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EFFECT OF THERMAL RADIATON ON MHD FREE CONVECTIVE OSCILLATORY FLOW BETWEEN TWO VERTICAL PARALLEL PLATES WITH PERIODIC PLATE TEMPERATURE AND DISSIPATIVE HEAT.

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ABSTRACT
This paper focuses on the effect of thermal radiation on unsteady free convection flow of a viscous incompressible electrically conducting fluid with dissipative heat between two long vertical plates when the temperature of one of the plates oscillates about a constant non zero mean temperature. The governing equations are solved by regular perturbation technique. The expressions for the velocity field, temperature field, the skin friction at the plate and Nusselt number are obtained in non-dimensional forms. Computations are performed for a wide range of the governing flow parameters, viz., the Hartmann number, Grashof Number for heat transfer, Prandtl Number and Radiation parameter. The effects of these flow parameters on the velocity, temperature, skin friction and Nusselt number are presented graphically and results are discussed.

Keywords: MHD, Thermal Radiation, Nusselt number, skin friction, free convection

1. INTRODUCTION:

The investigation of free convective flows between two long vertical plates is important because of their applications in the field of nuclear reactors, heat exchangers, cooling appliances in electronic instruments and in different engineering fields. Ostrach [1] first presented the fully develop free convection flow between two parallel plates at constant temperature. Aung [2] studied the exact solution for free convection in a vertical plate channel with asymmetric heating for a fluid of constant properties. Natural convection for heated iso-flux boundaries of the channel containing a low Prandtl number fluid was investigated by Compo et al. [3]. The above mentioned works are confined to fully developed steady flows. The flow may be unsteady when the current is periodic due to an off control mechanisms or due to partially rectified A.C voltage, there exists periodic heat inputs. Singh et al. [4] considered transient free convection flow between two long vertical plates maintained at constant but unequal temperatures. Jha et al. [5] extended the problem to the case of symmetric heating of the channel walls. Ahmed and Kalita [6] have studied MHD oscillatory free convective past a vertical plate in slip flow-regime with variable suction and periodic plate temperature.

Recent developments in hypersonic flights, missile re-entry, rocket combustion chamber plants for inter-planetary flight and gas cooled nuclear reactors focused on thermal radiation as a mode of energy transfer and emphasize the need for improved understanding of radiative transfer in these processes.

The effect of radiation on MHD flow, heat and mass transfer problems has become industrially more important. Many engineering processes occur at high temperatures, the knowledge of radiation heat transfer plays significant role in the design of equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering processes. At high operating temperature, the radiation effect can be quite significant.

Hossain and Takhar [7] studied the radiation effect using Rosseland approximation on mixed convection along a vertical plate with uniform plate temperature. Raptis[8] analyzed both the thermal radiation and free convection flow through a porous medium by using a perturbation technique. Raptis and Perdikis [9] considered the problem of thermal radiation and free convection flow past a moving plate. The interaction of radiation with laminar free convective heat transfer from a vertical plate was investigated by Cess [10]. Recently, Shivaiah and Rao [11], Rajput et al. [12] have studied the effect of thermal radiation on unsteady free convection heat and mass transfer. Very recently M.Mecili and E.Mezaache [13] have presented analytical solution for slip flow-heat transfer in microtubes including viscous dissipation and axial heat conduction.

The aim of the present investigation is to analyze the effect of thermal radiation on MHD free convective oscillatory flow between two vertical parallel plates with periodic plate temperature and dissipative heat. This work is an extension of the problem studied by Ahmed et al. [14] to the case when there is an imposed thermal radiation.

2.MATHEMATICAL ANALYSIS:

An unsteady flow of an incompressible viscous electrically conducting fluid between two long vertical parallel plates under the action of a transverse magnetic field is considered. $\bar{x}$ axis is taken along one of the plates in the vertically upward direction and $\bar{y}$ along the normal to the plates. Let $\vec{q}=\bar{u}\hat{i}+\bar{v}\hat{j}$ be the fluid velocity and $\vec{B}=B_0\hat{j}$ be the applied magnetic field. Since the plate is infinite in length in $\bar{x}$ direction, therefore all the quantities except possibly the pressure are to be independent of $\bar{x}$.

