EFFECT OF THERMAL RADIATION ON HEAT TRANSFER OVERAN UNSTEADY STRETCHING SURFACE IN A MICROPOLAR FLUID WITH VARIABLE HEAT FLUX

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ABSTRACT

Effect of thermal radiation on flow and heat transfer over an unsteady stretching surface in a micropolar fluid with variable heat flux is studied. The governing partial differential boundary layer equations are transformed into a system of ordinary differential equations containing the material parameter K, radiation parameter R, unsteadiness parameter A and Prandtl number Pr. These equations are solved numerically by mathematica program. Comparison of the numerical results is made with previously published results under the special cases, the results are found to be in good agreement. The effects of the unsteadiness parameter A, material parameter K, radiation parameter R and Prandtl number Pr on the flow and heat transfer are studied.

1. INTRODUCTION

The study of boundary layer flows over a stretching surface is important as it occurs in several engineering processes. Such processes are paper production, glass blowing wire drawing and glass fiber production. The dynamics of the boundary layer flow over a stretching surface originated from the pioneering work of Sakiadis [1] and [2] who initiated the study of boundary layer flow over a continuous solid surface moving with constant speed. Crane [3] extended it to analyze the steady two dimensional boundary layer flow caused by the stretching of elastic flat surface which moves in its plane with velocity varying linearly with distance from a fixed point. Many authors presented some mathematical results, and good amount of references can be found in the papers by Ali [4] and [5], Elbashbeshy [6], Ishak et al. [7] and Elbashbeshy and Bazid [8] . The studies carried out in these papers in the case steady state flow. The unsteady state problem over a stretching surface, which is stretched with a velocity that depends on time, is considered by Anderson et al. [9], Elbashbeshy and Bazid [10], Ishak et al. [11] and Elbashbeshy and Dalia [12]-[14]. The boundary layer flow and heat transfer of a Newtonian fluid over a stretching surface have been investigated by several authors [15]-[18]. The above investigations for the case of classical Newtonian fluids do not give satisfactory results if the fluid is a heterogeneous mixture such as liquid crystals, Ferro liquid and animal blood. In these fluids there are several constitutive equations, which do not obey the Newtonian laws. To overcome such a difficulty, Eringen [19] and [20] formulated the theory of micropolar fluids. Many researchers have considered with stretching surface in micropolar fluid by Elarabawy [21], Aldawody and elbashbeshy [22], Rahman et al. [23] and Ishak et al.[24] and [25].
The aim of the present paper is to discuss the unsteady boundary layer flow and heat transfer over a stretching surface in a micropolar fluid in the presence of the thermal radiation. Particular cases of the present results have been compared with Ishak et al. [25].

2. FORMULATION OF THE PROBLEM

Let us consider an unsteady, laminar, two-dimensional boundary layer flow of a viscous incompressible micropolar fluid over a continuous moving stretching surface in the presence of thermal radiation. The fluid is considered to be a gray, absorbing-emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x- axis is negligible with that in the y- axis. At time $t=0$, the surface is impulsively stretched with velocity $U_w(x,t)$ along the x-axis keeping the origin fixed in the fluid of ambient temperature T_∞.The stationary Cartesian coordinate system has its origin located at the leading edge of the surface with positive x-axis extended along the surface, while the y-axis is normal to the x-axis. The equations governing the flow in the boundary layer of an unsteady, laminar and incompressible micropolar fluid are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=(\frac{\mu+k}{\rho})\frac{\partial^2 u}{\partial y^2}+\frac{k}{\rho}\frac{\partial N}{\partial y} \tag{2}$$

$$\rho\, j\,(\frac{\partial N}{\partial t}+u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y})=\gamma\frac{\partial^2 N}{\partial y^2}-k\,(2N+\frac{\partial u}{\partial y}) \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k^{/}}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{4}$$

Subject to the boundary conditions

$$\begin{aligned} & y=0: u=U_w,\ v=0,\ N=-n\frac{\partial u}{\partial y},\ \frac{\partial T}{\partial y}=-\frac{q_w}{k^{/}} \\ & y\to\infty: u=0,\ N=0,\ T=T_\infty. \end{aligned} \tag{5}$$

