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The GRAY PREDICTION GM (1, 1) MODEL IN TRAFFIC FORECAST APPLICATION

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ABSTRACT

Since gray prediction has a big advantage, it is widely used in many fields. In this paper, gray prediction is used for population and GDP prediction, with GM (1, 1) model to predict the population and GDP of Changchun City in recent years. Explained in detail the prediction process, the error analysis, this model is applied to the population and GDP prediction and it is concluded that the results of the error are small with high credibility. Using the prediction results can be further trip generation prediction in four-stage method.

Keywords: Gray prediction, GM (1, 1), Population, GDP.

1. INTRODUCTION

Four-stage theory of transportation planning is divided into four steps, which are trip generation, trip distribution, model split, traffic assignment. Changchun City, Jilin province, Changchun City, Jilin Province to predict the track traffic passenger flow volume when using the four -stage method. Firstly, dividing Changchun City into several medium sized areas, then predicting trip generation .Trip generation prediction refers to the prediction of Two trip production, which are trip generation volume and trip attraction volume ,trip generation can be predicted using regression analysis method. Regression analysis establishes between the dependent variable and independent variables of a function expression, by looking at a relationship between dependent variables with more than one independent variable, and the statistical methods .The quantity of Trip generation forecast is regressed by population, GDP and land area and other factors .In this paper, we use the method of gray prediction to predict the number of population and GDP.

The gray system theory is put forward by Professor Deng Julong in 1982 [1],adopted the method of additive or subtraction, this theory has solved the abuse which time series short, the statistical data few, the information incompletely when establishes the model the malpractice, in many areas of our country have played a decisive role , was widely praised.

According to nature boundaries such as mountains, rivers and railway line in Changchun city, divide the populated areas into small ones, sparse population, and divide the areas covering too much into larger ones, then in Figure 1, traffic medium areas are divided into 40 in Changchun City through referring to administrative divisions, making use of the Trans CAD mapping.

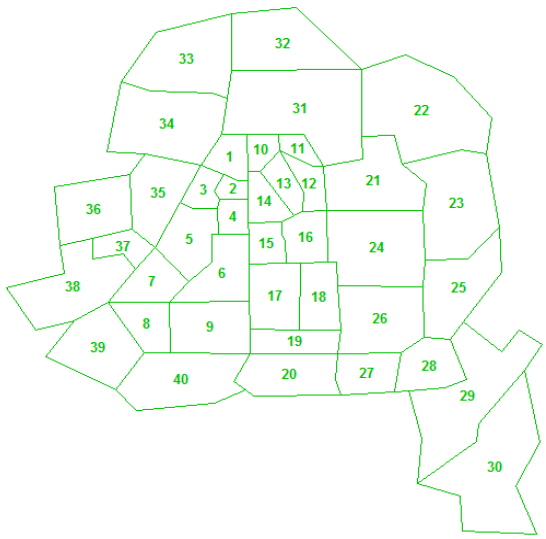

Figure1. Medium area traffic division

2. GM (1, 1) GRAY PREDICTION MODEL

2.1 GM (1, 1) gray prediction model is established

GM (1, 1) gray prediction model is that through the monotone sequence has been given, by means of once additive generates a group of new albinism differential equation. Set a monotone differential sequence:

$$x^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(N)\};$$

Get an accumulated generating sequence:

$$x^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(N)\};$$

GM (1, 1) differential equation model:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = \mu$$

a Called development index, μ called endogenous control index.

Set $\hat{b}$ as for the solution to parameter vector, using the least square method can find $\hat{b} = (B^T B)^{-1} B^T Y_N$ $\hat{X}^{(1)}(i)$, among them:

$$B = \begin{bmatrix} -\frac{1}{2}[x^{(1)}(1) + x^{(1)}(2)] & 1 \\ -\frac{1}{2}[x^{(1)}(2) + x^{(1)}(3)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}[x^{(1)}(n-1) + x^{(1)}(n)] & 1 \end{bmatrix},$$

$$Y_n = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$

Available prediction model is

$$\hat{X}^{(1)}(k+1) = \left[X^{(0)}(1) - \frac{\mu}{a} \right] e^{-ak} + \frac{\mu}{a}$$

Then using the subtraction method for inverse operation, restore the monotone sequence to obtain the forecast result.

