MHD FREE AND FORCED CONVECTION AND MASS TRANSFER FLOW PAST A POROUS VERTICAL PLATE

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ABSTRACT

A parametric study to investigate the effects of magnetic field, chemical reaction, thermal radiation, thermal diffusion (Soret effect) and diffusion – thermal (Dufour effect) on a free and forced convective fully developed boundary layer mass transfer flow of an electrically conducting viscous incompressible optically thick fluid past a semi-infinite vertical porous plate is presented. A magnetic field of uniform strength is assumed to be applied normal to the plate directed into the fluid region. The non-linear partial differential equations, governing the flow and heat and mass transfer have been transformed by a similarity transformation into a system of non-linear ordinary differential equations. The resulting system of ordinary non-linear differential equation is then solved numerically by adopting shooting method. The profiles of the dimensionless velocity, temperature and concentration distributions are demonstrated graphically for various values of the parameters involved in the problem.Finally, the corresponding local skin-friction co-efficient, local Nusselt number and local Sherwood number are also presented in tabular form. Some of the results of the present work are compared with that of Alam et al. [22] and found to be in good agreement

1. INTRODUCTION

MHD is concerned with the study of the interaction of magnetic fluids and electrically conducting fluids in motion. There are numerous examples of application of MHD principles, including MHD generators, MHD pumps and MHD flow meters, etc. Convection problems of electrically conducting fluid in presence of transverse magnetic field have got much importance because of its wide application in Geophysics, Astrophysics, Plasma Physics, and Missile Technology etc. MHD principles also find its application in medicine and Biology. The present form of MHD is due to the pioneer contribution of several notable authors like Alfven [1], Cowling [2], Ferraro and Pulmption [3], Shercliff [4] and Crammer and Pai [5].

Model studies on MHD heat and mass transfer problems have been carried out by many authors due to their applications in many branches of science and technology. Some of them are Singh and Singh [6], Singh et al. [7] and Ahmed [8].

In many times, it is observed that the foreign mass reacts with the fluid and in such a situation chemical reaction plays an important role in chemical industry. The study of the effect of chemical reaction on heat and mass transfer in a flow is of great practical importance to the Engineers and Scientist because of its almost universal occurrence in many branches of science and technology. In processes such as drying, distribution of temperature and moisture over agricultural fields and graves of fruit trees, damage of crops due to freezing, evaporation at the surface of a water body, energy transfer in a wet cooling tower, and flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. Many investigators have studied the effect of chemical reaction on different convective heat and mass transfer flows, of whom Apelblat [9], Andersson et al. [10], Muthucumaraswamy et al. [11], Kundasamy et al. [12], Rajeswari et al. [13] and worth mentioning.

Radiation is a process of heat transfer through electromagnetic waves. Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, and solar power technology and space vehicle re-entry. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role in space related technology. The effect of radiation on various convective flows under different conditions has been studied by many researchers including Hossain and Takhar[14], Ahmed and Sarmah [15] , Beg and Ghosh[16] and Kesavaiah et al. [17].

Thermal- diffusion (Soret effect) and diffusion-thermo (Dufour effect) concern with the methods of separating heavier gas molecules from lighter ones by maintaining temperature and composition gradients respectively over a volume of a gas containing particles of different masses. These methods are also used for separating the isotopes of an element. When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more intricate nature. It has been found that an energy flux can be generated not only by temperature gradient but by composition gradient as well. The energy flux caused due to composition gradient is called the Dufour or diffusion - thermo effect whereas the mass flux created by temperature gradient is termed as Soret or thermal -diffusion effect. In general, Soret and Dufour effects are of a smaller order of magnitude than the effects described by Fourier's or Fick's laws and are often neglected in heat and mass transfer processes. There are, however, exceptions. The Soret effect is utilized for isotope separation, and in mixtures between gases with very light molecular weight (H_2, H_e). For medium molecular weight (N_2, air), the Dufour effect is found to be of a considerable magnitude such that it can not be neglected as emphasized by Eckert and Drake [18]. In view of the importance of these above effects, several authors have carried out their reasonable works to investigate the thermal – diffusion and diffusion – thermo effects on various mass transfer related problems. Some of them are Kafoussias and Williams [19], Anghel et al. [20] Postenlnicu [21], Alam et al. [22], Ahmed [23], Ahmed and Sengupta [24] and Reddy and Reddy [25]. Recently Lorenzini et al.[26] have investigated the constructal design applied to the Geometric Optimization of Y-shaped cavities embedded in a conducting medium. The contribution of Bejan and Lorente[27] on Constructal Theory is worth mentioning. As far as the present authors are aware no attempt has been made till now to study the combined effect of magnetic field, chemical reaction, thermal radiation, thermal-diffusion and diffusion – thermo on a two dimensional boundary layer flow of an incompressible viscous electrically conducting fluid past a semi-infinite porous vertical plate with variable suction in presence of a uniform transverse magnetic field. Such an attempt has been made in the present work.

2. MATHEMATICAL FORMULATION

The equations governing the steady motion of an incompressible viscous electrically conducting radiating and chemically reacting fluid in presence of magnetic field are-

The continuity equation:

$$\operatorname{div} \vec{q} = 0 \tag{2.1}$$

The momentum equation:

$$\left(\vec{q}.\vec{\nabla}\right)\vec{q} = -\frac{1}{\rho}\vec{\nabla}p + \upsilon\nabla^2\vec{q} + \frac{\vec{J}\times\vec{B}}{\rho} + \vec{g} \tag{2.2}$$

The energy equation:

$$\left(\vec{q}.\vec{\nabla}\right)T = \alpha\nabla^2 T + \frac{D_M K_T}{C_S C_P}\nabla^2 C - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial n} \tag{2.3}$$

The species continuity equation:

$$\left(\vec{q}.\vec{\nabla}\right)C = D_M \nabla^2 C + \frac{D_M K_T}{T_M}\nabla^2 T + Q\left(C_\infty - C\right) \tag{2.4}$$

The Gauss's law of magnetism:

$$\vec{\nabla}.\vec{B} = 0 \tag{2.5}$$

The Ohm's law:

$$\vec{J} = \sigma\left[\vec{E} + \vec{q}\times\vec{B}\right] \tag{2.6}$$

The physical quantities involved in the above equations are defined in the Nomenclature.