Where x and y represent coordinate axes along the continuous surface in the direction of motion and normal to it, respectively. u and v are the velocity components along the x-axes and y-axes, respectively, t is the time, μ is the coefficient of dynamic viscosity ρ is the density, k is the vortex viscosity, N is the microrotation, $j=v/c$ is the micro-inertia density, γ is the spin gradient viscosity, T is the fluid temperature within the boundary layer, c_p is the specific heat of the fluid, $k^{/}$ is the thermal conductivity, q_r is radiative heat flux, T_w is the surface temperature, T_∞ is the free stream temperature and n is constant $(0\le n\le 1)$. It should be mentioned that the case $n=0$, we obtain $N=0$ which represents a no–spin condition i.e., the microelements in a concentrated particle flow close to the wall are not able to rotate (called the strong concentration). The case $n=0.5$ represents vanishing of the anti-symmetric part of the stress tensor (called weak concentration). The case $n=1$ is representative of turbulent boundary layer. We assume that γ is given by $\gamma=(\mu+0.5k)j=\mu(1+0.5K)j$, (See Ishak et al. [11]), where $K=k/\mu$ is the material parameter. It is assumed that the stretching velocity $U_w(x,t)$ and the heat flux $q_w(x,t)$ are of the form $U_w(x,t)=ax/(1-ct)$, $q_w(x,t)=bx/(1-ct)$. Where a, b and c are constants with $a>0, b>0$ and $c\ge 0$ (with $ct<1$) and both a and c have dimension time $^{-1}$. The radiative heat flux q_r, under Rosseland diffusion approximation [12], has the following form

$$q_r=-\frac{4\sigma_s}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

Where σ_s is the Stefan-Boltzman constant, and k^* is the absorption coefficient.
We assume that the temperature difference within the flow is sufficiently small so that T^4 can be expanded in a Taylor's series about T_∞ and neglecting higher orders we get:

$$T^4=4T_\infty^3 T-3T_\infty^4 \tag{7}$$

In view of equations (6) and (7), equation (4) reduces to:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma_s T_\infty^3}{3k^*\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

The equation of continuity is satisfied if we choose a stream function $\psi(x,y)$ such that: $u=\partial\psi/\partial y$, $v=-\partial\psi/\partial x$
The mathematical analysis of the problem is simplified by introducing the following dimensionless similarity variables

$$\begin{aligned} & \eta=(\frac{U_w}{vx})^{\frac{1}{2}}y,\quad \psi=(vxU_w)^{\frac{1}{2}}f(\eta), \\ & N=U_w(\frac{U_w}{vx})^{\frac{1}{2}}g(\eta), \theta(\eta)=\frac{k'(T-T_\infty)}{q_w}(\frac{U_w}{vx})^{\frac{1}{2}} \end{aligned} \tag{9}$$

Where η is the similarity variable. The transformed nonlinear ordinary differential equations are:

$$(1+K)f'''+ff''-f'^2+Kg'-A(f'+\frac{1}{2}\eta f'')=0 \tag{10}$$

$$(1+\frac{K}{2})g''+fg'-f\acute{g}-K(2g+f'')-A(\frac{3}{2}g+\frac{1}{2}\eta g')=0 \tag{11}$$

$$\theta''+k_0\Pr[f\theta'-f'\theta-\frac{A}{2}(\theta+\eta\theta')]=0 \tag{12}$$

Where primes denote differentiation with respect to η, $A=c/a$ is the unsteadiness parameter, $\Pr=\mu c_p/k'$ is Prandtl number, $R=16\sigma_s T_\infty^3/3k^*k^{/}$ is the thermal radiation parameter. Where $k_0=1/(1+R)$. It is worth mentioning here that when $k_0=1$, the thermal radiation effects not considered.
The boundary conditions (5) now become

$$\begin{aligned} & \eta=0: f=0, f'=1, g=-nf'', \theta'=-1 \\ & \eta\to\infty: f'=0,\quad g=0,\quad \theta=0. \end{aligned} \tag{13}$$

