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EFFECT OF WAVE NUMBER ON THE ONSET OF INSTABILITY IN COUPLE-STRESS FLUID AND ITS CHARACTERIZATION IN THE PRESENCE OF ROTATION

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ABSTRACT

Thermal instability of couple-stress fluid in the presence of uniform vertical rotation is considered. Following the linearized stability theory and normal mode analysis, the paper established the regime for all oscillatory and non-decaying slow motions starting from rest, in a couple-stress fluid of infinite horizontal extension and finite vertical depth in the presence of uniform vertical rotation and the necessary condition for the existence of "overstability" and the sufficient condition for the validity of the 'exchange principle' is derived, when the bounding surfaces of infinite horizontal extension, at the top and bottom of the fluid are rigid. Further, the stationary convection at marginal state with free horizontal boundaries is analyzed numerically and graphically, showing that the couple-stress parameter and rotation has stabilizing effect on the system. However, for the constant magnitude of couple-stress parameter and rotation, the wave number has a destabilizing effect for a value less than a critical value, which varies with the magnitude of the couple-stress parameter and rotation; and for higher value than the critical value of the wave number; it has a stabilizing effect on the system.

Keywords: Thermal convection, Couple-Stress Fluid, Rotation, PES, Taylor number.

1. INTRODUCTION

Right from the conceptualizations of turbulence, instability of fluid flows is being regarded at its root. The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in Geophysics, interiors of the Earth, Oceanography and Atmospheric Physics etc. A detailed account of the theoretical and experimental study of the onset of Bénard Convection in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar [1]. The use of Boussinesq approximation has been made throughout, which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. Sharma et al [2] has considered the effect of suspended particles on the onset of Bénard convection in hydromagnetics. The fluid has been considered here are all Newtonian in all above studies. With the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. Stoke [3] proposed and postulated the theory of couple-stress fluid. One of the applications of couple-stress fluid is its use to the study of the mechanism of lubrication of synovial joints, which has become the object of scientific research. According to the theory of Stokes [4], couple-stresses are found to appear in noticeable magnitude in fluids having very large molecules. Since the long chain hylauronic acid molecules are found as additives in synovial fluid, Walicki and Walicka [5] modeled synovial fluid as couple-stress fluid in human joints.

Sharma and Thakur [6] have studied the thermal convection in couple-stress fluid in porous medium in hydromagnetics. Sharma and Sharma [7] and Kumar and Kumar [8] have studied couple stress fluid under various conditions.

However, in all above studies the case of two free boundaries which is a little bit artificial except the stellar atmospheric case is considered. Banerjee et al [9] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee [10] established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al [11]. However no such result existed for non-Newtonian fluid configurations in general and in particular, for Couple-stress viscoelastic fluid configurations. Rana and Patial [12] and Das [13] had investigated the problem with different non-Newtonian fluids. Banyal [14] have characterized the oscillatory motions in couple-stress fluid.

Keeping in mind the importance of non-Newtonian fluids, the present paper is an attempt to characterize the onset of instability analytically, in a layer of incompressible couple-stress fluid heated from below in the presence of uniform vertical rotation opposite to force field of gravity, when the bounding surfaces of infinite horizontal extension, at the top and bottom of the fluid are rigid. It is shown that for the

configuration under consideration, if $\dfrac{T_A}{\left(\pi^2+\pi^4 F\right)^2}\leq 1$, then an

arbitrary neutral or unstable modes of the system are definitely non-oscillatory and, in particular the PES is valid, where T_A is the Taylor number and F is the couple-stress parameter. Further, the stationary convection at marginal state with free horizontal boundaries is analyzed numerically and graphically, showing that the couple-stress parameter and rotation has stabilizing effect on the system. However, for the constant magnitude of couple-stress parameter and rotation, the wave number has a destabilizing effect for a value less than a critical value, which varies with the magnitude of the couple-stress parameter and rotation; and for higher value than the critical value of the wave number; it has a stabilizing effect on the system.

2. FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Considered an infinite, horizontal, incompressible couple-stress fluid layer, of thickness d, heated from below so that, the temperature and density at the bottom surface z = 0 are T_0, ρ_0 respectively and at the upper surface z = d are T_d, ρ_d and that a uniform adverse temperature gradient $\beta\left(=\left|\dfrac{dT}{dz}\right|\right)$ is maintained. The fluid is acted upon by a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$. Let ρ, p, T and $\vec{q}(u,v,w)$ denote respectively the density, pressure, temperature and velocity of the fluid. Then the momentum balance, mass balance equations of the couple-stress fluid (Stokes [3]; Chandrasekhar [1] and Sharma and Sharma [6]) are

$$\frac{\partial \vec{q}}{\partial t}+\left(\vec{q}.\nabla\right)\vec{q}=-\frac{1}{\rho_0}\nabla p+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\left(\nu-\frac{\mu^{'}}{\rho_0}\nabla^2\right)\nabla^2\vec{q}+2\left(\vec{q}\times\vec{\Omega}\right) \quad (2.1)$$

$$\nabla.\vec{q}=0, \quad (2.2)$$

Where $\nabla^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}$ and $\varsigma=\dfrac{\partial v}{\partial x}-\dfrac{\partial u}{\partial y}$ denote the z-component of vorticity.

