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SORET EFFECTS IN A MHD FREE CONVECTIVE FLOW THROUGH A POROUS MEDIUM BOUNDED BY AN INFINITE VERTICAL POROUS PLATE WITH CONSTANT HEAT FLUX.

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ABSTRACT

An attempt has been made to investigate the Soret effects in a MHD free Convective Flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux and a magnetic field of uniform strength is applied perpendicular to the plate. The governing equations are solved by regular perturbation technique. The expressions for the velocity distribution, temperature field, skin friction, and species concentration are obtained and the effects of the different parameters namely Soret number Sr, Hartmann number M, Grashof number for heat transfer Gr, Grashof number for mass transfer Gm, Prandtl number Pr on these fields are demonstrated graphically and the results are discussed. Increasing the Soret number Sr increases the velocity profile, temperature and concentration.

Key Words: MHD, Electrically Conducting, Free Convection, Soret Effect, Permeability

1. Introduction:

In the last few years, it has been observed that a number of scholars have given much emphasis on free convection flow and heat transfer problems in the presence of magnetic field through a porous medium because of their possible applications in various branches of science and technology such as fiber and granular insulations , geothermal system etc. MHD convection flow problems are very significant in the field of steller and planetary magnetosphere, aeronautics, chemical engineering and electronics. Many authors present analytical solutions of such problems Gebhart and Pera [1], Soundalgekar [2], Acharya et. Al.[3] and Raptis and Kafousias [4] are some of them. The study through porous medium has got importance because of its occurrence in movement of water and oil inside the earth, flow of river through porous banks, chemical engineering for filtration and purification process, petroleum technology to study the movement of natural gas and in the fields of agriculture engineering to study the underground water resources. Study of problems of flow through porous medium is heavily based on Darcy's experimental law [5], Wooding [6] and Brinkman [7] have modified the Darcy's law, which are used by many authors on the study of convective flow in porous media. However in the above studies thermal diffusion effect was neglected. This assumption is justified when the concentration level is very low. The flux of mass caused due to temperature gradient is known as the Soret effect or the thermal diffusion effect. The experimental investigation of the thermal diffusion effect on mass transfer related problems was first performed by Charles Soret in 1879. There after this thermal diffusion is termed as the Soret effect in honour of Charles Soret. For isotope separation and in mixtures between gases with very light molecular weight (H_2, He) and the medium molecular weight (N_2, air), the Soret effect is utilized. Keeping in view of the importance of the thermo effect, Sattar and Alam [8] investigated the effect of free convection and mass transfer flow past an accelerated vertical porous plate taking into account the Soret effect.

Recently the effect of thermal diffusion as well as free convection and mass transfer flow through porous media studied by Ahmed and Kalita [9], The effect of Soret on Dufour in presence of heat and mass transfer on MHD free convection from a vertical plate in a porous medium studied by Ferdows and Chen [10] and natural convection in two porous media separated by a solid wall studied by Jafari et al. [11].

The aim of the present work is to investigate the Soret effects in MHD free convective flow through a porous medium bounded by an infinite vertical porous plate with constant heat flux. The work is an extension of the work done by Raptis and Kafousias [4].

2. Mathematical Analysis:

Let us consider the flow of a fluid of density ρ, viscosity μ and electrical conductivity σ through a porous medium of permeability K occupying a semi-infinite region of the space bounded by a porous infinite vertical plate. The porous medium is in fact an inhomogeneous medium, but for the sake of this analysis it is possible to describe the flow in terms of a homogeneous fluid with average dynamic properties which have a smoothing effect on the locally inhomogeneous continuum. Thus we can study the flow of a hypothetical homogeneous fluid under the action of the properly averaged external forces and so a complicated problem of flow through a porous medium reduces to the flow problem of a homogeneous fluid with some resistance. The velocity components of the fluid are $\bar{u}$ and $\bar{v}$ in the $\bar{x}$ and $\bar{y}$ directions

respectively, taken parallel and perpendicular to the porous plate. We assume that the fluid has constant properties except that the influence of the density variations with temperature and concentration are considered only in the body force term. Also the influence of the density variations in other terms of the momentum and energy equations and the variation of the expansion coefficients with temperature is negligible.
Basic equations are

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = g\beta(\bar{T}-\bar{T}_\infty) + g\bar{\beta}(\bar{C}-\bar{C}_\infty) + \upsilon\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\mu \bar{u}}{\rho \bar{k}} - \frac{\sigma B_0^2}{\rho}\bar{u} \tag{2}$$

$$\bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k}{\rho C_p}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \frac{\upsilon}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \tag{3}$$

$$\bar{v}\frac{\partial \bar{C}}{\partial \bar{y}} = D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + \frac{D_M K_T}{T_M}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \tag{4}$$

