CHEMICAL REACTION EFFECT ON FORCED CONVECTIVE HEAT AND MASS TRANSFER FLOW OF MICROPOLAR FLUID PAST A CONTINUOUSLY MOVING POROUS PLATE IN THE PRESENCE OF RADIATION

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ABSTRACT

This paper investigates the effect of chemical reaction and radiation on flow of a micropolar fluid on continuously moving plate with suction or injection. The plate is moving with a constant velocity in the chemically reacting micropolar fluid. The radiative heat flux and the viscous dissipation are taken into account in the energy equation. The partial differential equations governing the flow have been transformed into system of ordinary differential equation using similarity transformation and then solved numerically by fourth order Runge-Kutta method with shooting technique. The velocity, microrotation, temperature, and concentration are shown graphically for different value of suction or injection parameter and chemical reaction parameter. The rate of mass transfer for different values of chemical reaction parameter is also shown graphically and it shows that mass transfer is highly depending on chemical reaction parameter.

Keywords: Micro-rotation, Radiation, Mass Transfer, Heat Transfer, Chemical Reaction.

1. INTRODUCTION

The growth of the industries in the present era has given a chance to develop the new branch in the theory of fluid dynamics, which is used to describe the flow of non-Newtonian fluids such as liquid crystals, animal fluid, and some polymeric fluids. The theory of micro polar fluid was first derived by Eringen [1]. Micropolar fluid is one of the complex fluids with microstructure. It consists of rigid, randomly oriented particles suspended in a viscous medium. Due to the rotation of the particle, the governing equation of the flow contains micro rotation field in addition to velocity field. Eringen [2] extended this theory to thermo micropolar theory. The concept of continuous surface was introduced by Sakiadis [3]. In this paper the flow of Newtonian fluid past continuously moving plate is consider and similarity transformation is used to determine the numerical solution. Its heat transfer aspect analyzed by Tsou et al. [4] and showed that this flow physically attainable under laboratory condition. Ebert [5] revealed that under comparable flow condition polar fluid will exhibit a greater resistance than a Newtonian fluid. In the literature many authors studied the theory of micropolar fluid. Ahmadi [6] presented the theory of micropolar fluid and its application to the dynamics of low concentration suspension flow over a semi-infinite plate for variable micro-inertia. The excellent review can be found in Ariman et al. [7]. Recent development in many engineering process such as extrusion of plastic sheet, crystal growing, polymer sheet extruded continuously for a die etc. makes a boundary layer flow of micropolar fluid on a continuously moving surface as important area of research. The heat transfer of micropolar fluid past continuously moving plate was studied by Soundalgekar and Takhar [8] by considering the fluid medium at rest for constant micro-inertia. Many engineering areas such as nuclear power plant, gas turbines and space vehicles are occur at very high temperature and knowledge of radiation effect on the fluid flow is very essential. Perdikis and Raptis [9] analyzed the heat transfer of a micropolar fluid in the presence of radiation. Raptis [10] reported the effect of radiation on flow of a micropolar fluid past a continuously moving plate where it was shown that increasing radiation parameter has the effect of decreasing the temperature. Hassan and Arabawy [11] studied radiation effect on the flow of a micropolar fluid past a continuously moving plate with suction/injection. The problems mentioned above are concerned with heat transfer effects only.

Combined heat and mass transfer effect in moving fluid is also important in view of several physical problems. In process such as drying evaporation at the surface of water body, energy transfer in wet cooling tower and the flow in a desert cooler heat and mass transfer occur simultaneously. Recently few authors studied the problem of heat and mass transfer effects on micropolar fluid past continuously moving flat plate in the presence of radiation. The study of combined heat and mass transfer problems with chemical reaction are of important in many processes such as fluids undergoing exothermic and endothermic chemical reaction. In many chemical processes, chemical reaction takes place between the surface and the fluid which moves due to continuous movement of the surface. Chemical reaction can be classified into homogeneous and heterogeneous. If the rate of reaction is directly proportional to spices concentration, then it is said to be first order chemical reaction.

Compared to available research materials the problem of chemical reaction and mass transfer effects on flow of a micropolar fluid on continuously moving plate with suction or injection has remained unexplored. The objective of the present study is to analyze the effects of radiation and mass transfer flow of a micropolar fluid past continuously moving flat porous plate with first order chemical reaction by considering the mass diffusion process simultaneously for all aspects of the flow.

