SLIP EFFECTS ON MIXED CONVECTION FLOW ALONG A STRETCHING CYLINDER

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ABSTRACT

An analysis for the axi-symmetric laminar boundary layer mixed convection flow of a viscous incompressible fluid towards a stretching cylinder is presented. Instead of no-slip boundary condition, velocity slip is assumed at the boundary. Similarity transformations are used to convert the partial differential equations corresponding to the momentum and heat equations into highly non-linear ordinary differential equations. Numerical solutions of these equations are obtained by shooting method. It is found that for buoyancy aided flow, velocity increases with increasing mixed convection parameter whereas the temperature decreases in this case but opposite trend is noted in case of buoyancy opposed flow. Due to velocity slip, fluid velocity decreases initially but the temperature increases. The skin friction as well as the heat transfer rate at the surface is larger for a cylinder compared to those for a flat plate.

Key words: Boundary layer, mixed convection, stretching cylinder, heat transfer, velocity slip, similarity solution.

1. INTRODUCTION

The no-slip boundary condition (the assumption that a liquid adheres to a solid boundary) is one of the central tenets of the Navier–Stokes theory. However, there are situations wherein this condition does not hold. The non-adherence of the fluid to a solid boundary, known as velocity slip, is a phenomenon that has been observed under certain circumstances [1]. The fluids that exhibit boundary slip have important technological applications such as in the polishing of artificial heart valves and internal cavities. Partial velocity slip may occur on the stretching boundary when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. The slip flow problem of laminar boundary layer is of considerable practical interest. Microchannels which are at the forefront of today's turbomachinery technologies, are widely being considered for cooling of electronic devices, micro heat exchanger systems, etc. If the characteristic size of the flow system is small or the flow pressure is very low, slip flow happens. If the characteristic size of the flow system tends to the molecular mean free path, continuum physics is no longer suitable. In no-slip-flow, as a requirement of continuum physics, the flow velocity is zero at a solid-fluid interface and the fluid temperature instantly closest to the solid walls is equal to that of the solid walls. For viscous fluid, the slip flow condition has been employed by Navier [2] and then used in studies of fluid flow in rough and coated surfaces, and gas and liquid flow in microdevices. Unlike the no-slip case, the velocity does not vanish at stationary surfaces. Recently, many researchers viz. Wang [3], Andersson [4], Ariel et al. [5], Ariel [6], Abbas et al. [7], Mukhopadhyay [8], Bhattacharyya et al. [9, 10] etc. investigated the flow problems taking slip flow condition at the boundary.

The study of hydrodynamic flow and heat transfer over a stretching cylinders or flat plates has gained considerable attention due to its wide applications in industries and important bearings on several technological processes. Crane [11] investigated the flow caused by the stretching of a sheet. Other researchers such as Gupta and Gupta [12], Dutta et al. [13], Chen and Char [14] extended the work of Crane [11] by including the effect of heat and mass transfer analysis under different physical situations. Recently, various aspects of similar problem have been investigated by many authors such as Xu and Liao [15], Cortell [16,17], Elbashbeshy [18-21].

Lin and Shih [22, 23] considered the laminar boundary layer and heat transfer along cylinders moving horizontally and vertically with constant velocity and found no similarity solutions due to the curvature effect of the cylinder. Ishak and Nazar [24] showed that the similarity solutions could be obtained by assuming the cylinder stretched with linear velocity in the axial direction and claimed that their study might be regarded as an extension of the papers by Grubka and Bobba [25] and Ali [26], i.e. from a stretching sheet to a stretching cylinder. Recently, Ishak [27] discussed the mixed convection flow along a vertical cylinder in presence of surface heat flux. Off late, Mukhopadhyay [28] analyzed the solute transfer in case of boundary layer flow past a stretching cylinder in presence of slip. In this paper, by considering the effects of mixed convection and velocity slip at the boundary, a new dimension is added to the above mentioned study of Ishak and Nazar [24].

Since no attempt has been made to analyze the effects of velocity slip on boundary layer axi-symmetric mixed convection flow along a stretching cylinder, it is considered in this paper. Using similarity transformation, a third order ordinary differential equation corresponding to the momentum equation and a second order ordinary differential equation corresponding to the heat equation are derived.