Boundary conditions are

$$\begin{aligned} &\bar{y}=0: \bar{u}=0, \frac{\partial \bar{T}}{\partial \bar{y}} = -\frac{q'}{\bar{K}}, \bar{C}=\bar{C}_w \\ &\bar{y}\to\infty: \bar{u}\to 0, \bar{T}\to \bar{T}_\infty, \bar{C}\to\bar{C}_\infty \end{aligned} \tag{5}$$

where q' is the constant heat flux at the plate and $\bar{C}$ the species concentration at the plate.
From (1), we have

$$\bar{v} = -v_0 (v_0 > 0) \tag{6}$$

where v_0 is the constant suction velocity at the plate and the negative sign indicating that the suction velocity is directed towards the plate. We introduce now the following non dimensional quantities:

$$y = \frac{\bar{y}v_0}{\upsilon}, u = \frac{\bar{u}}{v_0}, T = \frac{\bar{T}-\bar{T}_\infty}{\frac{q'\upsilon}{kv_0}}, C = \frac{\bar{C}-\bar{C}_\infty}{\bar{C}_w-\bar{C}_\infty},$$

$$Gr = \frac{g\beta\upsilon^2 q'}{kv_0^4}, Ec = \frac{kv_0^3}{q'\upsilon C_p}, Gm = \frac{\upsilon g\bar{\beta}(\bar{C}_w-\bar{C}_\infty)}{v_0^3},$$

$$K = \frac{v_0^2\bar{K}}{\upsilon^2}, Sc = \frac{\upsilon}{D}, Sr = \frac{D_M K_T q'}{kT_M v_0(\bar{C}_w-\bar{C}_\infty)}, M = \frac{\sigma B_0^2\upsilon}{\rho v_0^2},$$

$$\Pr = \frac{\rho\upsilon C_p}{k}.$$

In view of equation (6) and the above non dimensional quantities, equations (2) –(4) reduces to the dimensionless form as

$$\frac{d^2u}{dy^2} + \frac{du}{dy} = -GrT - GmC + \frac{1}{k}u + Mu \tag{7}$$

$$\frac{d^2T}{dy^2} + \Pr\frac{dT}{dy} = -\Pr Ec\left(\frac{du}{dy}\right)^2 \tag{8}$$

$$\frac{d^2C}{dy^2} + Sc\frac{dC}{dy} + SrSc\frac{d^2T}{dy^2} = 0 \tag{9}$$

The boundary conditions [5] now become

$$\begin{aligned} &y=0: u=0, T'=-1, C=1 \\ &y\to\infty: u\to 0, T\to 0, C\to 0 \end{aligned} \tag{10}$$

where dashes represents differentiation with respect to y.
The system of equations [7], [8] and [9] are nonlinear and in order to obtained a solution we expand u, T and C in powers of the Eckert number Ec assuming that is very small. This is justified in low speed incompressible flows. Hence

$$\left.\begin{aligned} u(y) &= u_0(y) + Ecu_1(y) + o(Ec^2) \\ T(y) &= T_0(y) + EcT_1(y) + o(Ec^2) \\ C(y) &= C_0(y) + EcC_1(y) + o(Ec^2) \end{aligned}\right\} \tag{11}$$

Substituting [11] in [7], [8] and [9] and equating co-efficient of E^0, E^1 and neglecting higher order terms we have the following system of equations:

$$T_0'' + \Pr T_0' = 0 \tag{12}$$

$$C_0'' + ScC_0' + SrScT_0'' = 0 \tag{13}$$

$$u_0'' + u_0' - \left(\frac{1}{k} + M\right)u_0 = -GrT_0 - GmC_0 \tag{14}$$

$$T_1'' + \Pr T_1' = -\Pr u_0'^2 \tag{15}$$

$$C_1'' + ScC_1' + SrScT_1'' = 0 \tag{16}$$

$$u_1'' + u_1' - \left(\frac{1}{k} + M\right)u_1 = -GrT_1 - GmC_1 \tag{17}$$

The boundary conditions [10] become:

$$\left.\begin{aligned} &y=0: u_0=0, u_1=0, T_0'=-1, T_1'=0, C_0=1, C_1=0 \\ &y\to\infty: u_0\to 0, u_1\to 0, T_0\to 0, T_1\to 0, C_0\to 0, C_1\to 0 \end{aligned}\right\} \tag{18}$$