2. FORMULATION OF A PROBLEM

Consider a steady, two dimensional, incompressible mass transfer flows on a continuously moving flat porous plate with a constant velocity in the presence of radiation and first order chemical reaction in a micropolar fluid medium at rest as shown in the Fig 1. The origin of the coordinate system is placed at the place where the plate is drawn into the fluid medium (slot). The problem is described in the rectangular coordinate system. Here the x-axis is taken along the plate and y-axis is normal to it. The surface of the plate is maintained at a uniform temperature T_W and a uniform concentration C_W. The fluid is considered to be gray. Radioactive heat flux in the x-direction is negligible compared to the flux in the y-direction. Viscous dissipation is also taken in to account.

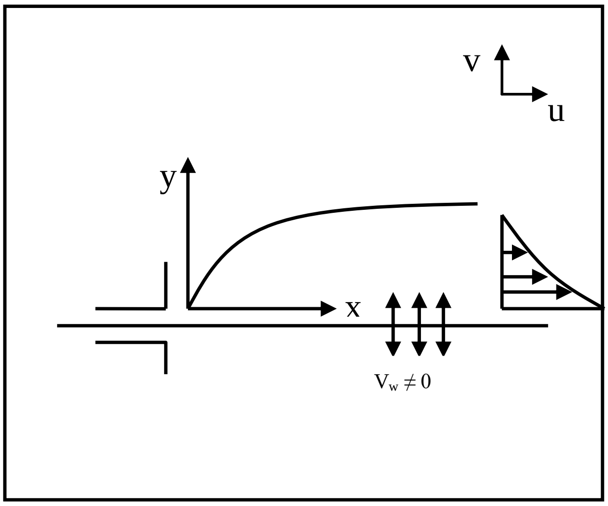

Figure1 Coordinate system and flow model

By the above assumption, the boundary layer governing the flow, angular velocity, and heat transfer of a micropolar fluid on a continuously moving plate is given by [10],

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \gamma\frac{\partial^2 u}{\partial y^2} + k_1\frac{\partial \sigma}{\partial y} \tag{2}$$

$$\gamma_s\frac{\partial^2 \sigma}{\partial y^2} - 2\sigma - \frac{\partial u}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right) + \frac{\gamma}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{4}$$

In addition to that species diffusion equation is also considered to analyze the mass transfer effet

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - R_C\left(C - C_\infty\right) \tag{5}$$

With the boundary condition

$$u = U_0,\ \ v = V_W,\ \ T = T_W,\ \ \sigma = 0,\ \ C = C_W \ \ at \ \ y = 0$$
$$u = 0,\ \ v = 0,\ \ T = T_\infty,\ \ \sigma = 0,\ \ C = C_\infty,\ \ as \ \ y \to \infty \tag{6}$$

where *u*, *v* are the velocity components along *x* and *y* co-ordinates respectively.

$\gamma = \frac{(\mu + s)}{\rho}$ is the apparent kinematics viscosity, μ is the dynamic viscosity, *s* is the gyro viscosity, ρ is the density of the fluid, σ is the micro-rotation component, $k_1 = \frac{s}{\rho}$ coupling constant, γ_s is the spin gradient viscosity, T is the temperature and C is the concentration of the fluid. C_p is the specific heat of the fluid at constant pressure, k is the thermal conductivity, q_r the radioactive heat flux, U_0 is the uniform velocity of the plate, V_W is the non zero velocity component of the wall. T_∞, C_∞ are the temperature and concentration of the ambient fluid.

Also Rosseland approximation [12] is used to describe the radioactive heat flux in the energy equation which leads to the radioactive heat flux $q_r = -\frac{4\sigma_1}{3k'}\frac{\partial T^4}{\partial y}$ where σ_1 is the Stefan Boltzmann constant and k' is the mean absorption co-efficient.

