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THE COMPARATIVE NUMERICAL ANALYSIS OF NATURE AND ARCHITECTURE: A NEW FRAMEWORK

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ABSTRACT

Maintaining or creating a visual relationship between the form of a building and its surrounding natural landscape is often cited as a crucial factor in producing designs that support psychological comfort or environmental sustainability. While multiple methods for the analysis of nature and architecture have developed over time, only a handful of past studies have ever attempted to quantitatively compare the geometric properties of nature to those of architecture. Fractal analysis provides one of the very few methods available to analyse and compare the geometry of diverse objects. The fractal dimension (D) of an object is a numerical value which reflects the volume and distribution of detail in an item. Of the many subjects analysed using this method, the forms of nature (such as coastlines, rivers and plant elements) have been successfully measured, as have built forms (such as houses, public buildings and cityscapes). However, despite the method's application to each subject area, few examples exist where fractal dimension data derived from nature are compared with equivalent architectural data. A primary reason cited for this situation is the disparity of methodological variables, in particular, representational approaches to the images used for comparison are presently disparate and uncategorised.

This paper responds to the existing lack of a comparable basis by analysing and categorising methodological examples from applications of fractal analysis to both natural and architectural cases. Specifically, the type of image delineation and the level of information contained in it are compared and ranked. Through this process, the paper provides a critical overview of the past application of fractal analysis to images, and thereby provides a starting framework for how the built and natural environments might be rigorously compared in the future.
Keywords: fractal dimension, landscape analysis, visual complexity.

1 INTRODUCTION

A visual connection, or geometric similarity, between the appearance of buildings and nature has been promoted as a benefit to psychological well-being [1] and as a factor in creating environmentally sustainable architecture [2, 3]. While many qualitative methods have been used to determine the visual difference between architecture and nature, there are a limited number of quantitative approaches to the issue. Fractal analysis offers a mathematical method for measuring and comparing visual complexity, and it has been used previously—in a limited way—to compare visual complexity (D) in buildings and landscapes [4]. However, such past attempts to compare nature and architecture using fractal analysis have not been entirely convincing. The disparity of the methodological variables employed in each approach has been cited as the primary reason that the fractal data derived from nature cannot be easily compared with equivalent architectural data. In particular, representational approaches to the analytical images are disparate, inconsistent and uncategorised.

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When comparing two very different subjects—natural objects and those designed and constructed—the disparity between the subjects and the essential purpose of the comparison comes into question. Fractal analysis is often used to make comparisons, for example, in biological sciences, to establish a difference in complexity between two forest types (natural data), or in the built environment to compare the design differences between the works of two architects (synthetic data). While each of these separate cases is well accepted (that is, the correlation between multiple natural forms and between different buildings) the comparison of data from two different sources, the natural and the synthetic, raises particular questions regarding the basis for any legitimate correlation. For example, how can we obtain data from static, constructed and designed forms, such as those that comprise the synthetic built environment, and compare this information with data sourced from the dynamic, natural and seemingly more random forms found in nature?

This present paper commences with a brief overview of fractal dimensions and fractal analysis. This is followed by a description of the application of fractal analysis to nature, to the built environment and then a review of the existing cases where the two are compared. The problems with these applications of the method are then examined. The second half of the paper seeks to remediate the existing lack of comparable information by categorising methodological examples from both natural and architectural fractal analysis methods. Specifically, the type of image delineation and the level of information contained in it are compared and ranked. Through this process, the paper provides a critical overview of the past application of fractal analysis to images of nature and architecture, and provides a starting framework for how the built and natural environments might be more rigorously compared in the future.

