



Performance of Angstrom Model under Algerian Climate

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ABSTRACT

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This study was conducted to estimate the monthly global solar irradiation in the Algerian territory using the Angström model, for seven sites in Algeria, namely: Algiers, Ain Bessem, Constantine, Oran, Djelfa, Mascara and Tamanrasset. The models are based on global solar irradiation in the horizontal surface as well as sunshine hours. To evaluate the results obtained by the different sites we calculated the following relative average algebraic differences: MAE, MBE and RMSE. The results show a remarkable agreement between the measured and computed values, especially in Tamanrasset site where the slope is in the order of 1. Obtained estimates can be generalized on the whole Algerian territory to provide ease way for researchers to get solar irradiation. However, optimization efforts have to be made to reduce mean error. Future works can be made based on our estimates to study effect of temperature and climatic parameters on the model. This work aim is achieved providing a data generator (software).

1. INTRODUCTION

The amount of solar irradiation received throughout the Algerian territory exceeds 2000 hours per year [1], which represents a high average compared to the countries of the world [2, 3].

To exploit this energy as a source of photovoltaic systems or thermal photos [4-7], it is necessary to know the quantity of the solar irradiation received by the different regions of the Algerian territory. The best source of this information is the data measured using the pyranometer.

The lack of means of measurement, the unavailability of meteorological stations all over and the surface area of the studied regions, all these present difficulties that obliged the researchers in the different countries of the world to develop models aiming to estimate global solar irradiation at any site. In this work the model is presented based on the linear Angström equation. Other investigations can be found in the referenced papers [8-38] for various solar irradiation models.

Through this recent study, a novel model has been established in order to estimate global solar irradiation from sunshine hours for seven Algerian locations. This model is based on the well-known Angstrom-PreScott linear regression. Statistical analysis of the results was performed using the root mean square error (RMSE), the mean bias error (MBE) and the mean absolute error (MAE) criteria.

The present paper is organized into three sections, the first describing used methods, the second gives information on studied sites and the third contains different obtained results a discussion.

2. STATISTICAL METHODS

In this work, we are interested in these methods. They have been used for the development of solar irradiation estimation models in Algeria. This method uses statistical relationships between solar irradiation and insolation [10-13], we present a review of different models used.

2.1 Linear relationships

Linear relationship between daily and monthly solar irradiation with sunstroke are considered the simplest and best models for estimating solar irradiation on a horizontal plane in sites where measuring stations are available, they can be applied in sites with the same climate and measured values are unavailable.

The constants of these mathematical relations are coefficients, which are based on the climate of the studied area; they are well represented by the Angström model.

2.2 Applied model

For the evaluation of the global solar irradiation on the chosen sites in our work, we chose the Angström model.

Angström model [14]: is the first model used, it is given by the relation (1):

$$\frac{G}{G_0} = a + b\left(\frac{S}{S_0}\right) \quad (1)$$

G_0 is calculated using Eq. (2) [15]:

$$G_0 = \frac{24}{\pi} I_0 \left(1 + 0.033 \cos \frac{360n}{365}\right) \times \left(\cos \lambda \cos \delta \sin \omega + \frac{2\pi}{360} w \sin \lambda \sin \delta\right) \quad (2)$$

where,

G is the average monthly global irradiation measured on a horizontal plane ($\text{MJ.m}^{-2}.\text{j}^{-1}$),

G_0 is the monthly average global irradiation outside the atmosphere on a horizontal plane ($\text{MJ.m}^{-2}.\text{j}^{-1}$),

I_0 is the solar constant, is equal to 1367 Wm^{-2} ,

λ is the latitude of the place,

w is the hour angle, and

δ is the declination, it is given by Eq. (3) [16]:

$$\delta = 23.45 \left[\sin \frac{360(284 + n)}{365} \right] \quad (3)$$

and ω is the sunset hour angle, defined as

$$\omega = \cos^{-1}(-\tan \lambda \tan \delta) \quad (4)$$

n represents the number of the day of the year from the first of January, S represents the duration of the effective sunshine, S_0 represents the maximum duration of sunshine or duration of the day, its expression is given by Eq. (5) [17-19]:

$$S_0 = \frac{2}{15} \omega \quad (5)$$

2.3 Statistical indicators statistical indicators

To determine the constants a and b of Equation (1), we calculated the rate (G/G_0) and (S/S_0) for each day and month for the different sites. To evaluate the results obtained by the different models, we calculated the following relative average algebraic differences:

- MAE (Mean Absolute Error): measures the average magnitude of errors in a series of forecasts, regardless of direction. It measures precision for continuous variables, its expression is given by Eq. (6) [20]:

$$MAE = \sum_1^N |G_{i,m} - G_{i,c}| / N \quad (6)$$

- MBE (Mean Bias Error): provides information on the long-term performance of expressions, offering information regarding more or underestimation of estimated data, given by Eq. (7) [22-23]:

$$MBE = \sum_1^N (G_{i,m} - G_{i,c}) / N \quad (7)$$

- Root Mean Square Error (RMSE): provides information on short-term performance by comparing the difference between estimated and measured values. The smaller the value, the better the model, given by the Eq. (8) [21, 22]:

$$RMSE = \left[\sum_1^N ((G_{i,m} - G_{i,c}))^2 / N \right]^{1/2} \quad (8)$$

m is the number of samples, $G_{i,m}$ and $G_{i,c}$ are the i -th measured and estimated values of global irradiation respectively.

3. STUDIED SITES

3.1 Geographical coordinates

For this study we have chosen seven sites: Algiers, Ain Bessem, Constantine, Oran, Djelfa, Mascara, and Tamanrasset. Measured values of solar irradiation have been provided from Metronome data source, in addition, they cover a considerable area of Algerian territory. Note that:

- Algiers is located in the Algerian north,
- Constantine is located in the Algerian east,
- Oran and Mascara are located in Algerian west,
- Djelfa and Ain Bessem are located in the middle south of Algeria, and
- Tamanrasset is situated in southernmost of Algeria.

Table 1. Geographical coordinates of the sites chosen

Station	Latitude (deg)(N)	Altitude(m)	Longitude (deg)
Oran	35.38	99	0.37W
Djelfa	34.68	1126	3.25E
Ain Bessem	36.31	629	3.67E
Tamanrasset	22.47	1378	5.31E
Alger	36.43	450	2.83E
Constantine	36.17	687	6.37E
Mascara	35.26	518	0.06E

In Table 1 is represented the geographical coordinates of studied sites.

3.2 Metronome data

Metronome is a complete meteorological reference including a catalog of meteorological data and calculation methods for solar applications and system design at any desired location in the world. It is based on more than 23 years of experience in the development of meteorological databases for energy applications. It brings together more than 20 measurement stations around the world. Table 2 presents the overall solar irradiance of studied sites.

Figure 1 shows the distribution of the average effective sunshine duration for a six-year period from the seven selected sites. It is more than 8 hours a day during the winter months and more than 12 hours a day during the months of June to July. The sites are therefore very favorable to the use of solar energy.

Table 2. Summarizes the monthly average of measured values of global solar irradiation

Month	Alger	Ain Bessem	Const.	Djelfa	Mascara	Oran	Tamanrasset
Measured values of global radiation solar G_m (M.J.m ⁻²)							
January	9.95	11.15	11.75	13.01	12.60	12.55	20.98
February	11.92	13.95	14.65	16.22	15.19	15.36	24.74
March	16.67	19.6	20.05	22.55	21.09	20.76	29.01
April	20.12	23.19	24.40	26.75	24.88	24.72	31.10
May	25.46	27.24	29.12	30.14	28.88	28.76	31.80
June	32.55	32.65	32.62	33.33	32.89	32.76	30.61
July	34.38	33.42	33.92	33.04	33.79	33.59	31.60
August	29.55	29.7	29.52	30.19	30.38	30.23	29.98
September	21.62	22.6	23.45	23.63	23.48	23.16	26.94
October	16.45	17.52	17.67	19.34	18.68	18.65	24.54
November	11.33	12.68	12.87	14.63	13.88	13.76	21.93
December	8.57	9.83	10.42	11.28	10.88	10.76	19.80

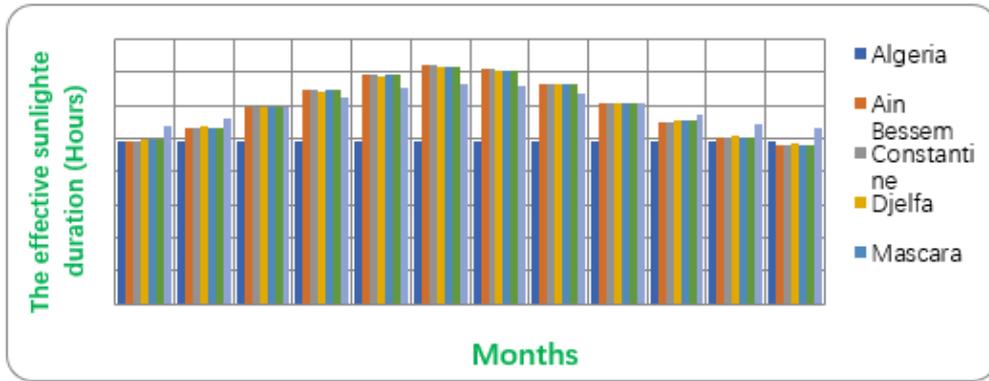


Figure 1. Distribution of the effective sunshine duration at the sites

4. RESULTS DISCUSSIONS

Using the different equations, the monthly average global solar irradiation was calculated and the different coefficients a and b were determined. Table 3 summarizes the expressions obtained for the different models as well as the statistical indicators of precision MAE, MBE and RMSE.