Using shooting method numerical calculations, up to desired level of accuracy, are carried out for different values of the dimensionless parameters of this problem and these serve the purpose of illustrating the results graphically. The results obtained are then compared with those of Ishak and Nazar [24], Grubka and Bobba [25] and Ali [26] who reported the results for some special cases of the present study. The analysis of the results obtained shows that the flow field is influenced appreciably by the mixed convection parameter and velocity slip parameter. Estimation of skin friction and heat transfer coefficient which are very important due to their application in industries are also presented in this analysis. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

2. EQUATIONS OF MOTION

Let us consider the steady axi-symmetric mixed convection flow of an incompressible viscous fluid along a stretching cylinder. The continuity, momentum and energy equations governing such type of flow are

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+g\beta(T-T_\infty)\,, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{\kappa}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{3}$$

where u and v are the components of velocity in the x and r directions respectively, $\nu=\mu/\rho$ is the kinematic viscosity, ρ is the fluid density , μ is the coefficient of fluid viscosity, κ is the thermal diffusivity of the fluid, T is the fluid temperature.

2.1 Boundary conditions

It is a well known fact that a viscous fluid normally sticks to a boundary, i.e., there is no slip of the fluid relative to the boundary. However, there are numerous situations where there may be a partial slip between the fluid and the boundary, e.g., the fluid may be particulate or that it could be a rarefied gas with a suitable value of the Knudsen number. For such fluids the motion is still governed by the Navier–Stokes equations, but the usual no-slip condition at the boundary is replaced by the slip condition.

The appropriate boundary conditions for the problem are given by

$$u=U(x)+B_1\nu\tfrac{\partial u}{\partial r},\, v=0,\, T=T_w(x) \text{ at } r=R \tag{4}$$

$$u\to 0,\, T\to T_\infty \text{ as } r\to\infty\,. \tag{5}$$

Here $U(x)=U_0\frac{x}{L}$ is the stretching velocity, $T_w(x)=T_\infty+T_0\frac{x}{L}$ is the prescribed surface temperature, U_0, T_0 are the reference velocity and temperature respectively, T_∞ is the ambient temperature , L is the characteristic length, B_1 is the velocity slip.

2.2 Method of solution

The continuity equation is automatically satisfied by the introduction of stream function ψ as

$$u=\frac{1}{r}\frac{\partial\psi}{\partial r} \text{ and } v=-\frac{1}{r}\frac{\partial\psi}{\partial x}\,.$$

Introducing the similarity variables as

$$\left.\begin{aligned}&\psi=(U\nu x)^{1/2}Rf(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}\\&\text{and } \eta=\frac{r^2-R^2}{2R}\left(\frac{U}{\nu x}\right)^{1/2}\end{aligned}\right\}, \tag{6}$$

and on substitution of (6) in equations (2), (3), (4) and (5), the governing equations and the boundary conditions reduce to

$$(1+2\mathrm{M}\eta)f'''+2\mathrm{M}f''+ff''+\lambda\theta-f'^2=0, \tag{7}$$

$$(1+2\mathrm{M}\eta)\theta''+2\mathrm{M}\theta'+\Pr(f\theta'-f'\theta)=0\,, \tag{8}$$

$$f'=1+\mathrm{B}f'',\ f=0,\ \theta=1 \text{ at } \eta=0\,, \tag{9}$$

$$f'\to 0,\ \theta'\to 0 \text{ as } \eta\to\infty \tag{10}$$

where the prime denotes differentiation with respect to η, $\mathrm{B}=B_1\sqrt{\frac{U_0\nu}{L}}$ is the slip parameter and $\mathrm{M}=(\frac{\nu L}{U_0R^2})^{1/2}$ is the curvature parameter, $\lambda=\frac{Gr_x}{\mathrm{Re}_x^2}=\frac{g\beta T_0L}{U_0^2}$ is the mixed convection parameter, $Gr_x=\frac{g\beta(T_w-T_\infty)x^3}{\nu^2}$ is the local Grashof Number and $\mathrm{Re}_x=\frac{Ux}{\nu}=\frac{U_0x^2}{L\nu}$ is the local Reynolds number. The no-slip case is recovered for $\mathrm{B}=0$.

One can note that if $\mathrm{M}=0$ (i.e. $R\to\infty$), the problem under consideration (with $\lambda=0$, $\mathrm{B}=0$) reduces to the boundary layer flow along a stretching flat plate considered by Ali [26], with $m=1$ in that paper. Moreover, when $\mathrm{M}=0$ (stretching flat plate) is subjected to (9) with $\lambda=0$, $\mathrm{B}=0$, the analytical solutions of Eqs. (7) and (8) are given by Crane [11] and Grubka and Bobba [25], respectively.

