AN OSCILLATORY THREE DIMENSIONAL FLOW PAST AN INFINITE VERTICAL POROUS PLATE WITH SORET AND DUFOUR EFFECTS

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ABSTRACT

The problem of an oscillatory three dimensional flow past an infinite vertical porous plate with Soret and Dufour effects is presented. Analytical solutions to the coupled non-linear equations governing the flow and heat and mass transfer are solved by regular perturbation technique. The expression for the velocity field, temperature field, species concentration, the Skin-friction, Nusselt number and Sherwood number at the plate are obtained in non-dimensional forms. The velocity distribution, temperature, chemical species concentration, Coefficient of Skin-friction, Nusselt number and Sherwood number at the plate are demonstrated graphically and the effects of different parameters viz. the Suction Reynolds number, the Grashof numbers, the Soret number and the Dufour number on these fields are discussed

1. INTRODUCTION

In the last few years, it has been observed that the investigation of the problems of laminar flow control has gathered a considerable importance in the fields of aeronautical engineering in view of its application to reduce drag and hence the vehicle power requirement by a substantial amount. Theoretical and experimental research reports have cited that the transition from laminar to the turbulent flow which causes the drag coefficient to increase may be prevented by suction of the fluid and the heat transfer from boundary layer to the wall. The development on this subject had been initiated by Lachmann in 1961[1].

Model studies on the problems of convection flows arising in fluids as a result of interaction of the force of gravity and density differences caused by simultaneous diffusion of thermal energy and chemical species have been carried out by many authors due to their applications in many branches of science and technology. Some of these are by Raptis and Kafousias[2] , Bejan and Khair [3] and Ahmed et al.[4].

Extensive work has been done on the effect of the three dimensional flow caused by the periodic suction perpendicular to the main flow when the difference between the wall temperature and free stream temperature gives rise to buoyancy force in the direction of free stream on heat transfer characteristics. In this regard, we may present the work done by Choudhary and Chand [5], Ahmed and Sarma [6] and Singh et al. [7]. The effect of transverse sinusoidal injection velocity distribution on three dimensional free convective Couette flow of a viscous incompressible fluid in slip flow regime under the influence of heat source was investigated by Jain and Gupta [8]. An analytical solution to the problem of the three-dimensional free convective flow of an incompressible viscous fluid past a porous vertical plate with transverse sinusoidal suction velocity taking in to account the presence of species concentration was obtained by Ahmed et al. [9].

It will be worthwhile to mention that, several authors have carried out their research works to investigate the effects of thermal diffusion on some mass transfer related problems. Some of them are Sattar and Alam [10], Singh et al. [11], Raju et al. [12], Ahmed and Sarmah [13] and recently by Ahmed and Goswami [14]. However in the above mentioned works, the Dufour effect was not taken in to account. The Dufour effect is the energy flux due to a mass concentration gradient. In view of the importance of the Dufour effect, a number of workers have carried out their research works, to investigate the Dufour effect on some mass transfer related flow problems. A few of them are Mortimer and Eyring [15], Alam et al. [16], Alam and Rahman [17], Ferdows et al [18], Motsa [19], Reddy and Rao [20], Shekar and Madhu [21] etc. The problem of Constructal design of cavities inserted into a cylindrical solid body was investigated by Lorenzini et al. [22].Very recently Biserni et al.[23] have studied the Geometric optimization of a convective T-shaped cavity on the basis of constructal theory.

As the present authors are aware till now, no attempt has been made to study analytically the Soret and Dufour effects simultaneously on an oscillatory three dimensional flow past an infinite vertical plate. Such an attempt has been made in the present work.

2. BASIC EQUATIONS

We now consider the unsteady, free and forced convection flow of an incompressible viscous fluid taking into account the species concentration, Dufour effect and Soret effect past a vertical porous plate with transverse sinusoidal suction velocity as mentioned earlier by making the following assumptions:

(1) All fluid properties except density in the buoyancy force term are constant.

(2) The viscous dissipative energy is negligible.

(3) $\bar{T}_w \rangle \bar{T}_\infty$ and $\bar{C}_w \rangle \bar{C}_\infty$.

We introduce a co-ordinate system $(\bar{x}, \bar{y}, \bar{z})$ with *X*-axis vertically upwards along the plate, *Y*-axis perpendicular to it directed into the fluid region and *Z*-axis along the width of the plate. Let $\vec{q} = \hat{i}\bar{u} + \hat{j}\bar{v} + \hat{k}\bar{w}$ be the fluid velocity at the point $(\bar{x}, \bar{y}, \bar{z})$.

The suction velocity distribution is taken as follows:

$$\bar{v}_w(\bar{z}) = -V_0 \left[1 + \varepsilon \cos \frac{\pi \bar{z}}{L} e^{i\bar{\omega}\bar{t}} \right] \quad (2.1)$$

which consists of a basic steady distribution $-V_0$ $(V_0 > 0)$ with a superimposed weak distribution $-\varepsilon V_0 \cos\left(\frac{\pi \bar{z}}{L}\right) e^{i\bar{\omega}\bar{t}}$, where L is the wave length of the periodic suction and ε is a small amplitude of suction velocity. Since the plate is infinite in length in *X*-direction, therefore all the quantities except possibly the pressure are assumed to be independent of $\bar{x}$.

With these assumptions and under usual boundary layer approximations, the equations governing the flow become

Equation of continuity:

$$\frac{\partial \bar{v}}{\partial \bar{y}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0 \quad (2.2)$$

Momentum equations:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{u}}{\partial \bar{z}} = g\beta(\bar{T} - \bar{T}_\infty) + g\bar{\beta}(\bar{C} - \bar{C}_\infty) + \upsilon\left(\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right) \quad (2.3)$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{v}}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}} + \upsilon\left(\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{z}^2}\right) \quad (2.4)$$

$$\frac{\partial \bar{w}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{w}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}} + \upsilon\left(\frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right) \quad (2.5)$$

Energy equation:

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{T}}{\partial \bar{z}} = \frac{k}{C_p \rho}\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right) + \frac{D_M K_T}{C_S C_p}\left(\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{z}^2}\right) \quad (2.6)$$

Species concentration equation:

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} + \bar{w}\frac{\partial \bar{C}}{\partial \bar{z}} = D_M\left(\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{C}}{\partial \bar{z}^2}\right) + \frac{D_M K_T}{T_M}\left(\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right) \quad (2.7)$$

The relevant boundary conditions are:

At $\bar{y} = 0$: $\bar{u} = 0, \bar{v} = \bar{v}_w, \bar{w} = 0, \bar{T} = \bar{T}_w, \bar{C} = \bar{C}_w$ (2.8)

At $\bar{y} \to \infty : \bar{u} = \bar{U}, \bar{v} = -V_0, \bar{w} = 0, \bar{T} = \bar{T}_\infty, \bar{C} = \bar{C}_\infty,$
$\bar{p} = \bar{p}_\infty$ (2.9)