We now consider a fully developed steady two-dimensional free and forced convective mass transfer flow of an incompressible viscous electrically conducting radiating having optically thick limit property and chemically reacting fluid past a semi-infinite vertical porous plate in presence of a transverse applied magnetic field taking into account the thermal – diffusion (Soret effect) and the diffusion thermo (Dufour) effects. Our investigation is restricted to the following assumptions.

(1) All the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the buoyancy force term.

(2) The magnetic Reynolds number is so very small to neglect the induced magnetic field.

(3) The surface of the plate is maintained at a constant temperature T_W , which is higher than the constant temperature T_∞ of the fluid far away from the plate.

(4) The surface of the plate is maintained at a uniform constant concentration C_W , of a foreign fluid which is higher than the constant concentration C_∞ of the fluid far away from the plate.

(5) The free stream velocity U_∞ parallel to the vertical porous plate is constant.

(6) The plate is electrically non-conducting**.**

(7) There is no applied electric field, and hence $\vec{E} = 0$.

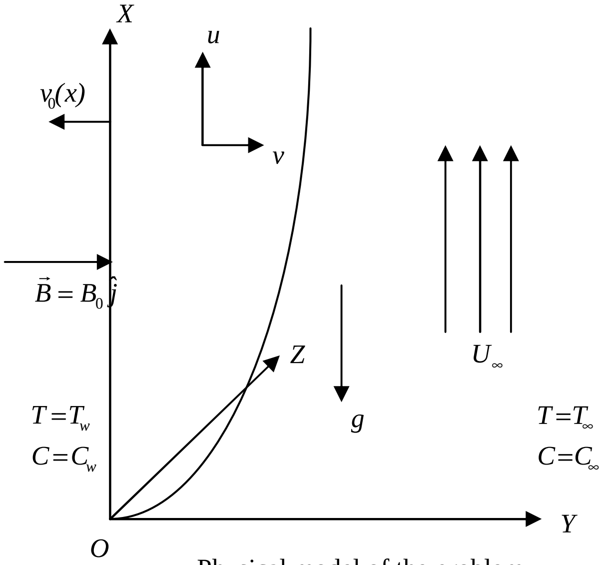
We now introduce a co-ordinate system (x, y, z) with X–axis vertically upwards along the plate, Y-axis normal to the plate directed into the fluid region and Z-axis along the width of the plate.

Let $\vec{q} = (u, v, o)$ denote the fluid velocity at a point (x, y, z) in the fluid and $\vec{B} = (0, B_0, 0)$ be the applied magnetic field.

With the foregoing assumptions and under usual boundary layer and Boussinesq of approximations, the governing equations reduce to:

The continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.7}$$



Physical model of the problem

The momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\bar{\beta}(C-C_\infty)$$

$$+\frac{\sigma B_0^2}{P}(U_\infty-u) \qquad (2.8)$$

The energy equation:

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{D_M K_T}{C_S C_p}\frac{\partial^2 C}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \qquad (2.9)$$

The species continuity equation:

$$u\frac{\partial C}{\partial x}+\upsilon\frac{\partial C}{\partial y}=D_M\frac{\partial^2 C}{\partial y^2}+\frac{D_M K_T}{T_M}\frac{\partial^2 T}{\partial y^2}+Q(C_\infty-C) \qquad (2.10)$$

For quantification, the thermal radiation effect from an optically thick layer in terms of the rediative heat flux q_r under Rosseland approximation is given by

$$q_r=-\frac{4\sigma_1}{3K_1}\frac{\partial T^4}{\partial y} \qquad (2.11)$$

Assuming the temperature differences within the flow to be sufficiently small, T^4 may be expressed as a linear function of the temperature T, and expanding T^4 in Taylor's series about T_∞ and neglecting the higher order terms, we thus derive

$$T^4 = f(T) \approx f(T_\infty)+(T-T_\infty)f'(\infty)$$

$$=T_\infty^4+4(T-T_\infty)T_\infty^3=4TT_\infty^3-3T_\infty^4 \qquad (2.12)$$

The equations (11) and (12) give,

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma_1}{3K_1}T_\infty^3\frac{\partial^2 T}{\partial y^2} \qquad (2.13)$$

The equation (2.9) yields,

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{D_M K_T}{C_S C_P}\frac{\partial^2 C}{\partial y^2}$$

$$+\frac{1}{\rho C_p}\frac{16\sigma_1}{3K_1}T_\infty^3\frac{\partial^2 T}{\partial y^2} \qquad (2.14)$$

The relevant boundary conditions for the problem are as follows:

$$u=0\,,\, v=-v_0(x)\,,\, T=T_w\,,\, C=C_w \text{ at } y=0 \qquad (2.15)$$

$$u=U_\infty\,,\, v=0\,,\, T=T_\infty\,,\, C=C_\infty \text{ at } y\to\infty \qquad (2.16)$$

In the equation (2.9), the terms $\frac{D_M K_T}{C_S C_P}\frac{\partial^2 C}{\partial y^2}$ and $\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}$ signify the diffusion –thermo effect and the thermal radiation respectively. On the other the last term and the last but one term on the right hand side of the equation (2.10) refer to the chemical reaction and Soret effect respectively.

