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STUDY OF CO_2 FLUID DENSITY CALCULATION MODEL BASED ON GRAYSCALE IMAGE

Dong Yong [1*, 2, 3], Wu Chuansheng [2], Lv Qinghe [1], Li Hongkui [1] and Guo Haimin [4]

*[1] Institute of Petroleum Engineering Technology, Zhongyuan Bureau of Petroleum Exploration, Puyang Henan，457001, China;
[2] Mechanical Postdoctoral Station, Wuhan University of Technology, Wuhan Hubei 430070, China;
[3] School of Information and Mathematic, Yangtze University, Jingzhou Hubei, 434023, China;
[4] Key Laboratory of Exploration Technologies for Oil and Gas Resources, Ministry of Education, Yangtze University, Wuhan Hubei, 434023, China

ABSTRACT

In order to get more accurate density calculation model of CO_2 under different phases which are liquid, gas and supercritical state, the paper applies the relative error to measure the precision of the calculation model based on six typical cubic calculation models and the basic data which is from the U.S. National Institute of Standards and Technology (NIST). It presents a new method based on grayscale image, which is to show the relative error matrix as grayscale image and binary image. It will help to analyze the distribution of relative error for different accuracy. The paper presents a new density calculation model based on composition rules, which improves the accuracy and keep things as simple as possible. The new model clears up relative error at basic data point. The results indicate that the new model is more accurate. The new method in the paper can also be applied to study calculate model of thermo physical parameter for other substances.

Keywords: state equation, density calculation, relative error, grayscale image, binary image.

1. INTRODUCTION

At present, CO_2 flooding is a popular technology to enhance oil recovery [1-3]. In order to improve the pertinence of injection, it needs to monitor the injection profile in the process of injection technology of CO_2 flooding. The tranditional water injection profile monitoring uses volume flow rate. There exists phase state change between liquid and supercritical state as CO_2 is injected from the ground surface to the downhole, the volume flow data in different depth have no comparability. The mass flow rates under different states always have comparability, so the mass flow rate can be considered as measurement standard when proceeds CO_2 injection profile monitoring. One key which is to get the mass flow rate of CO_2 is to obtain the density of CO_2 at measuring point. The empirical formula usually is used to get the physical parameters is usually determined by the formula [4-7], or by the experimental data [8-12], but it is difficult to keep the same computational accuracy for different phase states of CO_2 by using the empirical formula. Hence, it is necessary to study the accuracy of the calculation model of density of CO_2 under different temperatures and pressures, and then modified calculation model of density is put forward.

In order to show the influence rule on physical property parameter for the temperature or pressure, the existing literatures made the curve according to the physical property parameters and the corresponding temperatures or the pressures [2-12]. There is no method to show the influence rule on physical property parameter from both temperature and the pressure at present. Based on the NIST data, this paper measures the interpretation accuracy by using the relative error and proposes that the relative errors constitute the matrix. The matrix is transferred to grayscale image to be displayed intuitively and it can show the influence on the density of CO_2 from both temperature and pressure. In order to compare the accuracy between different interpretation models, this paper proposed the binarization method of relative error greyscale image. Based on the method, this paper proposed the interpretation model with higher accuracy. The numerical results showed that the method in this paper is efficient.

2. CALCULATION MODEL OF DENSITY

Because the cubic state equation is convenient to be calculated and it has high accuracy, it is widely used in the engineering field [1-3]. This paper compares and analyzes six typical cubic state equations which are EXP-RK [13], PR， PT， Tong， RT and SRK. Takes RK equation for example as follow, and shows in Eq.(1) which gives the calculation model of density. The other equations are deal with similarly.

$$p = \frac{R \cdot T}{V-b} - \frac{a}{\sqrt{T} \cdot V \cdot (V+b)} \quad (1)$$

Where

$$a = 0.42748\frac{R^2 \cdot T_c^{2.5}}{p_c}, \; b = 0.08664\frac{R \cdot T_c}{p_c}.$$

The parameters of Eq. (1) are described as follow:

p : pressure, MPa
T : temperature, K
V : molar volume, m3/mol
R : gas constant whose value relies on the formula system which is showed in literature [14]
T_c : critical temperature, K
p_c : critical pressure, MPa
ω : acentric factor, -
T_r : pseudoreduced temperature, -

The relation between density and volume is

$$\rho = \frac{M}{V} \quad (2)$$

The parameters of Eq.(2) are described as follow.