We introduce the following non-dimensional quantities:

$$y = \frac{\bar{y}}{L},\ z = \frac{\bar{z}}{L},\ u = \frac{\bar{u}}{V_0},\ v = \frac{\bar{v}}{V_0},\ w = \frac{\bar{w}}{V_0},\ U = \frac{\bar{U}}{V_0},$$

$$\theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_W - \bar{T}_\infty},\ \upsilon = \frac{\mu}{\rho},\ t = \frac{V_0 \bar{t}}{L},\ \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_W - \bar{C}_\infty},\ Pr = \frac{\mu C_p}{k},$$

$$Sc = \frac{\upsilon}{D_M},\ Sr = \frac{D_M K_T(\bar{T}_W - \bar{T}_\infty)}{\upsilon T_M(\bar{C}_W - \bar{C}_\infty)},\ Gr = \frac{L g \beta(\bar{T}_W - \bar{T}_\infty)}{V_0^2},$$

$$Gm = \frac{L g \bar{\beta}(\bar{C}_W - \bar{C}_\infty)}{V_0^2},\ Re = \frac{V_0 L}{\upsilon},\ p = \frac{\bar{p}}{\rho\left(\frac{\upsilon}{L}\right)^2},$$

$$p_\infty = \frac{\bar{p}_\infty}{\rho\left(\frac{\upsilon}{L}\right)^2},\ \omega = \frac{\bar{\omega} L}{V_0},\ Du = \frac{D_M K_T(\bar{C}_W - \bar{C}_\infty)}{C_S C_p(\bar{T}_W - \bar{T}_\infty)\upsilon}$$

The non-dimensional form of the equations (2.2), (2.3), (2.4), (2.5), (2.6) and (2.7) are

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (2.10)$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = Gr\theta + Gm\phi + \frac{\partial^2 u}{\partial y^2} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad (2.11)$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{Re^2}\frac{\partial p}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) \quad (2.12)$$

$$\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{Re^2}\frac{\partial p}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \quad (2.13)$$

$$\frac{\partial \theta}{\partial t} + v\frac{\partial \theta}{\partial y} + w\frac{\partial \theta}{\partial z} = \frac{1}{Pr\,Re}\left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) + \frac{Du}{Re}\left(\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right) \quad (2.14)$$

$$\frac{\partial \phi}{\partial t} + v\frac{\partial \phi}{\partial y} + w\frac{\partial \phi}{\partial z} = \frac{1}{Sc\,Re}\left(\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right) + \frac{Sr}{Re}\left(\frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2}\right) \quad (2.15)$$

with relevant boundary conditions:

$$y = 0 : u = 0, v = -\left(1 + \varepsilon \cos \pi z e^{i\omega t}\right), w = 0, \theta = 1, \phi = 1 \quad (2.16)$$

$$y \to \infty : u = U, v = -1, w = 0, \theta = 0, \phi = 0, p = p_\infty \quad (2.17)$$

3. METHOD OF SOLUTION

We assume the solutions of the equations (2.10) to (2.15) to be of the following forms

$$u = u_0(y) + \varepsilon u_1(y,z)\, e^{i\omega t} + 0(\varepsilon^2) \quad (3.1)$$

$$v = v_0(y) + \varepsilon v_1(y,z)\, e^{i\omega t} + 0(\varepsilon^2) \quad (3.2)$$

$$w = w_0(y) + \varepsilon w_1(y,z)\, e^{i\omega t} + 0(\varepsilon^2) \quad (3.3)$$

$$p = p_0(y) + \varepsilon p_1(y,z)\, e^{i\omega t} + 0(\varepsilon^2) \quad (3.4)$$

$$\theta = \theta_0(y) + \varepsilon \theta_1(y,z)\, e^{i\omega t} + 0(\varepsilon^2) \quad (3.5)$$

$$\phi = \phi_0(y) + \varepsilon \phi_1(y,z)\, e^{i\omega t} + 0(\varepsilon^2) \quad (3.6)$$

with $p_0 = p_\infty, w_0 = 0$

Substituting these in equations (2.10) to (2.15) and by equating the coefficients of the similar terms and neglecting ε^2, the following differential equations are obtained.

Zeroth-order equations:

$$\frac{dv_0}{dy} = 0 \quad (3.7)$$

$$v_0 \frac{du_0}{dy} = Gr\theta_0 + Gm\phi_0 + \frac{1}{Re}\frac{d^2 u_0}{dy^2} \quad (3.8)$$

$$v_0 \frac{d\theta_0}{dy} = \frac{1}{PrRe}\frac{d^2\theta_0}{dy^2} + \frac{Du}{Re}\frac{d^2\phi_0}{dy^2} \quad (3.9)$$

$$v_0 \frac{d\phi_0}{dy} = \frac{1}{ScRe}\frac{d^2\phi_0}{dy^2} + \frac{Sr}{Re}\frac{d^2\theta_0}{dy^2} \quad (3.10)$$

First-order equations:

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \quad (3.11)$$

$$-\frac{\partial u_1}{\partial y} + v_1 \frac{du_0}{dy} = Gr\theta_1 + Gm\phi_1 + \frac{1}{Re}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - i\omega u_1 \quad (3.12)$$

$$-\frac{\partial v_1}{\partial y} = -\frac{1}{Re^2}\frac{\partial p_1}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) - i\omega v_1 \quad (3.13)$$

$$-\frac{\partial w_1}{\partial y} = -\frac{1}{Re^2}\frac{\partial p_1}{\partial z} + \frac{1}{Re}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - i\omega w_1 \quad (3.14)$$

$$-\frac{\partial \theta_1}{\partial y} + v_1 \frac{d\theta_0}{dy} = \frac{1}{PrRe}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) + \frac{Du}{Re}\left(\frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2}\right) - i\omega\theta_1 \quad (3.15)$$

$$-\frac{\partial \phi_1}{\partial y} + v_1 \frac{d\phi_0}{dy} = \frac{1}{ScRe}\left(\frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2}\right) + \frac{Sr}{Re}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) - i\omega\phi_1 \quad (3.16)$$

with the boundary conditions:

$$y = 0 : u_0 = 0, u_1 = 0, v_0 = -1, v_1 = -\cos\pi z, w_0 = 0, w_1 = 0, \theta_0 = 1, \theta_1 = 0, \phi_0 = 1, \phi_1 = 0 \quad (3.17)$$

$$y \to \infty : u_0 = U, u_1 = 0, v_0 = -1, v_1 = 0, w_0 = 0, w_1 = 0, \theta_0 = 0, \theta_1 = 0, \phi_0 = 0, \phi = 0, p_1 = 0 \quad (3.18)$$

The solutions of the equations (3.7) to (3.10) under the boundary conditions (3.17) and (3.18)

$$v_0(y) = -1$$

$$\phi_0(y) = A_3 e^{-A_1 y} + A_4 e^{-A_2 y}$$

$$\theta_0(y) = A_8 e^{-A_1 y} + A_9 e^{-A_2 y}$$

$$u_0(y) = U + A_{15} e^{-R_e y} + A_{13} e^{-A_1 y} + A_{14} e^{-A_2 y}$$

The constants involved in the solutions are obtained but not presented here for the sake of brevity.