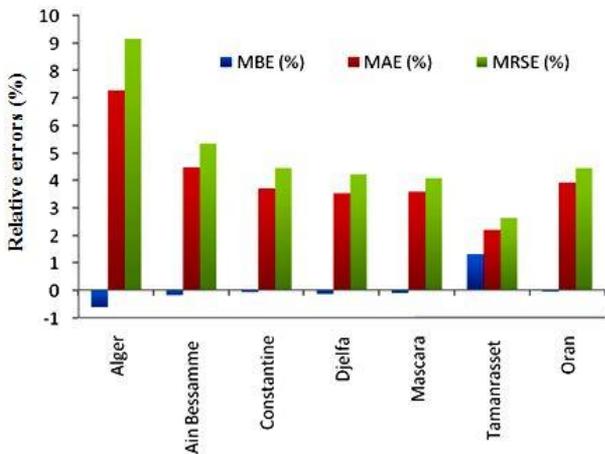


Figure 2. Relative errors between measured and calculated values by Angström model

Figure 2 shows the relative errors between the measured values and those calculated by the different models. We can observe that different methods provide good estimates of global irradiation. The maximum error was 9.137% obtained

in Algiers by RMSE. However, the best results are obtained by the model with an MAE of 1.312% for Oran, while the great value of MAE was 5.041% for Constantine. The calculated results of the RMSE show that the best model is the linear model.

In Figure 3 is shown the estimated variation of global solar irradiation during six years. The maximum value of global solar irradiation estimated by Angström model is obtained in the months of June and July for the different sites:

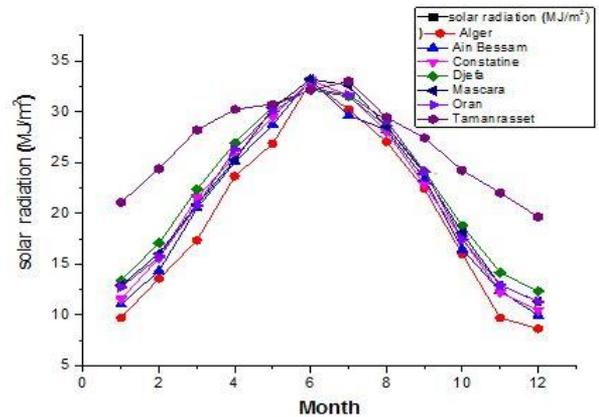
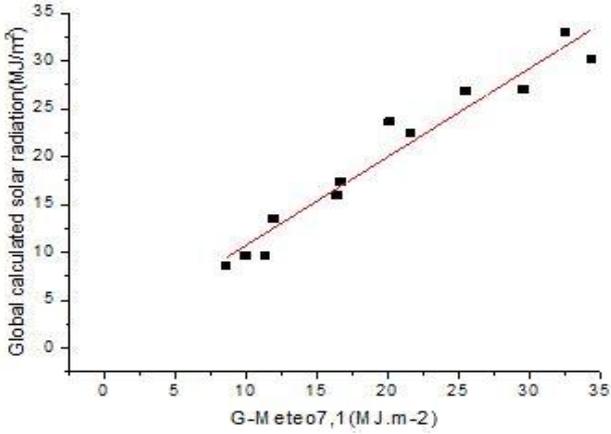


Figure 3. Estimated global solar irradiation

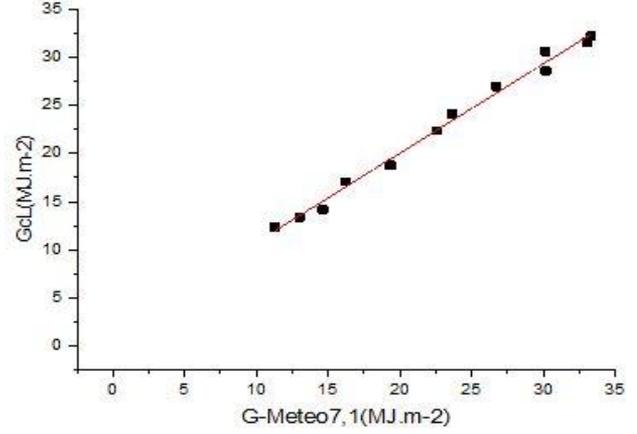
- Constantine: 33.13 (MJ.m⁻²),
- Djelfa: 32.18 (MJ.m⁻²),
- Mascara: 33.21 (MJ.m⁻²),
- Oran: 32.33 (MJ.m⁻²), and
- Tamanrasset: 33.02 (MJ.m⁻²).

