Mathematical modeling applied to renewable fishery management

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ABSTRACT

Nowadays, controlling dynamics of renewable resources such as fishery and forestry is the major environmental challenge. In this regard, this research study was aimed to find the facile tool by using mathematical modeling to study and monitor the dynamics of the system consisting of two regions: one is reserved region and the other is unreserved region. Holling type II functional response is considered to formulate the model. The boundedness of the solution of the model is discussed. The model has been analyzed by finding the existence of equilibrium points and also the conditions of stability and instability of the system has been derived. Finally, the reliability of the analytical model was confirmed with the numerical simulations.

1. INTRODUCTION

Renewable resources are under extreme pressure worldwide in spite of taking efforts to design regulation to control the excessive and unsustainable exploitation. Fish is a renewable but finite resource. With the rapid growth of industrialization and population, the exploitation of fisheries has increased significantly. Although exploitation of resources is essential for the growth and development of a country, unplanned exploitation eventually leads to the extinction of the resources. Consequently, this will affect the growth and survival of species depending on the resource. From this point of view, it is a major challenge to manage renewable resources for the sustainable development of the country.

Mathematical modeling can play a significant role in the efficient and sustainable management of renewable resources. It is mainly used to describe the real phenomena leading to design better prediction, prevention, management and control techniques. Several well documented mathematical models regarding real life problems can be found in [1-7].

During the last few decades, mathematical models regarding renewable fishery resource management have been described. Mathematical modeling in harvesting of fisheries was studied first by Clark [5]. Biswas et al. presented a model for fishery resource with reserve area [1]. They studied the dynamics of a fishery resource in a two patch environment: a free fishing zone and a reserved zone where fishing is not allowed. Dubey *et al.* [6] presented a model for fishery resource with reserve area. They studied the dynamics of a fishery resource in a two patch environment: a free fishing zone and a reserved zone where fishing is not allowed. An optimal harvesting strategy is also derived using Pontryagin's Maximum Principle. Chaudhury [9] analyzed the dynamic optimization of combined harvesting of a two species fishery. Kar [10] presented a model for fishery resource with reserved area and facing prey-predator interaction. He considered that predation takes place in the unreserved zone. Local and global stability, optimal harvesting policy are also discussed. Roy et al. [11] investigated the effects of two predators on a prey population. They considered different types of functional responses to formulate the model.

From the literature discussed above and to the best of our views, a model for the cultivation of black tiger prawn was proposed and the work of Biswas et al. [1] was extended in this study. The dynamics of single species fishery in reserved and unreserved zone were discussed in the work but the effect of predation that the species might face in the unreserved zone were not studied. Also, it was considered that fish population can migrate from unreserved to reserved zone and vice versa. The effect of predation on the fish production was aimed to study and the migration from reserved to unreserved area was also restricted in this present work. In this paper, a mathematical model of a prey predator fishery was proposed with the help of system of nonlinear differential equation. It was considered that the fish species in the unreserved area were related in prey predator relationship. Holling type II functional response was taken into account to study the interaction between prey and predator fish species. Harvesting was permissible only in the unreserved region. Predation and harvesting were restricted in the reserved regions. To analyze the model, the existence of equilibrium points, dynamical behavior of the points and also the stability and instability conditions were discussed. Finally, numerical simulations were carried out to verify the analytical result of our proposed model.

2. MODEL FORMULATION

We consider a three compartmental model of fishery consisting of two zones: one is reserved where only prey species can reside while the other is unreserved zone where both prey and predator species can reside. Harvesting is permissible in unreserved zone but it is prohibited in reserved

zone. The prey species can migrate from unreserved to reserved area while the vice versa is restricted. Keeping all these in view, the schematic diagram of the interaction among the fish species and predator in two zones has been shown in Figure 1.
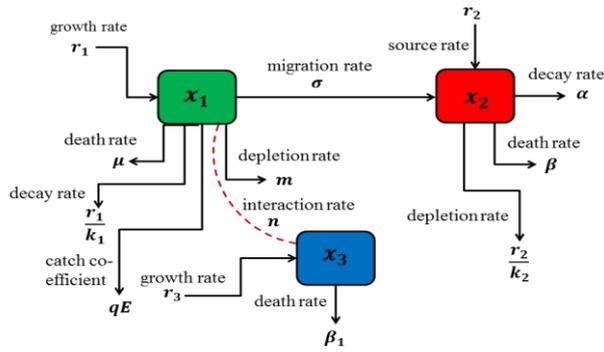