2 FRACTAL DIMENSIONS AND FRACTAL ANALYSIS

Fractal geometry began to inform a number of approaches to measuring and understanding non-linear and complex forms in the late 1970s. It was Benoit Mandelbrot's proposal—that natural systems frequently possess characteristic geometric complexity over multiple scales of observation—which initiated mathematical studies of fractal geometry [5]. Fractal geometry describes irregular or complex lines, planes and volumes that exist between whole number integer dimensions. This implies that instead of having a dimension, or D, of 1, 2, or 3, fractals might have a D of 1.51, 1.93 or 2.74 [5]. In *The Fractal Geometry of Nature*, Mandelbrot explains, develops and refines the applications of fractal geometry and provides an explanation of several methods used to calculate the dimensions of natural forms. These include calculations for measuring lengths and irregularity of images of rivers, lakes, trees and national boundaries as well as the fractal dimension of coastlines, the sky, clouds and galaxies. Mandelbrot also suggested that fractal analysis could be used to describe architecture, explaining that 'in the context of architecture: A Mies van der Rohe building is a scalebound throwback to Euclid, while a high period Beaux Arts building is rich in fractal aspects' [5: 23–24]. Just as Mandelbrot has used mathematical methods to calculate the fractal dimension of coastlines and compare them, so too could architecture be analysed for visual complexity. This idea was developed in detail by Carl Bovill [6] in his 1996 publication, *Fractal Geometry in Architecture and Design*.

Different fractal calculation methods exist, which include—but are not limited to—the area-perimeter method, the lacunarity method, multifractal analysis and box-counting. The different methods can produce varied outcomes, so for comparative purposes, the same method must be used when correlating the results. In order to eliminate this variable in the present study, we will investigate examples resulting from just one method—the box-counting method of fractal analysis—which has been developed to analyse nature and architecture (separately) in the past. However, even within this one analytical method, further discrepancies can occur as a consequence of a range of methodological and data quality variables [7]. Recently, the optimal settings to reduce the impact for the complete set of these factors has been determined for the first time for architecture [8]. Once these optimal settings have been applied, there is one remaining key variable which relates to the two-dimensional representation of the matter which is subject to analysis [9]. This is because—in very basic terms—the box-counting method starts with a two-dimensional image representing the subject matter, for example, a simple line drawing or a photograph. A series of grids are overlaid on the image and the number of boxes containing detail at changing scales are recorded. A comparison is made by plotting a log-log diagram for each grid size. By repeating this process over multiple grids of different scales, an estimate of the fractal dimension is produced. The method has been explained in past publications [6, 10, 11].

2.1 Applied fractal analysis

Mandelbrot's work has been adopted by many others as a method for providing a quantitative understanding of the natural world. Using fractal geometry to measure vegetation growth or decline is now a common method in botanical studies [12–14]. Others have added to the existing data of geographical [5, 15] and geological studies [16]. Makhzoumi and Pungetti [2] propose a fractal analysis as a method to interpret and understand the ecological landscape (1999). In a fractal study of Australian landscapes, Perry *et al*. [17: 15] reached the conclusion that 'different landscape types can be calculated by their mean fractal dimension'. Fractal geometry has also been used to analyse preferences for the visual complexity of natural landscapes [18–20].

In the built environment, fractal analysis 'can potentially be the tool that allows us to describe accurately [...] "organic" urban form' [21: 13]. Early research to specifically use the box-counting method in urban analysis can be traced to Batty and Longley [22], followed by others using the same method to measure differences in urban forms [19, 23–25]. The box-counting method for the analysis of buildings was initially applied by Bovill [6]. It has since been used to analyse archaeological [26, 27] and vernacular buildings [11, 28–30]. Researchers have also used the box-counting variation of fractal analysis to measure the visual properties of recent architecture [6, 10, 11, 31].

2.2 Comparing natural and built forms

In 1994, Bechhoefer and Bovill [28] used the box-counting method to compare indigenous buildings and natural land forms in Amasya, Turkey. They concluded that each of these features had similar fractal dimensions and thus, the topography must have either influenced the design of the buildings, or alternatively the features were shaped by larger environmental conditions. Robertson [21] noted the use of the box-counting method to calculate the fractal dimensions of urban landscapes in order to compare them with other urban areas or to integrate them into a wider region. The following year Bovill [6] suggested that one way of determining a successful regional building could be to assess whether its fractal dimensions were similar to those of the surrounding landscape or vegetation, proposing that the highly irregular coastline of Sea Ranch (California) is closely echoed in Moore, Lyndon, Turnbull and Whitaker's *Sea Ranch Condominium*. In this way, Bovill suggests that there is potentially mathematical evidence that this famous critical regionalist building is responding to its natural setting in much the same way that the landscape has been shaped by local environmental and climatic forces. Despite this suggestion, Bovill does not provide the full mathematical results of the proposed comparison [6].