If the temperature difference with in the flow are sufficiently small such that T^4 may be expressed as linear function of the temperature, then the Taylor's series T^4 about T_∞ after neglecting the higher order terms is given by

$$T^4 \approx 4T_\infty^3 - 3T_\infty^4 \tag{7}$$

In view of (6), (7), Eq. (4) becomes

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\gamma}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{16\sigma_1 T_\infty^3}{3K'\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{8}$$

Now, we introduce the following dimensionless similarity transform

$$\eta = y\sqrt{\frac{U_0}{2\gamma x}}, \qquad \psi = \sqrt{2\gamma U_0 x}\, f(\eta)$$

$$\sigma = \sqrt{\frac{U_0^3}{2\gamma\, x}}\, g(\eta), \quad \theta = \frac{T - T_\infty}{T_W - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_W - C_\infty} \tag{9}$$

where $f(\eta), g(\eta)$ are dimensionless stream functions. θ, ϕ are the dimensionless temperature and concentration respectively.

In view of Eq. (9), Eq. (2)-(5) and (8) are reduced to following ordinary differential

$$f' + ff'' + Kg' = 0 \tag{10}$$

$$Gg'' - 4g - 2f'' = 0 \tag{11}$$

$$(3N + 4)\theta'' + 3\Pr N\theta' + 3N\Pr Ec(f'')^2 = 0 \tag{12}$$

$$\phi'' + Scf\phi' + ScR\phi = 0 \tag{13}$$

The corresponding initial and boundary conditions in non-dimensional quantities are given by

$$\begin{aligned} &\eta = 0: \quad f(0) = F_W, \quad f'(0) = 1, \quad \theta(0) = 1, \quad g(0) = 0, \quad \phi(0) = 1 \\ &\eta = \infty: \quad f'(\infty) = 0, \quad \theta(\infty) = 0, \quad g(\infty) = 0, \quad \phi(\infty) = 0 \end{aligned} \tag{14}$$

In the above

$$K = \frac{k_1}{\gamma}, G = \frac{U_0\gamma_s}{\gamma x}, \Pr = \frac{\gamma\rho c_\rho}{k}, N = \frac{k'k}{4\sigma_1 T_\infty^3},$$

$$Ec = \frac{U_0^2}{c_p(T_W - T_\infty)}, F_W = -V_W\sqrt{\frac{2x}{\gamma U_0}}, R = -R_C\left(\frac{2x}{U_0}\right)$$

The interesting physical quantities are local skin friction coefficient, the local nusselt number and the local Sherwood number, which can be defined as follows

$$C_f = \frac{2\tau_W}{\rho U_0^2}, \quad NU_x = \frac{xq_W}{k(T_W - T_\infty)}, \quad Sh_x = \frac{xq_M}{D(C_W - C_\infty)}$$

Where

$$\tau_W = \left[(\mu + K)\frac{\partial u}{\partial y} + K\sigma\right]_{y=0},$$

$$q_W(x) = \left[\left(-k - \frac{16\sigma_1 T_\infty^3}{3k}\right)\left(\frac{\partial T}{\partial y}\right)\right]_{y=0}, q_M(x) = -D\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

Using similarity variable (9), we get

$$C_f\, Re_x^{1/2} = -2f''(0),$$

$$NU_x\, Re_x^{-1/2}\left(\frac{3N}{3N+4}\right) = \frac{-\theta'(0)}{\sqrt{2}},$$

$$Sh_x\, Re_x^{-1/2} = \frac{-\phi'(0)}{\sqrt{2}}$$

3. NUMERICAL SOLUTION

The flow Eq. (10) and (11) are coupled together with the energy and concentration Eq. (12) and (13). This set of non-linear ordinary differential equation with the boundary condition (14) is solved by using fourth order Runge-Kutta method along with Nactsheim–Swigert shooting technique [13] for the prescribed parameter *G, F_{w,} K, N, Pr, R,* and *Sc.* In the boundary condition (14), there are four asymptotic boundary condition and hence there are four unknown surface condition $f''(0), g'(0), \theta'(0), and\ \phi'(0)$. Values of these unknown surface conditions are obtained by Nactsheim-Swigert technique [13]. A computer program was set up for the above-mentioned procedure along with fourth order Runge-Kutta method to solve the equation (10) – (13) with boundary condition (14). A step size of $\Delta\eta$ = 0.01 was selected to satisfy the convergence criterion of 10^{-4} in all cases.