4. CROSS FLOW SOLUTION

We shall first consider the equations (3.11), (3.13) and (3.14) for $v_1(y,z)$, $w_1(y,z)$ and $p_1(y,z)$ which are independent of the main flow component u_1, temperature field θ_1 and concentration field ϕ_1.

The suction velocity distribution $v_w = -(1 + \varepsilon\cos\pi z e^{i\omega t})$ consists of a basic uniform distribution -1 with superimposed weak sinusoidal distribution $\varepsilon\cos\pi z e^{i\omega t}$. Hence the velocity components v, w and p are also separated into mean and small sinusoidal components v_1, w_1 and p_1.

We assume v_1, w_1 and p_1 as

$$v_1 = -\pi v_{11}(y)\cos\pi z$$

$$w_1 = v_{11}'(y)\sin\pi z$$

$$p_1 = Re^2 p_{11}(y)\cos\pi z$$

On substitution of the above, the equation (3.11) is satisfied and the equations (3.13) and (3.14) reduce to the following ordinary differential equations:

$$v_{11}'' + Re v_{11}' - (\pi^2 + i\omega Re)v_{11} = -\frac{Re}{\pi}p_{11}'$$

$$v_{11}''' + Re v_{11}'' - (\pi^2 + i\omega Re)v_{11}' = -\pi Re p_{11}$$

The corresponding boundary conditions are

$$y = 0 : v_{11} = \frac{1}{\pi}, \quad v_{11}' = 0$$

$$y \to \infty : v_{11} = 0, \quad v_{11}' = 0$$

The solutions of these equations are

$$v_{11} = \frac{1}{(\pi - A_{16})}\left[\frac{-A_{16}}{\pi}e^{-\pi y} + e^{-A_{16} y}\right]$$

$$p_{11} = \frac{-A_{16}}{\pi Re(\pi - A_{16})}\left[A_{17}e^{-\pi y} + A_{18}e^{-A_{16} y}\right]$$

where

$$A_{16} = \frac{Re + \sqrt{Re^2 + 4(\pi^2 + i\omega Re)}}{2}, A_{17} = -\pi Re - i\omega Re,$$

$$A_{18} = ReA_{16} - A_{16}^2 + \pi^2 + i\omega Re$$

Hence the solutions for the velocity components v_1, w_1 and pressure p_1 are as follows

$$v_1 = \frac{1}{(\pi - A_{16})}\left[A_{16}e^{-\pi y} - \pi e^{-A_{16} y}\right]\cos\pi z$$

$$w_1 = \frac{A_{16}}{(\pi - A_{16})}\left[e^{-\pi y} - e^{-A_{16} y}\right]\sin\pi z$$

$$p_1 = -\frac{R_e A_{16}}{\pi(\pi - A_{16})}\left[A_{17}e^{-\pi y} + A_{18}e^{-A_{16} y}\right]coz\pi z$$

5. Solution for first order flow, Concentration and temperature field

We now consider the equations (3.12), (3.15) and (3.16). The solutions for velocity components u, temperature field θ and concentration field ϕ are also separated into mean and sinusoidal components u_1, θ_1 and ϕ_1. To reduce the partial differential equations (3.12), (3.15)and (3.16) into ordinary differential equations, we consider the following assumptions for u_1, θ_1 and ϕ_1.

$$u_1 = u_{11}(y)\cos\pi z \quad (5.1)$$

$$\theta_1 = \theta_{11}(y)\cos\pi z \quad (5.2)$$

$$\phi_1 = \phi_{11}(y)\cos\pi z \quad (5.3)$$

Substituting the above expressions in equations (3.12), (3.15) and (3.16), the following ordinary differential equations are derived.

$$u_{11}'' + Re u_{11}' - (\pi^2 + i\omega Re)u_{11} = -\pi Re v_{11} u_0' - GrRe\theta_{11} - GmRe\phi_{11} \quad (5.4)$$

$$\theta_{11}'' + PrRe\theta_{11}' - (\pi^2 + i\omega PrRe)\theta_{11} = -\pi PrRe v_{11}\theta_0' - PrDu(\phi_{11}'' - \pi^2\phi_{11}) \quad (5.5)$$

$$\phi_{11}'' + ScRe\phi_{11}' - (\pi^2 + i\omega ScRe)\phi_{11} = -\pi ScRe v_{11}\phi_0' - SrSc(\theta_{11}'' - \pi^2\theta_{11}) \quad (5.6)$$

with boundary conditions:

$$y = 0 : \quad u_{11} = 0, \theta_{11} = 0, \quad \phi_{11} = 0 \quad (5.7)$$

$$y \to \infty : \quad u_{11} = 0, \theta_{11} = 0, \quad \phi_{11} = 0 \quad (5.8)$$

The solutions of the equations (5.4), (5.5) and (5.6) subject to the boundary conditions (5.7) and (5.8) are as follows:

$$\theta_{11} = B_{31}e^{-\lambda_3 y} + B_{29}e^{-\lambda_4 y} + A_{49}e^{-(\pi+A_1)y} + A_{50}e^{-(\pi+A_2)y} + B_1 e^{-(A_1+A_{16})y} + B_2 e^{-(A_2+A_{16})y} \quad (5.9)$$

$$\phi_{11} = B_{30}e^{-\pi y} + B_{31}B_{10}e^{-\lambda_3 y} + B_{29}B_{11}e^{-\lambda_4 y} + B_{12}e^{-(\pi+A_1)y} + B_{13}e^{-(\pi+A_2)y} + B_{14}e^{-(A_1+A_{16})y} + B_{15}e^{-(A_2+A_{16})y} \quad (5.10)$$

$$u_{11} = B_{54}e^{-\lambda_5 y} + B_{45}e^{-(\pi+Re)y} + B_{46}e^{-(Re+A_{16})y} + B_{47}e^{-(\pi+A_1)y} + B_{48}e^{-(\pi+A_2)y} + B_{49}e^{-(A_1+A_{16})y} + B_{50}e^{-(A_2+A_{16})y} + B_{51}e^{-\lambda_3 y} + B_{52}e^{-\lambda_4 y} + B_{53}e^{-\pi y} \quad (5.11)$$

The constants involved in the solutions are obtained but not presented here for the sake of brevity.
Now we have,

$$u = u_0(y) + \varepsilon\, e^{i\omega t}\, u_{11}(y)\cos\pi z \;= u_r + i\, u_i,$$

$$\theta = \theta_0(y) + \varepsilon\, e^{i\omega t}\, \theta_{11}(y)\cos\pi z \;= \theta_r + i\, \theta_i,$$

$$\phi = \phi_0(y) + \varepsilon\, e^{i\omega t}\, \phi_{11}(y)\cos\pi z \;= \phi_r + i\, \phi_i,$$

where u_r, θ_r, ϕ_r, u_i, θ_i, ϕ_i have been obtained but not shown here for brevity's sake.

