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NONLINEAR CONTROL FOR ELECTROMAGNETIC SUSPENSION SYSTEMS ON ELASTIC GUIDEWAY

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ABSTRACT

In order to improve the control behavior of the electromagnetic suspension (EMS) system of the low-speed maglev train, this paper establish the train-guideway coupling non-linear dynamic model composed of the single electromagnet and the elastic guideway. Hurwitz stability criterion is utilized to prove the open-loop instability of the train-guideway coupling system. The vibration information of the guideway is input into the controller design and involved in the calculation of the control strategy. Simulation results show the presented controller can eliminate the vibration of the guideway and reduce the exacting requirements of system stability on the guideway properties. Moreover, the train-guideway coupling system with the presented controller shows better dynamic performance.

Keywords: Low-speed maglev train 1, Dynamic model 2, Coupling vibration 3, Nonlinear control 4.

1. INTRODUCTION

The electromagnetic suspension (EMS) system [1] has been widely used in maglev passenger trains [2-3], magnetic bearing [4], bearingless motor [5-6], etc. The primary task of EMS systems is to eliminate the influence of gravity via electromagnetic forces， which can avoid contact, and thus, no friction. Due to its technological, comfortable and environmental attractions, the MAGLEV train has broad application and development prospects in the fields of the intercity transit and the urban traffic. Until now, the control of the EMS system, which is one of the key components of the MAGLEV train, still becomes the research focus. Due to the flexibility of the guideway, coupling effect will be generated between the vehicle and the guideway. If the performance of the EMS control system is less-powerful, strong coupling vibration may occur between the vehicle and the guideway endangering the stable suspension. Increasing the stiffness and damping of the guideway is the usual engineering application to eliminate this phenomenon. However, this will increase the cost of the MAGLEV line construction considerably. According to the statistics, the cost of guideway in the finished projects takes 60%-80% of the total MAGLEV system [7]

In recent years, much effort has been directed toward the area of dynamics and control of the EMS system. Golob and Tvornik simplified the EMS system to an electromagnet-ball system [8]. Xu et al. proposed a new nonlinear control method and implemented this method on the Shanghai Urban Maglev Test Line [9]. Tran and Kang proposed an arbitrary finite-time tracking control (AFTC) method to control the EMS systems with uncertain dynamics [10]. Ghosh et al. modeled the MAGLEV system and proposed 2-DOF PID controller to overcome open-loop unstable [11]. Unfortunately, guideway was all assumed as a rigid body in these studies, which took the vibration of the MAGLEV system as self-excited oscillation caused by the parameters of the control system. However, experiment results indicated that deformation of the guideway was no longer taken as external disturbance with the powerful coupling vibration. Therefore, it is necessary to model the EMS system considering the flexible guideway. Zhou et al. derived stability criterion of the MAGLEV train running over flexible guideway by the means of Lyapunov characteristic numbers [12]. Fang et al. proposed LQG controller scheme to avoid coulping vibration between the MAGLEV vehicle and track [13]. Unfortunately, these studies didn't eliminate the coulping vibration actively from the point of guideway control. Thus, the system may be unstable with the smaller stiffness and damping of the guide way.

In this paper, we are motivated to model the EMS system considering the guideway flexibility, which facilitate the study of vehicle-guideway coupling vibration. A novel controller scheme is presented to eliminate the coupling vibration effectively, which can maintain the system's stability and reduce the exacting requirements of system stability on the guideway properties, thus massively reduce the construction cost.

2. BASIC PRINCIPLE OF EMS SYSTEM

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The MAGLEV train, as a new kind of high-tech transportation methods, is levitated by the electromagnetic suspension (EMS) system. In this paper, we are focus on the core part to ensure stable suspension, which called EMS system composed by several single suspension modules with the same functions, and it is more versatile to analyze the dynamic model and control problem of single module of the EMS system [14-15] as presented in Fig.1.