Solving [12]-[17] under the boundary conditions [18] and putting them in [11], we have

$$\begin{aligned} u(y) = {} & A_{37}e^{-\lambda_1 y} - A_{38}e^{-\Pr y} - A_{39}e^{-Scy} + A_{40}e^{-2\lambda_1 y} + A_{41}e^{-2\Pr y} \\ & + A_{42}e^{-2Scy} - A_{43}e^{-A_1 y} + A_{44}e^{-A_{10}y} - A_{45}e^{-A_{12}y} \end{aligned} \tag{19}$$

$$\begin{aligned} T(y) = {} & \left(\frac{1}{\Pr} + EcA_{19}\right)e^{-\Pr y} - EcA_{13}e^{-2\lambda_1 y} - EcA_{14}e^{-2\Pr y} - EcA_{15}e^{-2Scy} \\ & + EcA_{16}e^{-A_1 y} - EcA_{17}e^{-A_{10}y} + EcA_{18}e^{-A_{12}y} \end{aligned} \tag{20}$$

$$\begin{aligned} C(y) = {} & (1 - A_1 + EcA_{27})e^{-Scy} + (A_1 + EcA_{20})e^{-\Pr y} + EcA_{21}e^{-2\lambda_1 y} \\ & + EcA_{22}e^{-2\Pr y} + EcA_{23}e^{-2Scy} + EcA_{24}e^{-A_1 y} + EcA_{25}e^{-A_{10}y} \\ & + EcA_{26}e^{-A_{12}y} \end{aligned} \tag{21}$$

The non-dimensional skin friction at the plate y=0 in the direction of free stream is given by

$$\tau_0 = -\frac{\mu}{\rho v_0^2}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0} = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = \tau_0^{\,0} + Ec\tau_0^{\,1}$$

where $\tau_0^{\,0} = -u_0'(0)$

and $\tau_0^{\,1} = -u_1'(0)$

The expression for $\tau_0^{\,0}, \tau_0^{\,1}$ are obtained but not presented here for the sake of brevity.
The non-dimensional temperature at the plate y=0 is given by

$$T(0) = T_0(0) + EcT_1(0)$$

The expression for $T_0(0)$, $T_1(0)$ are obtained but not presented here for the sake of brevity.
Therefore,

$$T(0) = \frac{1}{\Pr} + Ec(A_{19} - A_{13} - A_{14} - A_{15} + A_{16} - A_{17} + A_{18})$$

3. RESULTS AND DISCUSSION:

In order to get physical insight into the problem we have carried out numerical calculations, for the dimensionless velocity, temperature, concentration and skin friction at the plate and their behavior have been discussed for variations in the governing parameters.

The influence of the thermal Grashof number Gr on the velocity is presented in Figure1. The thermal Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Here, the positive values of Gr correspond to cooling of the plate.

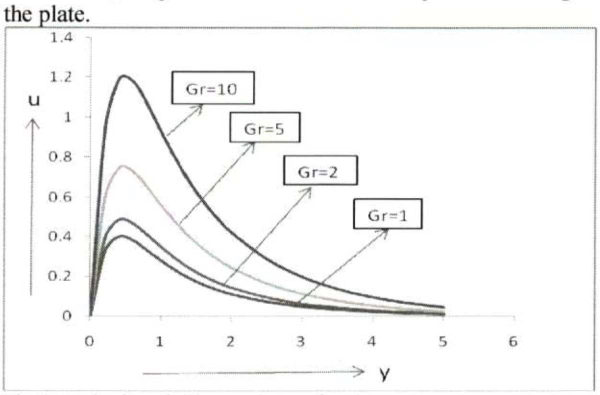

Fig.1: Velocity Field u against y for Gm=5, Pr=0.71, Sc=2, Sr=2, Ec=0.01, M=10, K=1.

Figure2, presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number Gm, while all other parameters are kept at some fixed values. The solutal Grashof number Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. It is seen that the fluid velocity increases due to increase in the species buoyancy force.