6. Coefficient of skin-friction at the plate

The non-dimensional coefficient of skin-friction τ at the plate in the direction of the free stream is given by:

$$\tau = -\frac{\mu \left.\dfrac{\partial \bar{u}}{\partial \bar{y}}\right]_{\bar{y}=0}}{\rho V_0^2} = -\frac{1}{Re}\left(\frac{\partial u}{\partial y}\right)_{y=0} = -\frac{1}{Re}\left[u_0'(0) + \varepsilon u_{11}'(0)\cos\pi z\, e^{i\omega t}\right] = \tau_r + i\,\tau_i$$

7. The co-efficient of heat flux

The heat flux at the plate at $\bar{y}=0$ in terms of Nusselt number Nu is given by:

$$Nu = \frac{L\left(\dfrac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0}}{(\bar{T}_W - \bar{T}_\infty)} = \left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \left[\theta_0'(0) + \varepsilon\theta_{11}'(0)\cos\pi z\, e^{i\omega t}\right] = Nu_r + i\, Nu_i$$

8. The co-efficient of mass flux

$$Sh = \frac{L\left(\dfrac{\partial \bar{C}}{\partial \bar{y}}\right)_{\bar{y}=0}}{(\bar{C}_w - \bar{C}_\infty)} = \left(\frac{\partial \phi}{\partial y}\right)_{y=0} = \left[\phi'(0) + \varepsilon\phi_{11}'(0)\cos\pi z\, e^{i\omega t}\right] = Sh_r + i\, Sh_i$$

The expressions for τ_r, τ_i, Nu_r, Nu_i, Sh_r, Sh_i have been obtained but not presented here for the sake of brevity.

9. Discussion of the results

In order to study the effects of Dufour number Du, suction Reynolds number Re, Soret number Sr, Schmidt number Sc, Grashof number for mass transfer Gm, Grashof number for heat transfer Gr and free stream velocity U, we have carried out the numerical calculations for u_r, θ_r, ϕ_r, τ_r, Nu_r and Sh_r which are respectively the real parts of the non dimensional velocity, temperature, chemical species concentration, skin friction, Nusselt number and Sherwood number at the plate surface and their profiles are demonstrated in graphs. In our investigation the value of the Prandtl number Pr is taken to be equal to 0.71 which corresponds to air. Since the water vapor is used as a diffusing chemical species of common interest in air therefore the value of Sc is taken to be 0.60 (water vapor). Through out our investigation, ω and t are chosen in such a way that $\omega t = \frac{\pi}{2}$ and the frequency of oscillation ε is considered as 0.001 and the remaining parameters namely Dufour number Du, suction Reynolds number Re, Soret number Sr, Schmidt number Sc, Grashof number Gm, Grashof number Gr and free stream velocity U are chosen arbitrarily.

Figures 1, 4, 7, 8, 9 and 10 demonstrate the behaviors of the real part of fluid velocity u namely u_r. Figures 1, 7, 8 and 9 respectively indicate an increase in u_r as each of Du, Gm, Gr and U rises. On the other hand, it is observed from figures 1, 4 and 10 that a rise in Sr or Sc or Re results in a fall in u_r. Clearly, the Dufour effect, the increasing buoyancy effect and the increasing free stream accelerate the main flow. Further the Soret effect, decreasing chemical molecular mass diffusivity and increasing suction retard the flow field u_r. It is worthwhile to mention that the product of Soret and Dufour numbers is taken to be constant for given values of D_M, K_T, C_S, C_P, υ and T_M. We fix this product as $Sr.Du=1$.

It is inferred from figures 2 and 3 that the Soret effect reduces the temperature field θ_r as well as the concentration field ϕ_r. However each of the temperature θ_r and species concentration ϕ_r registers a rise under Dufour effect.

The figures 5,6,11 and 12 indicate that a rise in Schmidt number Sc and suction Reynolds number Re leads to a fall in both θ_r and ϕ_r. It is seen both θ_r and ϕ_r exhibit reverse trend at large distances from the plate's surface.

It is observed from figures 13, 16, 17 and 18 that the magnitude of the skin friction τ_r at the plate registers a fall with an increase in each of Sc, Gm, Gr and U. Thus a decrease in chemical molecular mass diffusivity or increase in free stream velocity or rise in buoyancy effects (owing to temperature and species concentration differences) causes a fall in $|\tau_r|$.

It is noticed from figure 19 that the magnitude of the skin friction τ_r at the plate (i.e. $|\tau_r|$) increases under Dufour effect and decreases under Soret effect.

As observed from 14 and 15, a rise in Sc (decrease in chemical molecular mass diffusivity) raises both $|Nu_r|$ and $|Sh_r|$ (the magnitudes of the Nusselt number Nu_r and the Sherwood number Sh_r respectively).

Figures 20 and 21 exhibit a fall in both $|Nu_r|$ and $|Sh_r|$ under Dufour effect where as the Soret effect leads to a rise in each of $|Nu_r|$ and $|Sh_r|$.

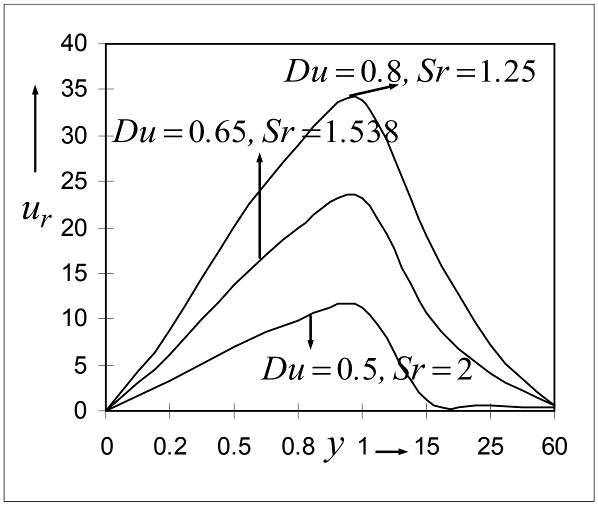

Figure 1: Velocity u_r against y, for variations of Dufour and Soret numbers when $Sc=0.22$, $Pr=0.71$, $Re=0.5$, $\varepsilon=0.001$, $Gr=10$, $Gm=5$, $\omega=0.5$, $t=1$, $U=0.5$ and $z=0.25$.