4. RESULTS AND DISCUSSION

To discuss the effect of chemical reaction *R* and suction or injection parameter F_w on the velocity, temperature, concentration, and angular momentum, the numerical solution is given as a graph for physical parameters such as *G, K, N,* and *Pr.* In order to assess the accuracy, the results are compared for skin friction parameter $f''(0)$ and gradient of microrotation $g'(0)$ with [9]. The heat transfer parameter $-\theta'(0)$ is also compared and it has excellent agreement with them. The comparison is shown in the following table 1-3. As a result, heat transfer increases for increasing Pr.

Table1 Comparison value of $f''(0)$ for K = 0.2, G = 2 at R=0.0

Fw	Hassan A.M. El. Arabawy [11] $f''(0)$	Present Value $f''(0)$
0.0	-0.616542	-0.62715
0.2	-0.7428	-0.75524
0.4	-0.87751	-0.89392
0.7	-1.09943	-1.17331

Table 2 Comparison value of $g'(0)$ for K = 0.2, G = 2 at R=0.0

Fw	Hassan A.M. El. Arabawy [11] $g'(0)$	Present Value $g'(0)$
0.0	0.35533	0.36134
0.2	0.38927	0.3969363
0.4	0.4222	0.418205
0.7	0.46892	0.48958

Table 3 Comparison value of $-\theta'(0)$ for *Pr*=0.733, *K* = 0.2, *G* = 2, *N* = 5.0, *Ec* = 0.02 at *R*=0.0

Fw	Hassan A.M. El. Arabawy [11] $-\theta'(0)$	Present Value $-\theta'(0)$
0.0	0.427013	0.42736
0.2	0.49267	0.49145
0.4	0.56506	0.566031
0.7	0.6868	0.68497

The concentration profile for different values of *R*, *Pr* = 0.7, *K* = 0.2, *G* = 2.0, *N* = 5.0, and *Sc* = 2.0 are shown in Figure (2). It is observed that with the increasing value of *R*, the concentration profile decreases. Moreover, it is observed that with smaller values of *R*, concentration profile gradually decreases, whereas larger values of *R* affects the concentration profile tremendously, it can be observed that the concentration changes positive to negative and then reaches to zero. This is because of the concentration of the fluid do not remains constant and is consumed continuously during the course of reaction, so the concentration decreases as *R* increases. From figure (3) we observed that the change in chemical reaction parameter has no effect on the temperature profile.

The concentration profile for different values of *Sc*, *Pr* = 0.733, *K* = 0.2, *G* = 2.0, *R*=2.0 and *N* = 5.0 are also shown in Figure (4). It is also observed that the increasing value of *Sc* the concentration profile decreases. Increasing *Sc* means that the species diffusion reduces and viscous force increases which cause a decrease in concentration as expected.