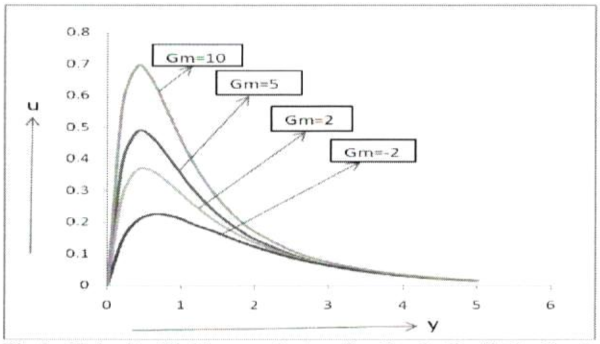

Fig.2: Velocity Field u against y for Gr=2, Pr=0.71, Sc=2, Sr=2, Ec=0.01, M=10, K=1.

Figure3, depicts the effect of permeability parameter K over the velocity profiles. It is noticed that an increase in permeability parameter K results in an increase in the velocity.

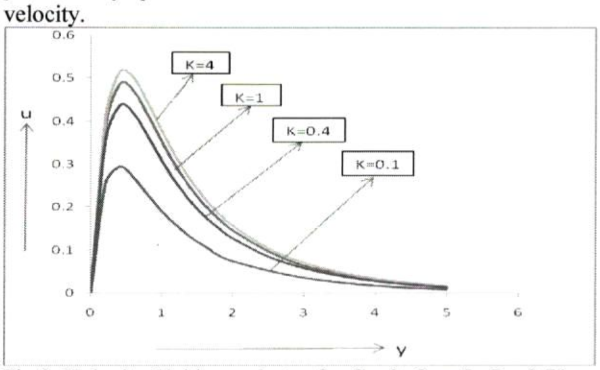

Fig.3: Velocity Field u against y for Gr=2, Gm=5, Pr=0.71, Sc=2, Sr=2, Ec=0.01, M=10.

For various values of Schmidt number Sc, the velocity profiles are plotted in Figure4. It is to be noted that Schmidt number Sc is the ratio of the kinematic viscosity with molecular diffusivity. This figure shows that an increase in Schmidt number Sc decreases the fluid velocity indicate the fact that the fluid motion is accelerated under the effect of mass diffusion.

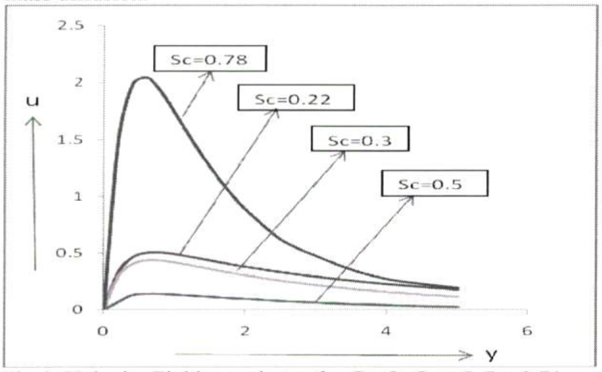

Fig.4: Velocity Field u against y for Gr=2, Gm=5, Pr=0.71, Sr=2, Ec=0.01, M=10, K=1.

Figure5, displays the effect of Soret number Sr over the velocity profiles. The Soret number Sr defines the effect of temperature gradients reducing significant mass diffusion effect. It is noticed that an increase in the Soret number Sr results in an increase in the velocity.

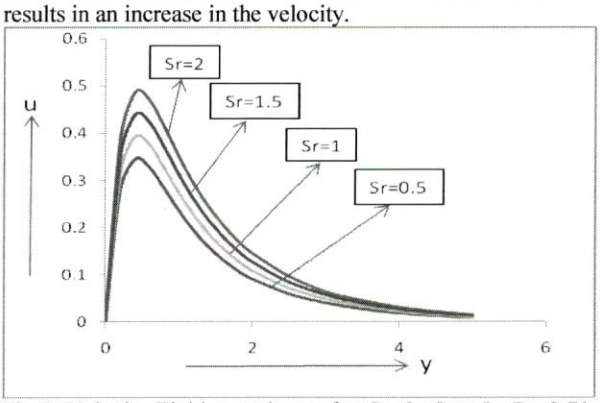

Fig.5: Velocity Field u against y for Gr=2, Gm=5, Pr=0.71, Sc=2, Ec=0.01, M=10, K=1.

Increasing the magnetic parameter M, decreases the velocity profiles as seen in Figure6. This result qualitatively agrees with the expectations as the magnetic field exerts a retarding force on the convective flow which serves to decelerate the flow.