2.2 Accuracy test

(1) Residual test [2]

Calculate according to prediction model $\hat{X}^{(1)}(i)$, and put the $\hat{X}^{(1)}(i)$ IAGO the $\hat{X}^{(0)}(i)$, then calculate the absolute residual and the relative residual sequence of the monotone sequence $X^{(0)}(i)$ and the IAGO sequence $\hat{X}^{(0)}(i)$.

Absolute residual:

$$\Delta^{(0)}(i) = | X^{(0)}(i) - \hat{X}^{(0)}(i) |, \ i = 1, 2, 3....n$$

Relative residual:

$$\Phi(i) = \frac{\Delta^{(0)}(i)}{X^{(0)}(i)} \times 100\%, \ i = 1, 2, 3....n$$

(2) Correlation degree test

Correlation coefficient:

$$\eta(i) = \frac{\min\{\Delta^{(0)}(i)\} + \rho \max\{\Delta^{(0)}(i)\}}{\Delta^{(0)}(i) + \rho \max\{\Delta^{(0)}(i)\}} (i = 1, 2, \cdots, \rho = 0.5)$$

Correlation degree:

$$r = \frac{1}{n} \sum_{1}^{n} \eta(i) \quad i = 1, 2, 3....n$$

Calculate the correlation coefficient and correlation degree of $\hat{X}^{(0)}(i)$ and $X^{(0)}(i)$. Once $\rho = 0.5$, $r > 0.6$ will be satisfied.

(3) Posterior-variance-test, calculate separately:

Mean for $x^{(0)}$:

$$\overline{X} = \frac{1}{n} \sum_{i=1}^{n} x^{(0)}(i)$$

Monotone sequence mean square error:

$$S_1 = \sqrt{\frac{\sum \left[X^{(0)}(i) - \overline{X}^{(0)} \right]^2}{n-1}}$$

Mean for residual:

$$\overline{\Delta} = \frac{1}{n} \left[\Delta^{(0)}(i) \right]$$

Residual mean square error:

$$S_2 = \sqrt{\frac{\sum \left[\Delta^{(0)}(i) - \overline{\Delta}^{(0)} \right]^2}{n-1}} \quad r = \frac{1}{n} \sum_{i=1}^{n} \eta_i(i) = 0.701 > 0.6$$

Variance ratio:

$$C = \frac{S_2}{S_1}$$

Small residual error probability:

$$P = P\{ | \Delta^{(0)}(i) - \overline{\Delta}^{(0)} | < 0.6745 S_1 \}$$

Table 1. Prediction precision grade table

Prediction precision grade	P	C
Wonderful	>0.95	<0.35
Qualified	>0.80	<0.45
Reluctant	>0.70	<0.50
Unqualified	≤ 0.70	≥ 0.65

3. POPULATION PREDICTION

There are populations of Changchun in recent years in the Jilin Province statistics yearbook, see Table2. As the monotone sequence $x^{(0)}$.

Table 2. Changchun population

Year	2002	2003	2004	2005	2006	2007
Population	7125055	7240845	7314959	7392561	7459463	7565065
Year	2008	2009	2010	2011	2012	2013
Population	7677089	7617663	7738000	7684651	7747324	7810503

Available prediction model:

$$\frac{dx^{(1)}}{dt} - 1.724 \times 10^{15} x^{(1)} = 3.165 \times 10^{7}$$

$$\hat{x}^{(1)}(i+1) = 0.580 \times 10^{-15} e^{1.726 \times 10^{15} t} - 1.836 \times 10^{-8}$$

Therefore, the prediction of the number of population from 2014 to 2016 as shown in Table 3.

Table 3. Predicting outcomes of future population

Year	2014	2015	2016
Population	7874235	7938421	8057456

Comparing the data of original population data and gray prediction is obtained as shown in Figure 2.