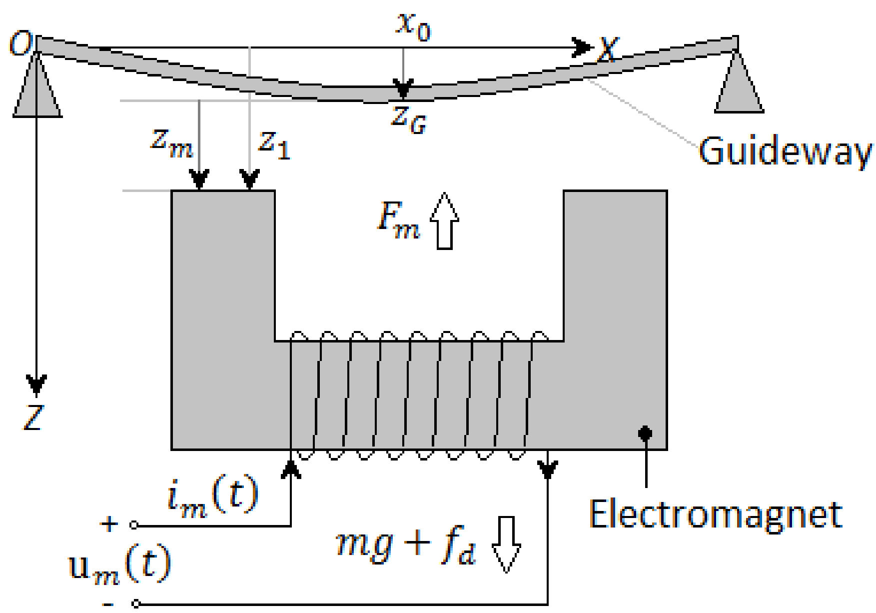

Figure 1. Basic structure of EMS module with flexible guideway

By some physical laws (Kirchhoff's law and Newton's law), the nonlinear electromagnetic force $F_m(t)$ and the electric equation of the electromagnet can be expressed as follows:

$$F_m(i_m, z_m) = \frac{\mu_0 N_m^2 a_m}{4}\left[\frac{i_m(t)}{z_m(t)}\right]^2,$$

$$u_m = i_m R_m + \frac{\mu_0 N_m^2 a_m}{2z_m}\dot{i}_m - \frac{\mu_0 N_m^2 a_m}{2z_m^2} i_m \dot{z}_m \quad (1)$$

Where, μ_0 denotes permeability of air, a_m , N_m denote the valid pole area and the number of turns of coil. i_m , u_m and R_m denote current, voltage and resistance of coil, respectively, z_m denotes the suspension airgap.

3. DYNAMICS MODEL OF EMS SYSTEM WITH FLEXIBLE GUIDEWAY

3.1 Model of Flexible Guideway

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The guideway is usually simplified as Bernoulli-Euler beam. x_0 denotes the electromagnet displacement from the origin of coordinates along the OX direction (see Fig.2), z_1 denotes the vertical displacement of electromagnet along the OZ direction. z_G denotes the vertical displacement of guideway. The vertical vibration of the guideway can be depicted as:

$$E_g I_g \frac{\partial^4 z_G(x,t)}{\partial x^4} + \delta \frac{\partial^5 z_G(x,t)}{\partial x^4 \partial t} + \rho g \frac{\partial^2 z_G(x,t)}{\partial t^2} = F\sigma(x) \quad (2)$$

Where, $E_g I_g$ denotes the bending stiffness, δ denotes the damping coefficient, ρ_g denotes the linear density of guideway; σ denotes position function. If $x = x_0$, σ and F are expressed as:

$$\sigma(x_0) = \begin{cases} 1 & x = x_0 \\ 0 & x \neq x_0 \end{cases}, \quad F = F_m(i_m, z_m) - mg - f_d \quad (3)$$

According to the theory of modal superposition, the solution of (2) can be expressed as follows:

$$z_G(x,t) = \sum \phi_i(x) \cdot q_i(t) \qquad (i = 1, 2 \cdots, \infty),$$

$$\phi_i(x) = \sqrt{\frac{2}{\rho_g l_g}} \sin\left(\frac{i\pi}{l_g} x\right) \quad (4)$$

