INTERNATIONAL JOURNAL OF HEAT AND TECHNOLOGY
Vol.33, No.2, 2015
http://dx.doi.org/10.18280/ijht.330214

NATURAL CONVECTION OF POWER LAW FLUIDS IN POROUS MEDIA WITH VARIABLE THERMAL AND MASS DIFFUSIVITY

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ABSTRACT

A numerical study for natural convection in heat and mass transfer with power-law fluids along an infinite vertical plate which is a porous medium is given in this paper. Be different from most classical works, the effects of power-law viscosity on temperature and mass are under consideration by supposing that both the thermal diffusivity and mass diffusivity vary as a power-law function. The partial differential equations are transformed into ordinary differential equations by similarity transformation and the numerical solution is obtained with shooting method. The effect of some physical quantities on heat and mass transfer characteristics are discussed, including the power law fluids' index n, the threshold pressure gradient parameter S and the buoyancy ratio N.

Keywords: Power law fluids; natural convection; shooting method; variable thermal diffusivity; variable mass diffusivity.

1. INTRODUCTION

In recent years, as the wide application of non-Newtonian fluids in engineering, such as in oil recovery, heat exchangers, material techniques and so on [1-3], more and more researches on the energy transport and mass transfer behavior of non-Newtonian fluids in porous media have been carried out. Non-Newtonian fluids are used to improve fluid rheological properties and produce damping devices firstly. The polymer fluids [4-5] are non-Newtonian fluids and Mowla and Naderi [6] found that polyisobutylene can make the level of reducing damp in oil recovery to 40%. Then they are also used in individual protection equipments and mechanical processing. Based on the reducing damp property, they can make energy saving obviously in the heating system. As non-Newtonian fluids perform better on heat transfer than Newtonian fluids, so they are used in air conditioner, heating and cooling devices. Non-Newtonian fluids are also common in our daily life, like blood, cell fluid, toothpaste and so on. Unlike Newtonian fluid, the constitutive equation of non-Newtonian fluids does not obey the linear relationship between stress and the rate of strain. The flow and heat transfer of these fluids have been solved by a number of diverse means when their constitutive equations vary greatly in complexity [7-9].

In 1960, Schowalter [8] and Acrivos et al. [9] successfully applied the boundary layer assumptions to the power-law fluids. The boundary layer equations were formulated, and the conditions for the existence of similarity solutions were established. Following the pioneering works of [8, 9], lots of work based on the boundary layer assumption have been done to investigate the flow and heat transfer of non-Newtonian fluid with different physical condition. Abel [10], Datti and Mahesha studied the flow and heat transfer in a power law fluid over a linear stretching sheet with variable thermal conductivity and non-uniform heat source. Chen [11] studied the effects of magnetic field and suction/injection on convection heat transfer of non-Newtonian power-law fluids past a power-law stretched sheet. Zheng et al. [12] proposed a new model by taking the effects of power-law viscosity on temperature field into account, they assumed that the temperature field is similar to the velocity field and the thermal diffusivity varies as a function of temperature gradient. Chen [13] studied magneto-hydrodynamic mixed convection of a power-law fluid on a stretching surface considering thermal radiation. Prasad et al. [14] studied MHD power-law fluid flow and heat transfer over a semi-infinite non-isothermal stretching sheet with internal heat generation or absorption. Pop and Na [15] performed an analysis for the MHD flow past a stretching permeable surface. Prasad et al. [16] did a research about hydromagnetic flow and heat transfer of anon-Newtonian power law fluid over a vertical stretching sheet and got the numerical solutions. Nanofluids which composed of nanoparticles and common fluids, is also a kind of non-Newtonian fluids. Nanofluids have been successfully applied in many practical problems. For example, nanofluids containing surfactant micelles can remedy the soil, remove oily soil and enhance oil recovery [17]. Also nanofluids has implication for cooling equipment and inkjet [18]. And they are widely applied in engineering with enhancement of heat transfer of common fluids. Niu et al. [19] presented a theoretical study on slip-flow and heat transfer of nanofluids choosing power law fluids as based fluids in a microtube. A numerical study on the problem of a steady boundary layer shear flow over a stretching/shrinking sheet in a nanofluid is presented by Yacob et al. [20].