The temperature of the plates $\bar{y}=0$ and $\bar{y}=h$ are assumed to be $\overline{T_0}$ and $\overline{T_0}+\varepsilon(\overline{T_0}-\overline{T_S})\cos\bar{\omega}\bar{t}$. Then under usual Boussinesq's approximation and closely following Narahari [15], the flow can be shown to be governed by the system of equations:

$$\frac{\partial\bar{u}}{\partial t}=g\beta(\bar{T}-\bar{T}_s)-\frac{\sigma B_0^2\bar{u}}{\rho}+\upsilon\frac{\partial^2\bar{u}}{\partial\bar{y}^2} \quad (1)$$

$$\rho C_p\frac{\partial\bar{T}}{\partial\bar{t}}=k\frac{\partial^2\bar{T}}{\partial\bar{y}^2}+\mu\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)^2+\sigma B_0^2\bar{u}^2-\frac{\partial q_r}{\partial y} \quad (2)$$

The radiative heat flux q_r under Rosseland approximation by Brewster [16] has the form:

$$q_r=-\frac{4\sigma_1}{3k_1}\frac{\partial\bar{T}^4}{\partial y} \quad (3)$$

Using the equation (3) in equation (2) we get

$$\rho C_p\frac{\partial\bar{T}}{\partial\bar{t}}=k\frac{\partial^2\bar{T}}{\partial\bar{y}^2}+\mu\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)^2+\sigma B_0^2\bar{u}^2+\frac{16\sigma_1\bar{T}_S^3}{3k_1}\frac{\partial^2\bar{T}}{\partial\bar{y}^2} \quad (4)$$

The symbols are defined in the nomenclature.
The relevant boundary conditions are:

At $\bar{y}=0:\bar{u}=0,\bar{T}=\bar{T}_0$ (5)

At $\bar{y}=h:\bar{u}=0,\bar{T}=\bar{T}_0+\varepsilon(\bar{T}_0-\bar{T}_S)\cos\bar{\omega}\bar{t}$ (6)

We introduce the following non-dimensional quantities:

$$u=\frac{\bar{u}h}{\upsilon},\theta=\frac{\bar{T}-\bar{T}_s}{\bar{T}_0-\bar{T}_s},y=\frac{\bar{y}}{h},t=\frac{\bar{t}\upsilon}{h^2},\Pr=\frac{\mu C_p}{k},M=\frac{\sigma B_0^2h^2}{\rho\upsilon},E=\frac{\upsilon^2}{h^2C_p\bar{T}_0-\bar{T}_s}$$

$$Gr=\frac{g\beta(\bar{T}_0-\bar{T}_S)h^3}{\upsilon^2},\omega=\frac{\bar{\omega}h^2}{\upsilon},N=\frac{kk_1}{4\sigma_1\bar{T}_s^3},\lambda=\frac{3N+4}{3N}$$

The non-dimensional forms of the equations (1) and (4) are:

$$\frac{\partial u}{\partial t}=Gr\theta+\frac{\partial^2u}{\partial y^2}-Mu \quad (7)$$

$$\frac{\partial\theta}{\partial t}=\frac{\lambda}{\Pr}\frac{\partial^2\theta}{\partial y^2}+E_c\left(\frac{\partial u}{\partial y}\right)^2+ME_cu^2 \quad (8)$$

Using (5) and (6), the corresponding non-dimensional boundary conditions become:

At $y=0:u=0,\theta=1$ (9)

At $y=1:u=0,\theta=1+\varepsilon\cos\omega t$ (10)

For ease of calculations, boundary condition (10) is modified as under:

At $y=1:u=0,\theta=1+\varepsilon e^{i\omega t}$ (11)

3.METHOD OF SOLUTION:

We assume the solutions of the equations (7) and (8) to be of the following forms:

$$u=u_0(y)+\varepsilon e^{i\omega t}u_1(y)+\ldots\ldots\ldots\ldots \quad (12)$$

$$\theta=\theta_0(y)+\varepsilon e^{i\omega t}\theta_1(y)+\ldots\ldots\ldots\ldots \quad (13)$$

where ε is a small positive quantity and <<1.