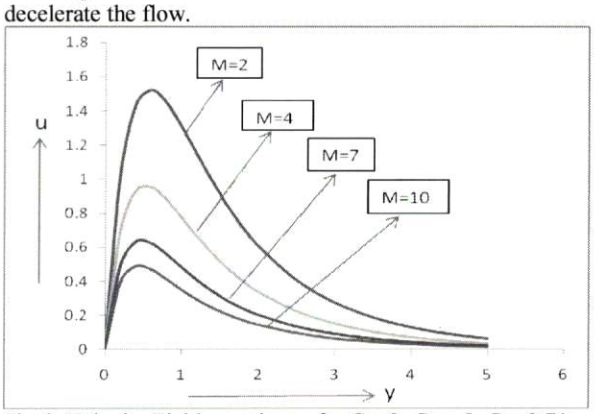

Fig.6: Velocity Field u against y for Gr=2, Gm=5, Pr=0.71, Sc=2, Sr=2, Ec=0.01, K=1.

Figure7 shows the influence of Prandtl number Pr on the velocity field. Prandtl number Pr is the ratio of viscous force to the thermal force. This figure shows that an increase in Prandtl number Pr decreases the fluid velocity indicates the fact that the fluid motion is accelerated under the effect of thermal diffusivity.

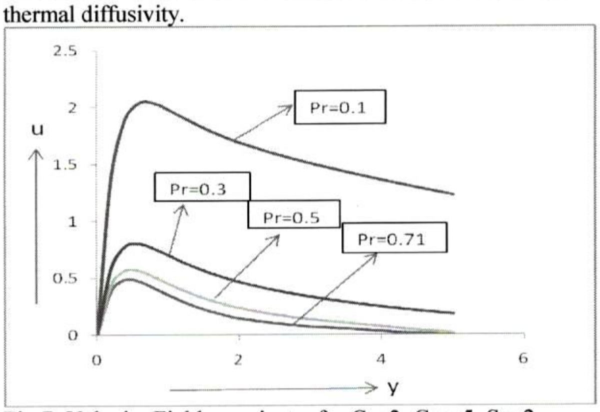

Fig.7: Velocity Field u against y for Gr=2, Gm=5, Sc=2, Sr=2, Ec=0.01, K=1, M=10.

Figure8 to 10 display the variations of temperature distribution against y under the influence of the parameters Sc, Sr, and Pr. It is observed from Figures 8 and 9 that temperature distribution increases with the increasing values of Schmidt Number Sc and Soret number Sr. Figure 10 depict that temperature distribution decreases with the increasing values of Pr.

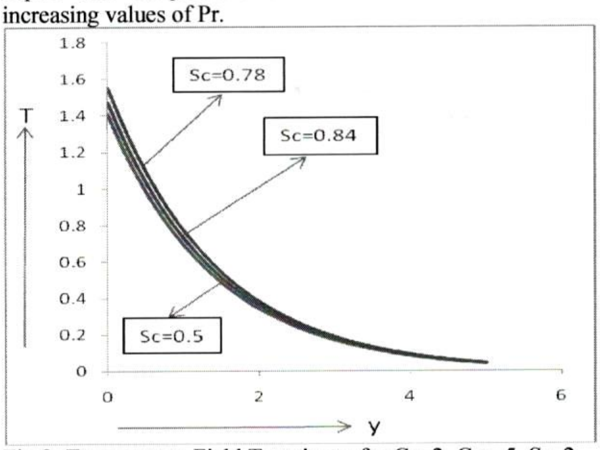

Fig.8: Temperature Field T against y for Gr=2, Gm=5, Sr=2, Ec=0.01, K=1, M=10, Pr=0.71.

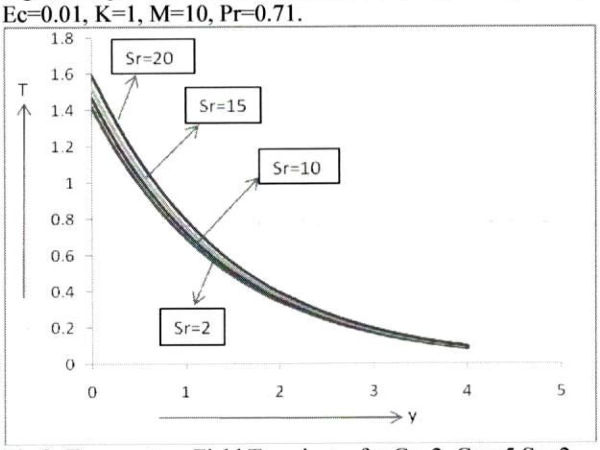

Fig.9: Temperature Field T against y for Gr=2, Gm=5,Sc=2, Ec=0.01, K=1, M=10, Pr=0.71.