3. NUMERICAL SOLUTIONS

The Eqs. (7)-(9) can be converted to a system of differential equations of first order, by using

$$\begin{aligned} & y_1=f, y_2=f', y_3=f'', y_4=g, y_5=g', y_6=\theta, \\ & y_7=\theta', y_1'=y_2, y_2'=y_3, \\ & (1+K)y_3'=y_2^2-y_1y_3-Ky_5+A(y_2+\frac{1}{2}\eta y_3), \\ & y_4'=y_5, \\ & (1+\frac{K}{2})y_5'=y_2y_4-y_1y_5+K(2y_4+y_3)+ \\ & A(\frac{3}{2}y_4+\frac{1}{2}\eta y_5), y_6'=y_7, \\ & y_7'=\frac{P_r}{1+R}(y_2y_6-y_1y_7+\frac{A}{2}(y_6+\eta y_7). \end{aligned} \tag{14}$$

Subjected to the initial conditions

$$y_1(0)=0,\ y_2(0)=1,\ y_3(0)=a,\ y_4(0)=-ny_3(0),$$
$$y_5(0)=b,\ y_6(0)=c,\ y_7(0)=-1. \qquad (15)$$

Where a, b and c are unknown to be determined as a part of the numerical solution. Using mathematica, a function (F) has been defined such that F [a, b, c]:=NDSolve [system (14) and (15)]. The value of a, b and c are determined upon solving the equations, $y_2(\eta_{\max})=0,\ y_4(\eta_{\max})=0$ and $y_6(\eta_{\max})=0$ to get the solution. NDSolve first searches for initial conditions that satisfy the equations, using a combination of Solve and a procedure much like Find Root. Once a, b and c are determined the system (14) and (15) is closed, it can be solved numerically using the NDSolve function.
To validate the numerical method used in this study, the Newtonian flow case K= 0, was considered and the results for Temperature gradient are compared with the numerical solution which is reported in Ishak [25]. The quantitative comparison is shown in table (1) and found to be in a good agreement.

Table 1- Comparison of θ(0) for a various values of pr and A at K =0, R =0, n= 0.5,

A	Pr	Elbashbeshy [6]	Ishak[25]	Numerical results
0	0.72	1.2253	1.2367	1.236655
0	1	1.0000	1.0000	1.00000
1	0.72		0.9116	0.911554
1	1		0.8591	0.859175

4. SKIN FRICTION COEFFICIENT, COUPLE STRESS AND NUSSELT NUMBER.

The parameters of physical and engineering interest for present are the skin friction C_f, dimensionless couple stress M_x and Nusselt number Nu_x which are defined as

$$C_f=\frac{2\tau_w}{\rho U_w^2},\ \tau_w=[(\mu+k)\frac{\partial u}{\partial y}+kN] \qquad (15)$$

where τ_w is the surface shear stress

$$\sqrt{\mathrm{Re}}C_f=2[1+(1-n)k]f''(0) \qquad (16)$$

$$M_x=\frac{2M_w}{\rho\nu U_w}=(2+k)g(0),\ M_w=\gamma[\frac{\partial N}{\partial y}]_{y=0} \qquad (17)$$

where M_w is the surface couple stress

$$Nu_x=xq_w/k'(T_w-T_\infty) \qquad (18)$$

$$q_w=-k'[\partial T/\partial y]_{y=0}-q_r|_{y=0} \qquad (19)$$

$$Nu_x/\sqrt{\mathrm{Re}}=-(1+R)/\theta(0) \qquad (20)$$

Table 2- Results of the local Nusselt number $(1+R)/\theta(0)$ for different values of unsteadiness parameter A, n and Prandtl number Pr at R=1 and K=1

pr	0.72		1		10	
A	n=0	n=0.5	n=0	n=0.5	n=0	n=0.5
0	1.1317	1.0899	1.4051	1.3596	5.2651	5.2063
0.8	1.2333	1.1988	1.4979	1.4579	5.4793	5.4138
1	1.2599	1.2270	1.5310	1.4934	5.5442	5.4795
1.2	1.2882	1.2569	1.5617	1.5257	5.6163	5.5530

Table 3- esults of the local Nusselt number $(1+R)/\theta(0)$ for different values of unsteadiness parameter A and material parameter (K) at n=0.5, P_r=0.72 and R=1

K	0		3		5	
A	n=0	n=0.5	n=0	n=0.5	n=0	n=0.5
0	1.0721	1.0032	1.2349	1.1834	1.2858	1.2403
0.8	1.1320	1.1630	1.3195	1.2627	1.3556	1.3088
1	1.1705	1.1912	1.3335	1.2970	1.3772	1.3310
1.2	1.2057	1.2210	1.3547	1.3220	1.40015	1.3549