Substituting these in equations (7) and (8) and by equating coefficients of the similar terms and neglecting ε^2, the following differential equations are obtained:

$$u_0''-Mu_0=-G_r\theta_0 \quad (14)$$

$$u_1''-(M+i\omega)u_1=-G_r\theta_1 \quad (15)$$

$$\lambda\theta_0''=-\left[E_cP_ru_0'^2+ME_cP_ru_0^2\right] \quad (16)$$

$$\lambda\theta_1''-i\omega P_r\theta_1=-2P_rE_c\left[u_0'u_1'+Mu_0u_1\right] \quad (17)$$

with boundary conditions:

At $y=0:u_0=0,u_1=0,\theta_0=1,\theta_1=0$ (18)

At $y=1:u_0=0,u_1=0,\theta_0=1,\theta_1=1$ (19)

Now using multiparameter perturbation technique, we make the following substitutions using E_c as the perturbation parameter:

$$\left.\begin{aligned}u_0&=f_0+E_cf_1\\u_1&=g_0+E_cg_1\\\theta_0&=h_0+E_ch_1\\\theta_1&=J_0+E_cJ_1\end{aligned}\right\} \quad (20)$$

Using (20) in (14) to (17) and equating the coefficients of E_c^0, E_c^1 and neglecting the higher powers of E_c, we obtain the following set of equations:

$$f_0''-Mf_0=-Grh_0 \quad (21)$$

$$f_1''-Mf_1=-Grh_1 \quad (22)$$

$$g_0''-(M+i\omega)g_0=-GrJ_0 \quad (23)$$

$$g_1''-(M+i\omega)g_1=-GrJ_1 \quad (24)$$

$$h_0''=0 \quad (25)$$

$$\lambda h_1''=-\Pr f_0'^2-M\Pr f_0^2 \quad (26)$$

$$\lambda J_0''-i\omega\Pr J_0=0 \quad (27)$$

$$\lambda J_1''-i\omega\Pr J_1=-2\Pr f_0'g_0'-2\Pr Mf_0g_0 \quad (28)$$

The relevant boundary conditions are:

At $y=0:f_0=0,f_1=0,g_0=0,g_1=0,h_0=1,h_1=0,J_0=0,J_1=0$ (29)

At $y=1:f_0=0,f_1=0,g_0=0,g_1=0,h_0=1,h_1=0,J_0=1,J_1=0$ (30)

The solutions of equations (21) to (28) under the boundary Conditions (29) and (30) as well as the results for u and θ are obtained but not presented here for the sake of brevity.

4.SKIN FRICTION:

The non-dimensional shear stress τ at the plate y=0 is given by :

$$\tau=\frac{h^2}{\upsilon}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0}=\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$=u_0'(0)+\varepsilon e^{i\omega t}u_1'(0)=\tau_r+i\tau_i$$

where τ_r,τ_i are respectively the real and imaginary parts of τ at y=0. Here $\bar{\tau}=\mu\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0}$ is the dimensional shear stress at the plate $\bar{y}=0$.

5.COEFFICIENT OF HEAT TRANSFER:

The heat flux from the plate to the fluid in terms of Nusselt number N_u is given by:

$$N_u = \frac{h\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0}}{\left(\bar{T}_0-\bar{T}_S\right)} = \left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$= \theta_0'(0)+\varepsilon e^{i\omega t}\theta_1'(0) = N_{u_r}+iN_{u_i}$$

where N_{u_r} and N_{u_i} are respectively the real and imaginary parts of N_u at y=0. Here $\overline{N_u}=k\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0}$ is the dimensional heat flux at the plate $\bar{y}=0$.

6.RESULTS AND DISCUSION:

In order to get physical insight into the problem, the numerical calculations are carried out from the solutions for the velocity field, temperature field, rate of heat transfer and shear stress by assigning some solicited values to the parameters involved in the problem. These numerical results have been displayed in figures.