3. NORMAL MODE ANALYSIS

Analyzing the disturbances into normal modes, we assume that the Perturbation quantities are of the form

$$\left[w,\theta,\varsigma\right]=\left[W(z),\Theta(z),Z(z)\right]\mathrm{Exp}\left(ik_x x+ik_y y+nt\right), \quad (3.1)$$

Where k_x,k_y are the wave numbers along the x and y-directions respectively $k=\left(k_x^{\,2}+k_y^{\,2}\right)^{\frac{1}{2}}$, is the resultant wave number and n is the growth rate which is, in general, a complex constant.

Using (3.1), equations (2.9), (2.10) and (2.11), on using (2.8), in non-dimensional form, become

$$\left(D^2-a^2\right)\left[\sigma+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]W$$
$$=-\frac{g\alpha d^2 a^2\Theta}{\nu}-\sqrt{T_A}\,dDZ , \quad (3.2)$$

$$\left[\left\{1-F\left(D^2-a^2\right)\right\}\left(D^2-a^2\right)-\sigma\right]Z=-\frac{\sqrt{T_A}}{d}DW , \quad (3.3)$$

$$\left(D^2-a^2-p_1\sigma\right)\Theta=-\frac{\beta d^2}{\kappa}W , \quad (3.4)$$

Where
$$a=kd,\sigma=\frac{nd^2}{\nu},p_1=\frac{\nu}{\kappa},F=\frac{\mu^{'}}{\rho_0 d^2\nu} ,$$

$$D=\frac{d}{dz} \text{ and } D_\oplus=dD \text{ and dropping } (\oplus) \text{ for convenience.}$$

Here $p_1=\dfrac{\nu}{\kappa}$, is the thermal prandtl number, F is the couple-stress parameter and T_A is the Taylor number.

Substituting $W=W_\oplus$, $\Theta=\dfrac{\beta d^2}{\kappa}\Theta_\oplus$ and $Z=\dfrac{\sqrt{T_A}}{d}Z_\oplus$ in equations (3.2), (3.3) and (3.4) and dropping $(\oplus)$ for convenience, in non-dimensional form becomes,

$$\left(D^2-a^2\right)\left[\sigma+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]W$$
$$=-Ra^2\Theta-T_A DZ , \quad (3.5)$$

$$\left[\left\{1-F\left(D^2-a^2\right)\right\}\left(D^2-a^2\right)-\sigma\right]Z=-DW , \quad (3.6)$$

$$\left(D^2-a^2-p_1\sigma\right)\Theta=-W , \quad (3.7)$$

Where $R=\dfrac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number.

Since both the boundaries rigid and are maintained at constant temperature, the perturbations in the temperature are zero at the boundaries. The appropriate boundary conditions with respect to which equations (3.5), (3.6) and (3.7) must be solved are

W = DW = 0, $\Theta = 0$ and Z = 0 at z = 0 and z = 1, (3.8)

Equations (3.5)-(3.7), along with boundary conditions (3.8), pose an eigenvalue problem for σ and we wish to characterize σ_i when $\sigma_r\geq 0$.

4. MATHEMATICAL ANALYSIS

We prove the following theorems:

Theorem 1: If R$\rangle$ 0 , F$\rangle$ 0, $T_A\rangle$ 0, $\sigma_r\geq 0$ and $\sigma_i\neq 0$ then the necessary condition for the existence of non-trivial solution (W,Θ,Z) of equations (3.5), (3.6) and (3.7) together with boundary conditions (3.8) is that

$$\frac{T_A}{\left(\pi^2 + \pi^4 F\right)^2} \rangle 1 .$$

Proof: Multiplying equation (3.5) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\sigma \int_0^1 W^* \left(D^2 - a^2\right) W dz + F \int_0^1 W^* \left(D^2 - a^2\right)^3 W dz$$

$$-\int_0^1 W^* \left(D^2 - a^2\right)^2 W dz$$

$$= -Ra^2 \int_0^1 W^* \Theta dz - T_A \int_0^1 W^* DZ dz , \qquad (4.1)$$

Taking complex conjugate on both sides of equation (3.7), we get

$$\left(D^2 - a^2 - p_1 \sigma^*\right)\Theta^* = -W^* , \qquad (4.2)$$

Therefore, using (4.2), we get

$$\int_0^1 W^* \Theta dz = -\int_0^1 \Theta \left(D^2 - a^2 - p_1 \sigma^*\right)\Theta^* dz , \qquad (4.3)$$