Table 4- Results of local Nusselt number $(1+R)/\theta(0)$ for different values of unsteadiness parameter A and thermal radiation parameter (R) at n=0, 0.5, P_r=0.72 and K=1

R	1		2		3	
A	n=0	n=0.5	n=0	n=0.5	n=0	n=0.5
0	1.1317	1.0899	1.2964	1.2336	1.4161	1.3580
0.8	1.2333	1.3769	1.4832	1.6109	1.6409	1.90451
1	1.2599	1.4282	1.5322	1.7275	1.7676	2.0080
1.2	1.2882	1.4252	1.5356	1.7388	1.8976	2.0177

5. RESULTS AND DISCUSSION

It is worth mentioning that for $K=0$ (Newtonian fluid), A=0 (steady state flow), k_o =1 (thermal radiation effects not considered) the problem is reduced to those considered by Elbashbeshy [6] and for K=0 & k_o =1 the problem is reduced those considered by Ishak et al. [11] and for k_o=1 the problem is reduced to those considered by Ishak et al. [25]. The quantitative comparison is shown in Table 1 and it is found to be in a very good agreement.

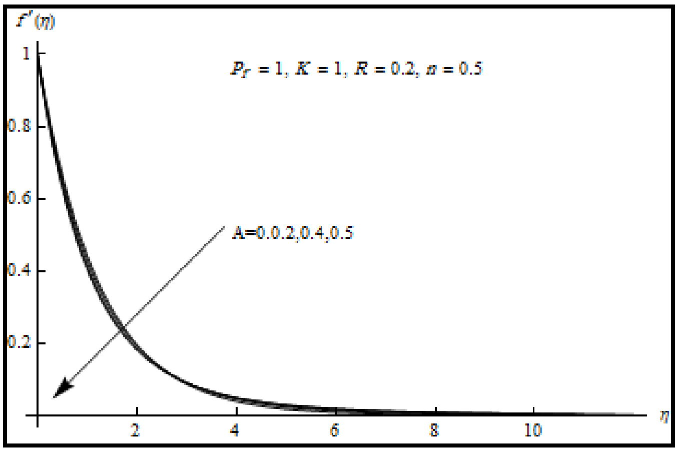

Figure 1 Velocity profiles $f'(\eta)$ for some values of A

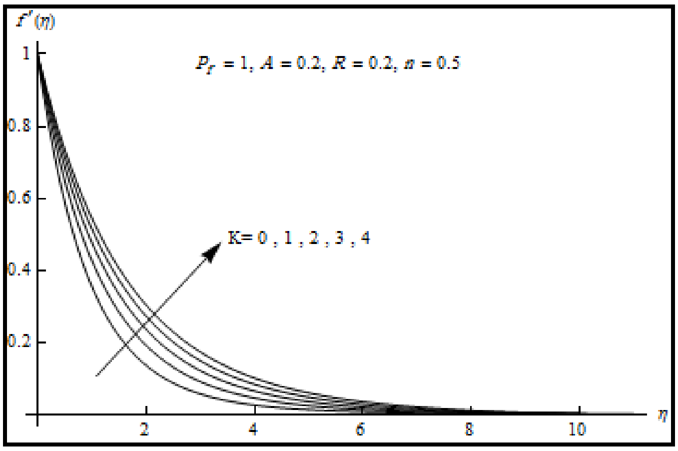

Figure 2 Velocity profiles $f'(\eta)$ for some values of K

Figure 1 and Figure 2 show the effects of the unsteadiness parameter A and material parameter K on the fluid velocity respectively. The effect of increasing A is to decrease the velocity of the fluid. The fluid velocity is incraesed due to increasing the value of the parameter K.