The equations (2.7), (2.8), (2.10) and (2.14) are coupled, parabolic and non-linear partial differential equations and hence it is very difficult to have an analytical solution. Therefore numerical technique is employed to obtain the required solution. Numerical computations are greatly fascinated by non-dimensionalization of the equations. In order to convert the partial differential equations (2.8), (2.14) and (2.10) from two independent variables (x, y) to a system of coupled, non-linear ordinary differential equations in a single variable η, we introduce the following similarity transformations, dimensionless variables and non-dimensional parameters:

$$\eta=y\sqrt{\frac{U_0}{2\upsilon x}}\,,\,\psi=\sqrt{2\upsilon x U_0}\,f(\eta)\,,\, u=\frac{\partial\psi}{\partial y}=U_0 f'(\eta)$$

$$\upsilon=-\frac{\partial\psi}{\partial x}=\sqrt{\frac{\upsilon U_0}{2x}}(\eta f'-f)\,,\,\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\,,$$

$$\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}\,,Re_x=\frac{2U_0 x}{\nu}\,,Gr=\frac{2g\beta x(T_w-T_\infty)}{U_0^2}\,,$$

$$Gm=\frac{2g\bar{\beta}x(C_w-C_\infty)}{U_0^2}\,,M=\frac{2\sigma B_0^2 x}{\rho U_0}\,,Pr=\frac{\upsilon}{\alpha}\,,Sc=\frac{\upsilon}{D_M}\,,$$

$$Sr=\frac{D_M K_T(T_w-T_\infty)}{\upsilon T_M(C_w-C_\infty)}\,,Df=\frac{D_M K_T}{C_s C_p\upsilon}\frac{(C_w-C_\infty)}{(T_w-T_\infty)}\,,$$

$$N=\frac{\alpha\rho C_p K_1}{4\sigma_1 T_\infty^3}\,,\quad \xi=\frac{2x\upsilon Q}{U_0 D_M}$$

The non dimensional form of the equations (2.8), (2.14) and (2.10) are as follows:

$$f'''+ff''-Mf'=-M-Gr\,\theta-Gm\,\phi \qquad (2.17)$$

$$\left(1+\frac{4}{3N}\right)\theta''+Pr\,f\theta'+Pr\,Df\,\phi=0 \qquad (2.18)$$

$$\phi''+Sc\,Sr\theta''+Sc\,f\phi'-\xi\phi=0 \qquad (2.19)$$

Subject to the boundary conditions:

$$f=f_w\,,\,f'=0\,,\,\theta=1\,,\,\phi=1 \text{ at } \eta=0 \qquad (2.20)$$

$$f'=1\,,\,\theta=0\,,\,\phi=0 \qquad \text{at } \eta\to\infty \qquad (2.21)$$

where $f_w=V_0\sqrt{\frac{2x}{\upsilon U_0}}$

3. METHOD OF SOLUTION

The non-linear ordinary differential equations (2.17) - (2.19) subject to the boundary conditions (2.20) and (2.21) are solved numerically using shooting iteration technique. In shooting method, the boundary value problem is converted to initial value problem by assigning some initial condition at the initial point of the interval. In order to check the accuracy of the missing initial condition, the value of the dependent variable at the terminal point is calculated and this value is compared with the given value there. An another missing initial condition is inserted provided a difference between the two values exists and this process is repeated until a suitable agreement between the calculated value and given value is achieved. In case of this type of iteration approach, it is required whether there is a systematic way of finding each succeeding value of missing initial condition. The boundary conditions (2.20) and (2.21) associated with the resulting governing equations are the two -point asymptotic classes. By the two point boundary conditions, it is meant that the dependent variable has values at two different values of the independent variable. In an asymptotic boundary condition, the first derivative or higher derivatives of the dependent variable approaches zero as the outer specified value of the independent variable is approached. In the numerical solution of a two-point asymptotic boundary value problem of boundary – layer type, the initial – value method is almost similar to an initial value problem. Therefore, for this type of solution, it is necessary to impose as many boundary conditions at the surface as were previously given at infinity. The system of the governing differential equations is then solved with these assumed surface conditions. If the required outer boundary is found to be satisfied, a solution is derived. However this cannot be treated as a general case. Hence, a method is to be devised so that new surface boundary conditions may be assumed for next trial integration. The asymptotic boundary value problems are further complicated by the fact that the outer boundary condition is specified at infinity. In the trial solution, infinity is numerically approximated by some large value of the independent variable. There is no priori rule of selecting these values. These values should be selected in such a way that the solution is allowed to asymptotically convergence and the procedure of integration is not so expensive in terms of computer time.

The shooting iteration method used in the present work is developed in such a way that the above criteria are fulfilled.

4. SKIN FRICTION

The skin friction at the plate in the direction of the free stream is given by

$$\tau_0 = \mu \frac{\partial u}{\partial y}\bigg]_{y=0} = \frac{\rho U_0^2 f''(0)}{\sqrt{Re_x}} \tag{4.1}$$

The local skin friction co-efficient C_f at the plate which signifies the surface shear stress is as defined as follows:

$$C_f = \frac{\tau_0 \sqrt{Re_x}}{\rho U_0^2} = f''(0) \tag{4.2}$$

5. NUSSELT NUMBER

The heat flux q_w at the plate is given by the Fourier law of heat conduction

$$q_w = -K^* \frac{\partial T}{\partial y}\bigg]_{y=0} \tag{5.1}$$

The equation (5.1) yields

$$2xq_w = -K^*\left(T_w - T_\infty\right)\theta'(0)\sqrt{Re_x} \tag{5.2}$$

where $K^* = k + \frac{16\sigma_1}{3K_1} T_\infty{}^3$

6. SHERWOOD NUMBER:

The mass flux M_w at the plate is determined by the Fick's law of mass diffusion.