And also some researchers pay attention to the study combining non-Newtonian fluids with convection. A numerical investigation on the convective heat transfer performance of nanofluids over a permeable stretching surface with the consideration of partial slip is presented by Das [21]. Huang et al. [22] presented a numerical study on the periodic unsteady natural convection flow and heat transfer in a square enclosure containing a concentric circular cylinder. Bhowmick et al. [23] investigated mixed convection based on boundary-layer flow of pseudo-plastic fluids from a horizontal circular cylinder with uniform surface heat flux. And in [25-29], Cheng did a lot of work about natural convection of non-Newtonian fluids. Later, Tai and Char [24] numerically studied the combined laminar free convection flow of non-Newtonian power-law fluids along a vertical plate within a porous medium in presence of radiation.

Most of the work mentioned before do not consider the effect of power law fluids on the thermal diffusivity and the mass diffusivity which means the thermal and mass diffusivity vary as a constant. In this paper the laminar boundary layer problem of natural convection from a vertical surface is considered by assuming that both the thermal and mass diffusivity vary as a power law function. The governing equations are solved by similarity transformation and shooting method.

2. FORMULATION OF THE PROBLEM

Consider the problem of natural convection along a vertical plate in a porous medium based on non-Newtonian power-law fluids [29]. The flow is laminar and steady-state. The x-coordinate is from the leading edge of the vertical plate, while the y-coordinate normal to the plate. T_∞ and C_∞ are the ambient temperature and the concentration far from the surface of the plate, respectively. And the governing equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u^n=\frac{K}{\mu_{eff}}[\rho g\beta_T(T-T_\infty)+\rho g\beta_C(C-C_\infty)-\alpha_0]\ if\ [\rho g\beta_T(T-T_\infty)+\rho g\beta_C(C-C_\infty)]>\alpha_0 \tag{2}$$

$$u=0\ if\ [\rho g\beta_T(T-T_\infty)+\rho g\beta_C(C-C_\infty)]\le\alpha_0 \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}(\alpha\frac{\partial T}{\partial y}) \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}(D\frac{\partial C}{\partial y}) \tag{5}$$

and the boundary conditions are:

$$-k(\frac{\partial T}{\partial y})_{y=0}=d_1x=q_w,-D(\frac{\partial C}{\partial y})_{y=0}=d_2x=m_w,v\big|_{y=0}=0,T\big|_{y=\infty}=T_\infty,C\big|_{y=\infty}=C_\infty,u\big|_{y=\infty}=0 \tag{6}$$

Here ρ, u, v, T and C are density of the fluid, x-components of the velocity, y-components of the velocity, temperature of the fluid and mass concentration of the fluid, respectively. The thermal diffusivity in this paper is $\alpha=\tilde{\alpha}\left|\partial T/\partial y\right|^{n-1}$ [12], and with the same idea the mass diffusivity is $D=\tilde{D}\left|\partial C/\partial y\right|^{n-1}$ [30]. In most classical work, α and D are constants. This idea comes from the assumption of the constitutive equation of power-law fluid for boundary layer problems. The constitutive equation of power-law fluid is:

$$\tau=\mu\left(\frac{1}{2}II\right)^{\frac{n-1}{2}}\left(\nabla\mathbf{u}+\nabla\mathbf{u}^T\right) \tag{7}$$

$$II=\left(\nabla\mathbf{u}+\nabla\mathbf{u}^T\right):\left(\nabla\mathbf{u}+\nabla\mathbf{u}^T\right)=\left\|\nabla\mathbf{u}+\nabla\mathbf{u}^T\right\|^2 \tag{8}$$

Here $\mathbf{u}=(u,v)$ is a vector and τ, $\left(\nabla\mathbf{u}+\nabla\mathbf{u}^T\right)$, μ are stress, strain and relative viscosity coefficient, respectively. For two-dimension problem, we get that

$$\nabla\mathbf{u}=\begin{pmatrix}\frac{\partial u}{\partial x} & \frac{\partial v}{\partial x}\\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y}\end{pmatrix},\nabla\mathbf{u}^T=\begin{pmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{pmatrix},\nabla\mathbf{u}+\nabla\mathbf{u}^T=\begin{pmatrix}2\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} & 2\frac{\partial v}{\partial y}\end{pmatrix}$$

$$II=\left\|\nabla\mathbf{u}+\nabla\mathbf{u}^T\right\|^2=4\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+4\left(\frac{\partial v}{\partial y}\right)^2 \tag{9}$$