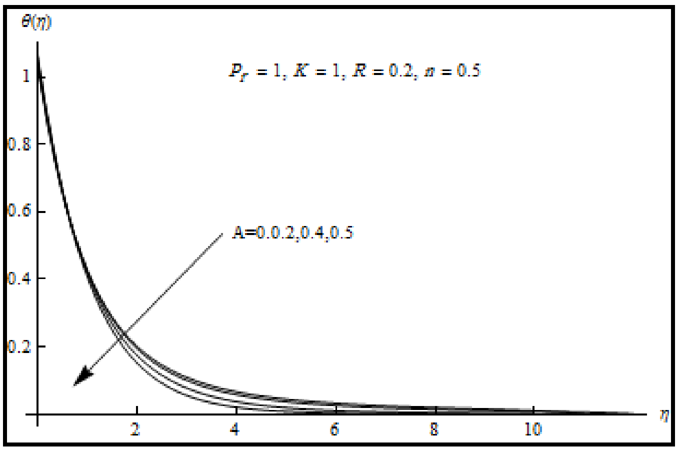

Figure 3 Temperature profiles $\theta(\eta)$ for some values of A

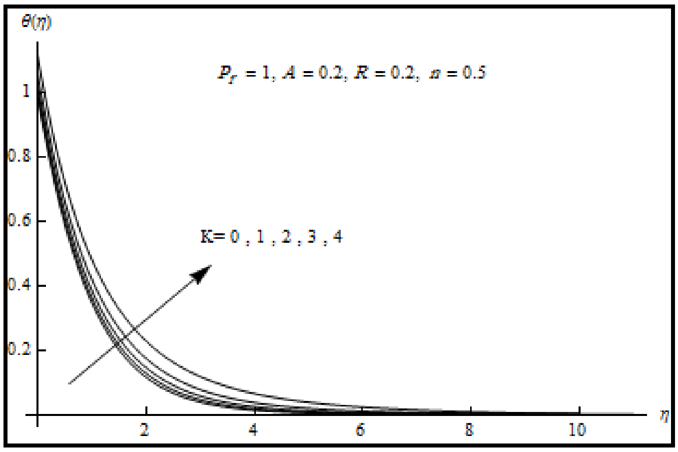

Figure 4 Temperature profiles $\theta(\eta)$ for some values of K

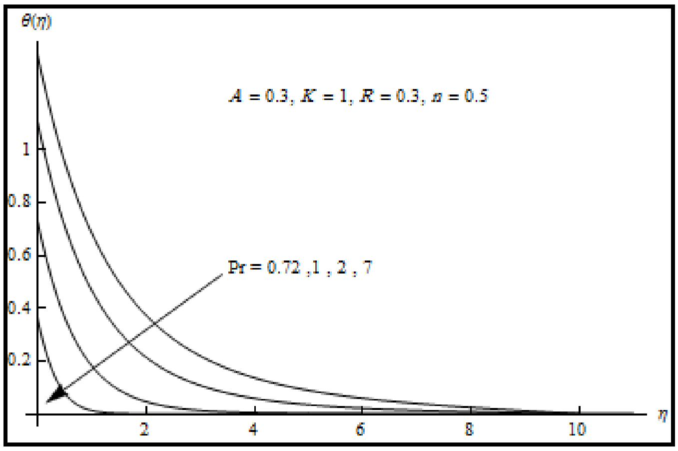

Figure 5 Temperature profiles $\theta(\eta)$ for some values of Pr

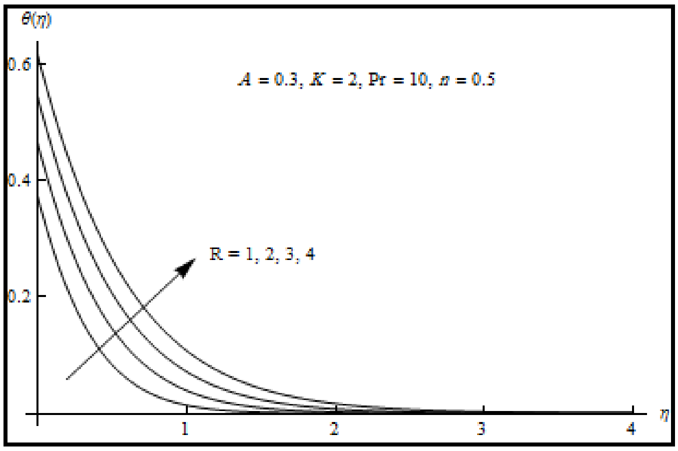

Figure 6 Temperature profiles $\theta(\eta)$ for some values of R

Figures (3)-(6) show the temperature profile for different values of the unsteadiness parameter (A), material parameter (K), Prandtl number (Pr) and thermal radiation parameter (R). Representative temperature profile is presented in Figure 3, for n=0.5, R=0.2, K=Pr=1 and different values of the unsteadiness parameter (A).The results show that the temperature decreases with the distance from the stretching surface. In addition, increasing the value of the unsteadiness parameter (A) tends to decrease the temperature within the boundary layer. Figure 4 shows that the effect of the material parameter (K) on the temperature.The temperature within the boundary layer increases with the increase of the material parameter (K), while in Figure 5, the temperature decreases with the increase of the Prandlt number (Pr), as the Prandtl number increases, viscous forces tend to suppress the buoyancy forces and cause the temperature in the thermal boundary layer to decrease.