Table 3. Developed models and values of MBE, MAE and RMSE

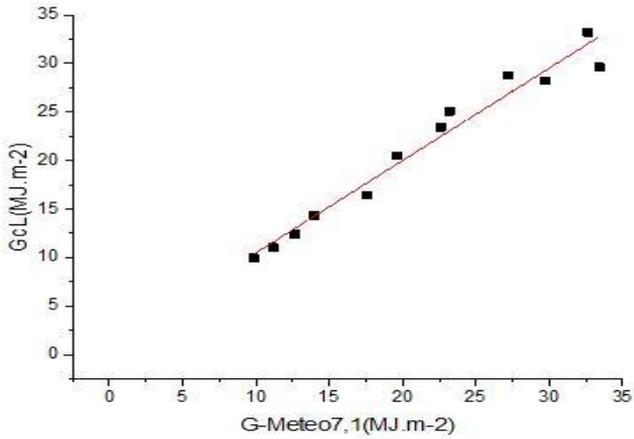
Site	Equation	MBE (%)	MAE (%)	RMSE (%)
Algiers	$G/G_0 = 0.061 + 0.947(S/S_0)$	-0.622	7.260	9.130
Ain Bessem	$G/G_0 = 0.447 + 0.427(S/S_0)$	-0.189	4.456	5.322
Constantine	$G/G_0 = 0.448 + 0.381(S/S_0)$	-0.075	3.694	4.429
Djelfa	$G/G_0 = 0.725 + 0.046(S/S_0)$	-0.147	3.511	4.203
Mascara	$G/G_0 = 0.396 + 0.525(S/S_0)$	-0.111	3.571	4.066
Oran	$G/G_0 = 0.47 + 0.392(S/S_0)$	-0.062	3.905	4.429
Tamanrasset	$G/G_0 = 0.573 + 0.316(S/S_0)$	0.130	2.188	2.623