That means the viscosity $\mu\left(\frac{1}{2}II\right)^{\frac{n-1}{2}}$ of power-law fluid is a power-law function of strain not a constant any more and that obeys the assumption of non-Newtonian fluids that no linear relation between stress and strain. n is power-law index of power-law fluid that Newtonian fluids with $n=1$, dilatant's fluids with $n>1$ and pseudo plastic fluids with $0<n<1$. Then thermal and mass diffusivity of power-law fluids is:

$$\alpha = \tilde{\alpha}\left(\|\nabla T\|^2\right)^{\frac{n-1}{2}},\ D = \tilde{D}\left(\|\nabla C\|^2\right)^{\frac{n-1}{2}}$$

$$\|\nabla T\|^2 = \left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2,\ \|\nabla C\|^2 = \left(\frac{\partial C}{\partial x}\right)^2 + \left(\frac{\partial C}{\partial y}\right)^2 \tag{10}$$

The constitutive equation of power-law fluids changes into the following formula for boundary layer problems:

$$\tau = \mu\left(\frac{\partial u}{\partial y}\right)^{n-1}\frac{\partial u}{\partial y} \tag{11}$$

So we choose the expressions of the thermal diffusivity and mass diffusivity of power-law fluids as the same as Equation (10). Parameters K, $\tilde{D}$, g, q_w, m_w, k^*, k, C_p, β_T, β_C, $\tilde{\alpha}$, α_0 and μ_{eff} are modified permeability, relative mass diffusivity parameter, gravitational acceleration, wall heat flux, wall mass flux, mean absorption coefficient, thermal conductivity, specific heat, coefficient of thermal expansion, coefficient of concentration expansion, relative thermal diffusivity, threshold pressure gradient of the fluid and effective viscosity.

According to the formulations and the similarity transformation, the stream function $\psi(x, y)$, similarity variable η, dimensionless temperature function $\theta(\eta)$ and dimensionless mass function $\varphi(\eta)$ are introduced as:

$$\psi = \left(\frac{\tilde{\alpha}^{n+1}K\rho g\beta_T d_1^{n^2}}{\mu_{eff}k^{n^2}}\right)^{\frac{1}{2n+1}}x^{\frac{n^2+n+1}{2n+1}}f(\eta)$$

$$\eta = \left(\frac{K\rho g\beta_T k^{n^2-n-1}}{\mu_{eff}\tilde{\alpha}^n d_1^{n^2-n-1}}\right)^{\frac{1}{2n+1}}x^{\frac{1-n^2}{2n+1}}y \tag{12}$$

$$T - T_\infty = \left(\frac{K\rho g\beta_T k^{n^2+n}}{\mu_{eff}\tilde{\alpha}^n d_1^{n^2+n}}\right)^{-\frac{1}{2n+1}}x^{\frac{n^2+2n}{2n+1}}\theta(\eta)$$

$$C - C_\infty = \frac{d_2}{D}\left(\frac{K\rho g\beta_T k^{n^2-n-1}}{\mu_{eff}\tilde{\alpha}^n d_1^{n^2-n-1}}\right)^{-\frac{1}{2n+1}}x^{\frac{n^2+2n}{2n+1}}\varphi(\eta) \tag{13}$$

Substituting Equations (12)-(13) into Equations (1), (2), (3), (4) and (5), we get the ordinary differential equations for the nonlinear boundary value problems:

$$f' = [\theta + N\varphi - S]^{1/n} \text{ if } (\theta + N\varphi) > S \tag{14}$$

$$f' = 0 \text{ if } (\theta + N\varphi) \le S \tag{15}$$

$$\frac{n^2+2n}{2n+1}f'\theta - \frac{n^2+n+1}{2n+1}f\theta' = (|\theta'|^{n-1}\theta')' \tag{16}$$

$$\frac{n^2+2n}{2n+1}f'\varphi - \frac{n^2+n+1}{2n+1}f\varphi' = B(|\varphi'|^{n-1}\varphi')' \tag{17}$$

$$f|_{\eta=0} = 0,\ \theta'|_{\eta=0} = -1,\ \varphi'|_{\eta=0} = -1$$
$$f'|_{\eta=\infty} = 0,\ \theta|_{\eta=\infty} = 0,\ \varphi|_{\eta=\infty} = 0 \tag{18}$$

where buoyancy ratio $N = (\beta_C k m_w)/(\beta_T D q_w)$, ratio of parameters of temperature and concentration $B = \left(d_2^{n-1}k^{n-1}\right)/\left(D^{n-2}\alpha d_1^{n-1}\right)$ and $S = \alpha_0\left(Kk^{n^2+n}/\left(\mu_{eff}\tilde{\alpha}^n\rho^{2n}g^{2n}\beta_T^{2n}d_1^{n^2+n}x^{(n^2+n)\lambda+n}\right)\right)^{1/2n+1}$ is dimensionless threshold pressure gradient parameter.