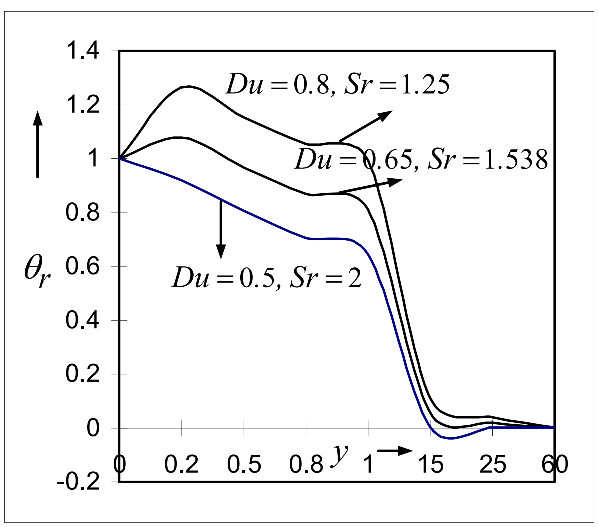

Figure 2: Temperature θ_r against y, for variations of Dufour and Soret numbers Du and Sr respectively, when $Sc=0.22$, $Pr=0.71$, $Re=0.5$, $\varepsilon=0.001$, $Gr=10$, $Gm=5$, $\omega=0.5$, $t=1$, $U=0.5$ and $z=0.25$.

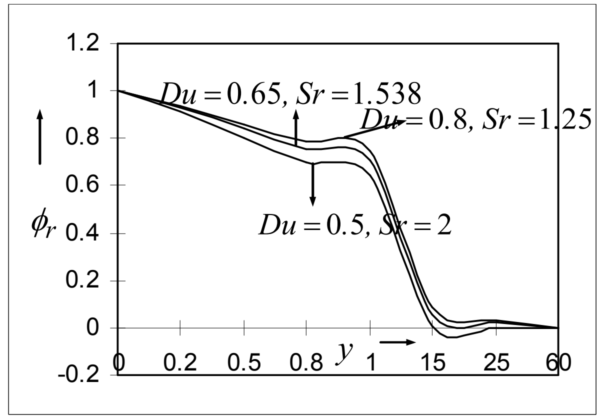

Figure 3: Concentration ϕ_r against y, for variations of Dufour and Soret numbers Du and Sr respectively, when $Sc=0.22$, $Pr=0.71$, $Re=0.5$, $\varepsilon=0.001$, $Gr=10$, $Gm=5$, $\omega=0.5$, $t=1$, $U=0.5$ and $z=0.25$.

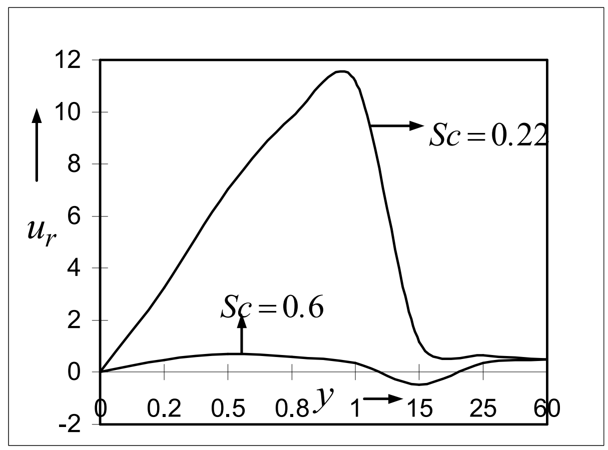

Figure 4: Velocity u_r against y, for variation of Schmidt number Sc when $Du=0.5$, $Sr=2$, $Pr=0.71$, $Re=0.5$, $\varepsilon=0.001$, $Gr=10$, $Gm=5$, $\omega=0.5$, $t=1$, $U=0.5$ and $z=0.25$.

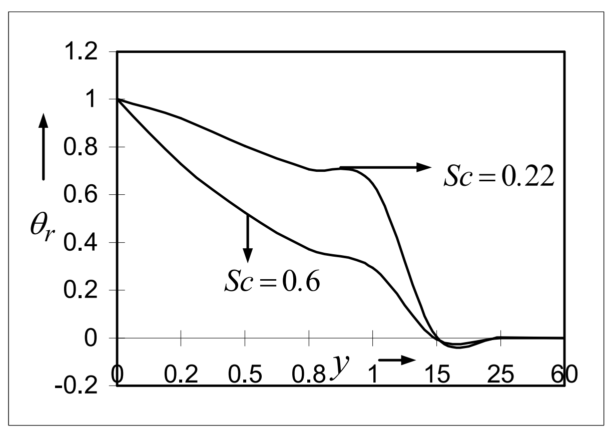

Figure 5: Temperature θ_r against y, for variation of Schmidt number Sc when $Du=0.5$, $Sr=2$, $Pr=0.71$, $Re=0.5$, $\varepsilon=0.001$, $Gr=10$, $Gm=5$, $\omega=0.5$, $t=1$, $U=0.5$ and $z=0.25$.

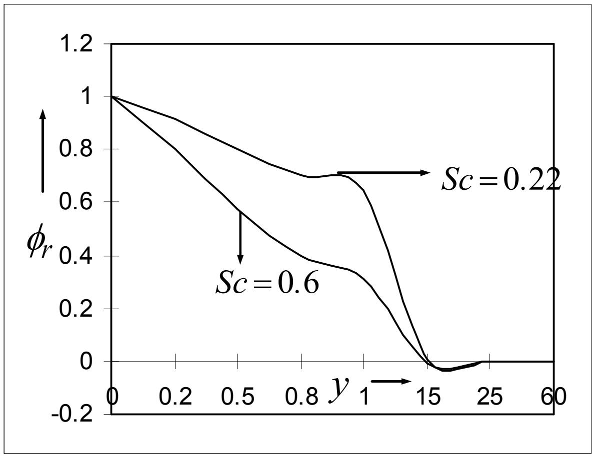

Figure 6: Concentration ϕ_r against y, for variation of Schmidt number Sc when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $Re = 0.5$, $\varepsilon = 0.001$, $Gr = 10$, $Gm = 5$, $\omega = 0.5$, $t = 1$, $U = 0.5$ and $z = 0.25$.

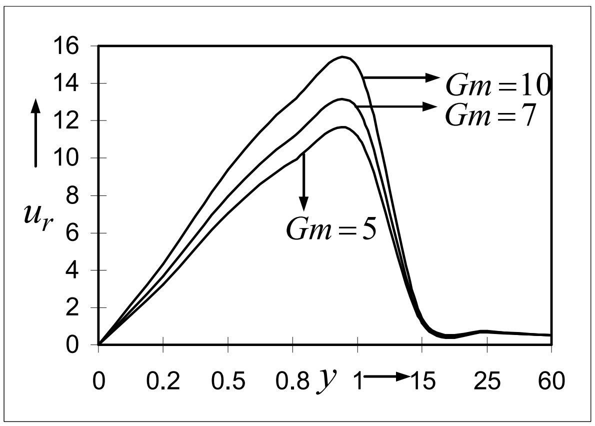

Figure 7: Velocity u_r against y, for variation of Grashof number Gm when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $Re = 0.5$, $\varepsilon = 0.001$, $Gr = 10$, $Sc = 0.22$, $\omega = 0.5$, $t = 1$, $U = 0.5$ and $z = 0.25$.