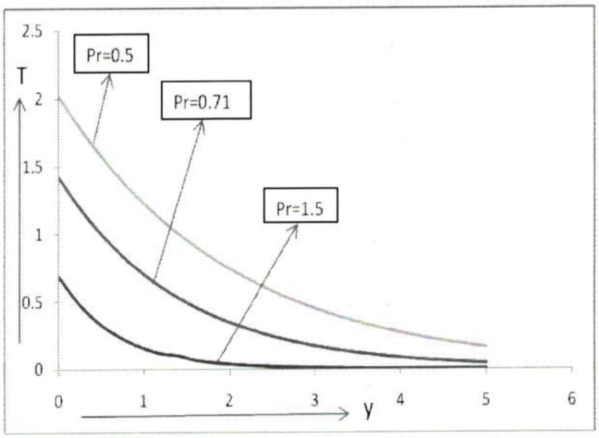

Fig.10: Temperature Field T against y for Gr=2, Gm=5,Sr=2, Sc=2, Ec=0.01, K=1, M=10.

The influence of Sr, M and Pr on the concentration profiles are plotted in figures 11 to 13. From Figure 11 it is noticed that an increase in the Soret number Sr results in an increase in the concentration in the boundary layer.

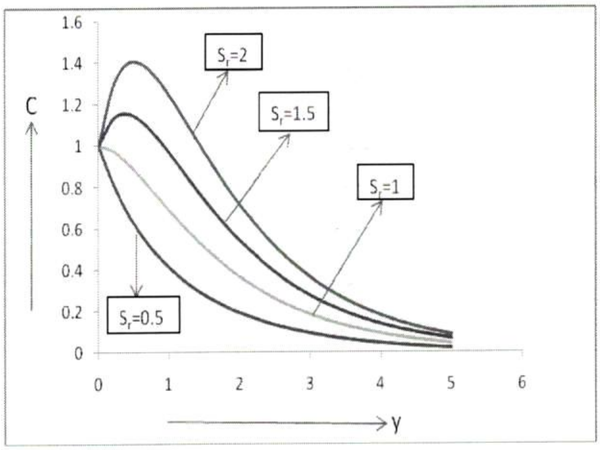

Fig.11: Species Concentration C against y for Gr=2, Gm=5, Sc=2, Ec=0.01, Pr=0.71, K=1, M=10.

Figures 12 and 13 shows that concentration profiles decreases with the increasing value of magnetic parameter M and Prandtl number Pr

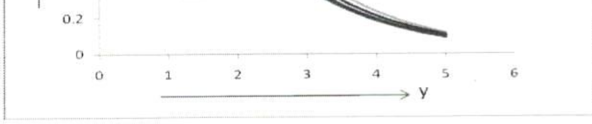

Fig.12: Species Concentration C against y for Gr=2, Gm=5, Sc=2, Sr=2, Ec=0.01, Pr=0.71, K=1.

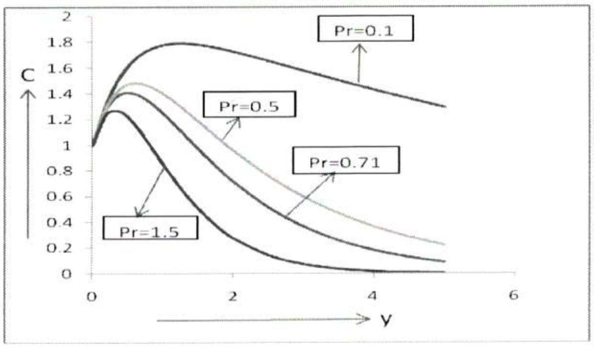

Fig.13: Species Concentration C against y for Gr=2, Gm=5, Sc=2, Sr=2, Ec=0.01, M=10, K=1.

The variation of the skin friction τ at the plate against Soret number Sr under the influence of K and M is shown in figures 14 and 15. It is observed from figure 14 that |τ| increases with the increasing values of K. Figure 15 illustrates that |τ| decreases with the influence of M. Hartmann number M decreases the Skin friction means viscous drag at the plate is suppressed owing to increase in the strength of the applied magnetic field.