$M_w = -D_M \frac{\partial C}{\partial y}\Big]_{\bar{y}=0}$ which yields

$$2xM_w = -D_M\left(C_w - C_\infty\right)\phi'(0)\sqrt{Re_x} \tag{6.1}$$

The local co-efficient of the rate of mass transfer at the plate in terms of the Sherwood number which embodies the ratio of convective to diffusive mass transport and simulates the surface mass transfer rate, is defined by

$$Sh = \frac{2xM_w}{D_M\left(C_w - C_\infty\right)\sqrt{Re_x}} = \phi'(0) \tag{6.2}$$

7. RESULTS AND DISCUSSION

In order to get clear insight of the physical problem, numerical computations for the representative velocity field, temperature field, concentration field and the co-efficient of the skin friction and the rates of heat and mass transfer in terms of Nusselt number and Sherwood number respectively at the plate have been carried out for different values of the magnetic parameter *M*, Soret number *Sr*, Dufour number *Df*, chemical reaction parameter ξ, radiation parameter *N*, free convection parameter Gr, and Gm, the Prandtl number Pr and Schmidt number Sc keeping the value of f_w fixed at 0.51. In the most of the cases the value of Pr is taken equal to 0.71 which corresponds physically to air and the value of Sc has been chosen to represent hydrogen at $Tm = 25^0 C$ and 1 atmosphere pressure. That is in the most of the cases of our parametric study, it is assumed that Hydrogen is diffused in air. Indeed, as we are interested in the investigation of the chemical reaction and thermal radiation also on the flow and transport characteristics, some arbitrary values of Pr and Sc are also considered. Throughout our investigation the values of the other parameters involved are chosen arbitrarily.

With the above mentioned flow parameters, the numerical results are illustrated for uniform wall temperature and species concentration in the figures 1-12 and tables 1-8, for the velocity, temperature and concentration profiles and the co-efficient of local skin friction C_f and the co-efficient of local rates of heat and mass transfer.

The figures 1-6 exhibit the behavior of the velocity field u due to variations of the parameters M, N, ξ, Df, Gm, Gr respectively. The figures 1, 2 and 3 show that an increase in the values of the parameters M, N, ξ leads to a decrease in the velocity field indicating the fact that the flow field is retarded due to the imposition of the transverse magnetic field, thermal radiation and chemical reaction. As such the magnetic field is an effective regularity mechanism for stabilizing the flow. On the other hand, it is observed from the figures 4, 5 and 6, that the flow is accelerated for the increasing values of Df, Gm, Gr respectively. The observations from the figures 1– 6 reveal the fact that the velocity boundary growth may be inhibited mainly under the effect of the transverse magnetic field. This phenomenon is consistent with the well known boundary layer theory and MHD principle. All the above figures further indicate that the velocity field u first increases from its zero value in a thin layer adjacent to the wall and there after it decreases asymptotically to its potential value as $\eta \to \infty$, establishing the fact that the buoyancy force has an significant role in controlling the flow field near the plate and its effect is almost nullified in the fluid region far away from the wall.

It is inferred from figures 7 and 10 that the temperature distribution θ falls down monotonically under the effects of thermal radiation and increasing Prandtl number. This result is in a good agreement to the fact that the growth of the thickness of the thermal boundary layer may be prevented with increasing Prandtl number.

The figures 8 and 9 establish that the fact that the effect of chemical reaction and Dufour effect have some contributions in raising the fluid temperature substantially. Further it is marked in figures 7–10 that the temperature field θ sharply and asymptotically decreases from its maximum value $\theta = 1$ at $\eta = 0$ to $\theta = 0$ as $\eta \to \infty$.

The variation of the concentration field ϕ under the effects of the parameters involved in the problem is presented in figures 11 and 12. These two figures establish the fact that there is a steady fall in the concentration of the fluid indicating reduction in the thickness of the concentration boundary layer due to chemical reaction. This phenomenon is substantially supported from physical reality. Like the temperature field, the concentration field ϕ also falls down asymptotically from its maximum value $\phi = 1$ at $\eta = 0$ to $\phi = 0$ as $\eta \to \infty$.

The tables 1–8 exhibit how the co-efficient of the skin friction C_f at plate, the co-efficient of the rate of heat transfer from the plate to the fluid in term of the Nusselt member Nu and the co-efficient of the rate of mass transfer from the plate to the fluid in term of the Schmidt number Sh are affected by the parameters entering into the problem under consideration for investigation. We infer from these tables that the viscous drag on the plate is increased under the effects of diffusion–thermo, free convections for both heat and mass transfer and for increasing Schmidt number. On the other hand the effects of chemical reaction, thermal diffusion, thermal radiation and the applied magnetic field contribute on steady fall in the internal friction on the plate due to viscosity.

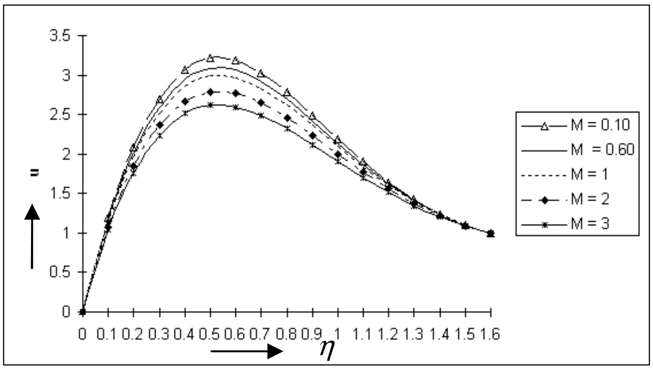


Figure 1: The velocity u versus η under the Hartmann number *M* for *Pr=0.71, Gr=15.00, Gm=10.00, fw=0.51, Sc=0.22, Sr=1.00, Df =1.00, N=1.00,* ξ *=1.00*

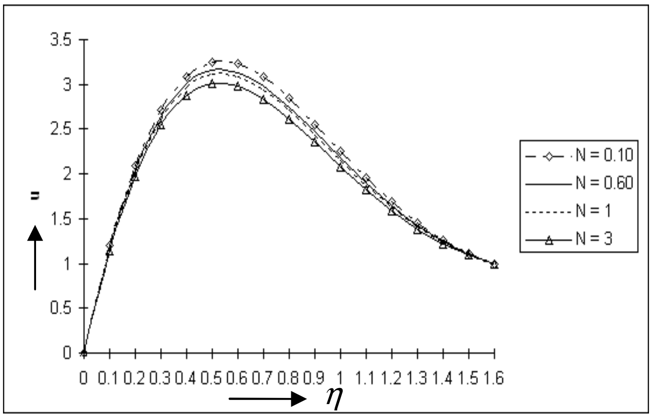


Figure 2: The velocity *u* versus η under the radiation parameter *N for M=0.50, Pr=0.71, Gr=15.00, Gm=10.00, fw=0.51, Sc=0.22, Sr=1.00, Df =1.00,* ξ *=1.00*