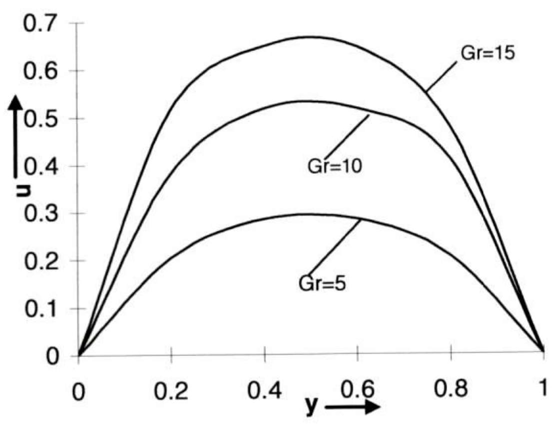

Fig1: velocity profiles for different Gr when M=10,Ec=0.1, N=-1, t=0.1, Pr=1, $\omega=0.5, \varepsilon=0.01$

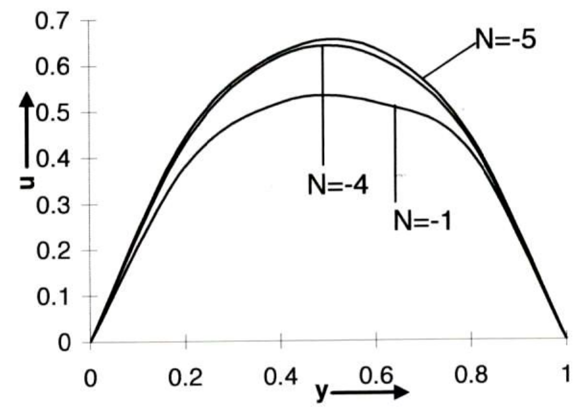

Fig2: velocity profiles for different N when M=10 Ec=0.1,Gr=10, t=0.1, Pr=1, $\omega=0.5, \varepsilon=0.01$

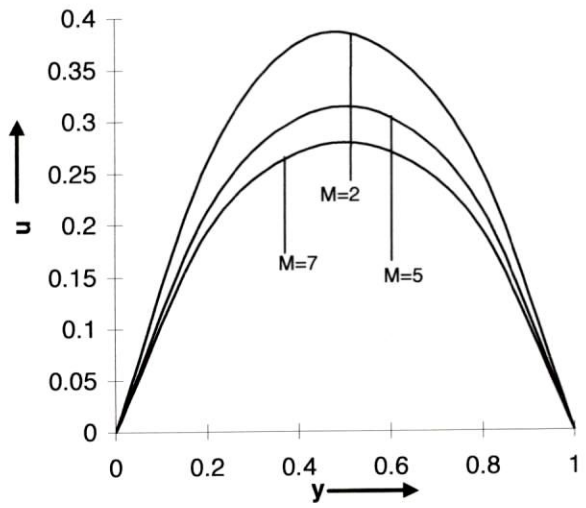

Fig3: velocity profiles for different M when Gr=5, Ec=0.1, N=-1,t=0.1, Pr=7, $\omega=0.2, \varepsilon=0.001$

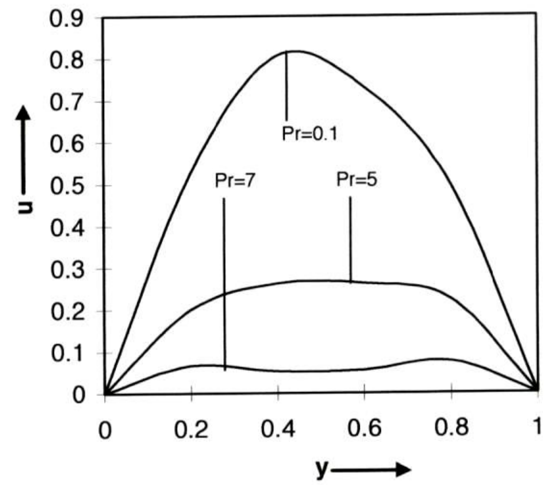

Fig4: velocity profiles for different Pr when M=5, Ec=0.1, N=-1, t=1, Gr=10, $\omega=0.2, \varepsilon=0.001$

