Heat and Technology
Vol. 32, n. 1-2, 2014

BÉZIER BASED SHAPE PARAMETERIZATION IN HIGH SPEED MANDREL DESIGN

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ABSTRACT

The optimization of bi-dimensional profiles of axisymmetric parts is one of the most commonly addressed problems in engineering. Shafts are a typical example of this basic shape. This work is concerned with the use of Genetic Algorithms (GAs), Finite Elements (FE) and rational Bézier curves for the optimization of high speed mandrels. The design variables of the problem are the weights of the nodes of the Bézier boundary curves used to define the finite element discretization. These values are generated by the GA and handled by a mesh generator which defines a candidate solution to the problem. The value of the natural frequencies for each individual is evaluated. For a given set of values of cross-sectional areas and resulting natural frequencies, the value of the fitness function of an individual is obtained. Is this case of a constrained optimization problem The binary-coded generational GA uses a Gray code, rank-based selection, and elitism. The paper briefly summarizes the basis of the GAs formulation and describes how to use refined genetic operators. The mixed pure cylindrical and Bézier shaped model boundary is discretized by using a beam FEM (Finite Element Method) model. Some selected parts of the boundary are modeled by using curves, in order to allow easy meshing and adaptation of the boundary to optimization process. A numerical examples is presented and discussed in detail, showing that the proposed combined technique is able to optimize the shape of the domains with minimum computational effort. The improvement in confront with the original multiple-cylinder shape is significant, without violating the restrictions imposed to the model.

Keywords: genetic algorithm, Bézier, FEA, natural frequencies, high speed winding mandrel.

1. Introduction

Shape optimization of continuum models has been always a subject of concern to design engineers. For a long time, this field of research has deserved great attention from the numerical analysis scientific community and many available optimization techniques have been developed and used in engineering analysis and design with hit-and-miss results. In this class of problems, aspects such as geometric definitions, mesh generation, analysis and displaying of results are usually of concern. Furthermore, other elements play a decisive role in the optimization process, such as numerical optimization programming. The problem can be divided into various tasks. The first step is to define the geometric and the analytical models. The geometric model is where the design variables are easily imposed and it allows an explicit integration with other design tools, such as CAD or CAM systems. On the other hand, the FE model is used to obtain the structural response of the part, subjected to external actions. Generally, the shape design optimization formula requires the evaluation of accurate stiffness values on the part, which is usually not difficult to obtain by the FE model.

Furthermore, the model requires a large number of design variables and tends to produce odd or impossible shapes. For this reason a large number of constraints must be added in order to generate smooth and feasible boundaries, which complicates the design task. The difficulty to implement the associated geometric model can be overcome by the integration with powerful design tools already developed for CAD and CAM systems.

Anyway, the success of any optimization methodology hinges on its ability to deal with complex problems, as is the case of shape optimization design. It is quite common in practice that the original methods are modified, combined and extended in order to tailor a procedure that matches best the features of the selected problem. For these reasons, emphasis has to be made on the selection and combination of various methods in order to choose the best solution.

The search for a robust and validated optimization algorithm, with good balance between computing-time and results, necessary to survive in many different environments, has led to the massive use of Genetic Algorithms (GAs). GAs have several many advantages over more traditional optimization methods. Among other considerations, they do not need further additional information than objective function values (fitness function). This information is used to determine the evaluate the design solution in the defined environment. Moreover, its ability to find out the quasi-optimum solution gives it a privileged position as a powerful tool in the solution of engineering problems.

All of these advantages have led to propose the combination of GAs with geometric modeling, by using Bézier curves, in order to solve non-conventional problems like shape optimization problems.

In CAD systems, the geometric modeling is carried out by using Bézier curves and B-splines curves and surfaces as well as the more traditional Coons patches.

Among these approaches, the Bézier curves are the best from the point of view of controllability. Other advantages that encourage the use of this technique are: the curve lies in a convex hull of the vertices; the curve does not depend on an affine-transformation; the tangents of the initial and final points are defined respectively by the first and the last edge of the polygon. The above characteristic allows the imposition of C1 continuity [1-4].