Figure 1. Compartmental fishery of prey predator fishery in different zones

Taking the diagram represented in Figure 1 into consideration, the mathematical model of the prey predator fishery can be written as

$$\frac{dx_1}{dt} = r_1 x_1 \left(1 - \frac{x_1}{k_1}\right) - (\mu + \sigma + qE) x_1 - \frac{m x_1 x_3}{a + x_1} \quad (1)$$

$$\frac{dx_2}{dt} = r_2 x_2 \left(1 - \frac{x_2}{k_2}\right) - \beta x_2 - \alpha x_2 + \sigma x_1 \quad (2)$$

$$\frac{dx_3}{dt} = r_3 x_3 \left(1 - \frac{x_3}{k_3}\right) + \frac{n x_1 x_3}{a + x_1} - \beta_1 x_3 \quad (3)$$

with $x_1(0) \geq 0$, $x_2(0) \geq 0$ and $x_3(0) \geq 0$. $\quad (4)$

Here, $x_1(t)$ denotes the biomass density of prey fish species (black tiger prawn) in unreserved zone, $x_2(t)$ represents the biomass density of same species in reserved zone and $x_3(t)$ denotes the biomass density of predator fish species in unreserved zone. In this model, it is considered that the predator consumes the prey population as an alternative food. Here, r_1, r_2 and r_3 represents the intrinsic growth rate of the prey and predator species in both zone respectively. k_1, k_2 and k_3 are the environment carrying capacity of prey and the predator species respectively. Therefore, $\frac{r_i x_i^2}{k_i}, (i = 1,2,3)$ is the amount by which the fish species decrease due to the interaction among themselves. Let E be the total effort applied for harvesting the prey population in the unreserved area and q is the catch ability coefficient, σ be the migration rate of the prey from unreserved to reserved area. We consider μ is the natural death rate of the species in the unreserved area. Therefore, $(\mu + \sigma + qE)x_1$ is the number of prey population that has been decreased from unreserved area due to fishing, migration and death rate. Let m be the depletion rate of prey species due to predation and n be the growth rate of the predator due to consumption. So, $\frac{m x_1 x_3}{a + x_1}$ is the depleted

number of prey due to the interaction with predator species. Here, a denotes the saturation constant. Let β be the death rate of prey in reserved area due to disease and α be the rate at which the prey population may be stolen due to insecurity. The term $(\alpha + \beta)x_2$ denotes the number by which the prey population decreases from the reserved area. In this prey predator system, we have considered Holling type II functional response to show the interaction between prey and predator species.

3. MODEL ANALYSIS

The model (1)-(3) had been analyzed in order to describe the dynamics of the fish species. For the analysis of the model the following studies were considered:

3.1 Boundedness of the model

To prove that the model system is biologically well posed the following Lemma was to be satisfied.

Lemma 1: The set

$$\Omega = \left\{ (x_1, x_2, x_3) : w(t) = x_1(t) + x_2(t) + x_3(t), 0 < w(t) < \frac{\delta}{\eta} \right\}$$

attracts all solutions initiating in the interior of the positive orthant, where η is a constant and

$$\mu = \frac{k_1}{4r_1}(r_1 + \eta - \mu - qE)^2 + \frac{k_2}{4r_2}(r_2 + \eta - \beta - \alpha)^2 + \frac{k_3}{4r_3}(r_3 + \eta)^2.$$

Proof: Let, $w(t) = x_1(t) + x_2(t) + x_3(t)$, $\eta > 0$ be a constant. Then we can write,

$$\frac{dw}{dt} + \eta w = (r_1 + \eta - \mu - qE)x_1 - \frac{r_1}{k_1} x_1^2 + (r_2 + \eta - \alpha - \beta)x_2 - \frac{r_2}{k_2} x_2^2$$
$$- (m - n)\frac{x_1 x_3}{a + x_1} + (r_3 + \eta - \beta_1)x_3 - \frac{r_3}{k_3} x_3^2$$

Since m is the depletion rate coefficient of prey due to its intake by the predator and n is the growth rate coefficient of predator due to its interaction with their prey, so it is assumed that $m \geq n$. Now η is chosen such that $0 < \eta < \beta_1$.

$$\frac{dw}{dt} + \eta w \leq \frac{k_1}{4r_1}(r_1 + \eta - \mu - qE)^2 + \frac{k_2}{4r_2}(r_2 + \eta - \alpha - \beta)^2$$
$$+ \frac{k_3}{4r_3}(r_3 + \eta)^2$$

By using differential inequality, we get,

$$0 < w(x_1(t), x_2(t), x_3(t)) \leq \frac{\delta}{\eta}(1 - e^{-\eta t}) + (x_1(0), x_2(0), x_3(0))e^{-\eta t}.$$

Taking limit as $t \to \infty$, we get, $0 < w(t) \leq \frac{\delta}{\eta}$.

3.2 Positivity of the solution of the model

Lemma 2: For $x_1(0) \geq 0, x_2(0) \geq 0, x_3(0) \geq 0$, the solutions $x_1(t), x_2(t), x_3(t)$ are all non-negative for all $t \geq 0$.