3. NUMERICAL METHOD FOR SOLUTION

The above equations (7) and (8) along with boundary conditions are solved by converting them to initial value problems. We set

$$f'=z,\ z'=p,\ p'=[z^2-fp-2\mathrm{M}p-\lambda\theta]/(1+2\mathrm{M}\eta) \tag{11}$$

$$\theta'=q,\ q'=-[\Pr(fq-z\theta)+2\mathrm{M}q]/(1+2\mathrm{M}\eta) \tag{12}$$

with the boundary conditions

$$f(0)=0,\ f'(0)=1+\mathrm{B}\gamma,\ f''(0)=\gamma,\ \theta(0)=1\,. \tag{13}$$

In order to integrate (11) and (12) as initial value problems, one requires a value for $p(0)$ i.e. $f''(0)$ and a value for $q(0)$ i.e. $\theta'(0)$ but no such values are given at the boundary. The suitable guess values for $f''(0)$ and $\theta'(0)$ are chosen and integration is carried out. Comparing the calculated values for f' and θ at $\eta = 10$ (say) with the given boundary conditions $f'(10) = 0$ and $\theta(10) = 0$ and adjusting the estimated values of $f''(0)$ and $\theta'(0)$, a better approximation for the solution is obtained. Taking the series of values for $f''(0)$ and $\theta'(0)$ and applying the fourth order classical Runge-Kutta method with step-size $\Delta\eta$=0.01, the above procedure is repeated until the results up to the desired degree of accuracy (10^{-5}) are obtained.

4. RESULTS AND DISCUSSION

4.1 Validation of the results

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For the verification of accuracy of the applied numerical scheme, the available results for forced convection case with prescribed wall temperature $T_w = T_\infty + T_0(\frac{x}{L})^N$ at $y = 0$ is considered. In this case the heat equation becomes $(1+2M\eta)\theta'' + 2M\theta' + \Pr(f\theta' - Nf'\theta) = 0$, N being the temperature exponent. Putting $N = 1$, equation (8) can be recovered.

Table 1 Values of $Nu_x \mathrm{Re}_x^{-1/2} = -\theta'(0)$ for several values of temperature exponent *N* for forced convection $(\lambda = 0)$ in flat plate $(M = 0)$ in the absence of slip $(B = 0)$ and $\Pr = 1$.

N	Ishak and Nazar [24]	Grubka and Bobba [25]	Ali [26]	Present study
0	0.5820	0.5820	0.5801	0.5821
1	1.0000	1.0000	0.9961	1.0000
2	1.3333	1.3333	1.3269	1.3332

A comparison of the results corresponding to the heat transfer coefficient [$-\theta'(0)$] for $\lambda = 0$ (i.e. for forced convection), $B = 0$ (in absence of velocity slip) and $M = 0$ (for stretching flat plate) for several values of temperature exponent *N* is made with the available published results of Ishak and Nazar [24], Grubka and Bobba [25], Ali [26] and is presented in Table 1. The results are found in excellent agreement.

4.2 Effects of different parameters on flow and heat transfer characteristics

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations. The results are given through a parametric study showing the influence of several non-dimensional parameters, viz. curvature parameter (M), mixed convection parameter (λ), velocity slip parameter (B) and Prandtl number (Pr). For the mixed convection case $N = 1$ is considered.

Let us first concentrate on the effects of curvature parameter M on velocity and temperature distribution. In Figure 1(a), horizontal velocity profiles are shown for different values of M. The horizontal velocity curves show that the rate of transport decreases with the increasing distance (η) of the sheet. In all cases the velocity asymptotically vanishes at some large distance from the sheet (at η=10). Though the velocity decreases initially but after a certain distance from the wall it increases with increasing values of M. Due to slip effects at the wall the velocity decreases initially. The velocity gradient at the surface is larger for larger values of M, which produces larger skin friction coefficient. Temperature is found to decrease with the increasing curvature parameter M [Figures 1(b)]. The thermal boundary layer thickness decreases as M increases, which implies increase in the wall temperature gradient and in turn, the surface heat transfer rate increases. Hence, the Nusselt number increases as M increases.

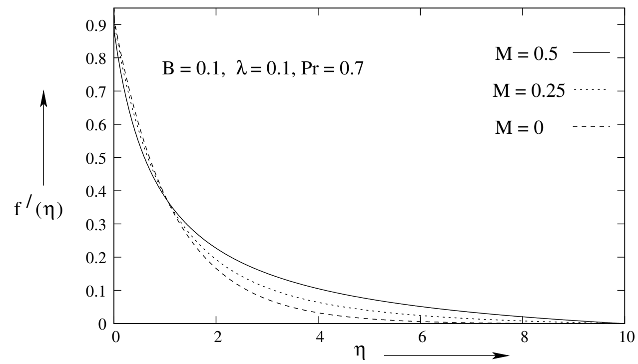

Figure 1(a) Variation of velocity $f'(\eta)$ with η for several values of curvature parameter M of the stretching cylinder.