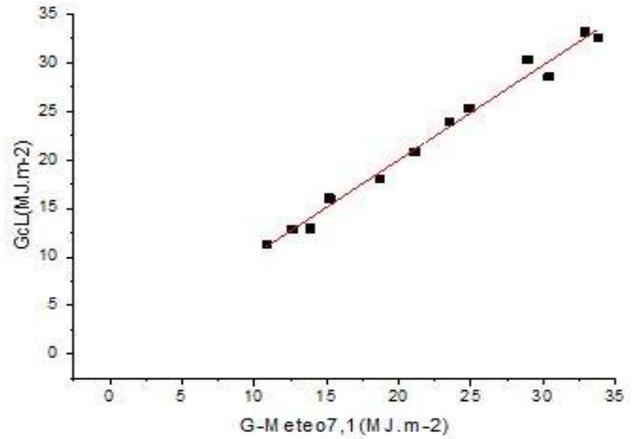
(a) Algiers



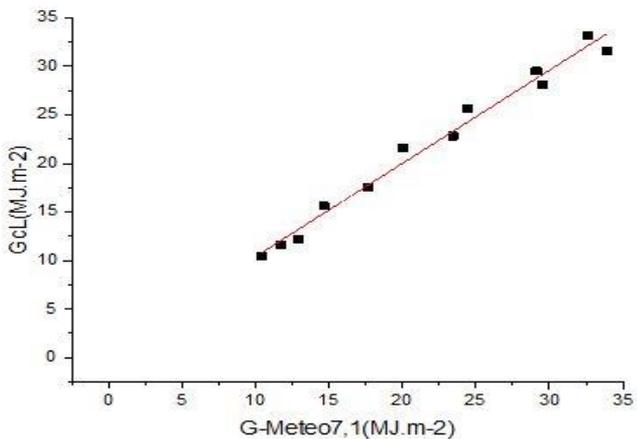
(d) Djelfa



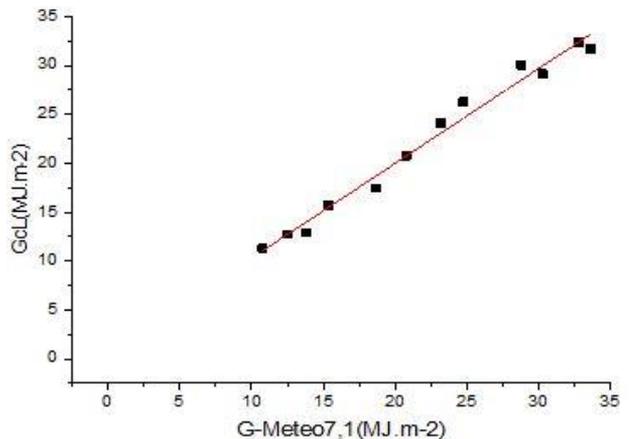
(b) Ain Bessem



(e) Mascra



(c) Constantine



(f) Oran

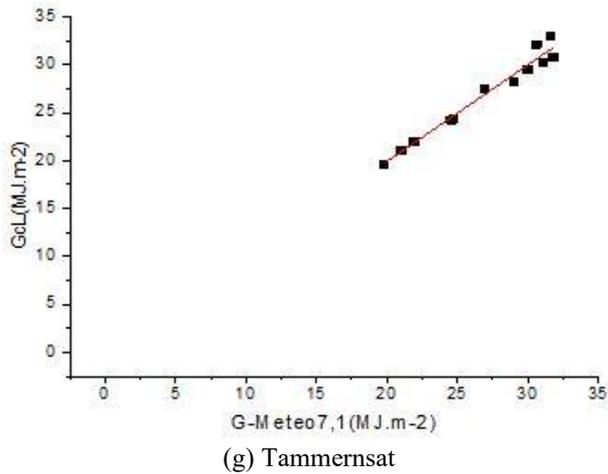


Figure 4. Comparison between measured and calculated global solar irradiation for deferent studied sites

To know the coefficients of connection between the measured and calculated values of each site, the results are presented in the Figure 4. Comparing the measured and calculated solar irradiation values of the seven regions, the order of the slope found is: Tamanrasset 1, Mascara and Oran 0.97, As Constantine and Ain Bassam 0.95. Algiers and Djelfa 0.92.

5. CONCLUSIONS

The current subject is relevant and important for industrial applications. This recent analysis was reported in order to estimate the monthly global solar irradiation in the Algerian territory using the Angström model, for seven sites in Algeria, namely: Algiers, Ain Bessem, Constantine, Oran, Djelfa, Mascara and Tamanrasset. The present model was based on global solar irradiation in a horizontal surface as well as sunshine hours. To investigate the data obtained by the different sites, we calculated the following relative average algebraic differences: MAE, MBE and RMSE. Three main findings are summarized as follows:

- The different methods, i.e., MAE, MBE and RMSE, provided good estimates of global irradiation.
- The maximum error was approximately 9.137 % obtained in Algiers by RMSE.
- The best results were obtained by the model with an MAE of 1.312% for Oran, while the great value of MAE was 5.041% for Constantine.
- The calculated results of the RMSE showed that the best model is the linear model.
- The maximum value of global solar irradiation estimated by Angström model is obtained in the months of June and July for the different sites investigated, i.e., 32.18, 32.33, 33.02, 33.13, and 33.21 MJ.m², respectively for Djelfa, Oran, Tamanrasset, Constantine, and Mascara.
- A remarkable concordance between global solar irradiation obtained from G-Meteo 7.1 and the values measured by the Angström model using calculated estimates, especially in Tamanrasset site where the slope is of the order of 1.
- Obtained estimates can be generalized on the whole Algerian territory to provide ease way for researchers to get solar irradiation. However, optimization efforts

have to be made to reduce mean error. Future works can be made based on our estimates to study effect of temperature and climatic parameters on the model.

- This work aim is achieved providing a data generator (software).

The problem solved by the authors is potentially interesting for researchers and engineers working on solar collectors and aerospace industry.

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NOMENCLATURE

G the average monthly global irradiation

G_0 measured on a horizontal plane ($\text{MJ.m}^{-2}.\text{j}^{-1}$).
the monthly average global irradiation outside the atmosphere on a horizontal plane ($\text{MJ.m}^{-2}.\text{j}^{-1}$).

I_0 the solar constant, is equal to 1367 Wm^{-2} .

N the number of the day of the year from the first of January.

S the duration of the effective sunshine.

S_0 the maximum duration of sunshine or duration of the day, its expression is given.

Greek symbols

Λ the latitude of the place, w: the hour angle.

Δ the declination, it is given by the equation.

ω the sunset hour angle.