Proof: For positivity, equation (2.1) can be written as,

$$\frac{dx_1}{dt} \geq -(\mu + \sigma + qE)x_1 \qquad \Rightarrow \frac{dx_1}{x_1} \geq -kdt \qquad \text{where,}$$

$k = (\mu + \sigma + qE) \Rightarrow x_1 \geq c_1 e^{-kt}$, where c_1 is an integrating constant. Applying the initial condition, at $t = 0, x_1(0) \geq 0$, we get, $x_1(0) = c_1$. Putting the value of c_1 in the equation, we get, $x_1(t) \geq x_1(0)e^{-kt}$. When $t \to \infty, x_1(t) \geq 0$. Therefore $x_1(t)$ is positive for all $t \geq 0$. Again equation (2.2) can be written as $\frac{dx_2}{dt} \geq -(\alpha + \beta)x_2 \Rightarrow \frac{dx_2}{x_2} \geq -Ldt$, where $L = (\alpha + \beta)$. Integrating we get, $x_2(t) \geq c_2 e^{-Lt}$, where c_2 is an integrating constant. Applying the initial condition, at $t = 0, x_2(0) \geq 0$, we get, $x_2(0) = c_2$. Putting the value of c_2, we get, $x_2(t) \geq x_2(0)e^{-Lt}$. When $t \to \infty$, $x_2(t) \geq 0$. Hence $x_2(t)$ is positive for all $t \geq 0$. To prove that $x_3(t)$ is positive, equation (2.3) is written as, $\frac{dx_3}{dt} \geq -\beta_1 x_3$ $\Rightarrow \frac{dx_3}{x_3} \geq -\beta_1 dt$. Integrating $x_3(t) \geq c_3 e^{-\beta_1 t}$, where c_3 is an integrating constant. Now, at $t = 0, x_3(0) \geq 0$.

So, $x_3(0) = c_3$.

Putting the value of c_3, we obtain, $x_3(t) \geq x_3(0)e^{-\beta_1 t}$. At $t \to \infty$, $x_3(t) \geq 0$. Therefore, $x_3(t)$ is positive for all $t \geq 0$. Hence this completes the proof.

3.3 Existence of equilibria

Let $(\bar{x}_1, \bar{x}_2, 0)$ be the positive solution of the equations (1) and (2)

$$\varphi_1 x_1 - \frac{r_1 x_1^2}{k_1} = 0 \tag{5}$$

$$\frac{dx_2}{dt} = \varphi_2 x_2 - \frac{r_2 x_2^2}{k_2} + \sigma x_1 = 0 \tag{6}$$

where, $\varphi_1 = r_1 - \mu - \sigma - qE$ and $\varphi_2 = r_2 - \alpha - \beta$. From (5), we get, $x_1 = 0, \bar{x}_1 = \frac{\varphi_1 k_1}{r_1}$. Putting the value of $\bar{x}_1$ in (6), we get, $\bar{x}_2 = \frac{\varphi_2 \pm \sqrt{\varphi_2^2 + \frac{4\sigma r_2 k_1 \varphi_1}{r_1 k_2}}}{\frac{2r_2}{k_2}}$

Again, let (x_1^*, x_2^*, x_3^*) be the positive solution of the equations

$$\varphi_1 x_1 - \frac{r_1 x_1^2}{k_1} - \frac{mx_1 x_3}{a + x_1} = 0 \tag{7}$$

$$\varphi_2 x_2 - \frac{r_2 x_2^2}{k_2} + \sigma x_1 = 0 \tag{8}$$

$$\varphi_3 x_3 + \frac{nx_1 x_3}{a + x_1} - \frac{r_3 x_3^2}{k_3} = 0 \tag{9}$$

where, $\varphi_3 = r_3 - \beta_1$. Equation (9) yields

$$x_3 = 0, \ a\varphi_3 + \varphi_3 x_1 - \frac{ar_3 x_3}{k_3} - \frac{r_3 x_1 x_3}{k_3} + nx_1 = 0$$

$$\Rightarrow x_1^* = \frac{ar_3 x_3^* - a\varphi_3 k_3}{\varphi_3 k_3 + nk_3 - r_3 x_3^*}.$$

Using the value of x_1^* in equation (8), we get

$$\frac{r_2 x_2^{*2}}{k_2} - \varphi_2 x_2^* - \sigma \left(\frac{ar_3 x_3^* - a\varphi_3 k_3}{\varphi_3 k_3 + nk_3 - r_3 x_3^*} \right) = 0.$$

This equation will have a positive solution if $\varphi_2 \geq 0 \Rightarrow r_2 - \alpha - \beta \geq 0$. Finally from equation (3.3), we obtain

$$x_3^* = \frac{a + x_1^*}{m} \left(\varphi_1 - \frac{r_1 x_1^*}{k_1} \right)$$

Hence the equilibrium points of the system are $P_1(0, 0, 0)$, $P_2(\bar{x}_1, \bar{x}_2, 0)$ and $P_3(x_1^*, x_2^*, x_3^*)$.