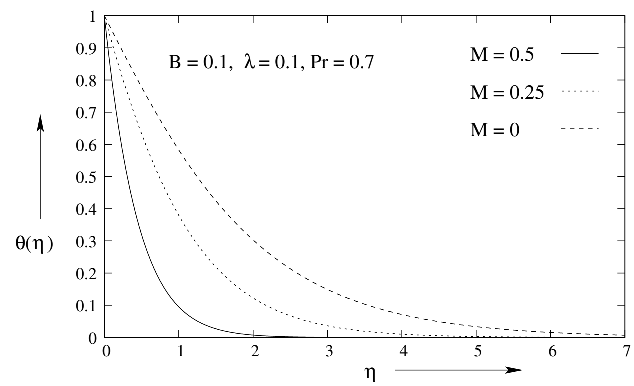

Figure 1(b) Variation of temperature $\theta(\eta)$ with η for several values of curvature parameter M of the stretching cylinder.

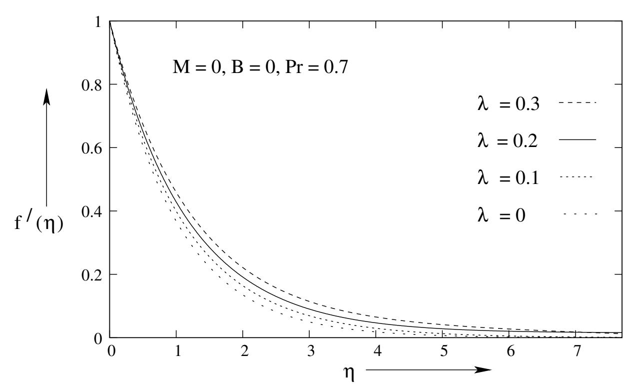

Figure 2(a) Variation of velocity $f'(\eta)$ with η for several values of mixed convection parameter λ for flat plate.

Figures 2(a)–(c) display the effects of the mixed convection parameter on velocity, temperature for flat plate and stretching cylinder. Figures 2(a) and 2(b) demonstrate the effects of mixed convection parameter (λ) on velocity profiles respectively for a stretching flat plate (i.e. for $M = 0$) and a stretching cylinder (for $M = 0.25$). With the increasing λ, the horizontal velocity is found to increase for buoyancy aided flow ($\lambda > 0$) [Figures 2(a), 2(b)]. It is noted that λ has a substantial effect on the solutions. λ=0 corresponds to the forced convection case. For $\lambda > 0$, there is a favorable pressure gradient due to the buoyancy forces, which results in the flow being accelerated.

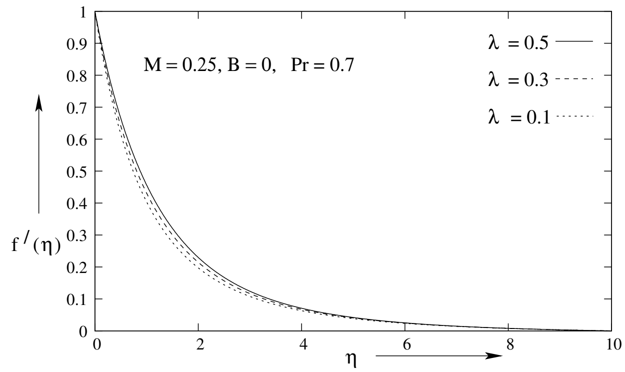

Figure 2(b) Variation of velocity $f^{\prime}(\eta)$ with η for several values of mixed convection parameter λ for stretching cylinder.

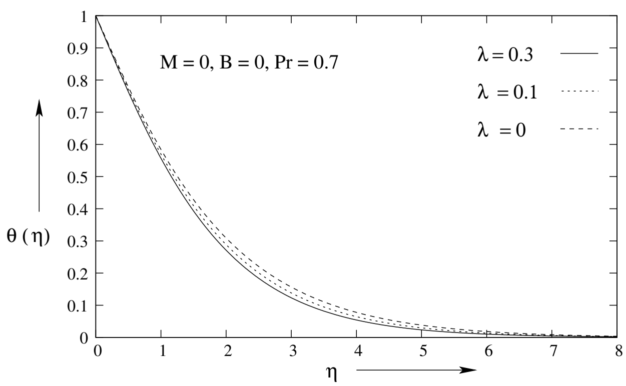

Figure 2(c) Variation of temperature $\theta(\eta)$ with η for several values of mixed convection parameter λ for flat plate.