Accepting Bovill's arguments, Bechhoefer and Appleby proposed that because 'the fractal dimension of vernacular housing is very similar to that found in nature' [32: 3] then perhaps new buildings in historic settings should be designed to match similar levels of visual complexity and thus provide a better contextual fit. In a more rigorous study, Stamps [19] produced computer-generated images of mountains and cityscapes with deliberately matching fractal dimensions, and tested peoples' preferences for which should match. He concluded that 'urban design decisions regarding skylines should not assume that matching [fractal dimensions] of skylines and landscapes is a good idea' [19: 170]. Taylor—in his study of preferences for fractal dimension ranges—was relieved by Stamps' findings, as he was concerned about the 'highly unfeasible prospect of having to match the […] designs of buildings to prevalent weather conditions' [21: 250]. Despite Stamps' reservations, interest in the relationship between buildings and landscapes continues in this field. For example, Lorenz reiterated Bovill's conclusions agreeing that 'the measured fractal dimensions of the environment, elevation and detail will be similar' [11: 47] and Bourchtein *et al*. used a rigorous mathematical methodology based on Bovill's work to compare streetscapes and landscapes in Brazil, concluding that 'the relationship between the visual complexity of built and natural landscapes is confirmed for the considered Brazilian settings' [33:11]. Other work connecting architecture and nature by way of fractal dimensions is limited in its presentation and use of quantitative data. Burkle-Elizondo and Valdéz-Cepeda, in their studies on the fractal dimensions of Mesoamerican pyramids, suggest that 'it is possible to identify the pyramids with particular mountains in the landscape' [27]. Yet, although they provide calculations for the pyramids, they do not undertake calculations of the surrounding mountains to provide any evidence for their claims.

3 IMAGE REQUIREMENTS FOR COMPARED FRACTAL DIMENSIONS

In 2009, the present authors [7] retested Bovill's results on the comparison of architecture and nature in Amasya to determine their validity, analysing the images using the same fractal analysis method, however using a more accurate computational version. These new results did not convincingly support Bovill's concept of a local ecology being reflected in the surrounding architecture, as the gap in the fractal dimensions of the images analysed was too large to provide compelling evidence. The Amasya data were further tested by others in 2014 [33] and both studies concluded that Bovill's results were limited by the images chosen for analysis. In particular, the selection of images from the original work lacked a clear rationale and the type of logical guidelines for the selection of data required to make such a disparate comparison. This problem can be traced to the often-overlooked fact that computational methods, like fractal analysis, do not measure 'nature' or 'architecture' *per se*, rather they extract measures from representations of natural objects, landscapes or buildings. Rather than rejecting the whole concept outright, our [9] conclusion proposed that the potential validity of a computational comparison of the complexity of nature and architecture could be further developed if a more rigorous methodology was applied during the data selected for analysis.

If two (or more) images are to be compared in a consistent and useful way, there are several practical elements that need to be considered. The images need to be similar in the *subject* they display (‘topic’) and, knowing the *attributes* of the subject (‘natural’ or ‘synthetic’), will clarify the practical ability to compare subjects. In particular, the images need to have a similar *presentation* (representation method) not only in their physical representation (a photograph or a line drawing for example) but also in their data type (a site plan or a perspective drawing for example), Fig. 1.