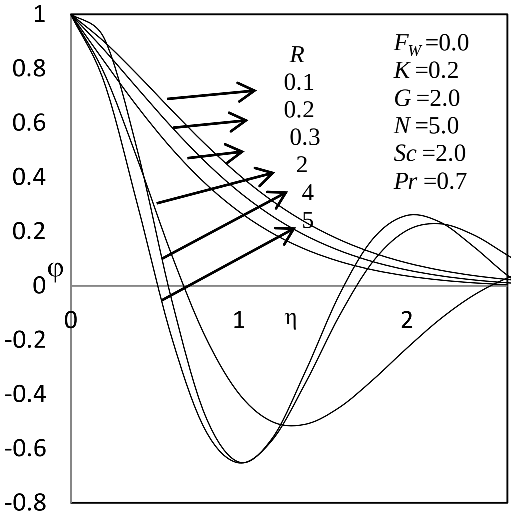

Figure 2 Concentration profile for different values of *R*

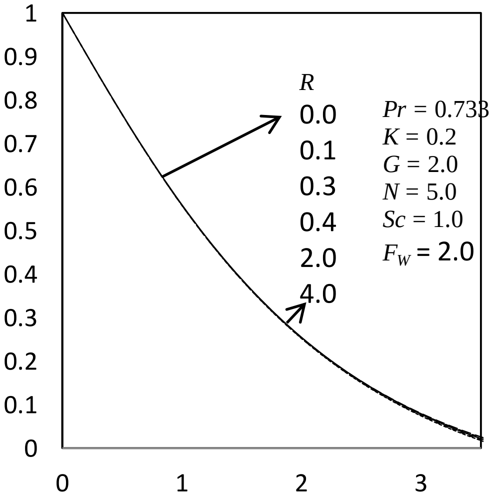

Figure 3 Temperature profile for different values of *R*

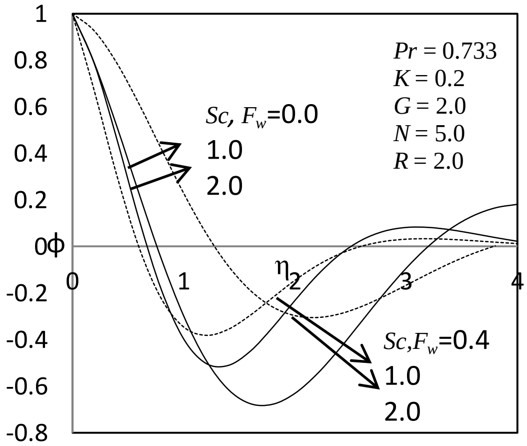

Figure 4 Concentration profile for different values of Sc

The dimensionless velocity component for different values of suction or injection F_w with K = 0.2, N = 5.0, Sc=2.0, R=2.0 and Pr = 0.733 is portrayed in Figure (5). It gives the effect of suction or injection parameter F_w on the velocity profile. It is also observed that for increasing value of F_w, the velocity field gradually decreases. If (F_w > 0), high resistance afford by the fluid and have a tendency to reduce the velocity of the flow, but wall injection (F_w < 0) produce opposite effect. This behavior is clearly seen from Figure (5).

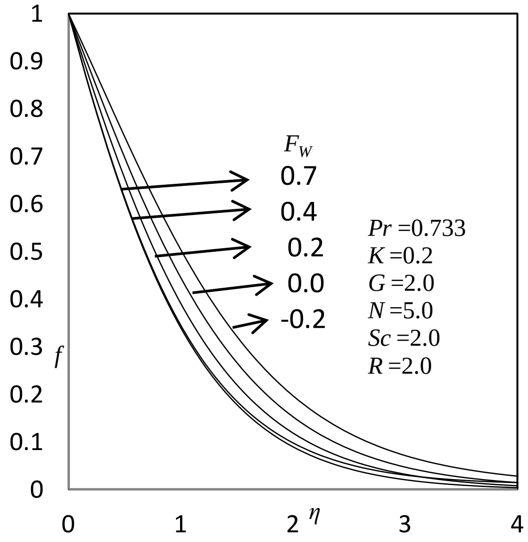

Figure 5 Velocity profile for different values of suction and injection

The effects of suction or injection parameter on the temperature profile for the value of Pr = 0.733 shown in Figure (6). Similar to velocity field, wall suction (F_w > 0) has a tendency to reduce the thermal boundary layer thickness and wall injection (F_w < 0) has a tendency to increase the thermal boundary layer thickness. Therefore, for increasing value of F_w, the temperature field gradually decreases.

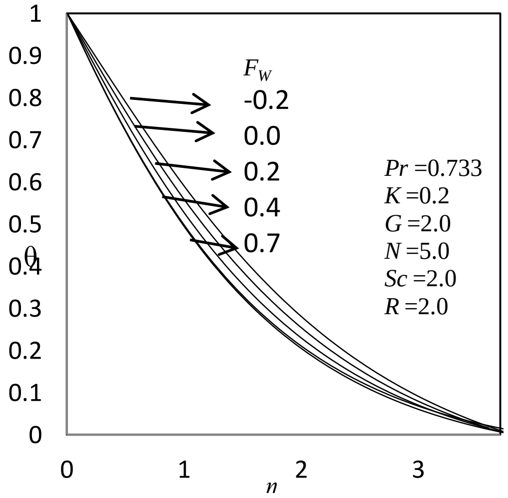

Figure 6 Temperature profile for different values of suction and injection

The temperature profile for different Pr values, R = 2.0, N = 5.0, Sc = 2.0, and F_w = 0 are shown in Figure (7). It shows that the temperature decreases with increasing value of Pr. For smaller value of Pr, thermal conduction is more, so the heat is able to diffuse more rapidly than higher values of Pr and also move away from the heat surface quickly than higher values of Pr.