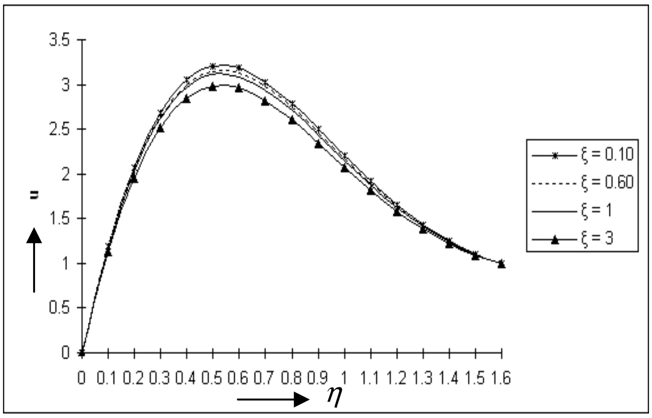


Figure 3: The velocity u versus η under the chemical reaction parameter ξ for *M=0.50, Pr=0.71, Gr=15.00, Gm=10.00, fw=0.51, Sc=0.22, Sr=1.00, Df=1.00, N=1.00*

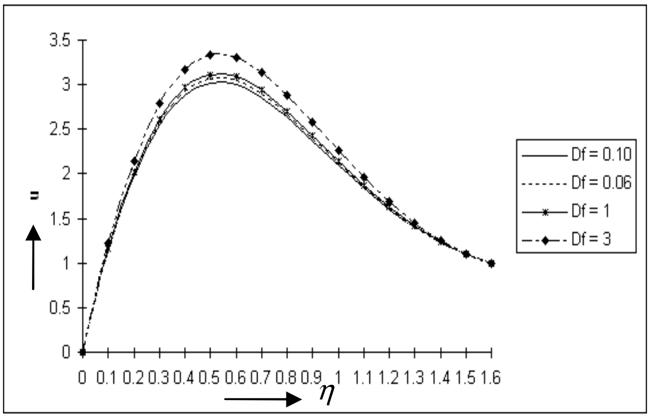


Figure 4: The velocity u versus η under the Dufour number *Df* for *M=0.50, Pr=0.71, Gr=15.00, Gm=10.00, fw=0.51, Sc=0.22, Sr=1.00, N=1.00,* $\xi = 1.00$

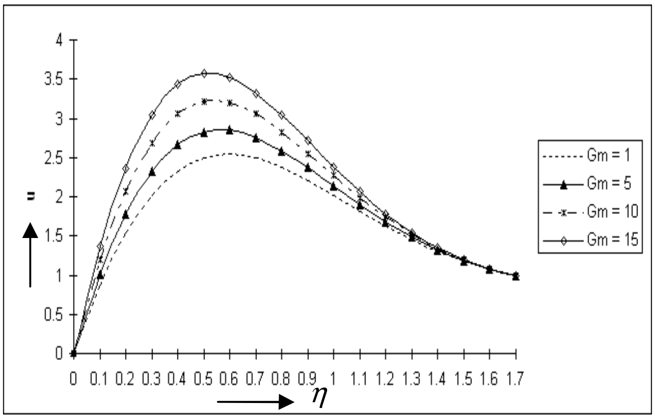


Figure 5: The velocity u versus η under the Solutal Graashof number *Gm* for *M=0.50, Pr=0.71, Gr=15.00, fw=0.51, Sc=0.22, Sr=1.00, Df=1.00, N=1.00,* $\xi = 1.00$

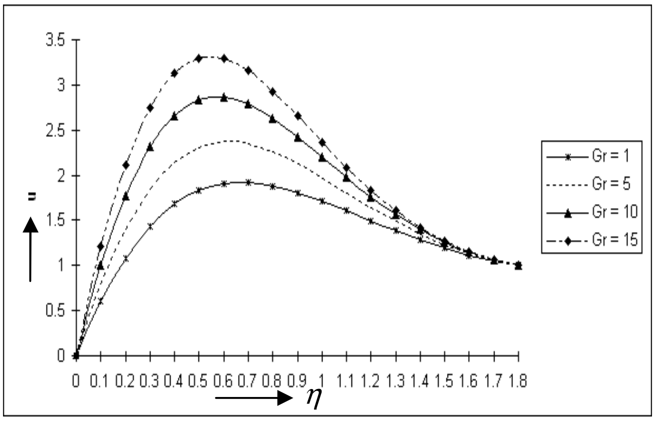


Figure 6: The velocity *u* versus η under the Thermal Grashof number *Gr* for *M=0.50, Pr=0.71, Gm=10.00, fw=0.51, Sc=0.22, Sr=1.00,Df=1.00, N=1.00,* $\xi = 1.00$

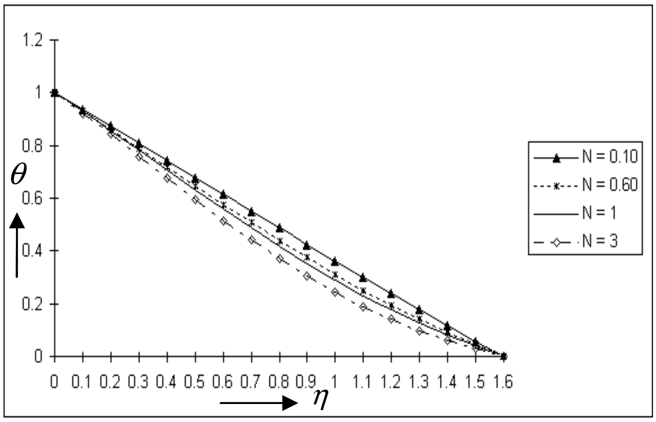


Figure 7: The temperature θ versus η under the radiation parameter *N* for *M=0.50, Pr=0.71, Gr=15.00, Gm=10.00, fw=0.51, Sc=0.22, Sr=1.00, Df=1.00,* $\xi = 1.00$