Also taking complex conjugate on both sides of equation (3.6), we get

$$\left[\left\{1 - F\left(D^2 - a^2\right)\right\}\left(D^2 - a^2\right) - \sigma^*\right]Z^* = -DW^* , \qquad (4.4)$$

Therefore, using (4.4), we get

$$\int_0^1 W^* DZ dz = -\int_0^1 DW^* Z dz = \int_0^1 Z \left\{\left(D^2 - a^2\right) - F\left(D^2 - a^2\right)^2 - \sigma^*\right\} Z^* dz \quad (4.5)$$

Substituting (4.3) and (4.5) in the right hand side of equation (4.1). Integrating the terms on both sides of the resulting equation for an appropriate number of times by making use of the appropriate boundary conditions (3.8), along with (3.6) and equating imaginary parts on both sides and cancelling $\sigma_i (\neq 0)$ throughout from imaginary part, we get

$$\int_0^1 \left\{|DW|^2 + a^2 |W|^2\right\} dz + Ra^2 p_1 \int_0^1 |\Theta|^2 dz = T_A \int_0^1 |Z|^2 dz , \qquad (4.6)$$

We first note that since W and Z satisfy $W(0) = 0 = W(1)$ and $Z(0) = 0 = Z(1)$ in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality [14]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz , \qquad (4.7)$$

$$\text{And } \int_0^1 |DZ|^2 dz \geq \pi^2 \int_0^1 |Z|^2 dz , \qquad (4.8)$$

Further, for $W(0) = 0 = W(1)$ and $Z(0) = 0 = Z(1)$, Banerjee et al [15] have shown that

$$\int_0^1 |D^2 W|^2 dz \geq \pi^2 \int_0^1 |DW|^2 dz$$

$$\text{And } \int_0^1 |D^2 Z|^2 dz \geq \pi^2 \int_0^1 |DZ|^2 dz , \qquad (4.9)$$

Further, multiplying equation (3.6) by Z^* (the complex conjugate of Z), integrating by parts each term of the resulting equation on the right hand side for an appropriate number of times and making use of boundary condition on Z namely $Z(0) = 0 = Z(1)$ along with (3.6), it follows that

$$\int_0^1 \left\{|DZ|^2 + a^2 |Z|^2\right\} dz + F \int_0^1 \left\{|D^2 Z|^2 + 2a^2 |DZ|^2 + a^4 |Z|^2\right\} dz + \sigma_r \int_0^1 |Z|^2 dz =$$

$$\text{Real part of } \left\{\int_0^1 Z^* DW dz\right\}$$

$$\leq \left|\int_0^1 Z^* DW dz\right| \leq \int_0^1 |Z^*||DW| dz \leq \int_0^1 |Z||DW| dz ,$$

$$\leq \left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}} , \qquad (4.10)$$

(Utilizing Cauchy-Schwartz-inequality),

This gives that

$$\int_0^1 |DZ|^2 dz + F \int_0^1 |D^2 Z| dz \leq \left\{\int_0^1 |Z|^2 dz\right\}^{\frac{1}{2}} \left\{\int_0^1 |DW|^2 dz\right\}^{\frac{1}{2}} , \quad (4.11)$$

Hence inequality (4.11) on utilizing (4.8) and (4.9), gives

$$\int_0^1 |Z|^2 dz \leq \frac{1}{\left(\pi^2 + \pi^4 F\right)^2} \int_0^1 |DW|^2 dz , \qquad (4.12)$$

Now R $\rangle$ 0 and $T_A \rangle$ 0, utilizing the inequalities (4.12), the equation (4.6) gives,

$$\left[1 - \frac{T_A}{\left(\pi^2 + \pi^4 F\right)^2}\right]\int_0^1 |DW|^2 dz + a^2 \int_0^1 |W|^2 dz + Ra^2 p_1 \int_0^1 |\Theta|^2 dz \, 0, \qquad (4.13)$$

Therefore, we must have

$$\frac{T_A}{\left(\pi^2 + \pi^4 F\right)^2} \rangle 1 . \qquad (4.14)$$

Hence, if $\sigma_r \geq 0$ and $\sigma_i \neq 0$, then $\dfrac{T_A}{\left(\pi^2 + \pi^4 F\right)^2} \rangle 1$, $\qquad (4.15)$

And this completes the proof of the theorem.