Thus, recent developments by the authors in shape optimization using the FEM, suggested the extension of some of the ideas previously presented to their application to the genetic optimization of bidimensional engineering.

For high-speed winding mandrel, in particular, it is very important to accurately predict natural frequencies and vibration modes of shafting at the design stage so as to minimize the likelihood of failure and improve the working speed range. Finite element (FE) analysis and experimental measurements are used to establish the natural frequencies and modes of multi-span shafting coupled by flexible coupling, and to assess the influence of leading design parameters of coupling, such as the intermediate shaft length, wall thicknesses, materials and bearings stiffness.

An efficient shape optimization scheme has been developed for designing axis symmetric structures. The FE and GA methods [24, 25] are coupled for this analysis. Selected sets of weights of master nodes on design boundaries are employed as design variables and assigned to move towards their normal directions. By interpolating the weight of master nodes, Bézier curves are constructed so that the remaining features on the design boundaries efficiently settle on the Bézier curves. A mesh optimization smoothing scheme is also applied to interior FE nodes to maintain the FE simulation quality. Applying these techniques a numerical implementation is presented to obtain the optimum design of an high speed mandrel for a winding machine. The results give the optimum shape of the mandrel showing a speed increment up to 20% after the shape optimization.

2. A review of GAs formulation

A detailed GAs formulation can be found in the works of Holland, Goldberg and Davis [21, 22, 23], among others. The first step in finding out the best design consists in the modeling of the basic features (attributes or characteristics) of the final product that will be optimized. These characteristics are summarized in the n dimensional vector $(a_1,...a_i....a_n)$. In our case these features are the 2D shape and the topological configuration of the various elements that form the final mechanical assembly. The model forms the core of the optimization process and is numerically evaluated in the objective function.

In GAs, the particular design (A) is represented by the set of its attributes ai, that form the phenotype.

The phenotype of a structure is formed by the interaction of the algorithm with:

$$A = a_1 * a_2 * a_3 * \cdots * a_c = \prod_{i=1}^{c} a_i \qquad a_i \in \Re \tag{1}$$

its environment and its genotype is obtained by encoding each ai into a particular code (Gray-code in our case). The transformation of the phenotype structure into a string of bits leads to the so-called chromosomes, and it represents, like in natural systems, the total genetic prescription for the construction and operation of some individual

$$A = \prod_{i=1}^{c} a_i = \prod_{i=1}^{c} (e : a_i \rightarrow \{0, 1\}) \tag{2}$$

GAs operate on populations of strings (structures) and progressively at any step t (t =0; 1; 2...) modifies their genotypes to obtain the best performance of their phenotype environment E that contains the constraints and boundary conditions.

The adaptation process is based a mechanic claimed to be similar to "natural selection and natural genetics". They combine the survival of the fittest together with string structures, with a structured yet randomized information, which is exchanged to form a search algorithm. In each generation, using bits and pieces of the fittest of the previous generation generates a new set of artificial individuals (strings). They efficiently exploit the already known information to evaluate a new search points set to look for improved performance (better fitness value). In order to use GAs it is only necessary to define an objective function or fitness function that measures the behavior of each individual inside the problem space. This function the unique available indication of the performance of each individual to solve the optimization problem in the constrained pre-defined solution space. With the population ranked according to fitness, a set of chromosomes are selected from the population as the current best. There exist several methods to choose the mating parents. In this paper, the Stochastic Sampling With Replacement (SSWR) has been used. In this method the mating probability of an individual is proportional to its fitness value. So that, individuals with higher fitness have a better coupling probability. The selected chromosomes are then reproduced through the binary crossover operator. Many procedures can be found in the technical literature to carry out this reproduction operation. In the present work the one break point (simple crossover) random point crossover is used, since this method leads to simple and reliable solutions.

In our implementation of the GA, there are no special schemes for saving "individuals" with low function evaluation. They are systematically removed from the population by the mechanics of the survival of the best.

In simple binary crossover, the string is binary-coded using a Gray code. Then an integer position n1 along the binary string is randomly chosen between 1 and the string length less one [1,{length-1}]. Then, two new strings are created by swapping all the characters between position $\{n_1+1\}$ and length.

