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On The Construction Of Numerous Magic Squares By Using A Fourth Order (4×4) Magic Squares

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ABSTRACT

Magic squares are much fascinating for the humanity. The fourth order magic square with entries 1,2,3......16 are considered in this paper. By using this we have constructed the 3936 different 4×4 new magic squares with the same magic sum. It is found that mesmerising passion and remarkable mathematical principles and procedures.

1. INTRODUCTION

Magic squares [1] have been considered a mathematical reaction providing entertainment and an interesting outlet for creating mathematical knowledge [2]. A new construction has been made for special magic squares by Nordgren [3]. A small note was given on franklin and complete magic square matrices by Nordgren [4] and Gupta [5] studied a generalized form of a 4 x 4 magic square.

2. THE CONSTRUCTION OF NUMEROUS MAGIC SQUARES BY USING FOURTH ORDER (4×4) MAGIC SQUARES

Let us construct fourth order magic square. The first step is Natural Square. If we reverse the first row and the last row, that will become Step (2). By reversing a row we mean that the number in the first cell goes to the last cell. For example is 1 2 3 4 row. If we reverse this row it becomes 4 3 2 1. Similarly the column can be reversed. If we reverse the first column and the last column that it becomes Step (3), which is the magic square.

Step (1) Natural Square				Step (2)				Step (3)			
1	2	3	4	4	3	2	1	16	3	2	13
5	6	7	8	5	6	7	8	9	6	7	12
9	10	11	12	9	10	11	12	5	10	11	8
13	14	15	16	16	15	14	13	4	15	14	1

This is the magic square as we require. The magic square sum $\frac{n(n^2+1)}{2} = 34.$

By observing the above magic square formation, we know that in this method three fourth of the numbers in the square change their place and the four numbers in the centre do not change their place. They remain as they are in the natural square. If we reverse the first and last columns, that is, rows and columns should be reversed in the same order. Half of the number of columns should be reversed.

For this kind of magic square construction, the number of fourth order magic squares construction carried out till now is 3936. If we with all the magic squares it will take a separate both form. So, they are written in short as far as possible.

3. NUMEROUS CONSTRUCTIONS OF MAGIC SQUARES

We should take a square to be the first square. For this we can take the magic square Step (3). That is the square No.1 is given below.

16	3	**2**	**13**
9	6	**7**	**12**
5	**10**	11	8
4	**15**	14	1

The magic sum of this square is 34. The magic sum of all the squares to be written hereafter is also 34. If the last two numbers in the first and second rows and fourth rows are mutually exchanged the following magic square No.2 is obtained.

16	3	5	10
9	6	4	15
2	13	11	8
7	12	14	1

In magic square No.1, 8 numbers changed their place in this square. If magic square No.1 is divided into small squares of 4 cells each and numbers in each small squares is written in the four rows, the following magic square No.3 is obtained.

16	3	9	6
2	13	7	12
5	10	4	15
11	8	14	1

Now we have prepared 3 magic squares. By connecting each square into 4 magic squares, we get 3 × 4 = 12 magic squares. For example from magic square No.1, we can prepare more magic squares. They are as follows in the magic square No.1, if the second and third rows mutually exchanged, we get the magic square No.4.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

In the magic square No.1, if the second and third columns are mutually exchanged, magic square No.5 is obtained.

16	2	3	13
9	7	6	12
5	11	10	8
4	14	15	6

In the magic square No.4, the second and third rows are mutually exchanged, magic square No.6 is obtained.

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

Similarly we can construct the 3 square into the 4 magic squares, totally 12. There are 24 magic square found out from the above 12 magic square. If we want to explain in detail how these 24 magic squares were found out, it will be a lengthy process.

These 36 magic squares are related closely. The 36 magic squares are regularly arranged and given below. Anyone can easily understand by were look at them.