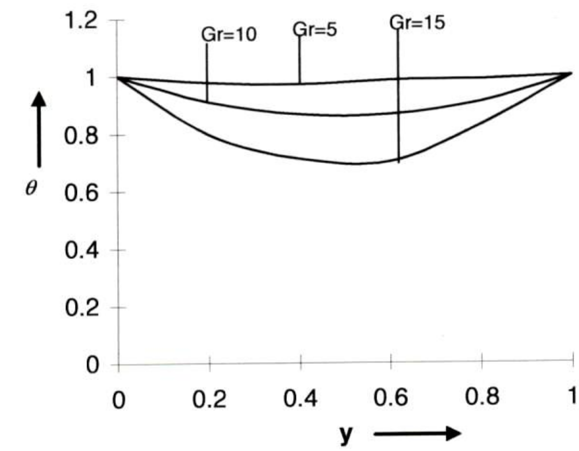

Fig5: Temperature θ against y for different Gr when M=10, Ec=0.1, N=-1,t=0.1,Pr=1, $\omega=0.5, \varepsilon=0.01$

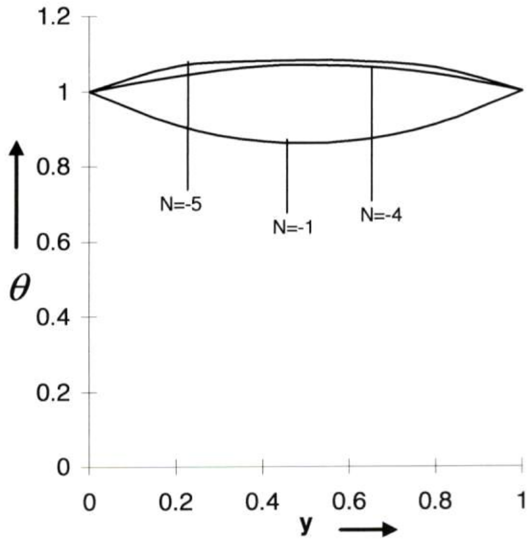

Fig6: Temperature θ against y for different N when M=10, Ec=0.1, t=0.1,Gr=10, Pr=1, $\omega=0.5, \varepsilon=0.01$

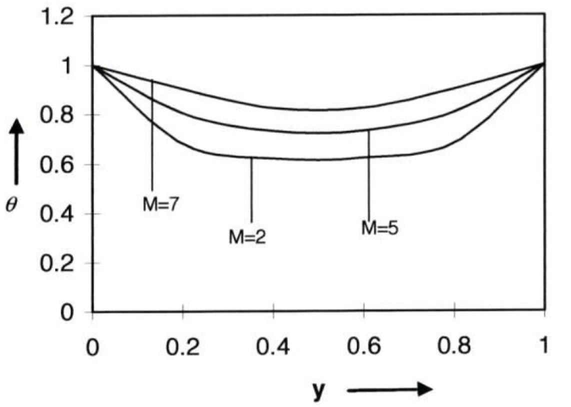

Fig7: Temperature θ against y for different M when N=-1, Ec=0.1, t=1,Gr=5, Pr=7, $\omega=0.2, \varepsilon=0.001$

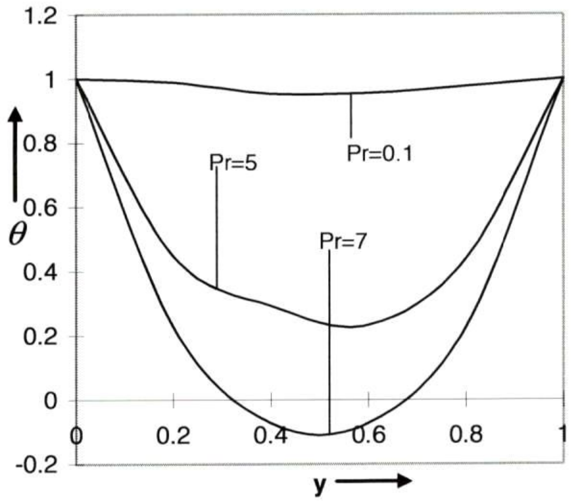

Fig8: Temperature θ against y for different Pr when N=-1, Ec=0.1, t=1,Gr=10, M=5, $\omega=0.2, \varepsilon=0.001$

