across elliptical fin is lesser than circular fin. Moreover, since the heat transfer coefficient on the front side of the fin is more than both sides, using elliptical fin is reasonable. Especially, when there is space restriction; elliptical fin can be a good candidate. With those aforementioned priorities, to the best knowledge of the authors, no formula may be found in the literature to predict annular elliptical fin efficiency.

Nagarani and Mayilsamy [4] performed some limited experiments to study the natural convection on specified elliptical fin geometry. Kundu and Das [5] proposed a semi-analytical method similar to sector method to approximate elliptical fin efficiency. However, this method ends to a series of Bessel function families.

In this study, it is decided to propose equivalent thermo-geometry parameters, to be used in circular fin efficiency formula to estimate elliptical fin efficiency. However, the exact solution of circular fin efficiency is a combination of modified Bessel functions which is very tedious. Several attempts have been made to approximate the exact solution with a simple correlation [6-9]; however, none of them has enough accuracy and simplicity. For this reason, at first step, a simple but accurate correlation has been proposed to approximate circular fin efficiency and at the next step, several numerical simulations have been performed to calculate elliptical fin efficiency in a vast range of thermo-geometry parameters. Finally, based on proposed circular fin efficiency formula and numerical results, equivalent parameters have been proposed to approximate elliptical fin efficiency by circular fin efficiency formula.

2. FORMULATION OF THE PROBLEM

An annular elliptical fin surrounding a circular tube is shown in Fig. 1. Fin thickness is $2t$ and tube radius is r_b. The major and minor radiuses of ellipse are r_1 and r_2, respectively. It is assumed that heat is exchanged with constant temperature ambient, T_∞ only by convection and also, the convection coefficient h is constant. Based on the above assumptions, the steady state energy equation for constant fin thermal conductivity, k, in polar coordinate system may be written as:

$$\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right)+\frac{1}{r}\frac{\partial^2\theta}{\partial \phi^2}=m^2\theta \tag{1}$$

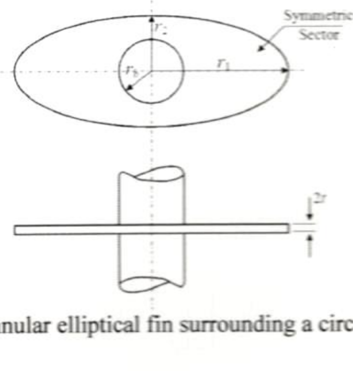

Fig. 1: annular elliptical fin surrounding a circular tube

Due to symmetric form of geometry and heat transfer, only a quarter of fin is required to be considered. Boundary conditions are assumed as follows:

- Outer periphery of the tube is assumed at constant temperature, T_b
- Heat transfer from symmetry lines and fin tip are zero.

In this regard, the boundary conditions may be present mathematically as (Fig. 2):

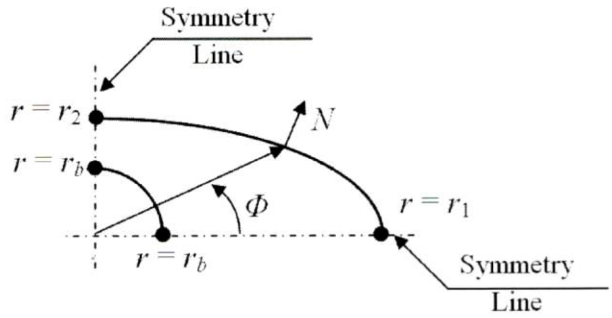

Fig.2: Assumed fin geometry in polar coordinate system

$$\theta = \theta_b \text{ at } r = r_b \ (0 \le \theta \le \pi/2) \tag{2a}$$

$$\partial\theta/\partial\phi = 0 \text{ at } \phi = 0 \ (r_b \le r \le r_1) \tag{2b}$$

$$\partial\theta/\partial\phi = 0 \text{ at } \phi = \pi/2 \ (r_b \le r \le r_2) \tag{2c}$$

$$\partial\theta/\partial N = 0 \text{ at } 0 \le \phi \le \pi/2 \quad \left(r = r_1 r_2 / \sqrt{(r_1\sin(\phi))^2 + (r_2\cos(\phi))^2} \right) \tag{2d}$$

In the preceding equations, θ is $(T - T_\infty)$, $m^2 = \frac{h}{kt}$ and N is normal vector.

Equation (1) is separable and can be solved by separation of variables method [10]. However, for elliptical fin, the forth boundary condition, Eq. (2d), is not separable. So, no analytical solution may be found for Eq. (1). Kundu and Das [5] proposed to divided the fin boundary to finite number of segments and approximate each segment as a circular arc to satisfy Eq. (2d) for each segment. It is somehow similar to sector method [8]. However, it is known that this method results in a series of Bessel function families. Even, with the existence of developed software, this series evaluation is elaborate and tedious.