3.3 Stability analysis at steady state

The Jacobean of the system is

$$J(x_1, x_2, x_3) =$$

$$\begin{pmatrix} r_1 - \frac{2r_1 x_1}{k_1} - (\mu + \sigma + qE) - \frac{amx_3}{(a + x_1)^2} & 0 & -\frac{mx_1}{a + x_1} \\ \sigma & r_2 - \frac{2r_2 x_2}{k_2} - \beta - \alpha & 0 \\ \frac{anx_3}{(a + x_1)^2} & 0 & r_3 - \frac{2r_3 x_3}{k_3} - \beta_1 \end{pmatrix}$$

$$\therefore J(0, 0, 0) = \begin{pmatrix} r_1 - (\mu + \sigma + qE) & 0 & 0 \\ \sigma & r_2 - \beta - \alpha & 0 \\ 0 & 0 & r_3 - \beta_1 \end{pmatrix}$$

Then the characteristics equation of the matrix with eigenvalue λ is

$$|J - \lambda I| = \begin{vmatrix} r_1 - (\mu + \sigma + qE) - \lambda & 0 & 0 \\ \sigma & r_2 - \beta - \alpha - \lambda & 0 \\ 0 & 0 & r_3 - \beta_1 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow r_1 - (\mu + \sigma + qE) - \lambda = 0, \ r_2 - \beta - \alpha - \lambda = 0 \text{ or } r_3 - \beta_1 - \lambda = 0$$

$$\Rightarrow \lambda_1 = r_1 - (\mu + \sigma + qE), \ \lambda_2 = r_2 - \beta - \alpha \text{ and } \lambda_3 = r_3 - \beta_1$$

The eigenvalue λ of the matrix determines the stability of the states. Depending on λ, the stability conditions are: 1. if the eigenvalue $\lambda > 0$, then the steady state is unstable, 2. if the eigenvalue $\lambda < 0$, then the system is stable.

In the following lemma, we show that $P_3(x_1{}^*, x_2{}^*, x_3{}^*)$ is locally asymptotically stable.

Lemma 3. The equilibrium point $P_3(x_1{}^*, x_2{}^*, x_3{}^*)$ is always locally asymptotically stable.

Proof: Let, the three functions be $u = u(x_1, x_2, x_3), v = v(x_1, x_2, x_3)$ and $w = w(x_1, x_2, x_3)$ of the system (2.1)-(2.3). Then the Jacobean at $P_3(x_1{}^*, x_2{}^*, x_3{}^*)$ is

$$J(x_1{}^*, x_2{}^*, x_3{}^*) =$$

$$\begin{pmatrix} r_1 - \dfrac{2r_1 x_1{}^*}{k_1} - (\mu + \sigma + qE) - \dfrac{amx_3{}^*}{\left(a + x_1{}^*\right)^2} & 0 & \dfrac{mx_1{}^*}{a + x_1{}^*} \\[3em] \sigma & r_2 - \dfrac{2r_2 x_2{}^*}{k_2} - \beta - \alpha & 0 \\[3em] \dfrac{anx_3{}^*}{\left(a + x_1{}^*\right)^2} & 0 & r_3 - \dfrac{2r_3 x_3{}^*}{k_3} - \beta_1 \end{pmatrix}$$

Then the characteristics equation of the matrix with eigenvalue λ is

$$|J - \lambda I| = 0$$

$$\begin{vmatrix} r_1 - \dfrac{2r_1 x_1{}^*}{k_1} - (\mu + \sigma + qE) - \dfrac{amx_3{}^*}{\left(a + x_1{}^*\right)^2} - \lambda & 0 & -\dfrac{mx_1{}^*}{a + x_1{}^*} \\[3em] \sigma & r_2 - \dfrac{2r_2 x_2{}^*}{k_2} - \beta - \alpha - \lambda & 0 \\[3em] \dfrac{anx_3{}^*}{\left(a + x_1{}^*\right)^2} & 0 & r_3 - \dfrac{2r_3 x_3{}^*}{k_3} - \beta_1 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \left(r_1 - \dfrac{2r_1 x_1{}^*}{k_1} - (\mu + \sigma + qE) - \dfrac{amx_3{}^*}{\left(a + x_1{}^*\right)^2} - \lambda \right) \left(r_2 - \dfrac{2r_2 x_2{}^*}{k_2} - \beta - \alpha - \lambda \right)$$
$$= 0$$
$$\left(r_3 - \dfrac{2r_3 x_3{}^*}{k_3} - \beta_1 - \lambda \right) + \left(\dfrac{mx_1{}^*}{a + x_1{}^*} \right) \left(\dfrac{anx_3{}^*}{\left(a + x_1{}^*\right)^2} \right) \left(r_2 - \dfrac{2r_2 x_2{}^*}{k_2} - \beta - \alpha - \lambda \right)$$

$$\Rightarrow (A_1 - \lambda)(A_2 - \lambda)(A_3 - \lambda) + A_4(A_2 - \lambda) = 0$$
$$\Rightarrow \lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3 = 0 \qquad (10)$$