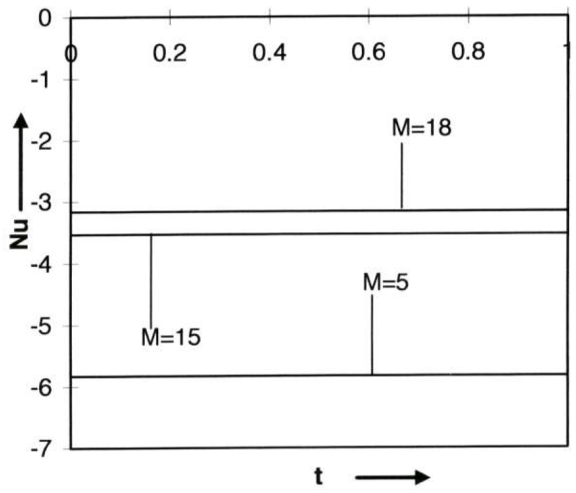

Fig9: Nusselt number Nu against time t for different M when Ec=0.1,N=-1,Gr=10,Pr=7, $\omega=0.2, \varepsilon=0.001$

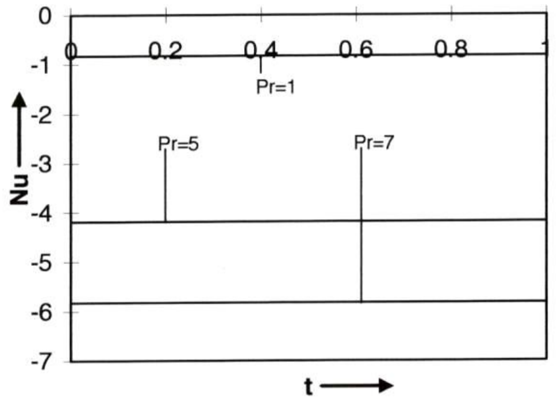

Fig10: Nusselt number Nu against time t for different Pr when M=5, Ec=0.1,N=-1,Gr=10, $\omega=0.2, \varepsilon=0.001$

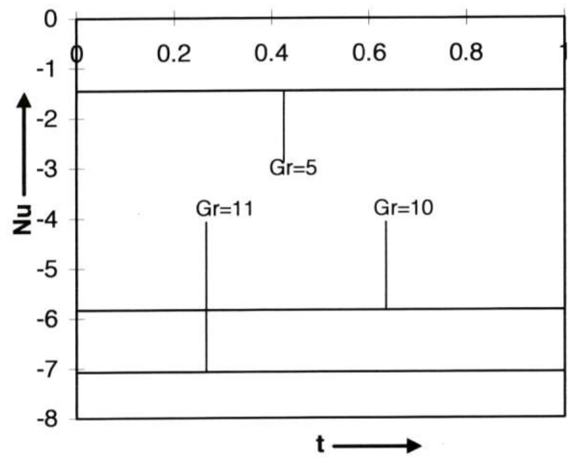

Fig11: Nusselt number Nu against time t for different Gr when M=5, Ec=0.1,N=-1,Pr=7, $\omega=0.2, \varepsilon=0.001$

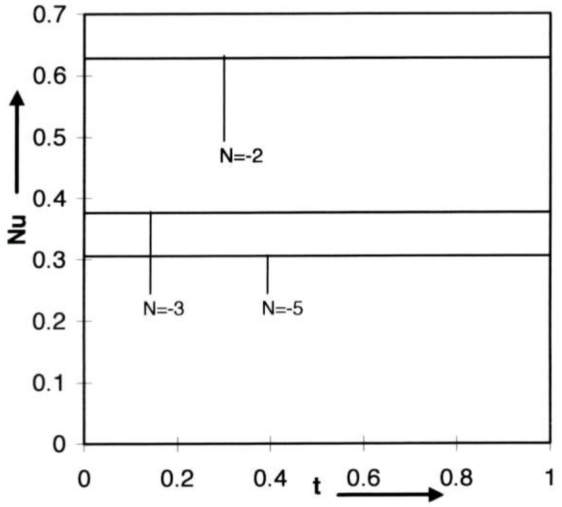

Fig12: Nusselt number Nu against time t for different N when M=10, Ec=0.1,Gr=10,Pr=1, ω=0.5, ε=0.01