This consideration is important because the presentation of the image provides us with at least, a twofold *data gradient* of information available from the source. For example, within a fractal dimension study of one subject, say a study of house facades on a street, the images compared need to relate to each other. Thus the housing analysis (subject) could be a collection of photographs (‘presentation’) of house frontages (‘types’). In this case, the fractal analysis would be based on synthetic images (‘attribute’). The data from the study would be compatible as the subject, attributes, presentation, and typology and hence data gradient, would all be similar. Another fractal study might be based on a tree species. The D values

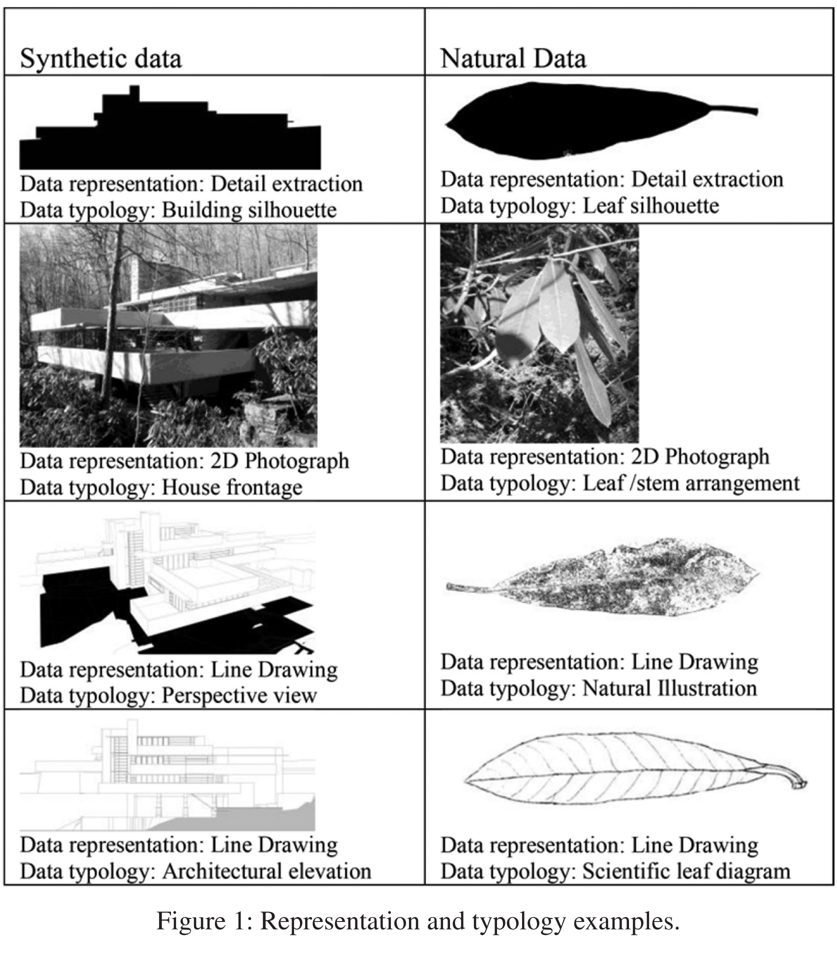

Figure 1: Representation and typology examples.

could be found from a set of line drawings ('presentation') of a natural ('attribute') leaf ('subject') using tracings ('typology'). Again, the data will correlate for a meaningful comparison. However, if we wished to compare the trees on the street with the houses, we need to approach the relationship of the subjects (houses/trees), their attributes (synthetic/natural), the method of representation (photograph/line drawing) and their typologies (house frontages/leaf tracings). Given the disparity of these image variables, the results of two studies would be difficult to compare unless some of the variables were made analogous. While we cannot change the natural/synthetic attributes in a proposed study, we could change the topic to the study of a particular streetscape. The analysis method would then need to select an appropriate representation and typology for all of the images to be analysed. While there are multiple presentation options commonly used for different applications of fractal analysis, no guidelines exist to assist with matching presentation methods. Thus, the following sections of this paper explore the existing representation options and thereafter propose an approach for matching data gradients to increase the likelihood of a more realistic quantitative comparison.