The mutation operator was not used since it proved to be unuseful in previous works [1-14].

The selection according to the fitness, combined with the crossover, provides GAs the necessary processing power.

There exist several new operators that can be used together with the basic reproduction (crossover and mutation operators) briefly introduced before. In the present work, a refined operator, called "elitism", is used to improve the results obtained with a simple genetic algorithm.

Unfortunately the crossover process can destroy the best member of the population, giving no offspring into the next generation. The elitist strategy fixes this potential loss by adding the best member of each generation into the succeeding generation. The elitist strategy increases the domination speed exerted by a superindividual on the population. With elitism GA may remain and converge onto a local maximum. However, it hugely improves the genetic algorithm convergence speed.

3. 2D Boundary modeling using Bezier curves

3.1 Rational Beziers curves

The Bézier curve can be defined as all points p(t) of the following parametric equation:

$$p(t) = (1 - t)^2 p_0 + 2\,t(1 - t)\,p_1 + t^2 p^2 \qquad (3)$$

In the formula, t is a number between 0 and 1, p_0 the point situated at one of both extremities of the curve, p_1 the control point, and p_2 the other extremity. A rational Bézier curves add the weights for more control on curve path

$$p(t) = \frac{(1-t)^2 p_0 w_0 + 2t(1-t) p_1 w_1 + t^2 p_2 w_2}{(1-t)^2 w_0 + 2t(1-t) w_1 + t^2 w_2} \qquad (4)$$

In our case only the weight w_1 is used and it has the effect to attract the curve toward the control pole p_1.
The main advantage of Bézier curves is the possibility to define the tangents at the extremities of the curve. This is important to assure C1 continuity and to control boundary condition on shaft geometry as assembly and manufacturability. The rational formulation allows to control the geometric boundary of the shaft by changing the weight w_1. An increase of w_1 attracts the curve toward p_1 as depicted in figures 8-9. The software implementation of Bézier curves is straightforward and efficient. Furthermore, Bézier curves are often implemented in CNC (Computerized Numerical Control) machines with powerful optimization algorithm that improve machining quality and reduce production time.

3.2 Description of the optimization process

At the beginning the boundary geometry is described as an assembly of parametric features. Geometry consistency of this first assembly is checked by the user and constrained are defined. The constrain are not only the boundary for the variation of the parameters, but also the functional and assembly condition. A first FE run is then performed on this first "prototype" shaft. Materials are also defined at this optimization stage. Then, a mesh generator is invoked to generate the elements geometric and elastic properties. The elements generated follow up the boundary defined by the Bézier curves and the other features associated to the model. Accordingly, a consistent FE model is obtained along with its stiffness and mass matrixes. After the "prototype" and its boundary conditions are checked, the GA algorithm is activated. This automatically generates the first population. This is ranked and the SSWR takes place. The best individual of the previous population is transferred directly to the next due to elitism. The process goes on until the last generation is calculated or an acceptable solution is found depending on the user's choice. During this process
The software also provides internal topology and geometric consistency controls that discard automatically those individuals (geometries) which display unfeasible configurations. Moreover, the evolutionary nature of the GA along with the FE analysis automatically removes those individuals displaying severe geometry distortions. In fact, they are severely penalized and are assigned the lowest fitness values. The FE analysis solver is then used to compute the eigenvalues. A natural frequency (and a rotational speed) is associated to each eigenvalue. It is then possible to determine the velocity range possible to the specific mandrel assembly. An individual of the GA is a mandrel assembly geometry. This is essential, since the decisions taken by the GA module are based on mandrel velocity range. It means that individuals (geometries) which violate the mandrel dynamic restrictions and show poor velocity range are penalized, in order to refine more and more the fitness of the population as a whole. The optimization cycle is repeated again and again until it reaches the number of generations previously defined or best element of the last 3 new generations are the same. During the run, a monitoring screen is available and a final report is issued. A numerical example is included herein, in order to show the ability of the proposed technique to deal with bidimensional domains discretized also by Bézier curves. As the optimization process carried out by GAs is not deterministic, the possibility to find slightly different solutions for the same problem with different runs will exist.