where,

$$a_1 = -(A_1 + A_2 - A_3), a_2 = (A_1 A_2 + A_2 A_3 + A_3 A_1 + A_4), a_3 = -(A_1 A_2 A_3 + A_2 A_4),$$

$$A_1 = r_1 - \dfrac{2r_1 x_1{}^*}{k_1} - (\mu + \sigma + qE) - \dfrac{amx_3{}^*}{\left(a + x_1{}^*\right)^2}, \quad A_2 = r_2 - \dfrac{2r_2 x_2{}^*}{k_2} - \beta - \alpha,$$

$$A_3 = r_3 - \dfrac{2r_3 x_3{}^*}{k_3} - \beta_1, \quad A_4 = \left(\dfrac{mx_1{}^*}{a + x_1{}^*} \right) \left(\dfrac{anx_3{}^*}{\left(a + x_1{}^*\right)^2} \right)$$

By Routh-Hurwitch criterion, all the eigenvalues of (3.7)

have negative real roots if and only if $a_1 > 0, a_3 > 0$, and $a_1 a_2 > a_3$.

Then the equilibrium point $P_3(x_1{}^*, x_2{}^*, x_3{}^*)$ is locally asymptotically stable.

4. NUMERICAL SIMULATIONS

In this section, some numerical simulations of our proposed model were presented to investigate the dynamical behavior of the model. To do these simulations, MATLAB ode45 solver was used. The description and values of all parameters used in our proposed model are presented in Table 1.

Table 1. Description of parameters and their values

| Symbols	Description of parameters	Values
r_1	intrinsic growth rate of black tiger prawn in unreserved area	5
r_2	intrinsic growth rate of black tiger prawn in reserved area	6
r_3	intrinsic growth rate of predator population in unreserved area	1.5
σ	migration rate	2.5
α	decay rate due to being stolen (illegal pouching)	0.2
β	death rate due to disease	0.9
β_1	death rate of predator	0.05
μ	death rate of prey in unreserved area	0.02
m	depletion rate of prey due to the predation	0.5
n	growth rate of predator due to predation	0.05
a	saturation constant	20
k_1	carrying capacity of black tiger prawn in unreserved area	100
k_2	carrying capacity of black tiger prawn in reserved area	200
k_3	carrying capacity of predator population in unreserved area	100
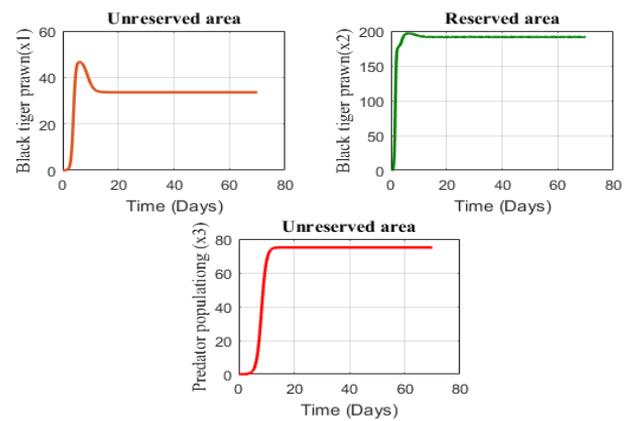
Figure 2. Variation of population with time (70 days)

A time interval of 70 days was considered to show the dynamics. The figures displayed depict the densities of fish and predator population in reserved and unreserved area and shows where the population increases and decreases within 70 days. Figures 4.3-4.14 show the variation of black tiger prawn and predator population in two zones for changing values of different parameters.

The Figure 2 represents that fish population first increases, then decreases and finally grows at a constant rate in the unreserved area and the species in the reserved area first increases, then shows a slight decrease and finally grows at a constant rate. The predator population shows an increasing effect for the above values of the parameter.
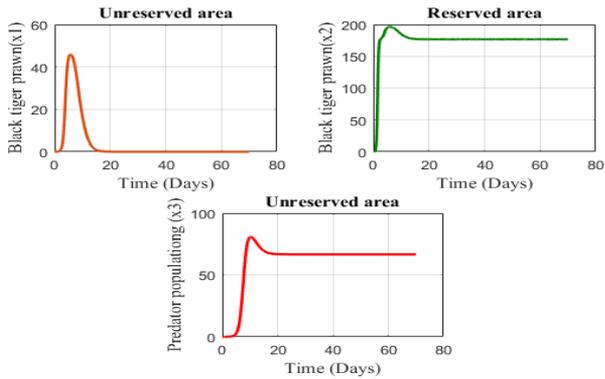

Figure 3. Variation of population with time (70) days for $m = 0.9, n = 0.5$ keeping all values same