1				2			
16	3	2	13	16	3	2	13
9	6	7	12	5	10	11	8
5	10	11	8	9	6	7	12
4	15	14	1	4	15	14	1

3				4			
16	2	3	13	16	2	3	13
9	7	6	12	5	11	10	8
5	11	10	8	9	7	6	12
4	14	15	1	4	14	15	1

5				6			
16	3	5	10	16	3	5	10
9	6	4	15	2	13	11	8
2	13	11	8	9	6	4	15
7	12	14	1	7	12	14	1

7				8			
16	5	3	10	16	5	3	10
9	4	6	15	2	11	13	8
2	11	13	8	9	4	6	15
7	12	14	1	7	12	14	1

9				10			
16	3	9	6	16	3	9	6
2	13	7	12	5	10	4	15
5	10	4	15	2	13	7	12
11	8	14	1	11	8	14	1

11				12			
16	3	9	6	16	3	9	6
2	7	13	12	5	4	10	15
5	4	10	15	25	7	13	12
11	8	14	1	11	8	14	1

13				14			
16	9	6	3	16	9	6	3
5	4	15	10	2	7	12	13
11	14	1	8	11	14	1	8
2	7	12	13	5	4	15	10

15				16			
16	9	6	3	16	9	6	3
1	13	12	7	5	10	15	4
11	14	1	8	11	14	1	8
5	10	15	4	2	13	12	7

17				18			
16	6	9	3	16	6	3	9
11	1	14	8	11	1	8	14
5	15	4	10	2	12	13	7
2	12	7	13	5	15	10	4

19				20			
16	10	5	3	16	5	10	3
7	1	14	12	9	4	15	6
9	15	4	6	7	14	1	12
2	8	11	13	2	11	8	13

21				22			
16	10	3	5	16	3	10	5
7	1	12	14	2	13	8	11
2	8	13	11	7	12	1	14
9	15	6	4	9	6	15	4

23				24			
6	7	12	9	6	12	7	9
10	11	8	5	15	1	14	4
15	14	1	4	10	8	11	5
3	2	13	16	3	13	2	16

25				26			
10	11	8	5	10	8	11	5
6	7	12	9	15	1	14	4
15	14	1	4	6	12	7	9
3	2	13	16	3	13	2	16

27				28			
7	6	12	9	7	12	6	9
11	10	8	5	14	1	15	4
14	15	1	4	11	8	10	5
2	3	13	16	2	13	3	16

29				30			
11	10	8	5	10	8	10	5
7	6	12	9	14	1	15	4
14	15	1	4	7	12	6	9
2	3	13	16	2	13	3	16

31				32			
6	4	15	9	6	15	4	9
13	11	8	2	12	1	14	7
12	14	1	7	13	8	11	2
3	5	10	16	3	10	5	16

33				34			
11	13	8	2	11	8	13	2
4	6	15	9	14	1	12	7
14	12	1	7	4	15	6	9
5	3	10	16	5	10	3	16

35				36			
7	13	12	2	10	4	15	5
4	10	15	5	13	7	12	2
14	8	1	11	8	14	1	11
9	3	6	16	3	9	6	16

From each of the 36 magic squares two other squares can be prepared. For example the magic square is given below.

16	3	2	13
9	6	7	12
5	10	11	85
4	15	14	1

In the same square (1), in the second row, if the first and last numbers and second and third numbers are mutually exchanged, and in the third row the first two numbers are mutually exchanged and the numbers in the fourth row are written in the reverse order, we get No.2

16	3	2	13
6	9	12	7
11	8	5	10
1	14	15	4

In the magic square No.1, if we change numbers in columns as we did in the rows, we get No.3 magic square.

16	6	11	1
9	3	14	8
5	15	2	12
4	10	7	13

If we construct each of 36 magic squares into 3, we get 36×3=108 magic squares. Each of 108 magic squares can be constructed into 4 magic squares as follows. In the magic square No.1, if the first row and second row are exchanged, square No.2 is obtained.

9	6	7	12
16	3	2	13
4	15	14	1
5	10	11	8

In the first magic square No.1, if the first column and second column are exchanged, Square No.3 is obtained.

3	16	13	2
6	9	12	7
10	5	8	11
15	4	1	14

In the Square No.3 if we exchanged rows as we did in Square No.1, Square No.4 is obtained.

If we construct the 108 magic squares into 4, one can get the 432 magic squares.

Besides these 432 squares, we have got 15 other squares, they are given below.