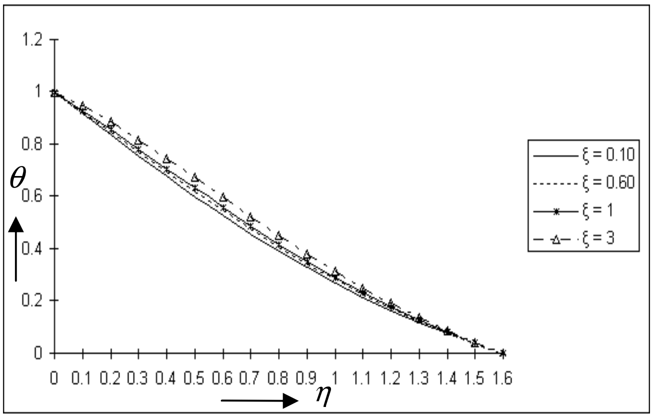


Figure 8: The temperature θ versus η under the chemical reaction parameter ξ for *M=0.50, Pr=0.71, Gr=15.00, Gm=10.00, fw=0.51, Sc=0.22, Sr=1.00, Df=1.00, N=1.00*

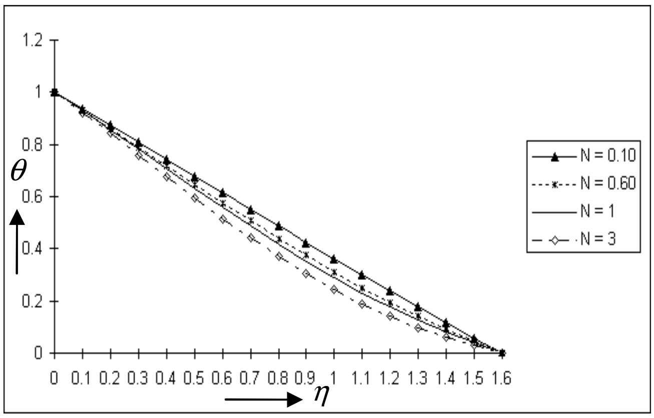


Figure 9: The temperature θ versus η under the radiation parameter *N* for *M=0.50, Pr=0.71, Gr=15.00, Gm=10.00, fw=0.51, Sc=0.22, Sr=1.00, Df=1.00,* $\xi = 1.00$

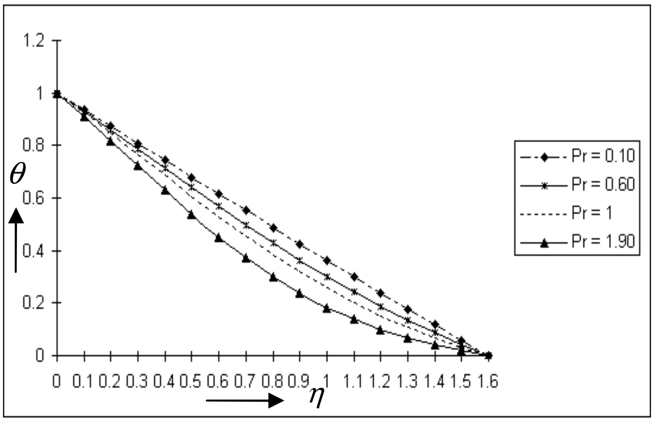


Figure 10: The temperature θ versus η under the Prandtl number *Pr* for *M=0.50,Gr=15.00, Gm=10.00,fw=0.51,Sc=0.22,Sr=1.00,Df=1.00, N=1.00* ,ξ *=1.00*

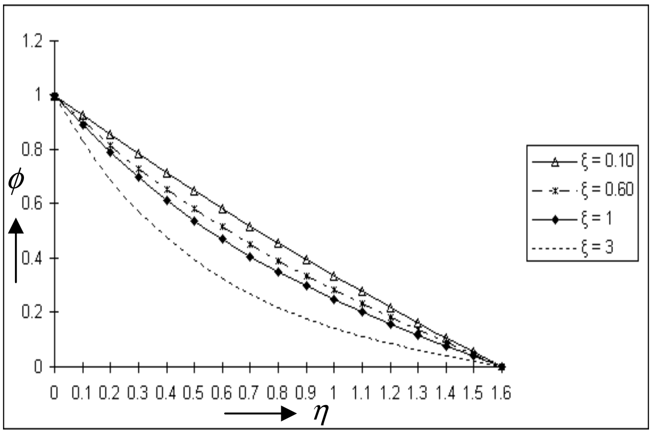


Figure 11: The concentration ϕ versus η under the chemical reaction parameter ξ for *M=0.50, Pr=0.71, Gr=15.00, Gm=10.00, fw=0.51, Sc=0.22, Sr=1.00, Df=1.00, N=1.00*

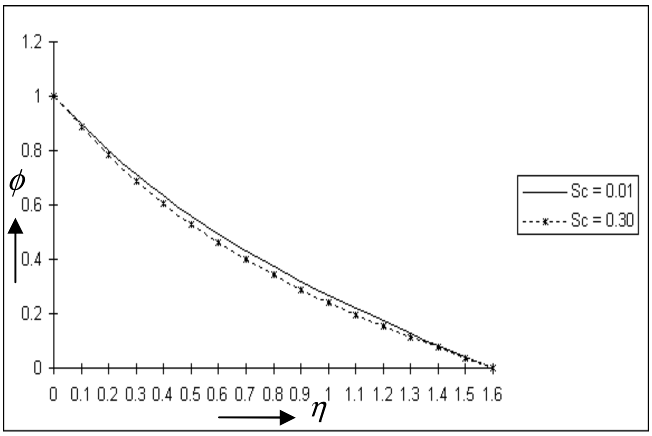


Figure 12: The concentration ϕ versus η under the Schmidt number *Sc* for *M=0.50, Pr=0.71, Gr=15.00, Gm=10.00, fw=0.51, Sr=1.00, Df=1.00, N=1.00,* ξ *=1.00*