Presented otherwise from the point of view of existence of instability as stationary convection, the above theorem can be put in the form as follow:-

Theorem 2: The sufficient condition for the validity of the 'exchange principle' and the onset of instability as a non-oscillatory motions of non-growing amplitude in a couple-stress fluid heated from below, in the presence of uniform vertical rotation is that $\dfrac{T_A}{\left(\pi^2 + \pi^4 F\right)^2} \leq 1$, where R is the thermal Rayleigh number and F is the couple-stress parameter, having top and bottom bounding surfaces rigid.

Or, the onset of instability in couple-stress fluid heated from below, in the presence of uniform vertical rotation, cannot manifest itself as oscillatory motions of growing amplitude if the thermal Rayleigh number R and the couple-stress parameter F satisfy the inequality $\dfrac{T_A}{\left(\pi^2 + \pi^4 F\right)^2} \leq 1$, for rigid horizontal boundaries of infinite horizontal extension. Which provide a significant improvement to the earlier result by Banyal [16, 17]. The result is also in accordance with corresponding configuration of Newtonian fluid when the couple-stress parameter F=0, by Gupta et al [10].

In the context of existence of instability in 'oscillatory modes' and that of 'overstability in the present configuration, we can state the above theorem as follow:-

Theorem 3: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a couple-stress fluid heated from below, in the presence of uniform vertical rotation, is that the thermal Rayleigh number R and the couple-stress parameter F, must satisfy the inequality $\dfrac{T_A}{\left(\pi^2 + \pi^4 F\right)^2} \rangle 1$, for rigid horizontal boundaries of infinite horizontal extension.

5. NUMERICAL ANALYSIS AND DISCUSSION

When both the boundaries are dynamically free and 'exchange principle' is valid the neutral state is characterized by $\sigma = 0$, then we seek the solution of equation (3.5) -- (3.7) which satisfy the appropriate boundary condition (3.8). Using the appropriate boundary condition (3.8), it can be shown that all the even order derivatives of W must vanish for z = 0 and z = 1 and hence the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z ,\qquad (5.1)$$

Where W_0 is a constant.

Eliminating Θ and Z between the equations (3.5)–(3.7) when $\sigma = 0$ and substituting the proper solution $W = W_0 \sin \pi z$, in the resultant equation, we obtain the dispersion relation

$$R_1 = \frac{(1+x)^3 \left\{1 + F_1(1+x)\right\}^2 + T_{A1}}{x\left\{1 + F_1(1+x)\right\}},\qquad (5.2)$$

Where $R_1 = \dfrac{R}{\pi^2}, x = \dfrac{a^2}{\pi^2}, T_{A1} = \dfrac{T_A}{\pi^4}$ and $F_1 = \pi^2 F$ are the

modified values of non-dimensional parameters. Here $\dfrac{dR_1}{dT_{A1}} = \dfrac{1}{x\left\{1 + F_1(1+x)\right\}}$, is always positive, showing the stabilizing effect of rotation on the system; and
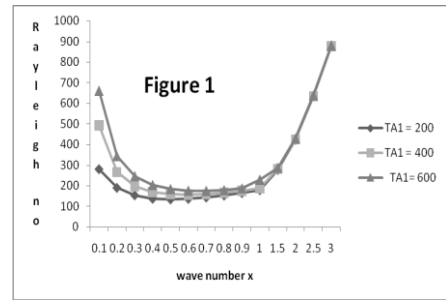
$$\frac{dR_1}{dF_1} = \frac{(1+x)}{x}\left[(1+x)^3 - \frac{T_{A1}}{\left\{1 + F_1(1+x)\right\}^2}\right],\qquad (5.3)$$

Therefore, for a rotating system, the couple-stress parameter have a stabilizing (or destabilizing) effect if

$$T_{A1} \langle(or)\rangle \left[(1+x)^3 \left\{1 + F_1(1+x)\right\}^2\right],\qquad (5.4)$$

However in the absence of rotation, the couple-stress parameter has the stabilizing effect on the system only.

The dispersion relation (5.2) is also analyzed numerically for the various values of F_1 and T_{A1}.

Figure 1. Variation of thermal Rayleigh number R_1 with wave number x when F_1= 10 and T_{A1} = 200, 400 and 600.