Where, $q_i(t)$ denotes the coordinate of i-orders mode , ϕ_i denotes the i-orders modal function, l_g denotes length of guideway. ω_i and ξ_i denote -order modal frequency and damping ratio, respectively. The motion differential equation of guideway in regular coordinate system is represented as:

$$\ddot{q}_i(t)+2\xi_i\omega_i\dot{q}_i(t)+\omega_i^2 q_i(t)=F\phi_i$$

$$\xi_i=\frac{\omega_i\delta}{2E_gI_g}$$

$$\omega_i=\left(\frac{i\pi}{l_g}\right)^2\sqrt{\frac{E_gI_g}{\rho_g}} \tag{5}$$

3.2 Nonlinear dynamic model of the coupling system

According to Newton's Second Law, the dynamic equation of electromagnet is defined as

$$m\ddot{z}_1=F=-F_m(i_m,z_m)+mg+f_d \tag{6}$$

From all the formulas above, nonlinear dynamic model of the coupling system with the n-orders modal of the guideway can be obtain as follows:

$$\begin{cases}\ddot{q}_i(t)+2\xi_i\omega_i\dot{q}_i(t)+\omega_m^2 q_i(t)=F\phi_i\; z_G(x,t)=\sum\phi_i(x)\cdot q_i(t), i=1,2,\cdots,n\\ F_m(i_m,z_m)=\dfrac{\mu_0N_m^2a_m}{4}\left[\dfrac{i_m(t)}{z_m(t)}\right]^2, u_m=i_mR_m+\dfrac{\mu_0N_m^2a_m}{2z_m}\dot{i}_m-\dfrac{\mu_0N_m^2a_m}{2z_m^2}\\ z_m(x,t)=z_1(t)-z_G(x,t)\quad m\ddot{z}_1=mg+f_d-F_m(i_m,z_m)\end{cases} \tag{7}$$

3.3 Linear model and stability analysis

With the load f_d, consider (z_N,i_N) as equilibrium point. z_N and i_N denotes suspension air gap and current in the coil at equilibrium point, respectively. Taylor expand $F_m(i_m,z_m)$ at equilibrium point as follows:

$$F_m(i_m,z_m)=F_m(i_N,z_N)+P_Ii_m-P_mz_m,$$

$$P_I=\frac{\mu_0N_m{}^2a_mI_N}{2z_N^2}$$

$$P_m=\frac{\mu_0N_m{}^2a_mI_N^2}{2z_N^3} \tag{8}$$

Taylor expand (1) at equilibrium point as:

$$u_m=u_N+i_mR_m+\frac{\mu_0N_m{}^2a_m}{2z_N}\dot{i}_m-\frac{\mu_0N_m{}^2a_mI_N}{2z_N^2}\dot{z}_m \tag{9}$$

Considering formulas above, select the state variables $X=\begin{bmatrix}q_1 & \dot{q}_1 & q_2 & \dot{q}_2 & \cdots & q_n & \dot{q}_n & z_1 & \dot{z}_1 & \ddot{z}_1\end{bmatrix}^T$, the output variable $Y=\begin{bmatrix}z_m & \ddot{z}_1\end{bmatrix}^T$ and $L=\dfrac{\mu_0N_m^2a_m}{2z_N}$, $\alpha_1=\dfrac{P_mR_m}{mL}$, $\alpha_2=\dfrac{\eta P_m}{m}$.

Dynamic model of the EMS system can be expressed as the following equation of state:

$$\dot{X}=AX+BU, Y=CX \tag{10}$$

Where, A,B,C are system matrices, the control matrix and output matrix of the system, which can be expressed as:

$$A=\begin{bmatrix}0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\ -\omega_1^2 & -2\xi_1\omega_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & \cdots & 0 & 0 & 0 & 0\\ 0 & 0 & -\omega_2^2 & -2\xi_2\omega_2 & \cdots & 0 & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & \cdots & -\omega_n^2 & -2\xi_n\omega_n & 0 & 0\\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0\\ -\alpha_1\phi_1 & -\alpha_2\phi_1 & -\alpha_1\phi_2 & -\alpha_2\phi_2 & \cdots & -\alpha_1\phi_n & -\alpha_2\phi_n & \alpha_1 & \alpha_2\end{bmatrix}$$

$$B=\begin{bmatrix}0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & -\dfrac{P_I}{mL}\end{bmatrix}^T$$

$$C=\begin{bmatrix}-\phi_1 & 0 & -\phi_2 & 0 & \cdots & -\phi_n & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 1\end{bmatrix} \tag{11}$$