3.1 A data selection methodology

3.1.1 Review of natural data presentation methods

Currently, the box-counting method of fractal analysis for use with both natural and synthetic objects utilises base data in several standard forms. Black and white binary photographs have been used to calculate the fractal dimension of top down, close-up views of plants [12] and pebbles [16], and D has been calculated from photographs framing abstract views for a range of plants, landforms and celestial bodies [13]. However, photographs generally produce a high fractal dimension due to the large amount of data in the image, a factor that is complicated by shadows, reflections, textures and depth of field. In contrast, a silhouette extracted from a photograph using edge-detection reduces this volume of 'noise' making the image processing more consistent and the process more repeatable [25], partially due to silhouette extraction by a software program without involving personal judgement [20]. Keller, Crownover and Chen [18] initiated much of the methodology in silhouette detection for fractal analysis, and the results of the study satisfied them that fractal dimension ranges of silhouettes can be used to distinguish between different elements in nature. Edge-detected photographic silhouettes are uses as raw data for the box-counting method and have been used to analyse vista outlooks of natural landscapes [20, 34], top down views of outlines of leaf collections [14] and aerial views of natural landscapes [34]. Osmond presents a novel approach by using a fisheye lens to photograph overhead landscapes, using edge detection to produce a hemispheric skyline for analysis [35]. Other linear types of images used for box-counting analysis include line drawings of landscape plans [36] and vegetation cover [37]. Nautical charts [38] and geographic maps [6, 15] are also examples of sources for line drawings of the natural forms analysed by box-counting. Other nature-based image data used for quantitative analysis includes (but is not limited to) line drawings of landscape views, botanical illustrations and contour plans.

3.1.2 Review of built environment data presentation methods

Akin to the fractal analysis of nature, various representations of the built environment have been used for the box-counting method. In everyday architectural use, the primary raw data used for representation are the orthographic drawing, which lends itself to fractal analysis, because the simple line drawings found in plans, elevations and sections are usually already binary (black/white) and are ideal sources for edge-detection processing. Elevations and plans are relevant subjects for analysis, as an elevation can be considered to provide a measure of the geometric complexity of the building as viewed from the exterior and the plan provides a measure of the complexity of the design as it is inhabited. In the past, box-counting has been applied to line drawings of plans [10, 31] simple line elevations [6] and detailed linear elevations [29]. Single line digitised tracings of site plans showing the outlines of buildings have been taken from the maps of both ancient [24] and contemporary cases [23]. As well as using digital drawings, box-counting analysis has been undertaken using photographs that have been converted to binary images of the built environment, including those used by Oleschko *et al.* [26] to calculate the D of aerial, top-down views of ancient buildings. Like the studies of D values of natural landscapes, edge-detected outlines from photographs are also used to study buildings, such as elevations extracted from photographs of vernacular housing [30] and building skylines [25].

From these examples, it can be seen that both natural and synthetic sources of information have shared raw data formats—photographs, edge-detected photographs and line drawings. Other sources, for both examples, could include 3D laser scanning and stereo photography. In a further alternative, Stamps [19] produced computer-generated images of imagined mountains and cityscapes to test peoples' preferences for which should match [19: 170]. Stamps' use of computer-generated images is indicative of the need for consistent parameters for constructing such a comparison. Computer generated images can be set at a similar scale, with a similar level of detail and are produced using an identical method. The data produced are consistent and straightforward to analyse. However, images produced entirely by computer generation do not solve the problem of extracting data for the comparison of real locations.

3.1.3 Comparing representation types

The new methodology proposed in this paper seeks to determine which of the above examples are compatible sources of raw data. It begins this process by categorising the different raw data sources in terms of the way in which they represent reality. The examples of previously analysed extracted synthetic and natural data provide a breakdown of not only *how*—via a particular presentation method (e.g. line drawing or photograph)—the subject is presented, but also *what*—via the decision on typology (e.g. a plan or a skyline)—is expressed in the subject. The data can first be classified by ranking its 'quality', relative to the realism of the image or data, a criterion which relates to our worldview. A ranking of 'high' quality is the closest level of visual reality; for example a high resolution, 3D colour laser scan might fulfil this criterion. A 'Medium' level of quality relates to images which are broadly connected to the visual properties of their originals, such as a silhouette extracted from a photograph. 'Low' quality images do not strongly correspond to forms as we perceive them. For example, we do not view a hillside as being covered in lines at regular intervals, despite what a contour plan suggests. The data can also be graded by its quantitative potential, or the degree to which accuracy of data in a representation reflects the original object (see Table 1). A 'high' quality of information might be found in a 3D laser scan, but also in a site drawing such as a contour plan, both of which are metrically accurate, but the former has a high level of realism, and the latter a low level. However, while a site plan has a 'low' level of realism of representation (as we rarely perceive our surroundings as abstracted plan views) they can provide a high quality of information when analysed, due to the amount of extractable data, and the relation of the information to the actual site.