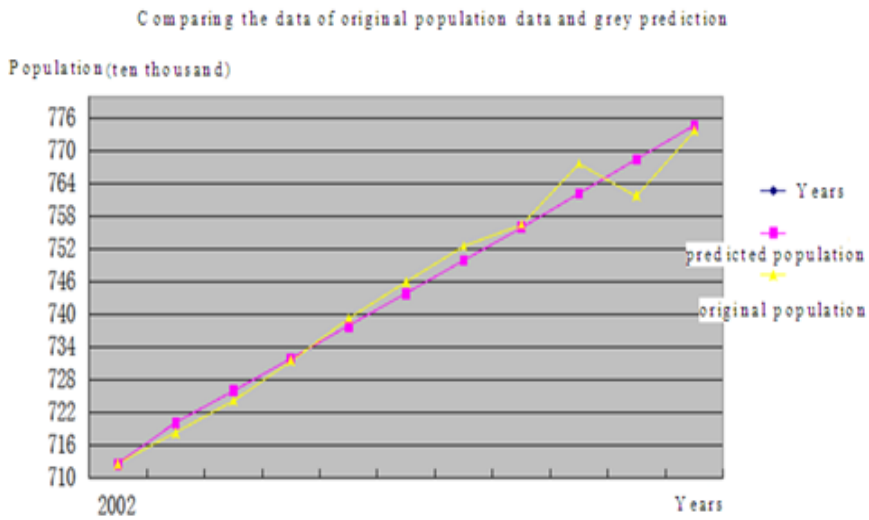

Figure 2. Comparing the data

Due to the sequence of absolute residuals:

$\Delta^{(0)}$ ={0,18834,19066,4161,13750,20473,25644,4242,54604,66988,9324}

Relative residual sequence:

Φ ={0,0.262%,0.263%,0.057%,0.186%,0.274%,0.341%,0.056%,0.711%,0.879%,0.120%}.

Relative error less than 0.879%, the model accuracy is very high.

Correlation degree test

η ={1,0.640,0.637,0.889,0.709,0.621,0.566,0.887,0.380,0.333,0.782},

Correlation degree: $\frac{1}{n}\sum_{i=1}^{n}\eta_i(i) = 0.74 > 0.6$, when therefore satisfies $\rho = 0.5$, satisfies the examination criterion.

Posterior-variance-test

Monotone sequence mean square error:

$$S_1 = \sqrt{\frac{\sum\left[X^{(0)}(i) - \bar{X}^{(0)}\right]^2}{n-1}} = 2.169 \times 10^{5}$$

Mean of residuals:

$$\bar{\Delta} = \frac{1}{n}\left[\Delta^{(0)}(i)\right] = 23708.6$$

Residual standard deviation:

$$S_2 = \sqrt{\frac{\sum\left[\Delta^{(0)}(i) - \overline{\Delta}^{(0)}\right]^2}{n-1}} = 0.2125 \times 10^5$$

Variance ratio:

$$C = \frac{S_2}{S_1} = 0.098$$

Small residual error probability:

$$P = P\{|\Delta^{(0)}(i) - \overline{\Delta}^{(0)}| < 0.6745 S_1\} = 1$$

Control precision grade table, the level of population prediction as well.

4. GDP PREDICTION

From the statistical yearbook of Jilin province shows that The GDP of Changchun City $x^{(0)}: \overline{x}^{(0)} = 224.863$ from 2004 to 2013, see Table 4.

Table 4. Gray prediction of data

Years	2004	2005	2006	2007	2008	2009
GDP (Billion)	33.83	62.0	134.9	159.	295.	227.8
Years	2010	2011	2012	2013		
GDP (Billion)	264.6	310.0	359.	402.5		

Educe prediction model:

$$\frac{dx^{(1)}}{dt} - 755561.771 x^{(1)} = -590.233231$$

$$\hat{x}^{(1)}(i+1) = 1.323 \times 10^{-6} e^{755561.771t} + 0.78 \times 10^{-3}$$

Table 5. Prediction results

Years	2014	2015	2016
GDP/Billion	490.20	568.27	658.77

Due to the sequence of absolute residuals:

$\Delta^{(0)}$ ={0,67.64,15.39,15.23,93.03,6.34,6.83,4.65,5.76,20.36}

Relative residual sequence:

Φ ={0,10.901%,11.408%,9.579%,31.536%,2.783%,2.581%,1.500%,1.604%,5.058%}

The maximum error is less than 31.563%, the good model Accuracy.