4. The numerical example

Various kinds of manmade "syntetic" fiber materials are evolving from continuous research and development. They already make up almost half of the world's textile products and have become a very natural part of our lives. The trend towards higher production speeds is for monofilaments is implemented in this paper. High speed monofilament-lines/plants increase the output and reduce the production costs multiple. Line speeds of up to 5000 meter per minute are now reliably achieved and further improvement are thoroughly searched. The take-up-winder (figure 1) is a critical component in the manufacturing process, it's duty is to wind up the bobbin. The winder mandrel is fundamental since it is at the end of the monofilament manufacturing process.

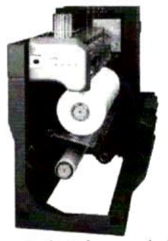

Figure 1: the take up winder

This increase in the speed of winding, however, involves an increase of the vibrations that may position itself in the region of frequencies of the technological process of spinning, leading to resonance phenomena. The objective of the optimization process is to find these frequencies and to move them from the work area by means of mandrel geometry optimization without modifying the wind-up-machine interface. In addition, it is necessary to with a margin of safety which takes into account the errors resulting from the transition from theory (mathematical model) to practice.

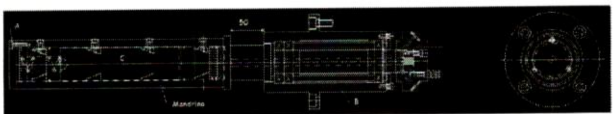

Figure 2 The winding mandrel geometry taken as example. Initial configuration.

The proposed spindle (mandrel) for filament winding shown in figure 2 must comply with geometric constraints and natural frequencies. Geometric constraints are: coupling with the bearing system (bearings and coupling to the machine body) and coupling with the filament collection interface (Bobbin). The bobbin has cardboard tube of internal diameter 110mm and external diameter 125 mm. A simplified model of this mandrel assembly is shown in figure 4, as schematized by the optimization software. The bobbin should be positioned on sections 1,2 &3 of figure 4 (external diameter 110 mm). The triangles show the position of the oblique roller bearings that guide the mandrel rotation. So sections from 8 to 13 should have the inner bearing ring diameter or slightly less. In fact, it is necessary to mount the bearings on the shaft. The problem is already geometrically over constrained leaving the possibility to vary only the inner diameter of section 1,2&3 and 8->10. For technological reasons the only feasible material is steel, with the well known density and elastic modulus. The dynamic objectives (i.e. natural frequencies) are related to the field of operation of the spindle, which must be adapted from time to time at the following Winding Speeds (WS) : WS= 2000, 3300, 4200, 5500 m/min. Taking into account the fact that in order to maintain a constant winding speed it is necessary to vary continuously the spindle rpm. In fact as the diameter of the spool increases, mandrel rpm decreases according to Table 1.

Bobbin mass (filament only) [kg]	WS=2,000 [m/min]	WS=3,300 [m/min]	WS=4,200 [m/min]	WS=5,500 [m/min]
0	5,093 [rpm]	8,403[rpm]	10,695[rpm]	14,005[rpm]
1	4,529[rpm]	7,473[rpm]	9,511[rpm]	12,455[rpm]
2	4,118[rpm]	6,796[rpm]	8,649[rpm]	11,327[rpm]
3	3,803[rpm]	6,275[rpm]	7,986[rpm]	10,458[rpm]
4	3,550[rpm]	5,858[rpm]	7,455[rpm]	9,763[rpm]
5	3,342[rpm]	5,514[rpm]	7,018[rpm]	9,190[rpm]
6	3,166[rpm]	5,225[rpm]	6,650[rpm]	8,708[rpm]
7	3,016[rpm]	4,976[rpm]	6,334[rpm]	8,294[rpm]
8	2,885[rpm]	4,760[rpm]	6,059[rpm]	7,934[rpm]
9	2,770[rpm]	4,570[rpm]	5,817[rpm]	7,618[rpm]
10	2,667[rpm]	4,401[rpm]	5,602[rpm]	7,336[rpm]
11	2,576[rpm]	4,250[rpm]	5,409[rpm]	7,084[rpm]
12	2,493[rpm]	4,113[rpm]	5,235[rpm]	6,855[rpm]
13	2,417[rpm]	3,989[rpm]	5,077[rpm]	6,648[rpm]
14	2,348[rpm]	3,875[rpm]	4,932[rpm]	6,458[rpm]
15	2,285[rpm]	3,770[rpm]	4,799[rpm]	6,284[rpm]
16	2,226[rpm]	3,674[rpm]	4,676[rpm]	6,123[rpm]
17	2,172[rpm]	3,584[rpm]	4,562[rpm]	5,974[rpm]
18	2,122[rpm]	3,501[rpm]	4,456[rpm]	5,835[rpm]