Physically $\lambda > 0$ means heating of the fluid or cooling of the surface (assisting flow), $\lambda < 0$ means cooling of the fluid or heating of the surface (opposing flow). Also, an increase in the value of λ can lead to an increase in the temperature difference $T_w - T_\infty$. This leads to an enhancement of the velocity due to the enhanced convection currents and thus an increase in the boundary layer thickness.

Variation of temperature $\theta(\eta)$ with η for several values of mixed convection parameter λ for flat plate is shown in Figure 2(c). Temperature decreases with increasing λ for buoyancy aided flow. An increase in the value of mixed convection parameter λ results in a decrease in the thermal boundary layer thickness and this results in an increase in the magnitude of the wall temperature gradient. This in turn produces an increase in the surface heat transfer rate.

The effects of slip parameter on velocity and temperature are exhibited in Figure 3(a)–Figure 3(d) for both flat plate and stretching cylinder. Due to slip, fluid velocity decreases initially but far away from the wall velocity is found to increase[Figure 3(a), Figure 3(b)]. When slip occurs, the flow velocity near the stretching wall is no longer equal to the stretching velocity of the wall. With the increase in B, such slip velocity increases and consequently fluid velocity decreases because under the slip condition, the pulling of the stretching wall can only be partly transmitted to the fluid. The temperature increases with increasing slip [Figure 3(c), Figure 3(d)]. All temperature profiles decay from the maximum value at the wall to zero in the free stream i.e. converge at the outer edge of the boundary layer.

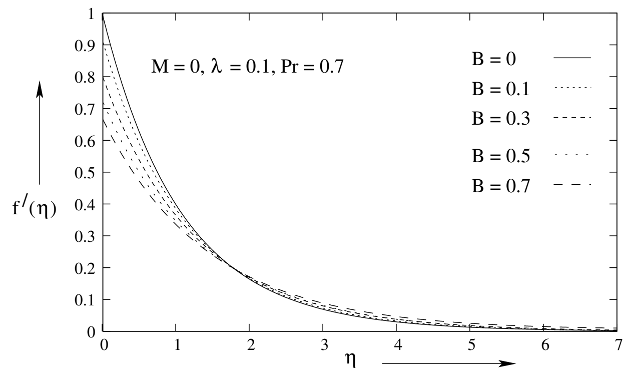

Figure 3(a) Variation of velocity $f^{\prime}(\eta)$ with η for several values of velocity slip parameter B for flat plate.

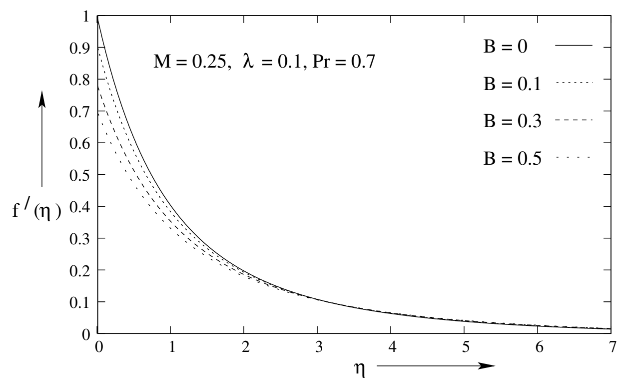

Figure 3(b) Variation of velocity $f^{\prime}(\eta)$ with η for several values of velocity slip parameter B for stretching cylinder.

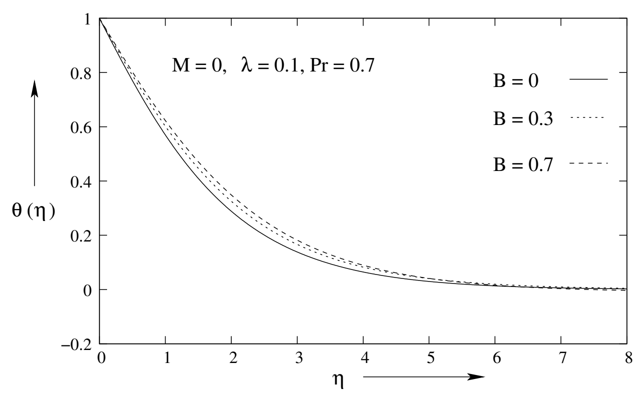

Figure 3(c) Variation of temperature $\theta(\eta)$ with η for several values of velocity slip parameter B for flat plate.