16	8	1	9	10	15	5	4
10	2	7	15	16	3	9	6
3	13	12	6	1	14	8	11
5	11	14	4	7	2	12	13

16	5	2	11
14	4	9	7
3	13	8	10
1	12	15	6

1	15	12	6
10	11	6	7
15	4	12	2
8	14	3	9

13	3	14	4
8	5	12	9
11	16	1	6
2	10	7	15

10	16	3	5
2	8	13	11
15	9	4	6
7	1	14	12

7	5	12	10
9	11	8	6
16	14	1	3
2	4	13	10

4	12	15	9
14	6	11	3
7	1	16	10
9	15	1	8

7	5	10	12
9	11	6	8
4	2	15	13
14	16	3	1

4	2	15	13
7	16	1	10
14	11	6	3
9	5	12	8

10	12	7	5
8	6	11	9
13	15	2	4
3	1	14	16

1	9	16	8
7	5	10	2
14	4	5	11
12	6	3	13

16	14	3	1
2	4	13	15
5	9	8	12
11	7	10	6

15	1	10	8
5	4	11	14
12	13	6	3
2	16	7	9

14	16	3	1
5	2	15	12
4	7	10	13
11	9	6	8

Each of the above magic squares can be converted into 4 magic squares can be converted into 4 magic squares. Magic square No.1 is converted into 4 magic squares as follows.

(1)				(2)			
16	8	1	9	2	10	15	7
10	2	7	15	8	16	9	1
3	13	12	6	11	5	4	14
5	11	14	4	13	8	6	12

(3)				(4)			
16	1	8	9	12	3	6	13
3	12	13	6	1	16	9	8
10	7	2	15	14	5	4	11
5	14	11	4	7	10	15	2

In the magic square No.1, first numbers in the first and second rows are exchanged mutually. Similarly, numbers in third row and fourth rows are exchanged and formed a row square, in that row square, the first and second column are exchanged mutually. Similarly, third and fourth column are exchanged then we get magic square (2).

In the magic square No.1, if the second row and third row are exchanged mutually and in the new magic square, thus formed, if the second and third column are mutually exchanged the magic square is obtained.

If we change the number in the magic square (3) as we did in square as well as in square No.1, the above the magic square (4) is obtained.

If we convert the 15 magic square 15 × 4 = 60 square as we did above, the total number of square we get in 432 + 60 = 492 squares into 8 magic squares by turning the whole square repeatedly and upside down. The magic square No.1, is converted into 8 magic square as follows:

1				2			
16	3	2	13	16	9	5	4
9	6	7	12	3	6	10	15
5	10	11	8	2	7	11	14
4	15	14	1	13	12	8	1

3				4			
13	12	3	16	13	12	8	1
12	7	6	9	2	7	11	14
8	11	10	5	3	6	10	15
1	14	15	4	16	9	5	4

5				6			
1	8	12	13	1	14	15	4
14	11	7	2	8	11	10	5
15	10	6	3	12	7	6	9
4	5	9	14	13	2	3	16

7				8			
4	15	14	1	4	5	9	16
5	10	11	8	15	10	6	3
9	6	7	12	14	11	7	2
16	3	2	13	1	8	12	13

In this magic square No.1, if the whole square is turned such that the numbers in the first column come in the first row magic squares is obtained.

In the magic square No. 1, if the whole square is turned such that numbers in the fourth column come in the first column magic square (3) is obtained.

In this magic square (3), if the whole square is turned such that the numbers in the first column came in the first row, magic square (4) is obtained.

In the magic square (4), if the whole square is turned such that the numbers in the first column in the first row magic square (6) is obtained.

In the magic square (1), if the whole square is turned such that the numbers in the fourth row come in the first row magic square (7) is obtained.

In the magic square (7), if the whole square is turned such the numbers in the first columns come in the row, magic square (8) is obtained. This means that the 492 magic square can be turned into 492 × 8 = 3936 magic square.

4. CONCLUSIONS

The construction of numerous magic squares of order 4 is presented in this paper. There are 3936 squares are constructed and possibility constructing the elaborated in this paper. This kind of exercise of need to be practiced with our kids and school children remove machine dependency also to improve the decision making and analytical ability for life problems. The further research in this process to explore to seek more possibilities of constructing fourth order magic square.

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