3. BETTER APPROXIMATION TO CIRCULAR FIN EFFICIENCY

The exact solution of Eq. (1) in circular case in which $\partial^2\theta/\partial\phi^2 = 0$ and $r_1 = r_2 = r_f$ is a combination of Bessel function families. So, it is not a good base to approximate the elliptical fin efficiency. The famous approximation to the exact solution of Eq. (1) has been proposed by Schmidt [6] based on longitudinal fin efficiency. Some modifications have been proposed [7, 8] to increase the accuracy of this approximation by price of more complexity and restrictions. The general form of longitudinal fin based approximation is:

$$\eta = \frac{\tanh(m\psi L)}{m\psi L}\cos(m\psi L s) \tag{3}$$

in which $L = r_f - r_b$ is fin length. The other parameters are presented in Table 3.

Table 3: Parameters for longitudinal fin approximation ($R_f = r_f / r_b$)

	s	ψ
Longitudinal fin	0	1
Schmidt [6]	0	$1 + 0.35\ln(R_f)$
Hong and Webb [7]	0.1	$1 + 0.35\ln(R_f)$
Perrotin and Clodic [8]	0.1	$1 + \left[0.3 + \left(0.26R_f^{0.3} - 0.3\right)\left(\frac{mL}{2.5}\right)^{1.5 - \frac{R_f}{12}}\right]\ln(R_f)$

However, it should be noted that in the limiting case in which $R_f = 1$, those approximations should be coincided with longitudinal fin efficiency formula, Eq. (3). So, the aforementioned modifications are not satisfactory. In this regard the following approximation is proposed for circular fin efficiency:

$$\eta = \left[\frac{\tanh(m\psi L)}{m\psi L}\right]^{\psi} \tag{4}$$

in which:

$$\psi = 1 + 0.17912\ln(R_f) \tag{5}$$

This formula is simple and accurate and in the limiting case, $R_f = 1$, returns to longitudinal fin efficiency formula. Comparison of relative errors of all expressions is presented in Fig. 3 in which, E is defined as difference between exact value and approximation of efficiency divided by exact value.

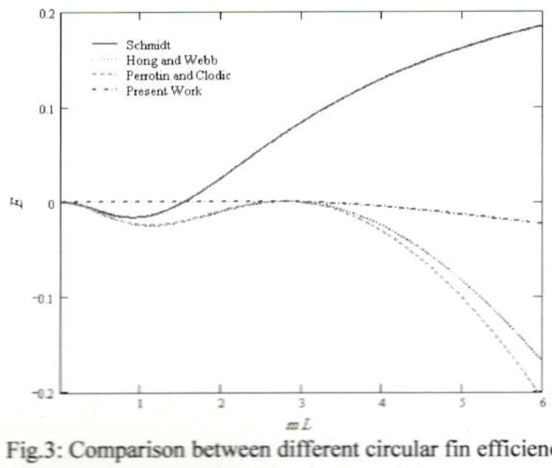

Fig.3: Comparison between different circular fin efficiency approximations for $R_f = 2.5$

4. APPROXIMATION OF ELLIPTICAL FIN EFFICIENCY

To calculate the elliptical fin efficiency, one hundred eighty models were simulated numerically using finite volume method. Those models cover a vest range of thermo-geometry parameters for them: $1 \le \frac{r_1}{r_2} \le 10$, $1.25 \le \frac{r_1}{r_b} \le 12.5$ and $0.6 \le m \le 2700$. Applied boundary conditions were the same as Eq. (2). A very fine structured mesh was used to be ensured about numerical results. It was observed that for r_1/r_2 near to unity, lines of constant temperature remain circular, approximately (Fig. 4a). However, by increasing radiuses ratio, temperature contours will no longer be circular (Fig. 4b). So, it is expected that usual methods such as sector method would not be applicable.

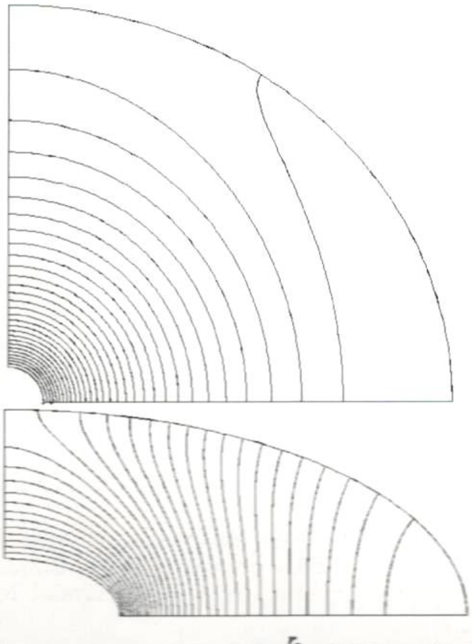

Fig.4: temperature contours, a) for $\frac{r_2}{r_1} = 0.9$ (approximately circular) b) for $\frac{r_2}{r_1} = 0.3$ (noncircular)

Based on numerical results, it was found out that one can use arithmetic mean of major and minor fin lengths instead of L in Eq. 4 and geometric mean of major and minor normalized radiuses instead of R_f to have elliptical fin efficiency. So, the following formulas will cover all longitudinal, circular and elliptical annular fins.