Figure 3 illustrates that for the increasing values of depletion rate and growth rate due to predation, the black tiger prawn in the unreserved area increases in the first 9 days, then decreases for after 9 days and at last it extinct while in the reserved area it shows a slight changing effect. The figure also shows that the predator population increases for the parameter values.
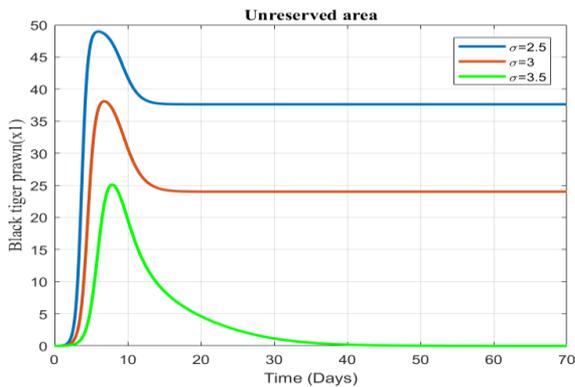
Figure 4. Variation of black tiger prawn inreserved area for different values of σ
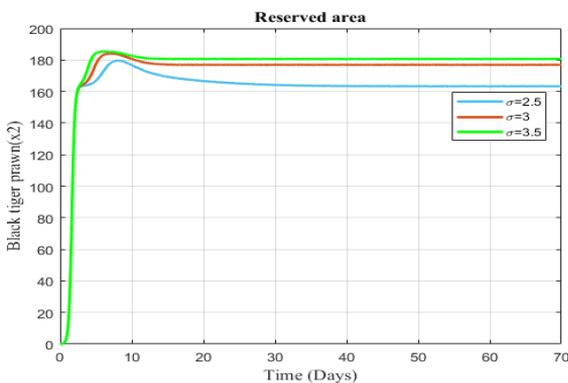
Figure 5. Variation of black tiger prawn in reserved area for different values of σ
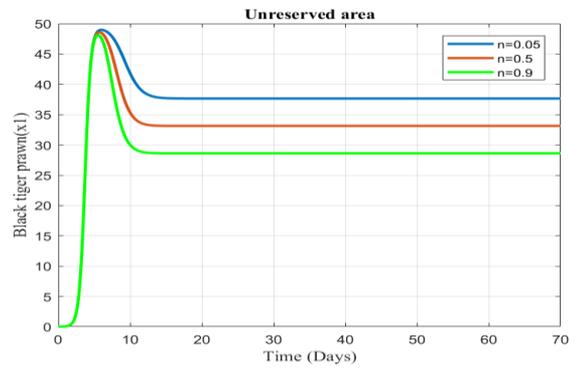
Figure 6. Variation of black tiger prawn in unreserved area for different values of n
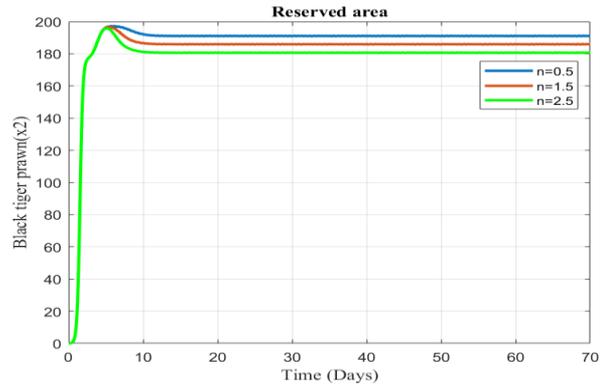
Figure 7. Variation of black tiger prawn in reserved area for different values of n
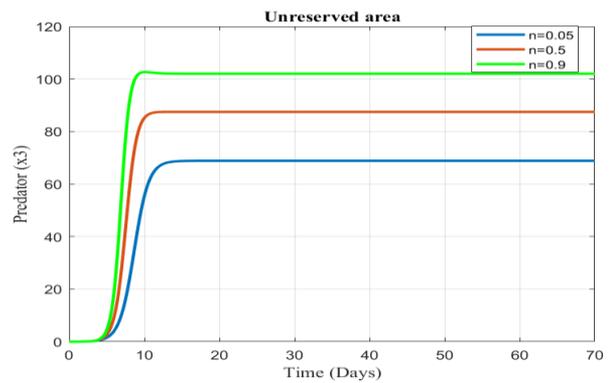
Figure 8. Variation of predator population in unreserved area for different values of n
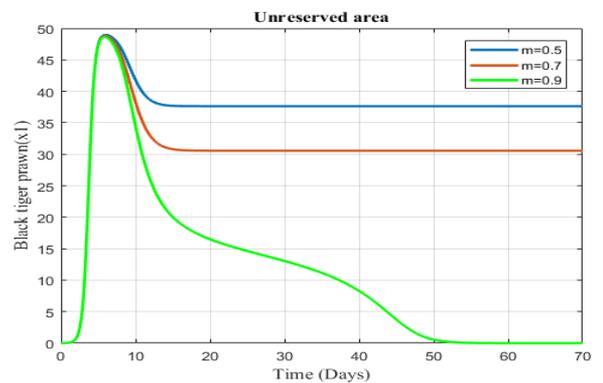