Correlation degree test

Sequence: the absolute residuals sequence of the $x^{(0)}$ and $\hat{x}^{(0)}$.

$\Delta^{(0)}(i)$ ={0,67.64,15.39,15.23,93.03,6.34,6.83,4.65,5.76,20.36}

Min $\Delta^{(0)}(i) = 0$; max $\Delta^{(0)}(i) = 93.03$.

Educe: η ={1,0.333,0.687,0.690,0.267,0.842,0.832,0.879, 0.854,0.624}.

Calculate correlation degree: r=0.701>0.6, when therefore

Satisfies $\rho = 0.5$, satisfies the examination criterion.

Posterior-variance-test

The mean for

Monotone sequence mean square error:

$$S_1 = \sqrt{\frac{\sum\left[X^{(0)}(i) - \overline{X}^{(0)}\right]^2}{n-1}} = 124.165$$

Mean for residuals:

$$\overline{\Delta} = \frac{1}{n}\left[\Delta^{(0)}(i)\right] = 23.523$$

Residual standard deviation:

$$S_2 = \sqrt{\frac{\sum\left[\Delta^{(0)}(i) - \overline{\Delta}^{(0)}\right]^2}{n-1}} = 32.130$$

Variance ratio:

$$C = \frac{S_2}{S_1} = 0.251$$

Small residual error probability:

$$P = P\{|\Delta^{(0)}(i) - \overline{\Delta}^{(0)}| < 0.6745 S_1\} = 1$$

Control precision grade table, this population forecast is good.

5.MEDIUM TRAFFIC AREA POPULATION DISTRIBUTION AND GDP DISTRIBUTION

Using the above predictions, making use of the Trans CAD software mapping, traffic medium area's population distribution (see Figure 4) and the corresponding distribution of GDP (see Figure 5) as follows:

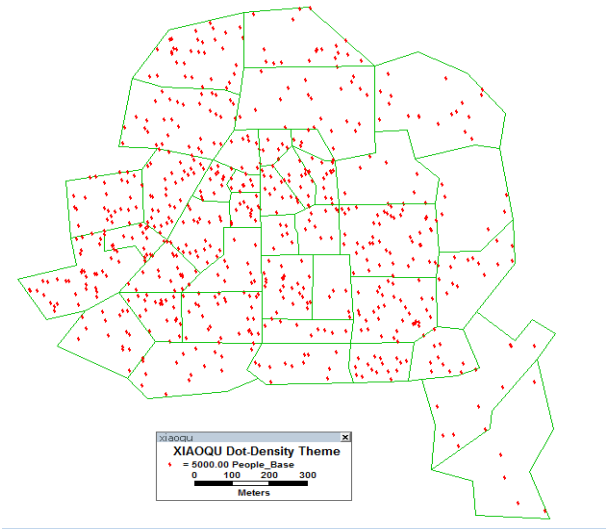

Figure 4. Distribution of population density map

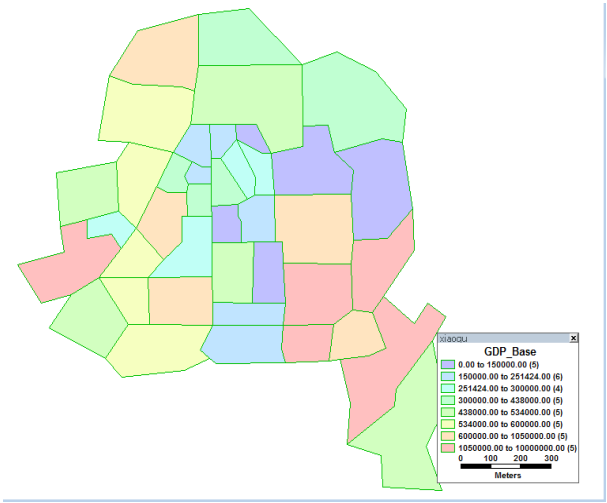

Figure 5. GDP color distribution map

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6. CONCLUSIONS

In this paper, based on the prediction of population number and GDP as an example, using GM (1, 1) model to forecast, the use of Trans CAD software, get the population distribution and GDP distribution of the medium traffic area, prepared for the prediction of traffic trip generation volume of four stages of research method.

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