It is also observed that (Figure 6) increasing the value of R have the tendency to increase the conduction effects and to increase the thermal boundary layer, so the fluid temperature is to increase

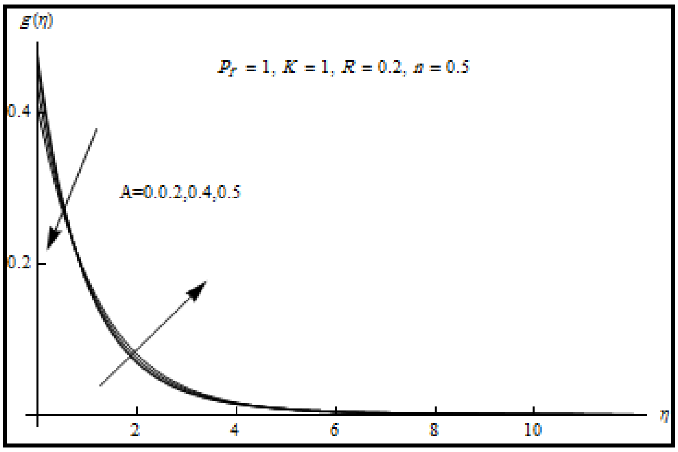

Figure 7 Angular velocity profiles $g(\eta)$ for some values of A

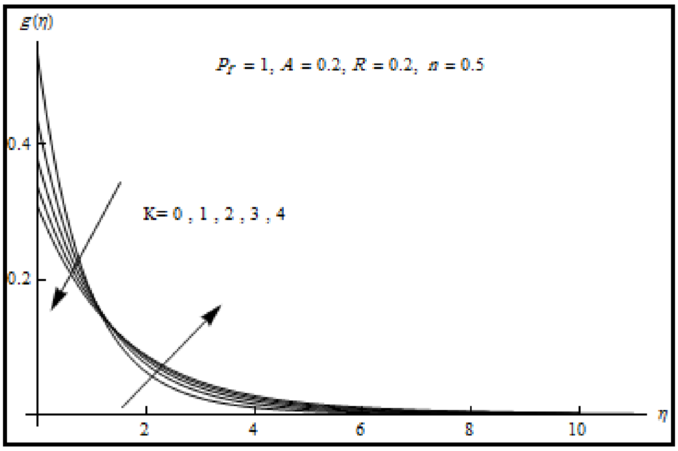

Figure 8 Angular velocity profiles $g(\eta)$ for some values of K

Figures (7) and (8) display the effects of the unsteadiness parameter (A) and the material parameter (K) on the micro rotation (angular velocity) profile. It is clear that as the unsteadiness parameter (A) and the material parameter (K) increase the angular velocity profile decreases in the region near of the surface and after a short distance from the surface these profiles overlap and then increase with increase of the unsteadiness parameter (A) and the material parameter (K).

In Tables (2)-(4) we have presented the local Nusselt number for various values of n, A, Pr, R, and K. These Tables show that the local Nusselt number is increased for all values of R, Pr, A, and K. This can be explained from the fact that as the Prandtl number increases the thermal boundary layer thickness decreases and the wall temperature gradient increases.

6.CONCLUSION

In this paper, we have studied the problem of the boundary layer flow of a micropolar fluid and heat transfer on an unsteady stretching surface in the presence of thermal radiation effect. The governing boundary layer equations were solved numerically .A discussion of the effects of the governing parameters; the unsteadiness parameter (A),material parameter (K), the Prandtl number (Pr) and the thermal radiation parameter (R) on the heat transfer characteristics in the cases n = 0, 0.5, Pr= 0.72, 1, 10, R=1, 2, 3, 4, K= 0, 1, 2, 3, 4 and A = 0, 0.2, 0.4, 0.5 has been done.We found that the heat transfer rate at the surface $(1+R)/\theta(0)$ increases with the increase of R, Pr, A, and K.

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