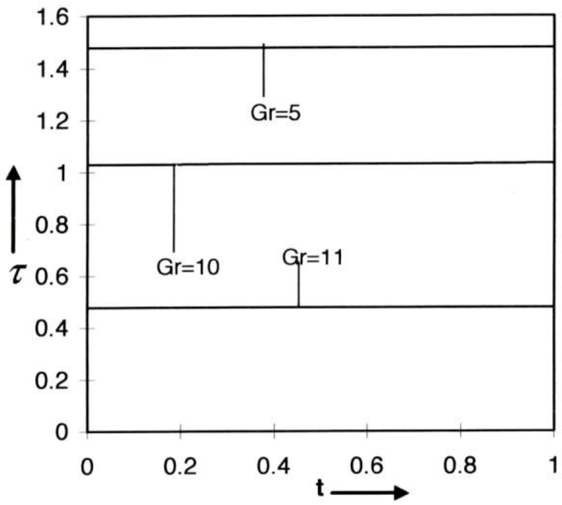

Fig15: Shear stress τ against time t for different Gr when Ec=0.1,N=-1,M=5,Pr=7, ω=0.2, ε=0.001

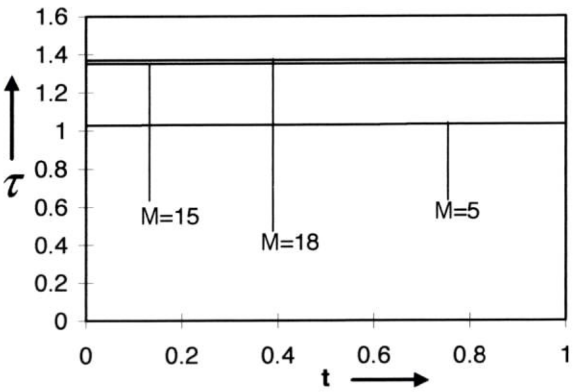

Fig13: Shear stress τ against time t for different M when Ec=0.1,N=-1,Gr=10,Pr=7, ω=0.2, ε=0.001

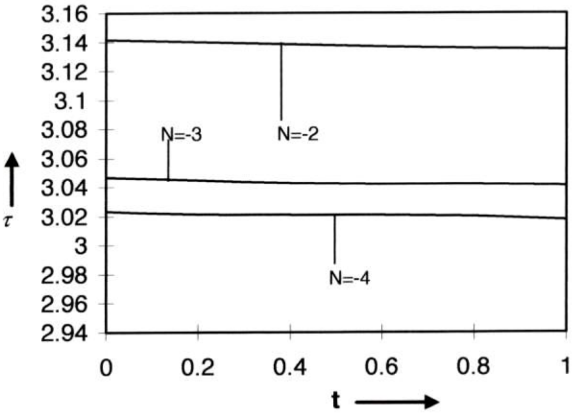

Fig16: Shear stress τ against time t for different N when Ec=0.1,Gr=10,M=10,Pr=1, ω=0.5, ε=0.01

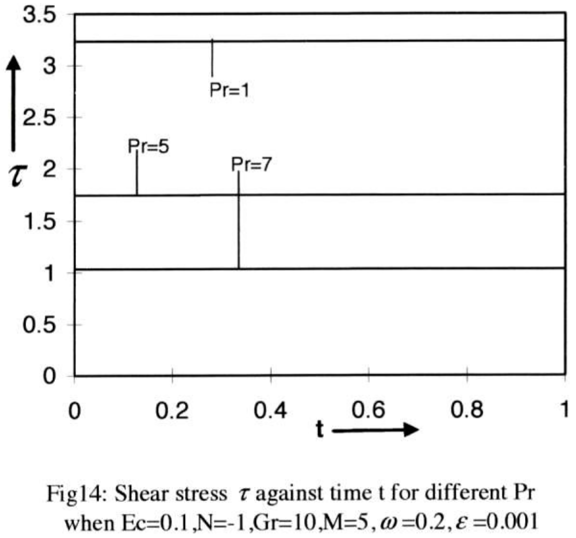

Fig14: Shear stress τ against time t for different Pr when Ec=0.1,N=-1,Gr=10,M=5, ω=0.2, ε=0.001