$$\eta = \left[\frac{\tanh(m\psi\bar{L})}{m\psi\bar{L}}\right]^{\psi} \tag{6}$$

$$\psi = 1 + 0.179121\ln(\bar{R}_f) \tag{7}$$

in which:

$$\bar{L} = \frac{L_1 + L_2}{2} = \frac{(r_1 - r_b) + (r_2 - r_b)}{2} \tag{8}$$

$$\bar{R}_f = \sqrt{R_{f1}R_{f2}} = \sqrt{\frac{r_1}{r_b}\frac{r_2}{r_b}} \tag{9}$$

Presented formula and numerical results in two limiting cases, $\bar{R}_f = 1.185$ and $\bar{R}_f = 11.85$ are shown in Fig. 5. A good agreement can be observed.

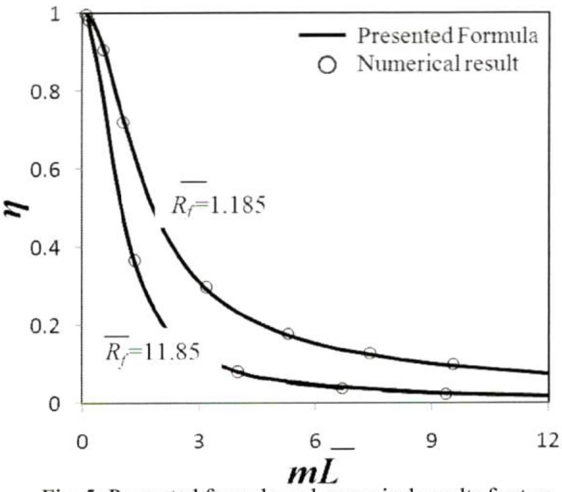

Fig. 5: Presented formula and numerical results for two limiting cases, $\bar{R}_f = 1.185$ and $\bar{R}_f = 11.85$

Also, comparison between numerical results and those obtained by Eq. 6, are depicted in Fig. 6.

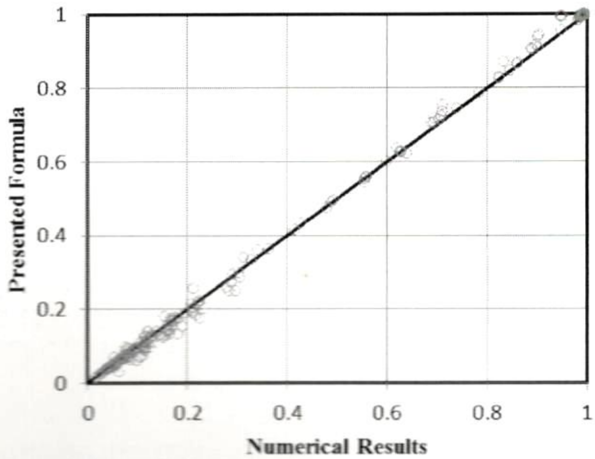

Fig. 6: Comparison between numerical results and predicted values

5. CONCLUSIONS

The efficiency of elliptical annular fin was studied at the present work. Since no analytical solution may be found to calculate the efficiency, numerical method was employed for a vast range of thermo-geometric parameters. At the first stage a new simple but accurate correlation was proposed to predict circular annular fin efficiency. In the next step, based on numerical results it was shown that the proposed formula for annular circular fin is still applicable if one uses arithmetic mean of elliptical fin lengths and geometric mean of elliptical fin radiuses.

6. NOMENCLATURE

h	Convection heat transfer coefficient (W/m^{2}K)
k	Fin thermal conductivity (W/mK)
L	Fin length, $(r_f - r_b)$(m)
$\overline{L}$	Arithmetic mean of major and minor fin lengths, $\frac{L_1 + L_2}{2}$ (m)
L_1	Major fin lengths, $(r_1 - r_b)$ (m)
L_2	Major and minor fin lengths, $(r_2 - r_b)$ (m)
m	Thermo-geometry parameters, $\sqrt{\frac{h}{kt}}$
N	Normal vector
r	Radial distance from tube center (m)
r_1	Fin major radios (m)
r_2	Fin minor radius (m)
r_b	Tube radius (m)
r_f	Circular fin radius (m)
R_f	Normalized fin radius, r_f/r_b
R_{f1}	Major normalized radiuses, r_1/r_b
R_{f2}	Minor normalized radiuses, r_2/r_b
$\overline{R}_f$	Geometric mean of major and minor normalized radiuses, $\sqrt{R_{f1}R_{f2}}$
s	Fin efficiency parameter, Table 3
t	Fin semi-thickness (m)
T_∞	Ambient temperature (K)
T_b	Fin base Temperature (K)

Greek symbols

η	Fin efficiency
ϕ	Angular position from horizontal symmetric line (rad)
ψ	Fin efficiency parameter, Table 3
θ	$(T - T_\infty)$(K)

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