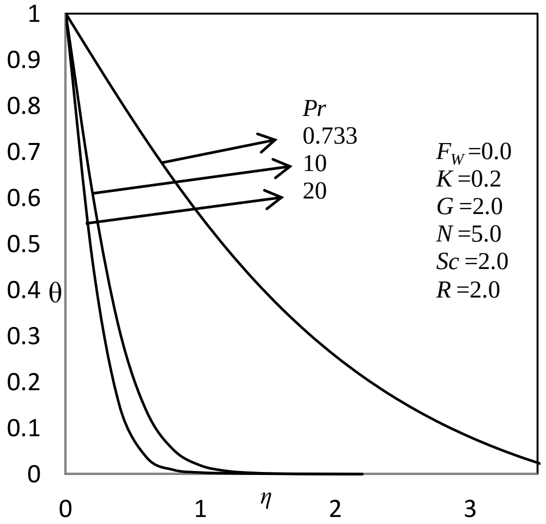

Figure 7 Temperature profile for different values of P

The effects of suction or injection on concentration profile for Sc = 2.0, Pr = 0.7, K = 0.2, G = 2.0, R = 2.0 and N = 5.0 are shown graphically in Figure (8). For increasing value of F_w, the concentration profile decreases. Angular velocity profiles for various F_w are presented in the fig (9). It suggested that micro rotation attains maximum and then reduced to zero.

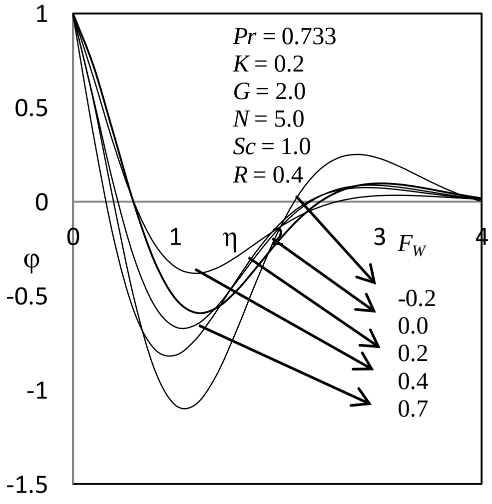

Figure 8 Concentration profile for different values of suction and injection

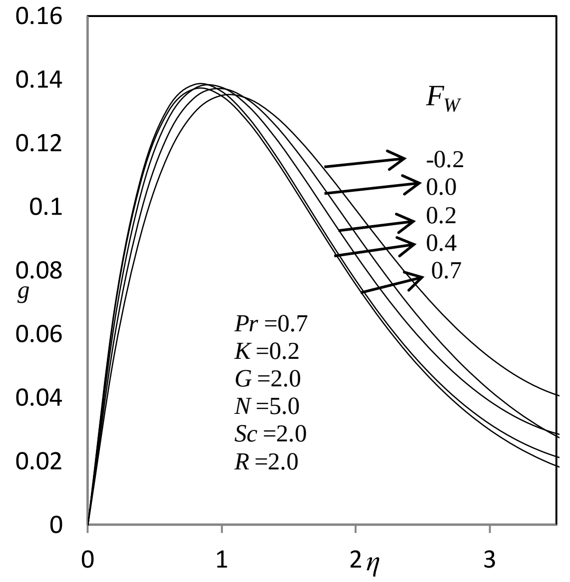

Figure 9 Micro-rotation profile for different values of F_W

Figure (10) depicts the effects of *Pr* on the heat transfer for various values of suction parameter F_w. It is observed that heat transfer increase quite rapidly with increasing *Pr*.

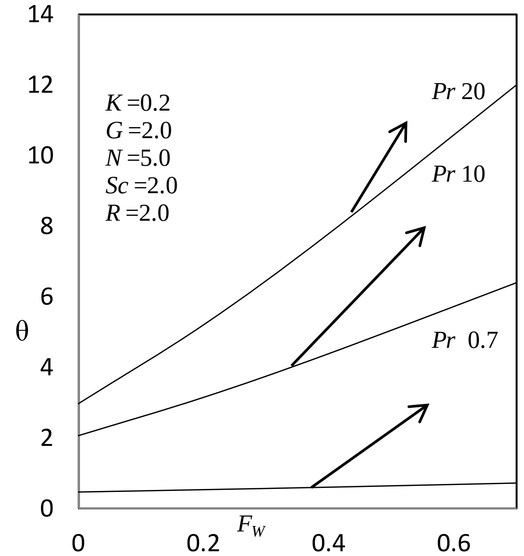

Figure10: Heat transfer parameter for different values of Pr

The rate of mass transfer for different value of chemical reaction parameter and different values of *Sc* are shown in Figure (11) and Figure (12). It shows that the mass transfer decrease for increasing value of *Sc* as well as *R*.