Considering the first-order modal of guideway, the transfer function $G_0(s)$ of airgap to electromagnet voltage with open-loop state can be expressed as: $G_0(s)=N_u(s)/D_e(s)$, where, $N_u(s)$ denotes the numerator of transfer function, $D_e(s)$ denotes the characteristic polynomial of the system, which can be expressed as follows:

$$\begin{aligned}D_e(s)&=mLs^5+m(R_m+2L\xi_1\omega_1)s^4\\&-(mLP_m\eta\phi_1^2-mL\omega_1^2-2mR_m\xi_1\omega_1+LP_m\eta)s^3\\&-(mR_mP_m\phi_1^2-mR_m\omega_1^2+2LP_m\eta\xi_1\omega_1+R_mP_m)s^2\\&-(LP_m\eta\omega_1^2+2R_mP_m\xi_1\omega_1)s-R_mP_m\omega_1^2\end{aligned} \tag{12}$$

Explicitly, the constant term $-R_mP_m\omega_1^2<0$ in. $D_e(s)$ .So, Hurwitz stability criterion is utilized to prove the open-loop instability of the system .The electromagnetic force must be adjusted to ensure stable suspension.

4. THE DESIGN OF CONTROLLER AND DYNAMIC ANALYSIS OF SYSTEM

The control law not only needs to control the vibration of electromagnet, but also the vibration of guideway. The vibration information is input into controller for calculating of the control law. The first-order mode of guideway is selected to describe the flexible vibration as $z_G=\phi_1q_1$.

The control objet is denoted in (10). The state feedback vector consisted of modal coordinate of the guideway, its differentiation, and the vertical displacement, velocity, acceleration of electromagnet that is $X_1=\begin{bmatrix}q_1 & \dot{q} & z_1 & \dot{z}_1 & \ddot{z}_1\end{bmatrix}^T$ .Feedback coefficient vector of controller denotes as $K_1=\begin{bmatrix}k_{11} & k_{12} & k_{13} & k_{14} & k_{15}\end{bmatrix}$, We design the following state feedback control law: $U=-K_1X_1=-k_{11}q_1-k_{12}\dot{q}_1-k_{13}z_1-k_{24}\dot{z}_1-k_{25}\ddot{z}_1$.

The performance index function J_1 is defined as:

$$J_1=\lim_{T\to\infty}\frac{1}{T}\int_0^T\left[Y_1^TQ_1Y_1+u_m{}^TR_{u1}u_m\right]dt \quad (13)$$

Where,

$$Y_1=\begin{bmatrix} z_m \\ \dot{z}_G \\ \ddot{z}_G \end{bmatrix},\ Q_1=\begin{bmatrix} q_z & 0 & 0 \\ 0 & q_v & 0 \\ 0 & 0 & q_a \end{bmatrix},\ R_{u1}=r_1\ . \quad (14)$$

q_z, q_v, q_a and r_1 represent the weighting coefficient of airgap, velocity , acceleration and voltage of electromagnet..

When voltage u_m satisfies $u_m=-K_1X_1=-R_{u1}^{-1}B_1^TPX_1$, the J_1 obtains the minimum value, and P is the solution of Riccati equation: $-PA_1-A_1^TP+PB_1R_{u1}^{-1}B_1^TP-Q_1=0$

Where, $R_{u1}=1$, matrix A_1, B_1 and Q_1 are presented as follows:

According to Table 1, the feedback coefficient vector of the designed controller could be calculated as

$$K_1=\begin{bmatrix}-1761.5 & 35.2 & -6093.9 & -304.8 & -14.1\end{bmatrix} \quad (15)$$