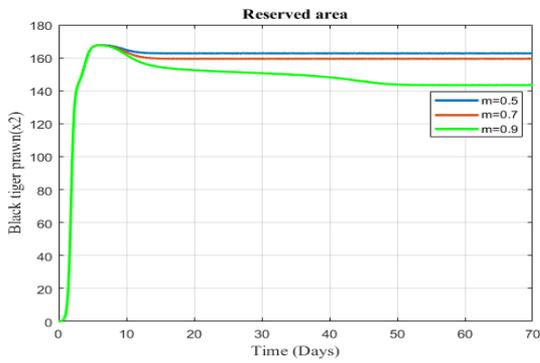**Figure 9.** Variation of black tiger prawn for different values of m

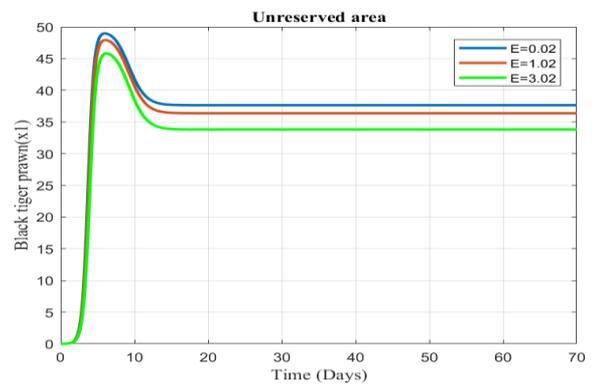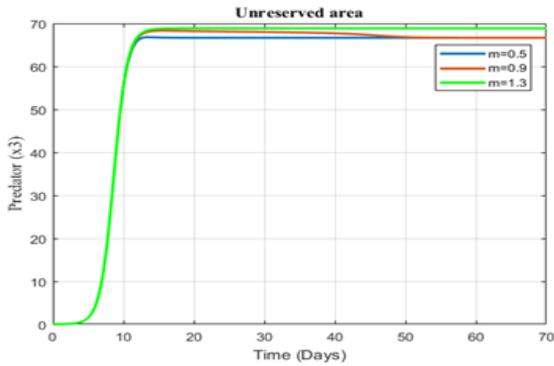**Figure 10.** Variation of black tiger prawn in reserved area for different values of m

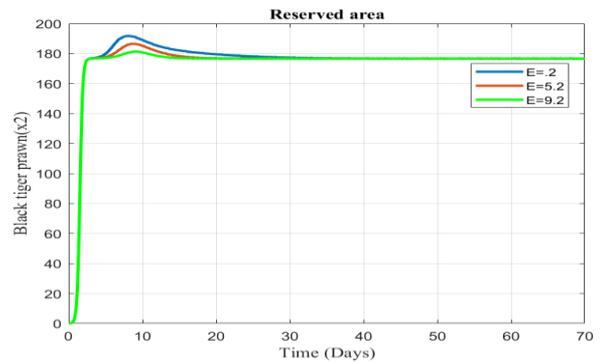**Figure 11.** Variation of predator in the unreserved area for different values of m

The change in the biomass density of black tiger prawn in the two zones for increasing values of σ has been represented in Figures 4 and 5. The fish population in the unreserved area decreases for the increasing value of migration rate while population in reserved area shows an increasing effect.

Figures 6-8 show the variation of fish and predator population for different values of consumption rate due to predation. It is seen from the Figures that black tiger prawn in both zones decreases and predator population increases due to the increasing values of n.

The variation of fish and predator population for different values of depletion rate due to predation is represented in Figures 9-11. It is seen from the figures that black tiger prawn in both zones decreases and predator population increases due to the increasing values of m. The fish population in unreserved area extinct after 50 days as depletion rate increases.

In Figures 12 and 13, we observe that fish population decreases from the unreserved area while the population increases in reserved area due to the increasing values of fishing or harvesting rate.

Figures 14 and 15 indicate the variation of black tiger prawn in reserved area for the changing values of the rate at which the species are stolen and death rate. Both the figures show a decreasing effect for the increasing values of α and β.