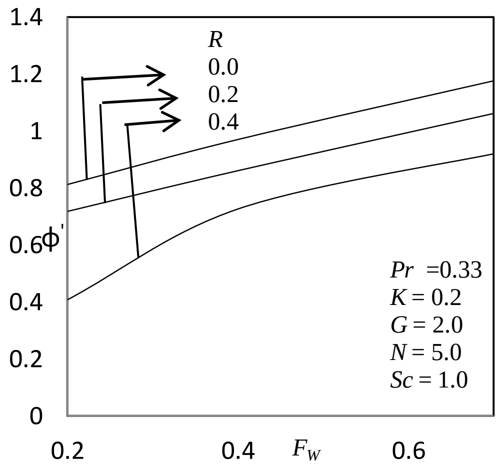

Figure 11 Rate of mass transfer against suction parameter for different value of chemical reaction parameter

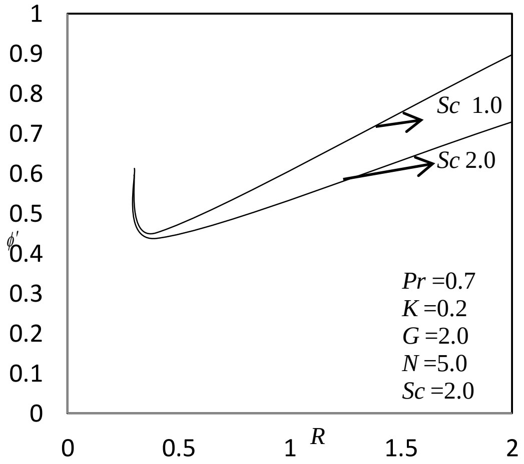

Figure12 Rate of mass transfer versus chemical reaction parameter R

5. CONCLUSION

A numerical study has been carried out to study the effect of chemical reaction on the flow of a micropolar fluid past continuously moving plate in the presence of mass transfer. The governing equations are transformed into system of non-linear ordinary differential equations by using similarity variables. It is solved numerically by using fourth order Runge-Kutta method along with Nactsheim–Swigert shooting technique [13]. Computation is carried out for the prescribed parameter F_w, *K*, *N*, *Pr*, *Sc*, *R* and *G*.

Conclusions of this study are as follows

(i) The velocity field decreases when the suction or injection parameter F_w increases.
(ii) The temperature field decreases as the suction or injection parameter F_w increases.
(iii) Also the temperature decreases due to increase in Prandtl number.

(iv) The concentration field decreases as the suction or injection parameter F_w increases and also concentration profile decreases at the increases of *Sc*.

(v) For increasing value of *R* the concentration profile decreases and for smaller values *R*, concentration profile gradually decreases, for large values *R* concentration profile changes positive to negative.

(vi) The heat transfer parameter increases as the *Pr* number is increased. The mass transfer parameter decreases with increase in *Sc* and *R*

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NOMENCLATURE

C	species concentration (kg / m^2)
C_p	specific heat (J / kg K)
D	binary diffusion coefficient
E_c	Eckert number
f	dimensionless stream function
F_w	suction or injection parameter
G	micro-rotation parameter
g	dimensionless micro rotation
k'	mean absorption coefficient
K_1	coupling constant
N	radiation parameter
Pr	Prandtl number
q_r	radio active heat flux (W /m^2)
R	chemical reaction parameter
Sc	Schmidt number
T	temperature
u	velocity in x direction (m/ s)
U_0	velocity of the plate (m/ s)
v	velocity in y direction (m/ s)
x	distance along the surface (meter)
y	distance normal to the surface (meter)

Greek Symbols

η	similarity Variables
γ	kinematic viscosity
γ_s	spin gradient viscosity
σ	micro-rotation component
σ_1	Stefan-Boltzmann constant
θ	dimensionless temperature
ϕ	dimensionless concentration

Subscripts

W	condition on the wall
∞	free stream condition