Table 1. Parameters values of the coupling system

physical quantity	Value	physical quantity	Value
Mass of electromagnet m/kg	750	Permeability of air $\mu_0/(H\cdot m^{-1})$	1.26×10^{-6}
Number of turns in the coil N_m	356	Leakage permeance η	0
Pole area of the coil a_m/m^2	0.021	Bending rigidity $E_gI_g/(N\cdot m^{-2})$	2.69×10^{10}
coil resistance R_m/Ω	1.0	Damping coefficient $\delta/(N\cdot s\cdot m^{-1})$	8.56×10^{5}
Stable suspension airgap z_N/m	0.01	Linear density $\rho/(kg\cdot m^{-1})$	2000

5. SIMULATION RESULTS

The first-order, first 3-orders, and first 5-orders modes of the guideway are utilized to describe the dynamic characteristic of guideway. The parameter values are shown in Table 1. The distance between the initial position of electromagnet and the equilibrium point is 5mm. The simulation time is 1 sec. The time domain response of system is calculated in MATLAB. The simulation results are shown in Fig.2-Fig.3.

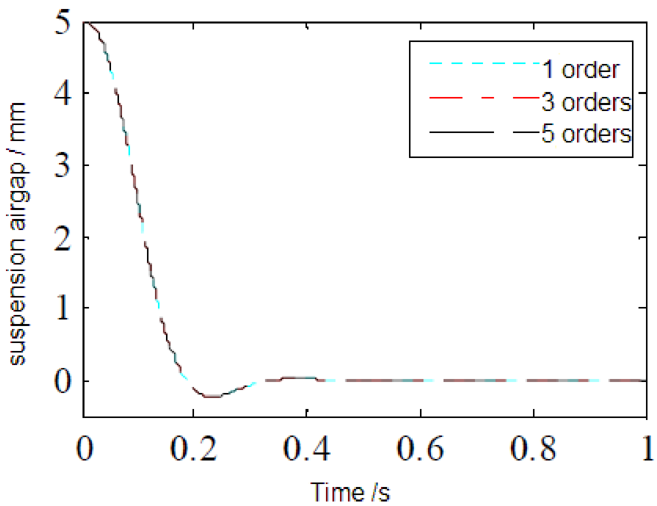

Figure 2. Suspension airgap

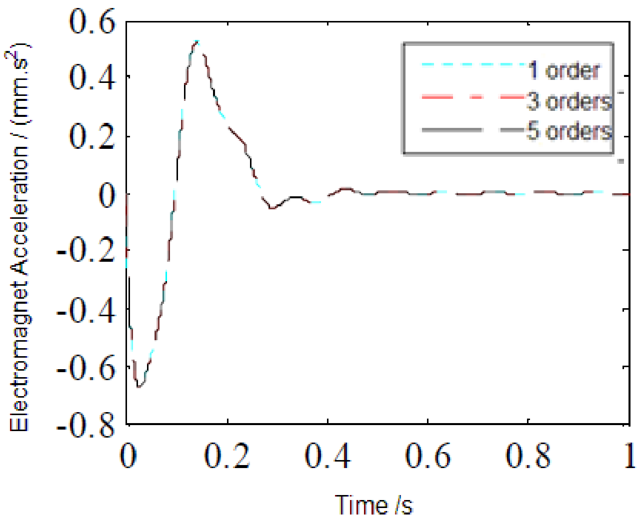

Figure 3. Vertical acceleration of the electromagnet

It can be seen from Fig.2-Fig.3 that if we set the stable region to 2%, the suspension airgap could completely enter the stable region in 0.28 sec and the maximum overshoot is about 2.5%. The vibration of electromagnet and guideway could attenuate quickly with the designed control law. The system is stable and shows excellent dynamic performance.

Since there is no obvious difference between the results of different modal orders of the guideway, it is reasonable to describe the vibration state of guideway with a low order mode in feedback state.

6. CONCLUSION

In this paper, a nonlinear mathematical model and novel control method have been presented for the nonlinear EMS system with flexible guideway. The open-loop instability of the coupling system has been proved by Hurwitz stability criterion. The vibration of guideway is introduced into the control system and the simulation results have been presented to demonstrate that the designed controller can not only ensure the electromagnet suspend steadily , but also eliminate the vibration of guideway, which means a lower request for the quality of guideway in maglev lines and the lower construction cost. Furthermore, future efforts will be directed at applying the proposed control strategy to the practice

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