The shooting method coupled with the secant method and the fourth order Runge-Kutta method (RK4) is employed to compute the solutions of Equations (14)-(18).

3. RESULTS AND DISCUSSION

With the computing of shooting method, the results are shown in figures. Figure 1 – Figure 2 show that the effects of power-law index n on the dimensionless temperature θ and the dimensionless concentration φ. When n increases, both θ and φ rises. Figure 3(a) – Figure 3(b) show that the effects of threshold pressure gradient parameter S on the dimensionless temperature θ and the dimensionless concentration φ for pseudo-plastic non-Newtonian fluid ($n = 0.6$). An increase in θ and φ happens straightly with the increasing S. The same phenomenon occurs with dilatant's fluid in Figure 4(a) – Figure 4(b). But the speed of the dimensionless temperature θ and concentration φ in Figure 4 when $S = 0.5$ converging to zero is much slower than $S = 0$. It means that threshold pressure gradient parameter S has an important influence on heat and mass transfer for dilatant's fluids. The effects of buoyancy ratio N on the dimensionless temperature θ and the dimensionless concentration φ are shown in Figure 5-6. The θ and φ are getting smaller with an increase in N.

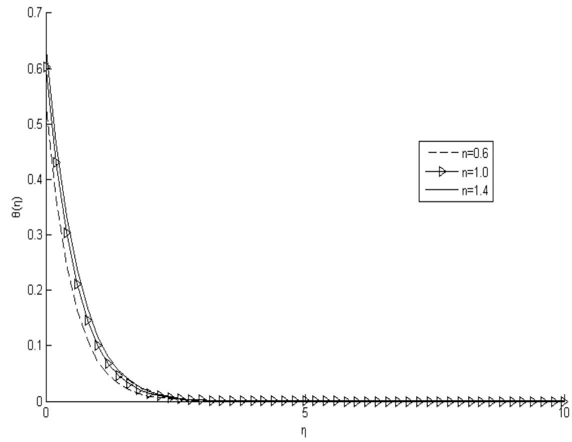


Figure 1. The effects of different values of n on the dimensionless temperature θ for $N = 2, S = 0, B = 2$

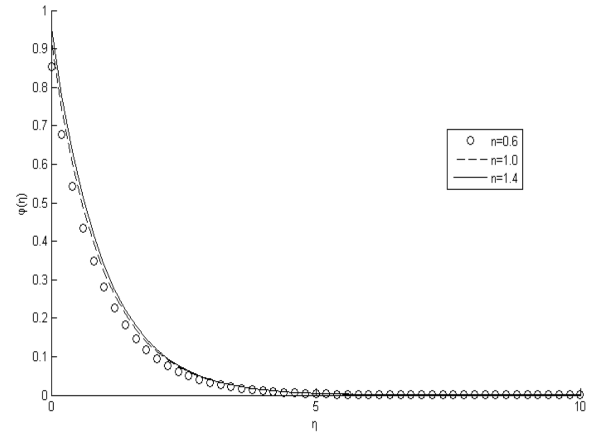
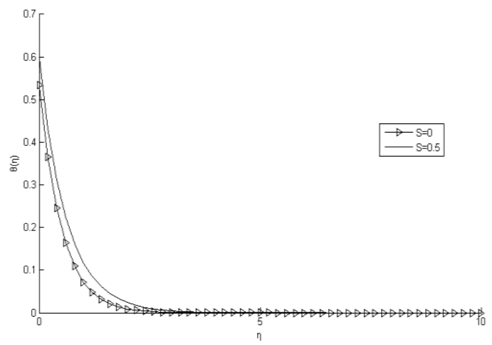
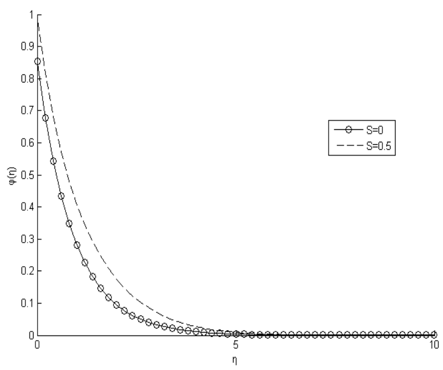


Figure 2. The effects of different values of n on the dimensionless concentration φ for $N = 2, S = 0, B = 2$

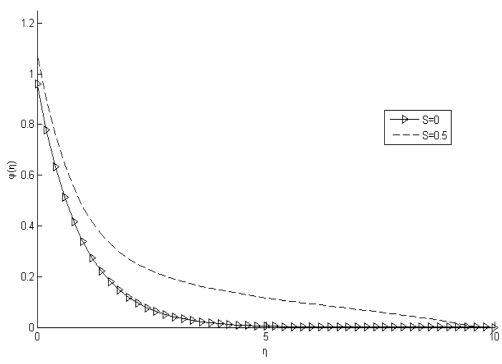


(a)