Table -1

The skin friction co-efficient *Cf* , Nusselt number *Nu* and the Sherwood number Sh under *Df for M=0.50 Pr=0.71 Gr=15.00 Gm=10.00 fw=0.51 Sc=0.22 Sr=1.00 , N=1.00* ξ *=1.00*

Df	*Cf*	*Nu*	*Sh*
0.10	11.375250	0.864415	1.139354
0.50	11.486044	0.794277	1.152486
1.00	11.630275	0.701488	1.169866
1.20	11.689820	0.662671	1.177140
1.50	11.781186	0.602507	1.188419
1.80	11.875072	0.539898	1.200165
2.00	11.939097	0.496728	1.208268
2.50	12.104321	0.383467	1.229551
2.80	12.207089	0.311600	1.243075
3.00	12.277154	0.261948	1.25242

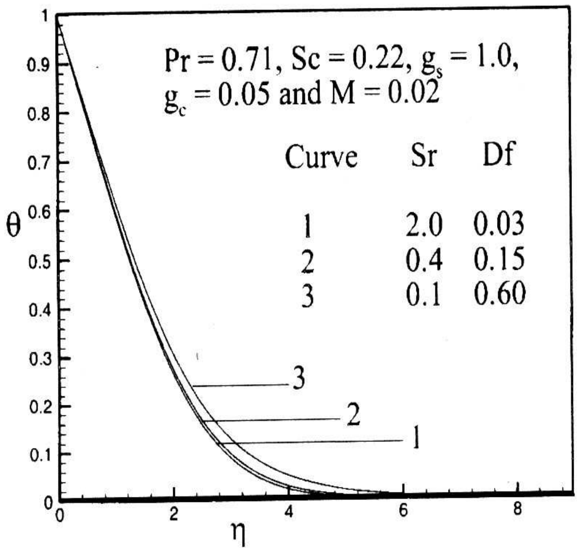


Figure 13. Temperature profiles for different values of *Sr* and *Df* of the paper by Alam et al. [22]

Table -2

The skin friction co-efficient *Cf* ,Nusselt number *Nu* and the Sherwood number *Sh* under *Gr* for *M=0.50,Pr=0.71 ,Gm=10.00,fw=0.51,Sc=0.22,Sr=1.00,Df=1.00,N=1.00,* ξ *=1.00*

Gr	*Cf*	*Nu*	*Sh*
01.00	05.727627	0.669091	1.176298
03.00	06.585573	0.679344	1.177978
05.00	07.421717	0.689109	1.179588
07.00	08.238006	0.698438	1.181135
09.00	09.036138	0.707371	1.182625
11.00	09.817601	0.715946	1.184063
12.00	10.202499	0.720110	1.184765
13.00	10.583710	0.724196	1.185454
14.00	10.961371	0.728208	1.186133
15.00	11.335619	0.732148	1.186802

Table -3

The skin friction co-efficient Cf , Nusselt number Nu and the Sherwood number Sh under Gm for $M=0.50, Pr=0.71 ,Gr=15.00, fw=0.51, Sc=0.22, Sr=1.00, Df=1.00, N=1.00\ \xi=1.00$

Gm	Cf	Nu	Sh
01.00	08.204772	0.699239	1.181207
03.00	08.913775	0.706866	1.182497
05.00	09.614893	0.714305	1.183758
07.00	10.308503	0.721566	1.184994
09.00	10.994965	0.728661	1.186205
10.00	11.335619	0.732148	1.186802
11.00	11.674609	0.735597	1.187393
12.00	12.011972	0.739009	1.187979

Table-4

The skin friction co-efficient Cf , Nusselt number Nu and the Sherwood number Sh under ξ for $M=0.50\ Pr=0.71, Gr=15.00\ , Gm=10.00, fw=0.51, Sc=0.22\ ,Sr=1.00\ , Df=1.00\ , N=1.00$

ξ	Cf	Nu	Sh
0.10	11.905527	0.831755	0.748441
0.40	11.802663	0.784522	0.900980
0.80	11.683467	0.727701	1.084874
1.00	11.630275	0.701488	1.169866
1.20	11.580736	0.676532	1.250882
1.50	11.512455	0.641177	1.365827
1.80	11.450457	0.608000	1.473876
2.00	11.412175	0.586944	1.542542
2.20	11.376070	0.566651	1.608790
2.50	11.325575	0.537504	1.704064
2.80	11.278983	0.509743	1.794935
3.00	11.249854	0.491927	1.853314

Table-5

The skin friction co-efficient Cf , Nusselt number Nu and the Sherwood number Sh under Sr for $M=0.50, Pr=0.71\ Gr=15.00\ \ Gm=10.00\ ,fw=0.51, Sc=0.22\ ,\ Df=1.00\ N=1.00,\ \xi=1.00$

Sr	Cf	Nu	Sh
0.10	11.656624	0.696642	1.160349
0.50	11.645263	0.698721	1.164459
0.80	11.636379	0.700358	1.167665
1.00	11.630275	0.701488	1.169866
1.50	11.614334	0.704460	1.175598
1.80	11.604276	0.706350	1.179207
2.00	11.597348	0.707658	1.181687
2.50	11.579211	0.711111	1.188166
2.80	11.567728	0.713317	1.192257
3.00	11.559803	0.714847	1.195075

Table-6

The skin friction co-efficient Cf , Nusselt number Nu and the Sherwood number Sh under Sc for $M=0.50, Pr=0.71, Gr=15.00, Gm=10.00, fw=0.51, Sr=1.00, Df=1.00, N=1.00,\ \xi=1.00$