In figure 1: R_1 is plotted against modified wave number x when $F_1 = 10$ and $T_{A1} = 200$, 400 and 600 respectively. It is clear that the rotation postpone the onset of convection in a couple-stress fluid heated from below as the Rayleigh number increases with the increase in the magnitude of rotation. Further, for the constant value of the magnitude of rotation; the Rayleigh number decreases when the modified wave number x is less than a certain critical value (which is x_c= 0.6, 0.7 and 0.8 when T_{A1} =200, 400 and 600 respectively) and increases thereafter. Therefore, for a constant value of the magnitude of rotation, the wave number has destabilizing effect for $x \leq x_c$, and stabilizing effect when $x \geq x_c$, on the system and this critical value of wave number vary with the change in the magnitude of rotation.
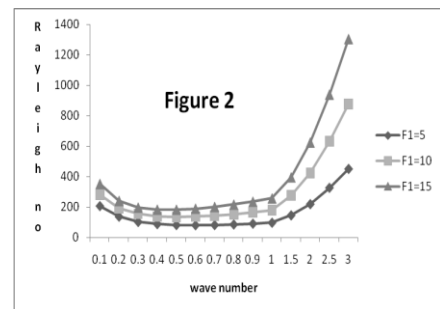
Figure 2. Variation of thermal Rayleigh number R_1 with wave number x when T_{A1} = 200 and F_1 = 5, 10 and 15.

In figure 2: R_1 is plotted against wave number x when $T_{A1} = 200$ and $F_1 = 5$, 10 and 15 respectively. It is clear that the couple-stress parameter postpone the onset of convection in a couple-stress fluid heated from below because the Rayleigh number increases with the increase in the magnitude of couple-stress parameter. Further, for the constant value of the magnitude of couple-stress parameter, the Rayleigh number decreases when the modified wave number x is less than a certain critical value (which is $x_c = 0.6$, 0.5 and 0.5 when $F_1 = 5$, 10 and 15 respectively) and increases thereafter. Therefore, for a constant value of couple-stress parameter, the wave number has destabilizing effect for respective values when $x \leq x_c$ and stabilizing when $x \geq x_c$, on the system and this critical value of wave number vary with the change in the magnitude of couple-stress parameter.

6. CONCLUSION

In this paper, the effect of wave number and uniform vertical rotation on a couple-stress fluid heated from below is investigated and the immediate conclusions of the theorems proved above; and numerical and graphical discussion, are as follows:

(a). The necessary condition for the onset of oscillatory motions and "overstability", for configuration under consideration, is that the inequality (4.17) must be satisfied. Thus the sufficient condition for the non-existence of oscillatory motions and hence the validity of 'exchange principle' is that $\dfrac{T_A}{\left(\pi^2 + \pi^4 F\right)^2} \leq 1$, for the configuration under consideration, which provides a significant improvement to the earlier Which provide a significant improvement to the earlier result by Banyal [16]. The result is also in accordance with corresponding configuration of Newtonian fluid when the couple-stress parameter F=0, by Gupta et al [10].

(b). It is observed from figure1 that of the rotation has the stabilizing effect on the onset of instability in the present configuration, from figure 2, the couple stress parameter in the absence of rotation has the stabilizing effect on the onset of instability in the present configuration. However, in the presence of rotation couple-stress parameter may have stabilizing or destabilizing effect as per the conditions given by (5.4), the result which is in accordance with Sharma and Sharma [6]

(c). Further, the stationary convection at marginal state for the constant magnitude of couple-stress parameter and uniform vertical rotation, the wave number has a destabilizing effect for a value less than a critical value, which varies with the magnitude of couple-stress parameter and magnetic field, and for higher value than the critical value of the wave number; it has a stabilizing effect on the system when both the horizontal boundaries are free.

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NOTATIONS

a	Dimensionless wave number, $[-]$	
F	Couple-Stress parameter, $[-]$	
g	Acceleration due to gravity, $\left[m/s^2\right]$	

k	Wave number, $[1/m]$	α	Coefficient of thermal expansion, $[1/K]$
k_x, k_y	Wave numbers in x and y-directions, $[1/m]$	β	Uniform temperature gradient, $[K/m]$
n	Growth rate, $[1/s]$	Θ	Perturbation in temperature, $[K]$
T_A	Taylor number, $[-]$	κ	Thermal diffusivity, $[m^2/s]$
R	Rayleigh number, $[-]$	ν	Kinematic viscosity, $[m^2/s]$
$\Omega(0,0,\Omega)$	Rotation vector having components $(0,0,\Omega)$,	ν'	Kinematic viscoelasticity, $[m^2/s]$
T	Temperature, $[K]$	∇, ∂, D	Del operator, Curly operator and Derivative with respect to z (=d/dz).
$q(u,v,w)$	Components of velocity after perturbation,		