Figures 1, 2, 3 and 4 display velocity profile versus y for different values of thermal Grashof number Gr, radiation parameter N, Hartmann number M and Prandtl number Pr. It is observed that velocity increases from zero to its maximum value and then leads to zero as y increases. Figure 1 depicts that velocity increases owing to an increase in the value of Gr. The thermal Grashof number Gr signifies the relative effects of the thermal Buoyancy force to viscous force. Here, the positive values of Gr correspond to cooling of the plate. Figure 2 shows that velocity decreases with the increasing values of N. In figure 3, we notice that an increase in M causes the velocity to fall. It is because that the application of transverse magnetic field will result a resistive type force (Lorentz's force), similar to drag force. Physically, fluids with higher Prandtl number have high viscosity and hence moves slowly. This behaviour is evident from figure 4.

Figures 5, 6, 7 and 8 represent the temperature profiles against y for different values of Gr, N, M and Pr. It is marked in figures 5,6 and 8 that the temperature profile decreases with the increasing values of Gr, N and Pr. That is the thickness of the thermal boundary layer decreases with increasing G, N and Pr. The temperature profile under the effect of M is given in figure 7 which shows that, increasing values of M increase temperature profile.

Figures 9,10, 11 and 12 deal with the effects of M, Pr, Gr, and N over the rate of heat transfer in terms of Nusselt number Nu. We observe from figures 9 and 12 that magnitude of the Nusselt number decreases with the increasing M and N. Figures 10 and 11 show that |Nu| increases with the increasing values of Pr and Gr while figure 12 indicates that Nusselt number increases with the increasing values of N

Variations of shear stress τ against time t for various values of M, Pr, Gr and N are plotted through figures 13, 14, 15, and 16. The shear stress τ is increased by Hartmann number M and radiation parameter N as observed from figures 13 and 16. Figure 14 and 15 show that τ is decreased under the influence of Pr and Gr.

7. CONCLUSIONS:

1.The radiation parameter N decelerates the velocity profile and decreases the temperature distribution in the boundary layer.

2.The increase in radiation parameter N leads to increase the rate of heat transfer.

3.An increase in the value of radiation parameter N leads to increase the shear stress τ .

4.An increase in value of magnetic parameter M leads to fall in the velocity which is consistent with the law of physics.

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NOMENCLATURE:

Symbol	Quantity	SI unit
$\vec{g}$	Acceleration due to gravity	m/s^2
u	Velocity	m/s
$\bar{t}$	Time	s
C_p	Specific heat at constant pressure.	J/kg-K
k	Thermal conductivity of the fluid	W/m-K
μ	Coefficient of viscosity	$N\text{-}m/s^2$
M	Hartmann number	
E_c	Eckert number	
Gr	Grashof number for heat transfer.	
Pr	Prandtl number	
N	Radiation parameter	
k_1	Mean absorption coefficient	
$\bar{T}_s$	Temperature at the static condition	
B_0	Strength of the applied magnetic field.	Tesla
P	Fluid pressure	N/m^2
T	Temperature	K

Greek symbol	Quantity	SI unit
θ	Non-dimensional temperature.	
σ	Electrical conductivity	$(ohm\text{-}m)^{-1}$
ε	Small reference parameter,	
ρ	Density of the fluid	kg/m^3
υ	Kinematic viscosity	m^2/s
β	Coefficient of volume expansion	K^{-1}
σ_1	Stefan-Boltzmann constant	
ω	Dimensionless frequency	Hz
$\bar{\omega}$	Frequency of oscillation	