Sc	Cf	Nu	Sh
0.10	11.628894	0.727459	1.122136
0.20	11.630043	0.705941	1.161659
0.30	11.631191	0.683155	1.203756
0.40	11.632266	0.659010	1.248651
0.50	11.633178	0.633405	1.296600
0.60	11.633822	0.606227	1.347894
0.70	11.634066	0.577349	1.402866
0.80	11.633755	0.546624	1.461904
0.90	11.632702	0.513887	1.525455
1.00	11.630685	0.478946	1.594041

Table-7

The skin friction co-efficient Cf , Nusselt number Nu and the Sherwood number Sh under N for $M=0.50, Pr=0.71, Gr=15.00,\ Gm=10.00,\ fw=0.51,\ ,Sc=0.22, Sr=1.00 ,Df=1.00,\ \xi=1.00$

N	Cf	Nu	Sh
0.10	11.991333	0.636216	1.162532
0.50	11.785865	0.671849	1.166873
0.80	11.683358	0.691127	1.168867
1.00	11.630275	0.701488	1.169866
1.50	11.531854	0.721374	1.171672
1.80	11.488611	0.730388	1.172452
2.00	11.464327	0.735525	1.172886
2.50	11.415230	0.746074	1.173761
2.80	11.391734	0.751200	1.174178
3.00	11.377962	0.754229	1.174423

Table-8

The skin friction co-efficient Cf , Nusselt number Nu and the Sherwood number Sh under M for $Pr=0.71, Gr=15.00, Gm=10.00, fw=0.51,\ ,Sc=0.22, Sr=1.00 ,Df=1.00, N=1.00\ \ \xi=1.00$

M	Cf	Nu	Sh
0.10	11.910067	0.706214	1.170552
0.50	11.630275	0.701488	1.169866
0.80	11.436852	0.698170	1.169387
1.00	11.315167	0.696060	1.169083
1.50	11.034371	0.691120	1.168375
1.80	10.880657	0.688369	1.167983
2.00	10.783829	0.686619	1.167735
2.50	10.559919	0.682514	1.167153
2.80	10.437012	0.680224	1.166830
3.00	10.359447	0.678764	1.166625

8. COMPARISON OF RESULTS

To compare the results, the work by Alam *et al.* [22] is considered. Their work concerns with the combined effects of Thermal diffusion and Diffusion-thermo on a steady two-dimensional MHD mixed convection and mass transfer flow past a semi infinite vertical plate. On the other hand, the present Paper deals with two dimensional boundary layer flow of an incompressible viscous electrically conducting fluid past a semi-infinite porous vertical plate with variable suction in presence of a uniform transverse magnetic field with Thermal diffusion, Diffusion-thermo, Chemical reaction and Thermal radiation past an infinite vertical porous plate. Comparing fig 9 with fig 13(fig 9 of the work of Alam et.al. [22]), we observe that the two figures uniquely indicate that an increase in Dufour number causes an increase in fluid temperature. Hence there is a good agreement between the results obtained by Alam et al. [22] and the present authors.

9. NOMENCLATURE

Symbol	Quantity	SI unit
B_0	Strength of the applied magnetic field	Tesla
$\vec{B}$	Magnetic induction vector	----
C_S	Concentration susceptibility	$(Kmol)^2 s^2$
C	Dimensional concentration	$\frac{Kmol}{m^3}$
C_p	Specific heat at constant pressure	$J / kg \times K$
C_w	Species concentration near the plate	$\frac{Kmol}{m^3}$
C_∞	Species concentration in the free stream	$\frac{Kmol}{m^3}$
C_f	Local skin friction co-efficient	---
D_M	Co-efficient of mass diffusion	$m^2 s^{-1}$
Df	Dufour number	-
f_w	Dimensionless suction velocity	-
$\vec{g}$	Acceleration due to gravity	m/s^2
Gr	Thermal Grashof number	-
Gm	Solutal Grashof number	-
$\hat{i}, \hat{j}, \hat{k}$	Unit vectors along the coordinate axes	---
$\vec{J}$	Current density	---
k	Thermal conductivity	$\frac{\mathrm{W}}{\mathrm{mK}}$
K_1	Absorption co-efficient	
K_T	Thermal diffusion ratio	$Kmol$
M	Local Hartmann number	
M_w	Mass flux from the plate to the fluid	$\frac{kmol}{m^2 s}$
N	Radiation parameter	---
Nu	Nusselt number	-
Pr	Prandtl number	-
p	Pressure	Pa (Pascal)
q_w	Heat flux from plate to the fluid	$\frac{\mathrm{W}}{\mathrm{m}^2}$
q_r	Radiative flux	W/m^2
$\vec{q}$	Velocity vector	---
Q	Constant first order homogeneous reaction rate	$1/\mathrm{s}$
Re_x	Local Reynolds number	-
Sh	Sherwood number	-
Sc	Schmidt number	-
Sr	Soret number	-
T_w	Temperature at the plate	K or ^0C
T_∞	Temperature in the free stream	K or ^0C
T	Dimensional temperature	-
T_M	Mean fluid temperature	K or ^0C
U_o	Free stream velocity; m/s	
(u, v, o)	Velocity components	m/s
$v_0(\eta)$	Suction velocity	m/s
(x, y, z)	Cartesian co-ordinates	m

Greek Symboles	Quantity	SI unit
δn	An element of the normal to the surface	m
ρ	Density	kg / m^3
υ	Kinematic viscosity	m^2 / s
α	Thermal diffusivity	m^2 / s
β	Co-efficient of volume expansion for thermal expansion	K^1
$\bar{\beta}$	Co-efficient of volume expansion for mass transfer	$\frac{1}{Kmol}$
σ	Electrical conductivity;	$(\mathrm{ohm} \times \mathrm{meter})^{-1}$
σ_1	Stefan-Boltzamann constant	
η	Similarly variable	----
ξ	Chemical reaction parameter	---

ψ	Stream function	----	
θ	Dimensionless temperature	$K\ or\ {}^0C$	
ϕ	Dimensionless species Concentration	-	
τ_0	Local skin friction at the plate	(Pascal) $\frac{N}{m^2}$	

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