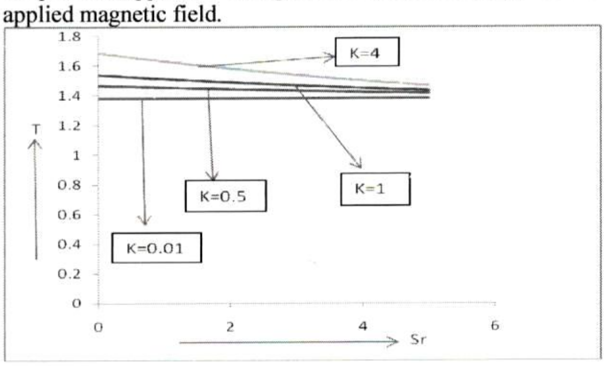

Fig.14: Skin Friction τ against Sr for Gr=2, Gm=5, Ec=0.01, Sc=2, Pr=0.71, M=10.

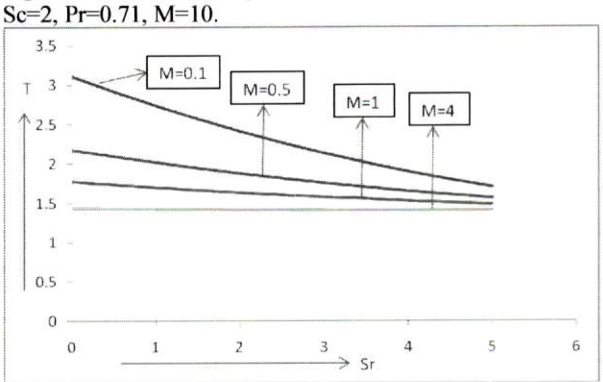

Fig.15: Skin Friction τ against Sr for Gr=2, Gm=5, Ec=0.01, Sc=2, K=1, Pr=0.71.

Figures 16 to 20 demonstrate the variation of Gm, Gr, K, Sc, M, Pr against Sr on temperature at the plate. For Gm > 0, temperature at the plate increases with increasing values of Gm whereas if Gm <0 the opposite behavior is observed in the figure 16. Figure 17 illustrates that for cooled plate Gr accelerates the temperature and for heated plate Gr decelerates the temperature at the plate. Figures 18 and 19 depict that temperature at the plate rises under the influence of K and Sc. The magnetic parameter M decelerate the temperature at the plate as observed from figure 20.

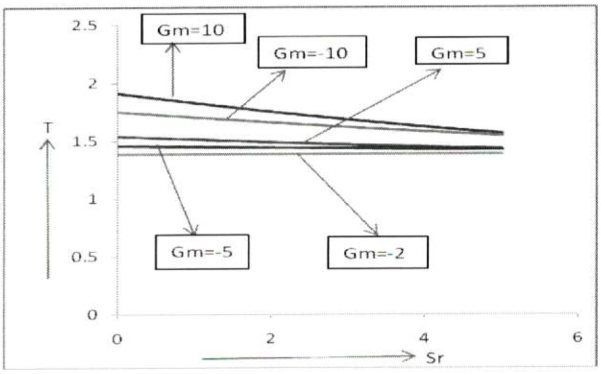

Fig.16: Temperature Field T against Sr for Gr=2, Ec=0.01, Pr=0.71, Sc=2, K=1, M=10.

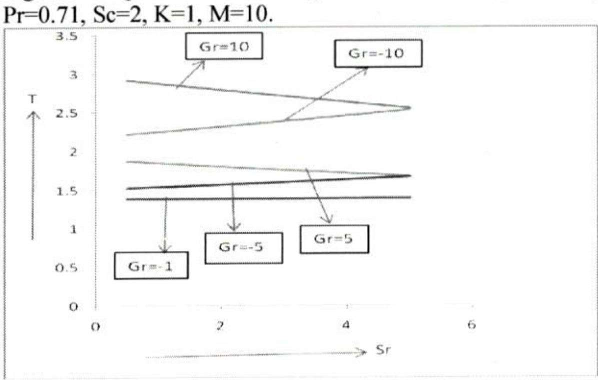

Fig.17: Temperature Field T against Sr for Gr=2, Ec=0.01, Pr=0.71, Sc=2, K=1, M=10.

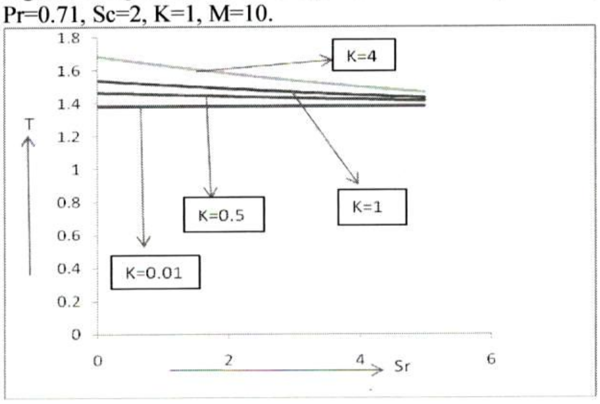

Fig.18: Temperature Field T against Sr for Gr=2, Gm=5, Ec=0.01, Pr=0.71, Sc=2, M=10.