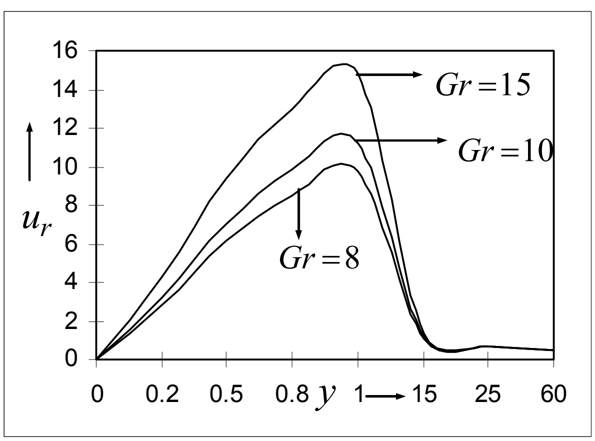

Figure 8: Velocity u_r against y, for variation of Grashof number Gr when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $Re = 0.5$, $\varepsilon = 0.001$, $Gm = 5$, $Sc = 0.22$, $\omega = 0.5$, $t = 1$, $U = 0.5$ and $z = 0.25$.

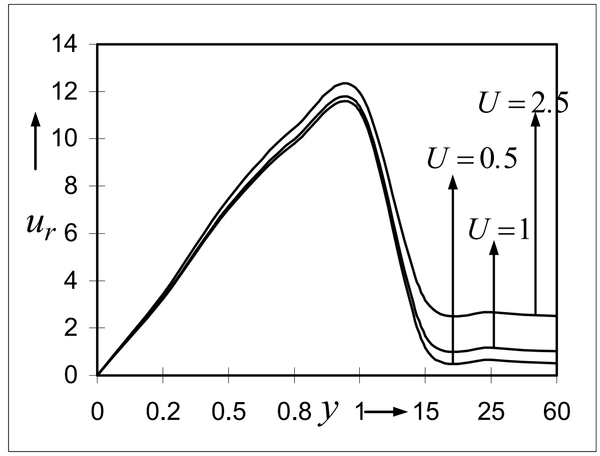

Figure 9: Velocity u_r against y, for variation of U when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $Re = 0.5$, $\varepsilon = 0.001$, $Gm = 5$, $Sc = 0.22$, $\omega = 0.5$, $t = 1$, $Gr = 10$ and $z = 0.25$.

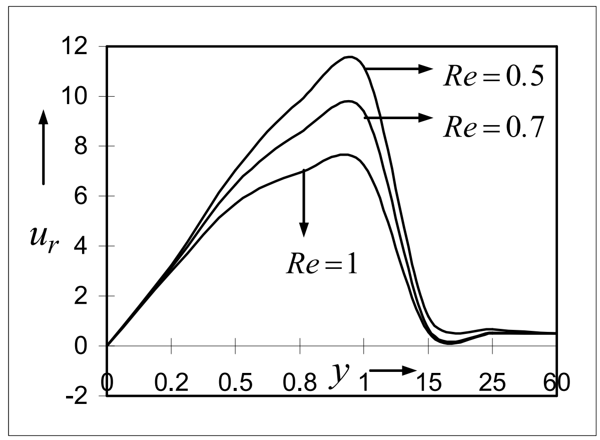

Figure 10: Velocity u_r against y, for variation of suction Reynolds number Re when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $U = 0.5$, $\varepsilon = 0.001$, $Gm = 5$, $Sc = 0.22$, $\omega = 0.5$, $t = 1$, $Gr = 10$ and $z = 0.25$.

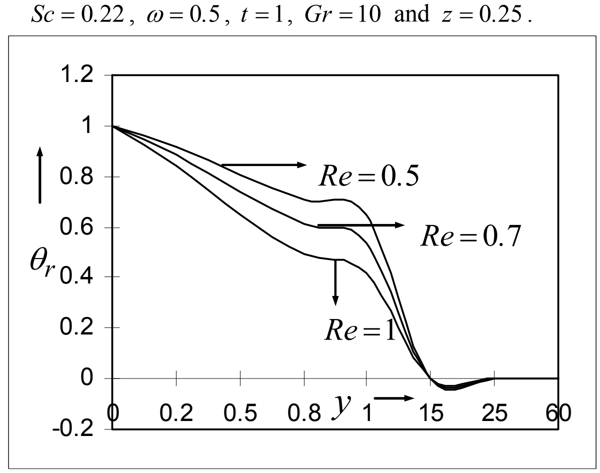

Figure 11: Temperature θ_r against y, for variation of suction Reynolds number Re when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $Sc = 0.22$, $\varepsilon = 0.001$, $Gr = 10$, $Gm = 5$, $\omega = 0.5$, $t = 1$, $U = 0.5$ and $z = 0.25$.

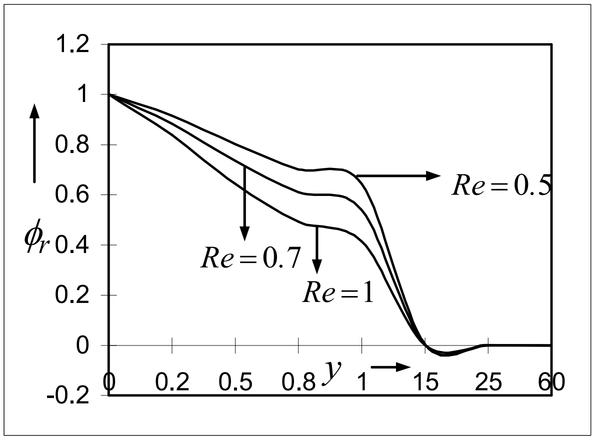

Figure 12: Concentration ϕ_r against y, for variation of suction Reynolds number Re when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $Sc = 0.22$, $\varepsilon = 0.001$, $Gr = 10$, $Gm = 5$, $\omega = 0.5$, $t = 1$, $U = 0.5$ and $z = 0.25$.

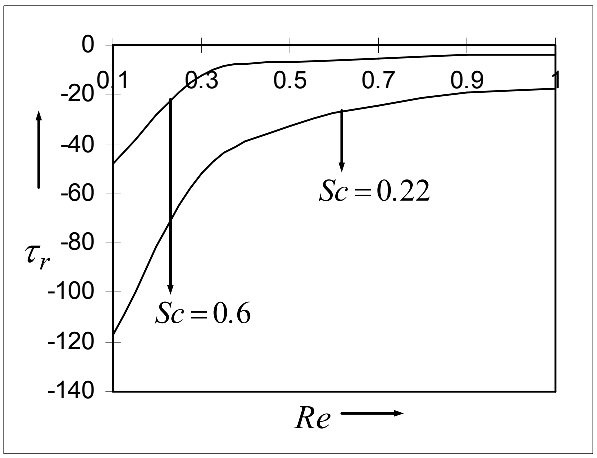

Figure 13: Skin friction τ_r against Reynolds number Re, for variation of Schmidt number Sc when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $\varepsilon = 0.001$, $Gr = 10$, $Gm = 5$, $\omega = 0.5$, $t = 1$, $U = 0.5$ and $z = 0.25$.