Table 1: winding data. Since WS is constant, as the filament piles up on the bobbin the mandrel should slow down

As a result, the natural frequencies (NF) of the spindle must comply with the following limits: relation (5) at the beginning of the winding process and relation (6) at the end. So the mandrel should have the feasible operating range of equation (7):

$$(NF_i < 5{,}093) \text{ AND } (NF_{i+1} > 14{,}005) \quad (5)$$

$$(NF_j < 2{,}122) \text{ AND } (NF_{j+1} > 5{,}835) \quad (6)$$

$$(NF_i < 2{,}122) \text{ AND } (NF_{i+1} > 14{,}005) \quad (7)$$

Wanting a safety margin of at least 15% of the critical natural frequencies, relation 7 becomes (8)

$$(NF_i < 1{,}800) \text{ AND } (NF_{i+1} > 16{,}100) \quad (8)$$

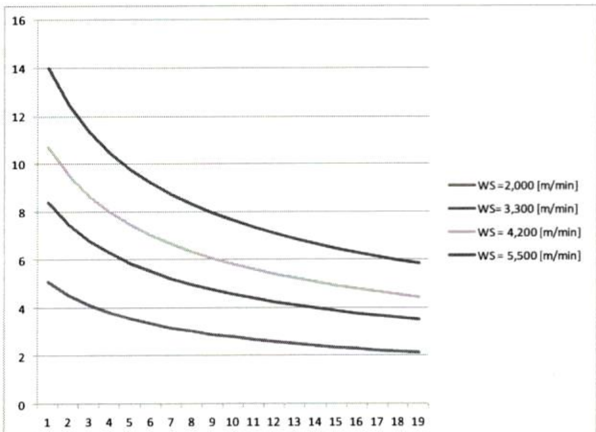

Figure 3: rpm vs. WS with different bobbin mass (filament only) (da sistemare deve partire da 0 e deve finire a 16 kg, non Vf ma WS).

A proper software has been implemented for the calculation of the natural frequencies. The model of the mandrel of figure 2 is shown in figure 4 and 5.

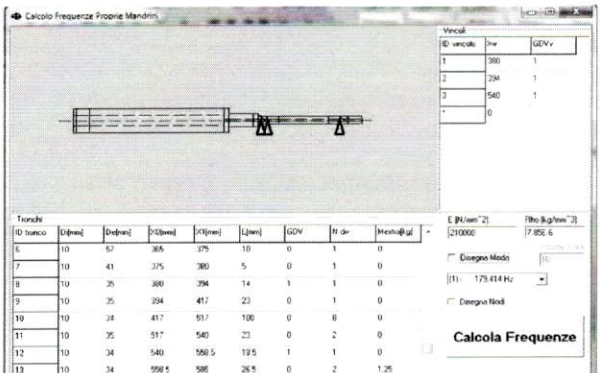

Figure 4: input data for the mandrel of fig 2

Figure 4 shows the software User Interface (UI). The mandrel is subdivided in cylindrical parts that are inputted by specifying the internal and external diameters (D_i and D_e), the starting and ending coordinates (X_1 and X_2) and the length (L). GDV is for the Degree Of Freedom (DOF) of the node: 0 stands for free, 1 for sliding hinge (1 DOF constrained, 1 DOF free) and 2 fixed hinge (2 DOF constrained). This type of input is necessary since the shafts can be overlapped in some designs. An extra mass can be added in the middle of the single cylinder (Mextra). N_{div} gives the number of FE elements for each cylinder. A 2D geometric model is sufficient since the mandrel is axisymmetric [15-20].