The effect of Prandtl number (Pr) on temperature profiles is exhibited in Figure 4(a) for flat plate (i.e. for $M = 0$). Temperature is found to decrease with increasing Pr. An increase in Prandtl number reduces the thermal boundary layer thickness. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. Fluids with lower Prandtl number will possess higher thermal conductivities (and thicker thermal boundary layer structures) so that heat can diffuse from the wall faster than higher Pr fluids (thinner boundary layers). Hence Prandtl number can be used to increase the rate of cooling in conducting flows. Figure 4(b) presents the behaviour of thermal field for increasing Pr for stretching cylinder ($M = 1$). It is very clear that the effects of Pr is much more prominent for flat plate compared to stretching cylinder.

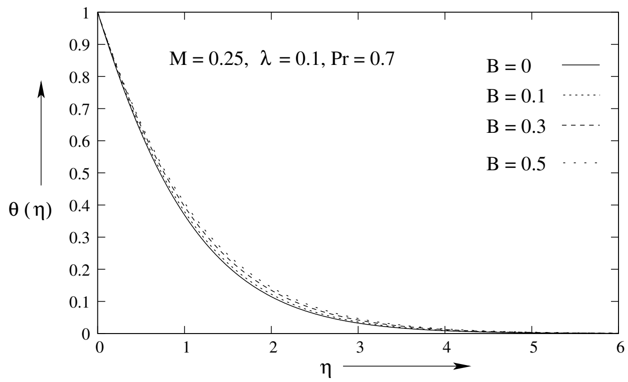

Figure 3(d) Variation of temperature $\theta(\eta)$ with η for several values of velocity slip parameter B for stretching cylinder.

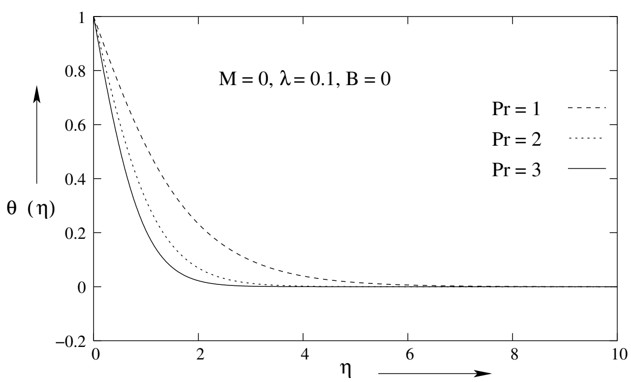

Figure 4(a) Variation of temperature $\theta(\eta)$ with η for several values of Prandtl number Pr for flat plate.

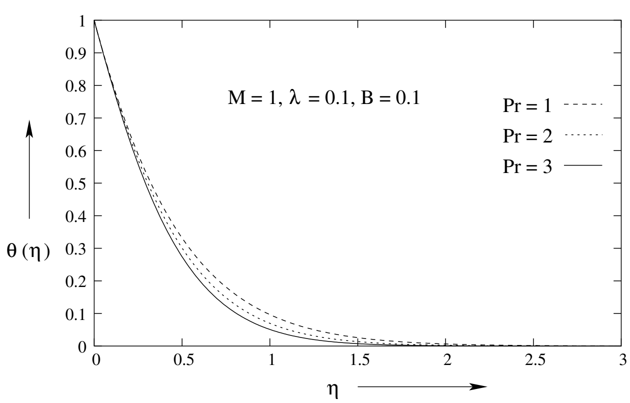

Figure 4(b) Variation of temperature $\theta(\eta)$ with η for several values of Prandtl number Pr for stretching cylinder.

The magnitude of skin-friction decreases with increasing mixed convection parameter λ and increases with the increasing curvature parameter M of the cylinder [Figure 5(a)]. From the figure it is very clear that shear stress at the wall is negative here. Physically, negative sign of $f''(0)$ implies that surface exerts a dragging force on the fluid and positive sign implies the opposite. This is consistent with the present case because a stretching cylinder which induces the flow, is considered here. On the other hand magnitude of skin-friction decreases with increasing slip parameter B [Figure 5(b)]. Slip condition reduces the momentum transfer from the sheet to the fluid. Rate of heat transfer $|\theta'(0)|$ is found to decrease with increasing slip parameter B but increases with increasing curvature parameter M [Figure 5(c)].