M : molar gas mass, kg/mol
V : molar volume, m3/mol
ρ : gas density, kg/m3

Substituting Eq.(2) into Eq. (1) and simplifying, we get

$$ab\rho^3 + (RMT^{1.5}b - aM + T^{0.5}Mpb^2)\rho^2 + RM^2T^{1.5}\rho - T^{0.5}pM^3 = 0 \quad (3)$$

This is a cubic equation about ρ, which has three roots in the complex field. According to the root, we can determine the fluid phases under the given temperature and pressure [15]. We calculate the minimum positive root which is the fluid density from Eq.(3).

3. THE STANDARD DATA OF CO2 FLUID DENSITY

In order to measure the accuracy of result of each formula, we choose the data [16] from the U.S. National Institute of Standards and Technology to be the standard. The standard data gives the following pressure: 4 MPa, 8 MPa, 12 MPa, 16 MPa and 20 MPa. The temperature increases from 10 degrees to 200 degrees with step 10 degrees. Partial data is showed in the Table 1.

Table 1. CO_2 density data given by NIST (kg/m3)

	10˚C	20˚C	……	200˚C
4 MPa	108.41	97.49	……	46.43
8 MPa	903.13	827.71	……	96.14
12 MPa	935.23	878.10	……	148.77
16 MPa	959.85	911.37	……	203.46
20 MPa	980.18	937.04	……	258.82

From Table 1, the density data difference which is under different temperature and pressure is large. Hence, it is not appropriate to use the absolute error to measure the difference between the calculation result and the NIST data, but can use the relative error. The definition of relative error is

$$RE = \frac{|\rho_{cal} - \rho_{NIST}|}{\rho_{NIST}} \quad (4)$$

Where, ρ_{cal} is the density which obtained from calculation and ρ_{NIST} is the density from the NIST data.

In order to intuitively represent the relative error under different temperature and pressure, this paper firstly forms a relative error matrix according to the relative position of the relative error in Table 1. The relative matrix is showed as follow.

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,5} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,5} \\ \cdots & \cdots & \cdots & \cdots \\ a_{20,1} & a_{20,2} & \cdots & a_{20,5} \end{bmatrix} \quad (5)$$

Where $a_{i,j}$ is the relative error value corresponding to the temperature $i \times 10℃$ and pressure $j \times 4MPa$.

The relative error matrix is displayed visually as a grayscale image which is called relative error grayscale image. The algorithm is showed as follow.

STEP 1 We choose the maximum and the minimum from A, and denote them M and m, respectively.

STEP 2 Map.

$$b_{i,j} = round\left[\frac{a_{i,j} - m}{M - m} \times 255\right]$$

STEP 3 Let $a_{i,j} = b_{i,j}$, and display the matrix.

The relative error grayscale image of the equation in this paper is showed in Figure 1.

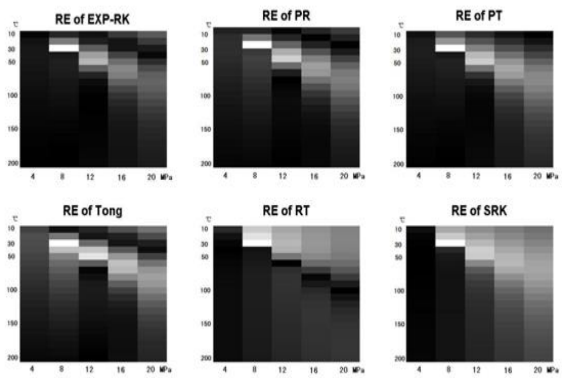

Figure 1. The relative error grayscale image of various models under different gray scales

In Figure 1, the smaller the element value is, the more pure the black is, and then the smaller the relative error. The pure white represents the corresponding maximum relative error of each model. The gray means that the relative error is between minimum and maximum relative error. From the Figure 1, we can get that the accuracy of all models is lower when the temperatures are 30 degrees and between 40 and 50 degrees with the corresponding pressures 8 MPa and 12 MPa. We also get the opposite results for the RT model when the temperatures are 60 degrees, 80 degrees, and 100 degrees with the corresponding pressures 12 MPa, 16 MPa, and 20 MPa. We guess that maybe the calculation accuracy can be advanced after we combine the RT model and the preceding four models which are EXP-RK, PR, PT and Tong because of the complementary relationship which they may form.

There is not comparability between different gray images in Figure 1. For example, the elements which locate in the third row and the second column in the first gray image are pure white, and it just means that the relative error is maximum under the current temperature and pressure for the EXP-RK model. Although the elements which locate in the same position in the last grayscale image are gray, it doesn’t mean that the relative error of SRK model is smaller than the relative error of EXP-RK model under the same temperature and pressure.

Figure 2 shows the relative error grayscale images of each model under the same gray scale. We can see that the RT model can be the effective supplement for the preceding four models from the Figure 2 and the RT model improves the calculation accuracy of density of the preceding four models. The algorithm is showed as follow.