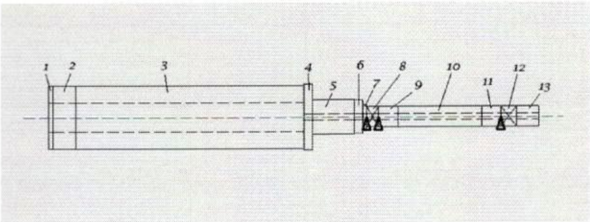

Figure 5: mandrel geometry as schematized by the software

Figure 5 shows how the software outputs in real time the model inputted in figure 4. It is then possible to control immediately the consistency of input data. The numbers indicate the cylinders for the 2D geometrical model of the mandrel.

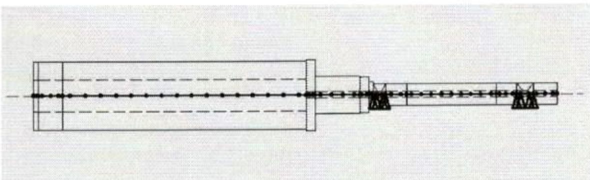

Figure 6: mesh generated for the mandrel of figure 1xxx

By performing the calculation for the mandrel with an empty bobbin the first natural frequency is at 179.41 Hz (10,764 rpm -figure 7) well below the required minimum value of 16,100 rpm (see equation 8).

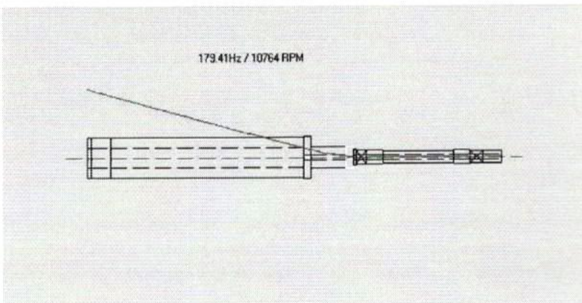

Figure 7: first natural frequency of the original mandrel

At first a simple manual optimization was tried by increasing D_i of the II and III cylinder from 49.5 to 88. In this case the first natural frequency increases from 179.41 Hz (10,764 rpm) to an insufficient 244.5 Hz (14,670 rpm).

Not being able to further lighten the traits 2 and 3 without affecting the functionality of the winding system. It is possible to modify the traits 4 and 5. The lengths cannot be varied, for reasons relating to the winding system. The simple variation of the diameters of 4 and 5 proved to be not sufficient. It was then decided to change the geometry of the section 5 itself, from cylindrical to curved with variable thickness, by filleting the spindle between the traits 4 and 6 with a cubic Bézier as shown in figure 7. The rational Bézier curve can have depict infinite paths by changing the weight of intermediate control point P_1 (see figure 9). It is then easy to implement a GA algorithm that given a Bézier filleting curve, the point P_1 position and the initial and final tangent to the profile optimizes the Bézier and the variable thickness of trait 5 of the mandrel. The aim is to obtain the first "empty bobbin" natural frequency above the required value of 16,100 rpm (see equation 8) [26-28].