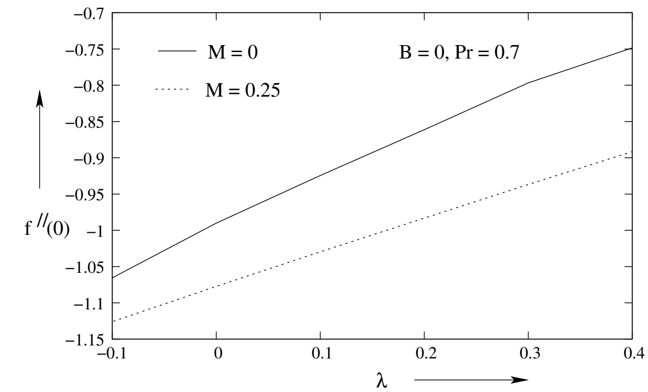

Figure 5(a) Skin friction coefficient $f''(0)$ against mixed convection parameter λ for two values of curvature parameter M.

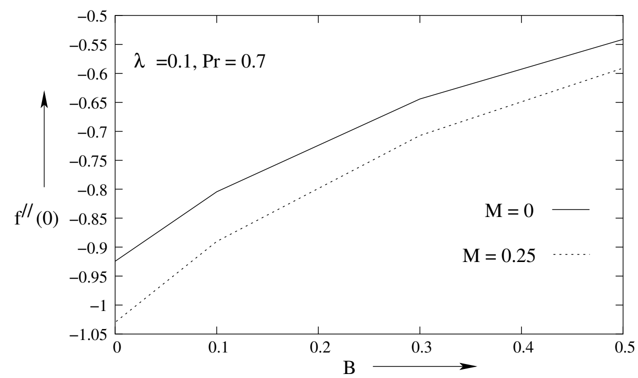

Figure 5(b) Skin friction coefficient $f''(0)$ against velocity slip parameter B for two values of curvature parameter M.

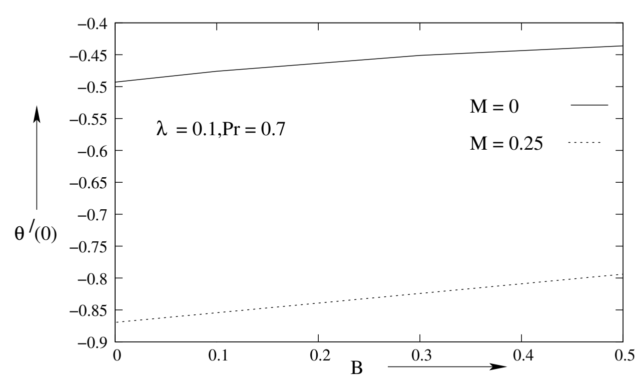

Figure 5(c) Heat transfer coefficient $\theta'(0)$ against velocity slip parameter B for two values of curvature parameter M.

5. CONCLUSIONS

The present study gives the numerical solutions for steady mixed convection boundary layer flow and heat transfer along a stretching cylinder with slip at the boundary. From this investigation the following observations are made:

(i) The rate of transport is considerably reduced with increasing values of curvature parameter M.
(ii) Due to slip, velocity decreases initially but increases away from the wall.
(iii) Temperature increases with increasing values of slip at the boundary.
(iv) The surface shear stress and the heat transfer rate at the surface increase as the curvature parameter increases.