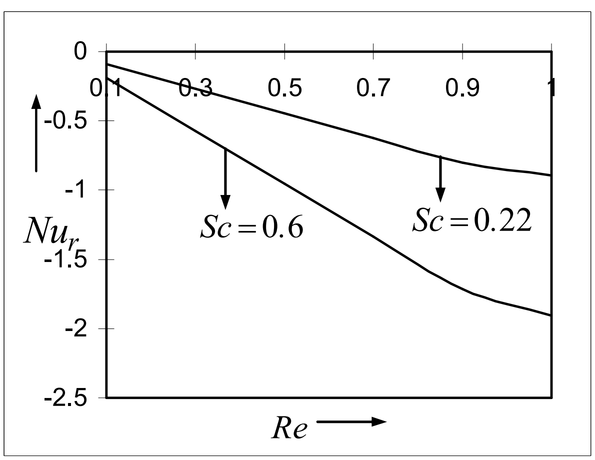

Figure 14: Nusselt number Nu_r against Reynolds number Re, for variation of Schmidt number Sc when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $\varepsilon = 0.001$, $Gr = 10$, $Gm = 5$, $\omega = 0.5$, $t = 1$, $U = 0.5$ and $z = 0.25$.

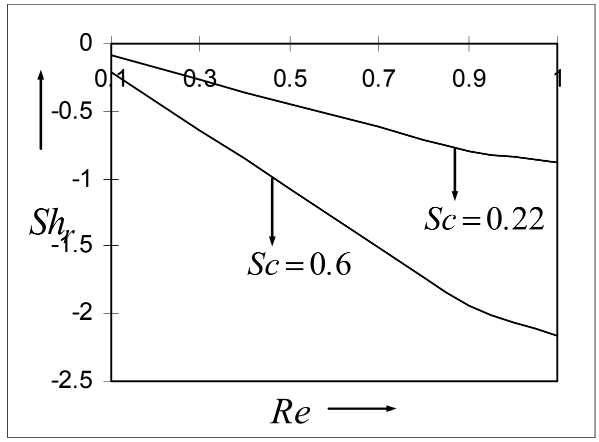

Figure 15: Sherwood number Sh_r against Reynolds number Re, for variation of Schmidt number Sc when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $\varepsilon = 0.001$, $Gr = 10$, $Gm = 5$, $\omega = 0.5$, $t = 1$, $U = 0.5$ and $z = 0.25$.

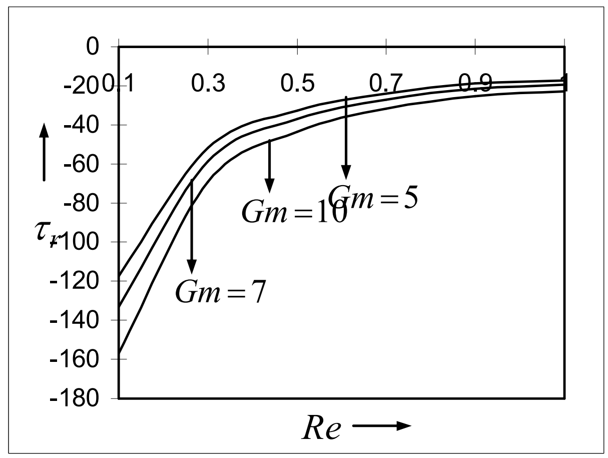

Figure 16: Skin friction τ_r against Reynolds number Re, for variation of Grashof number Gm when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $\varepsilon = 0.001$, $Gr = 10$, $\omega = 0.5$, $t = 1$, $U = 0.5$, $Sc = 0.22$ and $z = 0.25$.

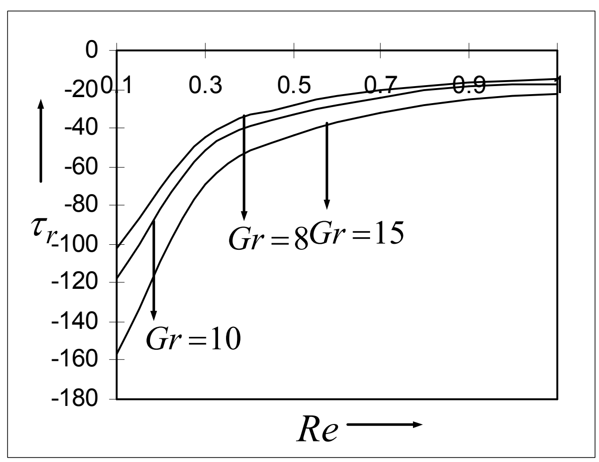

Figure 17: Skin friction τ_r against Reynolds number Re, for variation of Grashof number Gr when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $\varepsilon = 0.001$, $Gm = 5$, $\omega = 0.5$, $t = 1$, $U = 0.5$, $Sc = 0.22$ and $z = 0.25$.

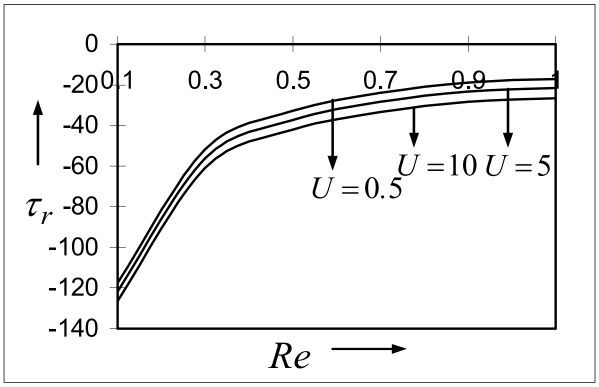

Figure 18: Skin friction τ_r against Reynolds number Re, for variation of U when $Du = 0.5$, $Sr = 2$, $Pr = 0.71$, $\varepsilon = 0.001$, $Gm = 5$, $Gr = 10$, $\omega = 0.5$, $t = 1$, $Sc = 0.22$ and $z = 0.25$.