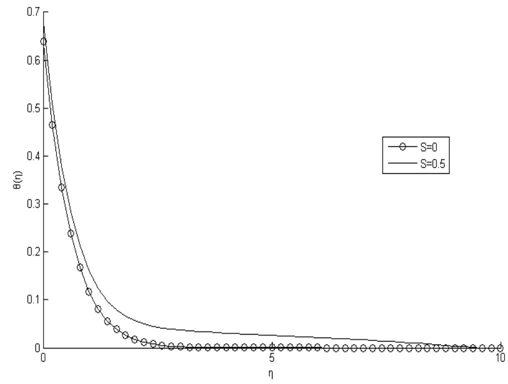


(b)

Figure 3. The effects of different values of S on the dimensionless temperature θ and φ for $n = 0.6, N = 2, B = 2$

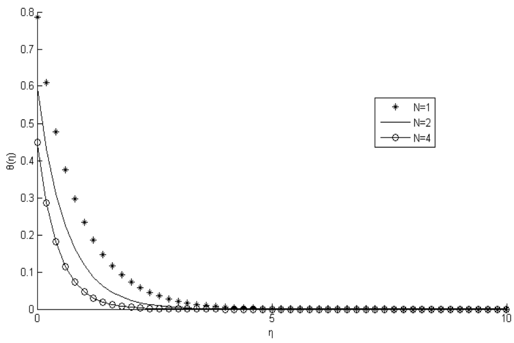


(a)

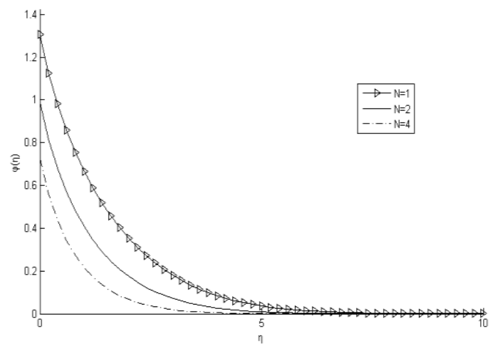


(b)

Figure 4. The effects of different values of S on the dimensionless concentration θ and φ for $n = 1.4, N = 2, B = 2$

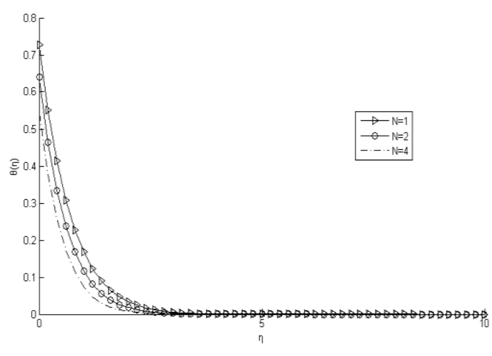


(a)

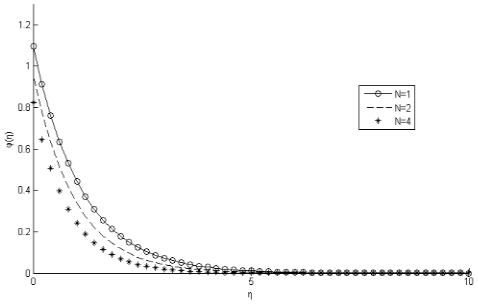


(b)

Figure 5. The effects of different values of N on the dimensionless temperature θ and φ for $n = 0.6, S = 0.5, B = 2$



(a)



(b)

Figure 6. The effects of different values of N on the dimensionless temperature θ and φ for $n = 1.4, S = 0, B = 2$

4. COCLUSION

This paper presents a numerical study on the problem of the rheology of power law fluids along an infinite vertical plate in porous media using the similarity transformation and shooting method. And we also take the situation that thermal and mass diffusivity vary as power law function of the gradient of temperature and concentration into account which is different from classical works. The effects of different parameters on temperature and concentration of the non-Newtonian fluids have been discussed. Some of the important findings of the paper are:

(1) The increasing of power-law index parameter of power law fluids results in the increasing of the temperature and concentration.

(2) Threshold pressure gradient parameter has a much more important influence on heat and mass transfer for dilatant's fluids compared with other parameters.

Data and figures obtained through our studies will be used to support techniques in material and chemical industries.

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