STEP 1 Denote the corresponding relative error matrices of the six models $A_1, A_2, \cdots, A_6$. Choose the maximum and minimum from A_i and denote them M_i and m_i respectively.

STEP 2 Denote $M = \max\limits_{1 \le i \le 6}\{M_i\}$, $m = \min\limits_{1 \le i \le 6}\{m_i\}$.

STEP 3 Map.

$$b_{i,j} = round\left[\frac{a_{i,j} - m}{M - m} \times 255\right]$$

STEP 4 Let $a_{i,j} = b_{i,j}$, and display the matrix A .

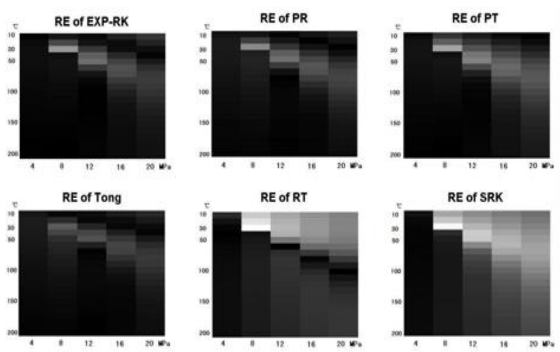

Figure 2. The relative error grayscale image of various models under the same gray scale

Although we can roughly see the degree of precision of each model from Figure 2, we can’t determine the compound mode of these models in detail because the resolving power of human eyes to different gray scales is limited. The algorithm is showed as follow.

STEP 1 Give one relative error level which is also called threshold value η .

STEP 2 Compare the elements in matrix A and η , and map as follow.

$$b_{i,j} = \begin{cases} 0, & if \quad a_{i,j} \le \eta \\ 255, & if \quad a_{i,j} > \eta \end{cases}$$

And let $a_{i,j} = b_{i,j}$.

STEP 3 Display the corresponding grayscale image of A which is the binary image.

Take η to be 1%, 2%, 3%, 4%, 5%, 6% and 7%. Give the binarization grayscale image. For the sake of simplicity, we only list the cases which are showed in Figure 3 to Figure 5.

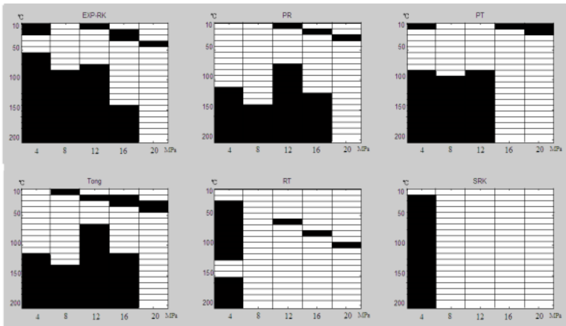

Figure 3. Binarization image of relative error when η =1%

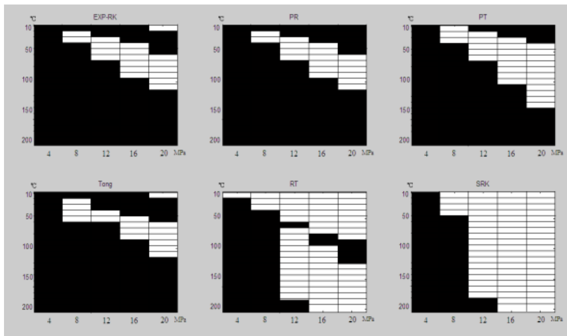

Figure 4. Binarization image of relative error when η =3%

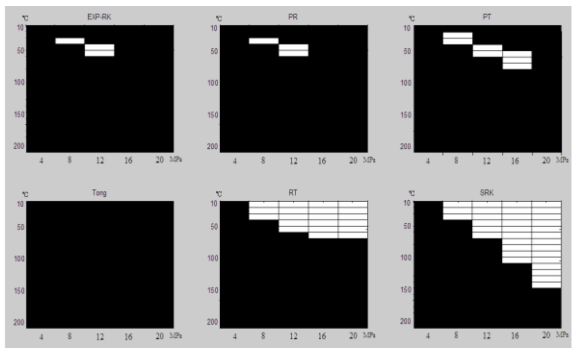

Figure 5. Binarization image of relative error when η =7%

Though analyzing the above relative error binarization image, for all models, the lower the pressure and the higher the temperature are, the higher the calculation accuracy is. In the range of the referred temperature and pressure, no formula is overall superior. The literature [15] indicates that combining the EXP-RK model and PR model can improve the calculation accuracy of density, but the modified method in literature [15] is not the most superior by analyzing Figure 3 to 5.