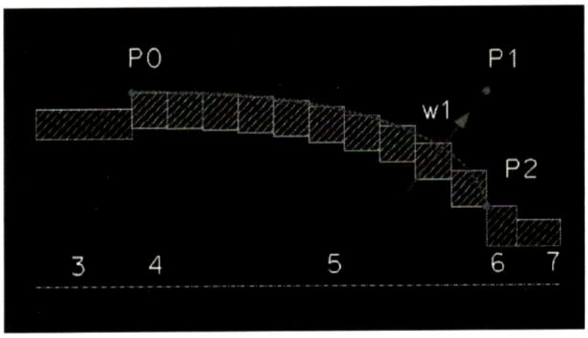

Figure 8 A cubic Bézier curve (red) replaces cylinders 4 and 5

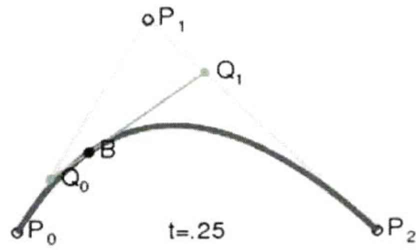

Figure 9: the weight of point P1 control the curve geometry

The GA software automatically discretizes the Bézier filleted trait with multiple cylindrical traits for FE calculations.

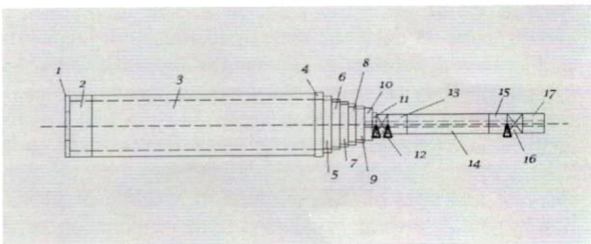

Figure 10: a GA item generated mandrel

The GA then optimizes the shaft by waring the weight of the Bézier curve and the thicknesses of the cylinders that discretize the Bézier-shaped fillet.

The number of this cylinders was fixed to 10. The mandrel of figure 11 has a first natural frequency with the empty bobbin of 16,432 rpm. The requirements of equation 8 are then satisfied.

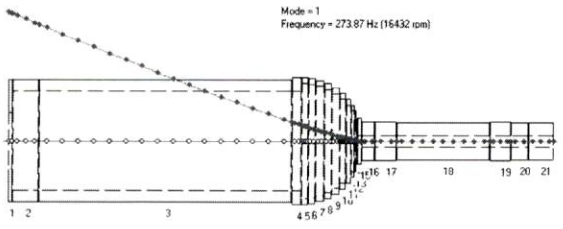

Figure 11: GA optimized mandrel

GA optimization parameters for the mandrel natural frequencies optimization are shown in table 2:

DATA	VALUE
Population size	99
Number of generations	30
Selection scheme	SSWR
Crossover scheme	Uniform crossover
Mutation probability	0.015
Other operators	Elitism
Crossover probability	0.78

Table 2: GA parameters

5. Conclusions

A new numerical technique to optimize the shape of 2D engineering models has been presented and discussed herein. The new method combines three powerful tools in shape optimization such as FE, Rational Bézier for geometric modeling and GAs. As shown in the numerical example, the use of this approach improves the final results since the boundary of the model to be optimized can be defined and modified by using an easy and effective tool such as rational Bézier curve modeling. Since few design variables are required and the GA is very efficient, a real time solution is possible. An automatic mesh generator was used to generate the analytical model from the geometric model through the reduced set of geometric parameters. The FE method has proven to be an excellent analysis technique in this kind of problems, with the limit to approximate the Bézier curve with cylinders. Very accurate values natural frequencies can be obtained when using the FE analysis. The repeated structural analysis accounts for the main computational cost in this optimization processes. In addition, the major contributor to the cost and time of the optimization of large problems is usually the derivative calculation. The annihilation of this computational effort is achieved by using GAs. In common practical overconstrained mechanical assemblies, too many design variables are not a common problem in the process of shape optimization. In this work, it has been shown that the use of few design variables lead to fast convergence and to good accuracy.
The use of the powerful GAs as an optimization technique improves the performance of the approach, due to its great advantage as compared with other optimization techniques. GA reaches an optimum or quasi-optimum solution, even in the presence of complex domains and multiple boundary conditions as well. In the "evolutionary" optimization approach there are implicit processes involved in the evolution of the individuals that form the initial population. These implicit mechanisms lead them to produce improved individuals as the generations follow up. So, the evolutionary approach [29-37] cannot be compared with traditional optimization techniques in terms of the number of objective function evaluations due to the fact that they behave in a different way to obtain the optimum or quasi-optimum solution.

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