















$\alpha = 0.01$  and  $0.1(\text{m}^{-1})$  with common input values  $D_{x_0} = 2.18(\text{m}^2/\text{day})$ ,  $D_{y_0} = 0.218(\text{m}^2/\text{day})$  and  $t = 14(\text{day})$ . It may be observed that heterogeneity plays a significant role on concentration distribution. concentration attenuates with position and time. The concentration pattern decreases with heterogeneity parameter and position but after a certain distance it becomes constant for all time.

## 5. CONCLUSION

Analytical solutions are obtained for spatially dependent solute dispersion for varying input point source defined by Heaviside function in a two-dimensional semi-infinite porous medium with an appropriate realistic initial and boundary conditions. At the initial stage the aquifer domain is considered not solute free. Dispersion coefficient is considered proportional to square of groundwater velocity in both directions (longitudinal and lateral). The effects of various parameters are significantly observed on the concentration profiles. Laplace Integral Transformation Technique (LITT) is employed to get the analytical solutions. LITT is simpler, more viable and commonly used in assessing the stability of numerical solutions in more realistic dispersion problems. Two transformations have been used to obtain the analytical solutions. The effects of solute transport parameters on concentration profiles are evaluated in different time domains and demonstrated with help of graphs. The obtained analytical solutions may be helpful in predicting the concentration levels in the aquifer at any position and time and also useful for verifying the accuracy of numerical solutions.

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## NOMENCLATURE

$C$	Solute concentration, $\text{kg m}^{-3}$
$C_0$	Reference solute concentration, $\text{kg m}^{-3}$



$C_i$	Initial solute concentration, $\text{kg m}^{-3}$	$t_1$	Beginning time of source activation, s
$u$	Longitudinal groundwater velocity, $\text{ms}^{-1}$	$t_2$	Ending time of source activation, s
$v$	Lateral groundwater velocity, $\text{ms}^{-1}$	$s$	Laplace parameter
$u_0$	Initial longitudinal groundwater velocity, $\text{ms}^{-1}$	$c_1$ & $c_2$	Arbitrary constant
$v_0$	Initial lateral groundwater velocity, $\text{ms}^{-1}$	$p, q$ & $r$	Parameter of time function
$D_x$	Longitudinal dispersion coefficient, $\text{m}^2\text{s}^{-1}$	$\bar{C}$	Laplace transform of $C$
$D_y$	Lateral dispersion coefficient, $\text{m}^2\text{s}^{-1}$	$u(t-t_1)$ } $u(t-t_2)$ }	Heaviside function
$D_{x_0}$	Initial longitudinal dispersion coefficient, $\text{m}^2\text{s}^{-1}$	$D_0, \gamma_0, w_0,$ } $U_0, U_1, U_2,$ }	
$D_{y_0}$	Initial lateral dispersion coefficient, $\text{m}^2\text{s}^{-1}$	$\alpha, \beta, \delta$ }	
$x$	Longitudinal space variable, m		
$y$	Lateral space variable, m		
$X$	New longitudinal space variable, m		
$Y$	New lateral space variable, m		
$Z$	New space variable, m		
$a$	Heterogeneous parameter, $\text{m}^{-1}$		
$t$	Time variable, s		