Some arbitrary data are assumed for describing the phase diagram of the system. Using the Maple2018 software, we have analyzed the stability analysis of the fishery model. The phase diagram of the model in presence and absence of predator in both reserved and unreserved area has been analyzed for the system. Figures 16 and 17 describe the phase diagram of the system.
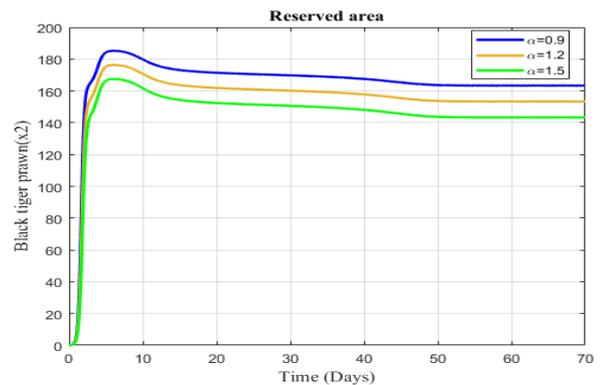

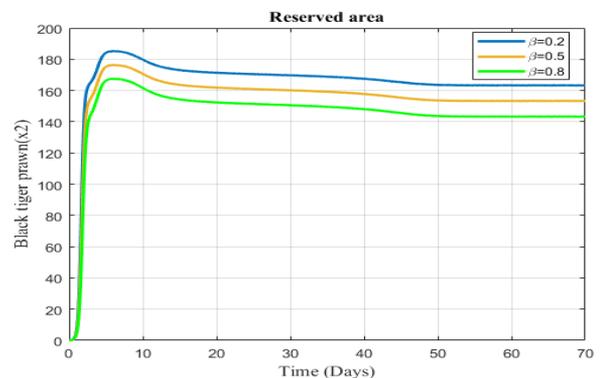**Figure 12.** Variation of black tiger prawn in unreserved area for different values of E

Figure 13. Variation of black tiger prawn in reserved area for different values of E

Figure 14 Variation of black tiger prawn in the reserved area for different values of α

Figure 15. Variation of black tiger prawn in reserved area for different values of β
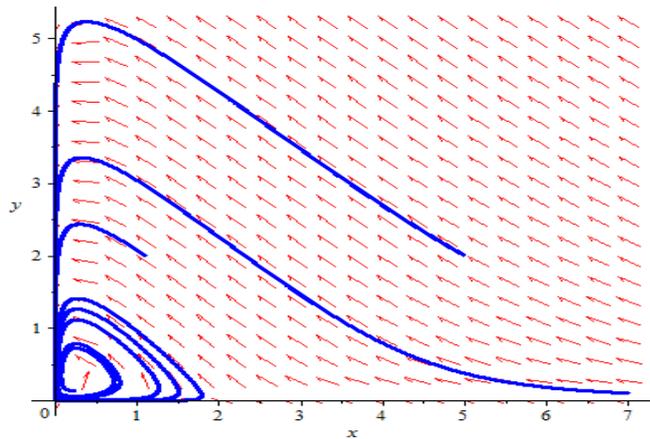
Figure 16. Phase space diagram of the system in the presence of predator in the unreserved area with the parameter values $r_1 = 1$, $k_1 = 7$, $m = 6$, $n = 6$, $a = 7$, $k_3 = 9$, $r_3 = 0.2$, $\beta = 0.4$, $\mu = 0.2$, $\sigma = 0.3$, qE $= 0.2$
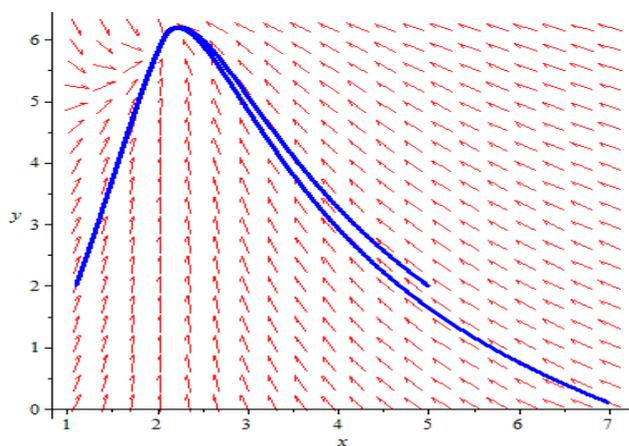

Figure 17. Phase space diagram of the system in the absence of predator in the unreserved and reserved area with the parameter values $r_1 = 1$, $k_1 = 7$, $k_2 = 9$, $r_2 = 0.5$, $\mu = 0.2$, $\sigma = 0.4$, qE $= 0.2$, $\beta = 0.2$, $\alpha = 0.1$

5. CONCLUTION

Considering two ecosystems, a mathematical model of fishery has been formulated in this paper. We have analyzed the behavior of the model, focusing on the parameters that are mainly responsible for the production and reduction of black tiger prawn. The numerical results reveal that the high mortality rate, fishing rate and predation rate of black tiger prawn in the unreserved area have a decreasing effect on the species in the reserved area. It also shows that the number of black tiger prawn increases in the reserved area due to the high migration from the unreserved area. When predation increases, the number of black tiger prawn in both reserved and unreserved area reduces by a significant amount. It is also evident from the simulations that the production of black tiger prawn in the reserved area decreases due to death from different diseases and for being stolen in absence of security. The main conclusion based on the result is that it is possible to maximize the production of black tiger prawn by proper management and thus it can play a significant role in the economy of a country.

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