Table 1: Correlating data.

Data Representation		Data Gradient		
Natural	Synthetic	Realism of representation	Quantitative potential	Degree of representation
3D laser scan with colour values (3D Imaging)	3D laser scan with colour values (3D Imaging)	High	High	"aactual"
Stereo photography (3D Imaging)	Stereo photography (3D Imaging)	High	High	
Photograph (2D)	Photograph (2D)	Medium/High	High	
Detail extraction (eg. Silhouette lines)	Detail extraction (eg. Silhouette lines)	Medium	Medium	
Drawings (eg. natural illustration)	Drawings (eg. perspective views)	Medium	Medium	
Drawings (scientific illustrations)	Drawings (plans, elevations)	Low	High	
Drawings (contour plans, site lines, coastlines)	Drawings (site sections, urban layouts, landscape plans)	Low	High	"abstract"

4 DISCUSSION

3D laser scans and stereo photography offer both the highest quality of representation and information, however, they are difficult to obtain without specialist hardware and specific software. 2D photographs are the most available data type identified in this study, which capture an "actual" building or natural form, and offer a medium to high quality of representation and information, but even these are limited to a single viewpoint. Line drawings extracted from photographs can be considered to represent a high level of "abstract" data, a conclusion which is supported by the ongoing work of Hagerhall *et al.* [20] who reason that a silhouette can be extracted without personal judgement by a software program and that past research by themselves and others has included silhouettes so the collection of data can be increased by continuing work in this area. Generally the categorisation of the data types by degree of representation—ranked from "abstract" to "actual"—reflects the ranking of the quality of image representation and information, except in the case of the more "abstract" data. In this latter case, the quality of realistic representation is likely to be low, while the quality of information can be high.

If the hypothetical study of trees and houses in a streetscape, posed previously in this paper, is examined in this new context, the guideline would provide a method to review the original approach for constructing a comparison between the data (Table 2)

While some of the details correlate above, according to the proposed methodology, this does not provide a "reasonable" comparison between the synthetic and natural data types. While the quantitative potential is analogous, level of realism does not match and the degree of representation of the synthetic data tends towards abstract, while for the natural, this is the inverse. Referring back to the guidelines presented in Table 1, changing the data for the houses to a set of line drawing elevations, the two subjects would match their data levels on all aspects as shown in Table 3.

Table 2: Non-correlating data.

House/tree study	Subject #1	Subject #2
	Houses	Trees
Attribute	Synthetic	Natural
Data Representation	Photograph (2D)	Line Drawing
Data Typology	House frontages	Scientific illustrations
Realism	Medium/high	Low
Quantitative potential	High	High
Degree of representation	↑ actual	↓ abstract

Table 3: Correlating data.

Streetscape study	Subject #1	Subject #2
	Houses	Trees
Attribute	Synthetic	Natural
Data Representation	Line Drawing	Line Drawing
Data Typology	Delineated elevations	Scientific illustration
Realism	Low	Low
Quantitative potential	High	High
Degree of representation	↓ abstract	↓ abstract

5 CONCLUSION

There are several types of data which can be used to analyse both buildings and nature. By applying the new methodological approach outlined in this paper to the various data types available, data may be categorised into similar types and a suggested framework is provided by which natural and synthetic images may be more effectively compared. This methodology could now be tested by using it in the early stages of design analysis, to compare synthetic and natural data. This testing of this proposed methodology will be the subject of future research, with equivalent synthetic and natural data analysed and compared. Additional parameters, including the effect of the scale, quality and detail present in the data will require further consideration.

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