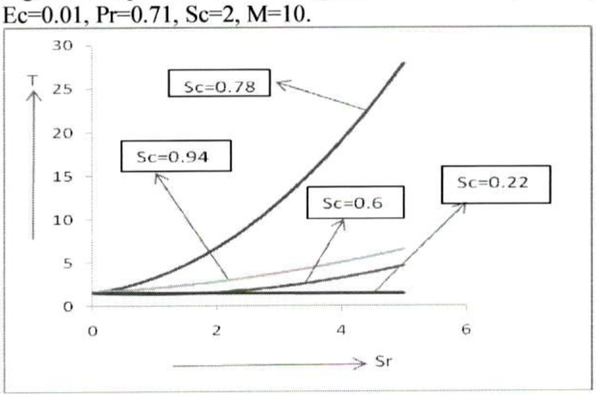

Fig.19: Temperature Field T against Sr for Gr=2, Gm=5, Ec=0.01, Pr=0.71, K=1, M=10.

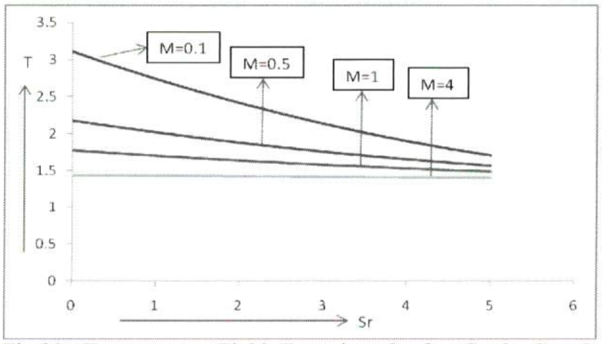

Fig.20: Temperature Field T against Sr for Gr=2, Gm=5, Ec=0.01, Sc=2, Pr=0.71, K=1.

Conclusions:

The conclusions of the study are as follows:

i. The velocity increases with the increase of thermal Grashof number, solutal Grashof number and Soret number.

ii. The velocity decrease with the increase of Schmidt number, Hartmann number and Prandtl number.

iii. An increase in the thermal Grashof number solutal Grashof number, Schmidt number and Soret number lead to a rise in the temperature profiles whereas it decreases as increasing values of Hartmann number and Prandtl number.

iv. Soret number increases the concentration profiles whereas Hartmann number and Prandtl number decreases the concentration profiles.

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Nomenclature:

B_0 is the strength of the applied field.
$\bar{C}$ is the species concentration.
$\bar{C}_\infty$ is the species concentration in free stream.
$\bar{C}_W$ is the species concentration at the plate.
Cp is the specific heat at constant pressure. [J/kg K]
Gr is the Grashof number for heat transfer.
Gm is the Grashof number for masst transfer.
k is the thermal conductivity. [W/mK]
K is a permeability parameter.
Sr is the Soret number. Sc is the Schmidt number.
$\bar{T}$ is the temperature in the boundary layer. [K]
$\bar{T}_W$ is the temperature at the plate.
$\bar{T}_\infty$ is the fluid temperature in the free stream.
$\bar{U}$ is the free stream velocity.
U is the non dimensional free stream velocity.
Ec is the Eckert number. M is the Hartmann number.
D_M is the co-efficient of chemical molecular diffusivity.
T_M is the mean fluid temperature.
K_T is the thermal diffusion ratio.
D is the co-efficient of mass molecular diffusivity.
g is the acceleration due to gravity. [m/s^2]
$(\bar{u}, \bar{v}, \bar{w})$ are the component of the fluid velocity $\bar{q}$.
(u, v, w) are the non dimensional components of the fluid velocity.
v_0 is the mean suction velocity.
$(\bar{x}, \bar{y}, \bar{z})$ is the co-ordinate system.

Greek Symbols:

β is the co-efficient volume expansion for heat transfer.
$\bar{\beta}$ is the c-efficient of volume expansion for mass transfer.
υ is the kinematic viscosity. [m^2/s]
σ is the electrical conductivity. [s/m]
ρ is the density of the fluid. [kg/m$^{3]}$
μ is the co-efficient of viscosity. [N s/m^2]
and other symbols have their usual meaning.