6. REFERENCES

1. A. Yoshimura and R. K. Prudhomme, Wall slip corrections for Couette and parallel disc viscometers, *J. Rheol.,* Vol. 32, pp.53-67, 1998.
2. C. L. M. H. Navier, Sur les lois du mouvement des fluids, *Men. Acad. R. Sci. Inst. Fr.,* Vol. 6, pp. 389-440, 1827.
3. C. Y. Wang, Flow due to a stretching boundary with partial slip – an exact solution of the Navier-Stokes equations, *Chem. Eng. Sci.,* Vol. 57, pp. 3745-3747, 2002.
4. H. I. Andersson, Slip flow past a stretching surface, *Acta Mech.,* Vol. 158, pp. 121-125, 2002.
5. P. D. Ariel, T. Hayat and S. Asghar, The flow of an elastico-viscous fluid past a stretching sheet with partial slip, *Acta Mech.,* Vol. 187, pp. 29-35, 2006.
6. P. D. Ariel, Two dimensional stagnation point flow of an elastico-viscous fluid with partial slip, *Z. Angew. Math. Mech*., Vol. 88, pp. 320-324, 2008.
7. Z. Abbas, Y. Wang, T. Hayat and M. Oberlack, Slip effects and heat transfer analysis in a viscous fluid over an oscillatory stretching surface, *Int. J. of Numer. Meth. Fluids*, Vol. 59, pp. 443-458, 2009.
8. S. Mukhopadhyay, Effects of slip on unsteady mixed convective flow and heat transfer past a porous stretching surface, *Nucl. Eng. Des.*, 2011 doi:10.1016/j.nucengdes.2011.05.007.
9. K. Bhattacharyya, S. Mukhopadhyay and G. C. Layek, MHD boundary layer slip flow and heat transfer over a flat plate, *Chin. Phys. Lett.*, Vol. 28(2), 024701, 2011.
10. K. Bhattacharyya, S. Mukhopadhyay and G.C. Layek, Slip Effects on unsteady boundary layer stagnation-point flow and heat transfer towards a stretching sheet, *Chin. Phys. Lett.*, Vol. 28(9), 094702, 2011.
11. L. J. Crane, Flow past a stretching plate, *Z. Angew Math. Phys.*, Vol. 21, pp. 645-647, 1970.
12. P.S. Gupta and A.S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, *Can. J. Chem. Eng.*, Vol. 55, pp. 744-746, 1977.
13. B. K. Datta, P. Roy and A. S. Gupta, Temperature field in the flow over a stretching sheet with uniform heat flux, *Int. Comm. Heat Mass Transfer* , Vol. 12, pp. 89-94, 1985.
14. C. K. Chen and M. I. Char, Heat transfer of a continuous stretching surface with suction or blowing, *J. Math Anal. Appl.*, Vol. 135, pp. 568-580, 1988.
15. H. Xu and S. J. Liao, Series solutions of unsteady magnetohydrodynamics flows of non-Newtonian fluids caused by an impulsively stretching plate, *J. Non-Newtonian Fluid Mech.*, Vol. 129, pp. 46-55, 2005.
16. R. Cortell, Flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction/blowing, *Fluid Dyna. Res.*, Vol. 37, pp. 231-245, 2005.
17. R. Cortell, Effects of viscous dissipation and work done by deformation on the MHD flow and heat transfer of a viscoelastic fluid over a stretching sheet, *Phys. Lett. A*, Vol. 357, pp. 298-305, 2006.
18. E. M. A. Elbashbeshy and M. A. Bazid, Heat transfer over a continuously moving plate embedded in a non-Darcian porous medium, *Int. J. Heat Mass Transfer*, Vol. 43, pp. 3087-3092, 2000.
19. E. M. A. Elbashbeshy, Heat transfer over an exponentially stretching continuous surface with suction, *Arch. Mech.*, Vol. 53, pp. 643- 651, 2001.
20. E. M. A. Elbashbeshy and M. A. Bazid, The mixed convection along a vertical plate with variable surface heat flux embedded in porous medium, *Appl. Math. Comp.,* Vol. 125, pp. 317-324, 2002.
21. E. M. A. Elbashbeshy and M. A. Bazid, Heat transfer over a stretching surface with internal heat generation, *Appl. Math. Comput.,* Vol. 138, pp.239-245, 2003.
22. H. T. Lin, and Y. P. Shih, Laminar boundary layer heat transfer along static and moving cylinders, *J. Chin. Inst. Eng.*, Vol. 3, pp. 73-79, 1980.
23. H. T. Lin, and Y. P. Shih, Buoyancy effects on the laminar boundary layer heat transfer along vertically moving cylinders, *J. Chin. Inst. Eng.*, Vol. 4, pp. 47-51, 1981.
24. A. Ishak and R. Nazar, Laminar boundary layer flow along a stretching cylinder. *Euro. J. of Sci. Resc.*, Vol. 36(1), pp. 22-29, 2009.
25. L. G. Grubka and K. M. Bobba, Heat transfer characteristics of a continuous stretching surface with variable temperature, *ASME J. Heat Transfer*, Vol. 107, pp. 248-250, 1985.
26. M. E. Ali, Heat transfer characteristics of a continuous stretching surface, *Heat Mass Transfer*, Vol. 29, pp. 227-234, 1994.
27. A. Ishak, Mixed convection boundary layer flow over a vertical cylinder with prescribed surface heat flux, *J. Phys. A Math. Theor.*, Vol. 42, 195501, 2009.
28. S. Mukhopadhyay, Chemically reactive solute transfer in boundary layer slip flow along a stretching cylinder, *Front. Chem. Sci. Eng.*, Vol. 5(3), pp. 385-391, 2011.