4. THE CONSTRUCTION OF NEW CALCULATION MODEL

This paper adopts the following combining rule. Firstly, we choose the higher accuracy model to be combined. Secondly, use the less model which is to be combined as far as possible. From Figure 3, we should choose EXP-RK model preferentially. Hence, the compound mode which is proposed in this paper is showed in Table 2.

Table 2. Optimization of compound mode of model

MPa / ℃	4	8	12	16	20
10	1 (1%)	4(1%)	1 (1%)	3 (1%)	3 (1%)
20	1 (1%)	4(5%) 2(5%)	4 (1%)	1 (1%)	3 (1%)
30	5 (1%)	4(7%)	4 (3%)	1 (1%)	4 (1%)
40	5 (1%)	1(2%)	4 (6%)	4 (3%)	1 (1%)
50	5 (1%)	1(2%)	4 (7%)	4 (5%)	1 (3%)
60	1 (1%)	1(2%)	5 (2%)	4 (6%)	1 (5%)
70	1 (1%)	1(2%)	4 (1%)	4 (5%)	1 (5%)
80	1 (1%)	1(2%)	1 (1%)	5 (1%)	1 (5%)
90	1 (1%)	1(1%)	1 (1%)	5 (2%)	1 (5%)
100	1 (1%)	1(1%)	1 (1%)	1 (3%)	5 (1%)
110	1 (1%)	1(1%)	1 (1%)	1 (2%)	5 (2%)
120	1 (1%)	1(1%)	1 (1%)	1 (2%)	1 (3%)

Where, 1, 2, 3, 4, 5 and 6 represent the EXP-RK model, PR model, PT model, Tong model, RT model and SRK model, respectively. When the temperature exceeds 120 degrees, EXP-RK model is preferentially chosen to be the calculation model. The percent indicates the relative error.

When the temperature is 20 degrees the pressure is 8 MPa, PR model and Tong model can be chosen. Because the errors of the two models are the same, Tong model is chosen based on the combining rule. From Table 2, we can get that the combined model refers the four models which are EXP-RK, PT, RT, Tong models. There are two points whose maximum relative error is 7%. The relative error of the point whose ratio is 57.5% doesn’t exceeds 1%.

Table 2 shows that when the temperature exceeds 120 degrees, the calculation accuracy of the EXP-RK model is higher than the other models. But the accuracy occurs the decreasing trend as the pressure is increasing.

Table 2 only gives the calculation model of density at same points with special pressure and temperature. For any point (p,T) where $4 \le p \le 20$ and $10 \le T \le 200$, we determine the calculation model of density according to the nearest neighbor rule. The algorithm is showed as follow:

STEP 1 Comparing with 4, 8, 12, 16 and 20, determine the position of p. Denote the two pressure points which are nearest to p as p_1 and p_2. Comparing with 10, 20, …, 20, determine the position of T. Denote the two temperature points which are nearest to T as T_1 and T_2.

STEP 2 Calculate the distance between the points (p,T) and (p_i,T_j), where $i=1,2$ and $j=1,2$.

$$d_{i,j} = \sqrt{(p-p_i)^2 + (T-T_j)^2}\ ,\ (i=1,2;\ j=1,2) \tag{6}$$

Take the minimum distance value, and denote the corresponding pressure and temperature as (p^*,T^*). If the distances equal, then take randomly.

STEP 3 Use the corresponding calculation model of the point (p^*,T^*) in Table 2 to calculate the density value at the point (p,T).

Compare the result of the model in this paper and the one in the literature [15], and show the comparison result in Table 3. The comparison result shows that the method of this paper is more accuracy.

5. CONCLUSION

(l) Based on the basic data of the NIST, this paper gives the more reasonable calculation model of density through comparing the errors of 6 cubic calculation model of density.

(2) The method which displays the gray image of the relative error and binarization grayscale image can be used to analyze the calculation model of various physical parameter for other matter, and can expect to construct more reasonable calculation model.

Table 3. The comparison of calculation results

temp ˚C	pressure MPa	points N	Relative error	
			literature [15](%)	This paper(%)
10	4~20	5	1.52	0.53
20	4~20	5	2.04	1.36
30	4~20	5	3.68	2.46
40	4~20	5	2.64	1.89
50	4~20	5	3.60	2.85
60	4~20	5	3.26	2.23
70	4~20	5	2.78	2.26
80	4~20	5	2.33	1.56
90	4~20	5	1.90	1.63
100	4~20	5	1.54	0.79

110	4~20	5	1.26	0.92
120	4~20	5	1.06	1.06
total	4~20	60	2.30	1.63

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