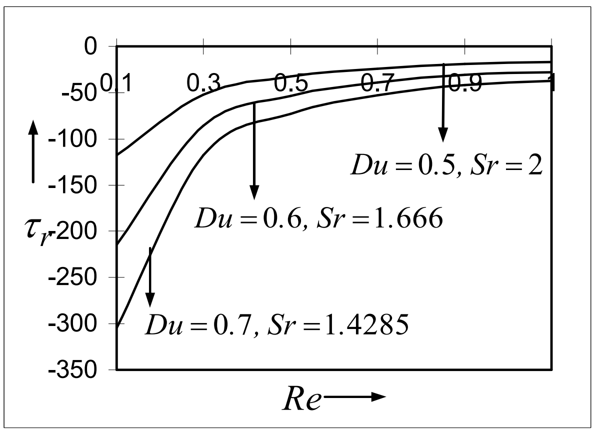

Figure 19: Skin friction τ_r against Reynolds number Re, for variations of Dufour and Soret numbers Du and Sr respectively, when $Pr = 0.71$, $\varepsilon = 0.001$, $Gm = 5$, $Gr = 10$, $\omega = 0.5$, $t = 1$, $Sc = 0.22$, $U = 0.5$ and $z = 0.25$.

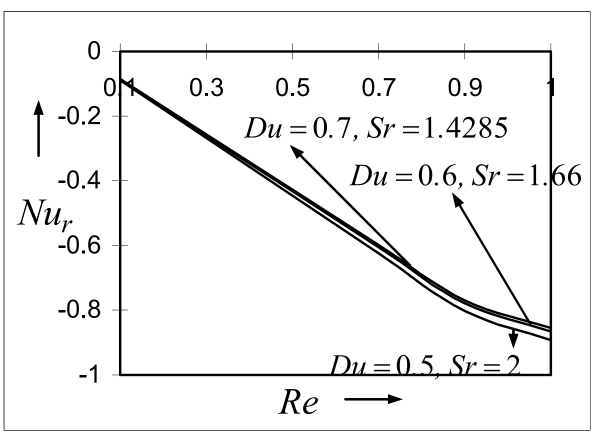

Figure 20: Nusselt number Nu_r against Reynolds number Re, for variations of Dufour and Soret numbers Du and Sr respectively, when $Pr = 0.71$, $\varepsilon = 0.001$, $Gm = 5$, $Gr = 10$, $\omega = 0.5$, $t = 1$, $Sc = 0.22$, $U = 0.5$ and $z = 0.25$.

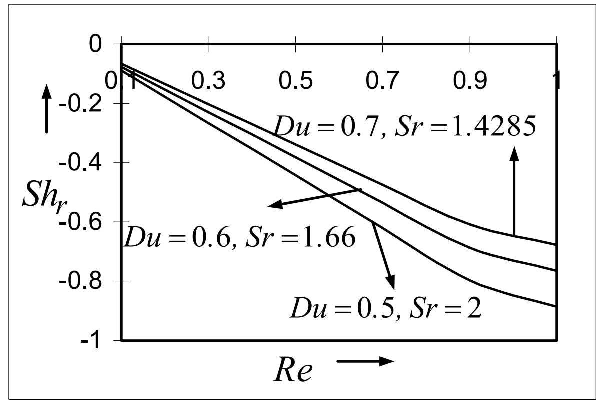

Figure 21: Sherwood number Sh_r against Reynolds number Re, for variations of Dufour and Soret numbers Du and Sr respectively, when $Pr = 0.71$, $\varepsilon = 0.001$, $Gm = 5$, $Gr = 10$, $\omega = 0.5$, $t = 1$, $Sc = 0.22$, $U = 0.5$ and $z = 0.25$.

10. CONCLUSIONS

Based on our flow model, our investigation may be summarized to the following conclusions:

(i) A decrease in chemical molecular mass diffusivity, Grashof numbers for heat and mass transfer (buoyancy effects) and free stream velocity reduces the magnitude of the skin friction τ_r at the plate.

(ii) A decrease in chemical molecular mass diffusivity causes the magnitudes of the Nusselt number Nu_r and Sherwood number Sh_r at the plate to rise.

(iii) An increase in Dufour effect or a decrease in Soret effect leads to a rise in the magnitude of the skin friction τ_r at the plate.

(iv) The magnitudes of the Nusselt number Nu_r and the Sherwood number Sh_r at the plate, register a fall with a rise in Dufour effect or a fall in Soret effect.

(v) The thermal and the species concentration boundary layers are analogous to one another. Further, the profiles for the Nusselt number Nu_r and the Sherwood number Sh_r are very similar to one another indicating the fact that the heat and the mass transfer processes are analogous to one another.

(vi) The suction is useful in controlling the rates of heat and mass transfer at the plate and in minimizing the skin friction at the plate surface.

11. NOMENCLATURE

Symbol	Quantity	SI unit
$\overline{C}$	Species concentration	$\frac{Kmol}{m^3}$
C_P	Spefic heat at constant pressure	$J / kg \times K$
C_S	Concentration susceptibility	$(Kmol)^2 s^2$

$\overline{C}_\infty$	Species concentration in the free stream	$\frac{Kmol}{m^3}$
D_M	Co-efficient of chemical molecular mass diffusivity	$m^2 s^{-1}$
D_T	Co-efficient of chemical thermal diffusivity	$\frac{Kmol}{m\,K\,s}$
Du	Dufour number	-
Gr	Grashof number for heat transfer	-
Gm	Grashof number for mass transfer	-
g	Acceleration due to gravity	m/s^2
K_T	Thermal Diffusion ratio	$Kmol$
k	Thermal conductivity	$\frac{W}{mK}$
Pr	Prandtl number	-
$\overline{p}$	Pressure	Pa (Pascal)
p	Non dimensional pressure	-
p_∞	Non dimensional pressure in the free stream	-
Re	Suction Reynolds number	-
Sc	Schmidt number	-
Sr	Soret number	-
$\overline{T}$	Temperature in the boundary layer	K or ^0C
T_M	Mean fluid temperature	K or ^0C
$\overline{T}_\infty$	Temperature in the free stream	K or ^0C
$\overline{t}$	Time	s (second)
t	Non dimensional time	-
$\overline{U}$	Free stream velocity	m/s
U	Non dimensional free stream velocity	-
$(\overline{u},\overline{v},\overline{w})$	Components of the fluid velocity	m/s
(u,v,w)	Non dimensional components of the fluid velocity-	
u_r, u_i	The real and imaginary parts of u	
Nu	Nusselt number at the plate	-
Nu_r, Nu_i	The real and imaginary parts of Nu	-
Sh	Sherwood number at the plate	-
Sh_r, Sh_i	The real and imaginary parts of Sh .	-

Greek Symboles	Quantity	SI unit
β	Coefficient volume expansion for heat transfer	K^1
$\overline{\beta}$	Co-efficient of volume expansion for mass transfer	$\frac{1}{Kmol}$
υ	Kinematic viscosity	m^2/s
ρ	Fluid density	kg/m^3
θ	Non dimensional temperature	K or ^0C
θ_r, θ_i	The real and imaginary parts of θ	-
ϕ	Non dimensional chemical species concentration	-
ϕ_r, ϕ_i	The real and imaginary parts of ϕ	-
ω	Frequency parameter	H_z
ε	Amplitude of the suction velocity	ms^{-1}
τ	Coefficient of skin friction at the plate	-
τ_r, τ_i	The real and imaginary parts of τ	-

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