

AN ANALYSIS OF MIXED CONVECTIVE ELASTICO- VISCOUS FLUID PAST A VERTICAL POROUS PLATE IN PRESENCE OF INDUCED MAGNETIC FIELD AND CHEMICAL REACTION

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ABSTRACT

The flow of an MHD elasto-viscous fluid past an infinite vertical porous plate with constant heat flux and chemical reaction in presence of heat source has been studied with the consideration of induced magnetic field. The mechanism of heat and mass transfer has been considered. The elasto-viscous fluid flow is characterized by Walters liquid (Model B'). Analytical solutions to the coupled non-linear equations governing the flow are obtained by using regular perturbation technique. The expressions for velocity field, temperature field, concentration field, induced magnetic field, shearing stress at the plate are derived analytically. The rate of heat transfer and rate of mass transfer of the fluid flow in terms of Nusselt number and Sherwood number at the plate are also obtained in non-dimensional forms. The results are discussed graphically for the various values of elastic-viscous parameter. The importance of this problem is noticed in the field of chemical engineering and geophysical applications.

Keywords: Mixed convection, Perturbation technique, induced magnetic field, Walters liquid (Model B')
2010 Mathematics subject classification: 76A05, 76A10

1. INTRODUCTION

Many natural phenomena and technological problems are susceptible to MHD visco-elastic fluid flow analysis. Such flows play important roles in chemical engineering, turbo-machinery, aerospace technology, polymer industries, paper industries etc.

The convection problem in porous medium has important applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. The phenomena of mass transfer are also significant in theory of stellar structure and observable effects are detectable at least on the solar surface. The thermal physics of hydromagnetic problems with mass transfer is of interest in power engineering and metallurgy. The basis for these models was the early experimental work of Raptis and Kafousias [1] on Magnetohydrodynamics free convective flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Bejan and Khair [2] have reported a pioneering work on heat and mass transfer in a porous medium. Acharya et al. [3] have investigated the magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux. MHD effects on heat and mass transfer in flow of a viscous fluid with induced magnetic field have discussed by Singh and Singh [4]. Postelincus [5] was analyzed influence of a magnetic field on heat and mass transfer by a natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Singh et al. [6] have noticed MHD free convection mass transfer flow past a flat plate. In light of these facts, a number of problems

have been studied by Choudhury and Dey [7, 8], Choudhury and Das [9, 10], Choudhury et al. [11, 12, 13] etc by considering visco-elastic fluid flow phenomenon. Visco-elastic fluid exhibits both the viscous and elastic characteristics. The analysis of heat and mass transfer in MHD visco-elastic fluid flows now forms an integral part of the research activities in inter disciplinary fields.

The objective of the study is to investigate the MHD mixed convective flow with heat source and chemical reaction of a visco-elastic fluid characterized by Walters liquid (Model B') over a vertical porous plate in presence of induced magnetic field. The velocity field and the shearing stress at the plate are obtained and illustrated graphically to observe the visco-elastic effects in combination with other flow parameters.

The constitutive equation for Walters liquid (Model B') is

$$\sigma_{ik} = -p g_{ik} + \sigma'_{ik}, \quad \sigma'^{ik} = 2\eta_0 e^{ik} - 2k_0 e'^{ik} \quad (1.1)$$

where σ^{ik} is the stress tensor, p is isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i , v_i is the velocity vector, the contravariant form of e^{ik} is given by

$$e'^{ik} = \frac{\partial e^{ik}}{\partial t} + v^m e'_{,m}{}^{ik} - v^k_{,m} e'^{im} - v^i_{,m} e'^{mk} \quad (1.2)$$

It is the convected derivative of the deformation rate tensor e^{ik} defined by

$$2e^{ik} = v_{i,k} + v_{k,i} \quad (1.3)$$

Here η_0 is the limiting viscosity at the small rate of shear which is given by

$$\eta_0 = \int_0^\infty N(\tau) d\tau \text{ and } k_0 = \int_0^\infty \tau N(\tau) d\tau \quad (1.4)$$

$N(\tau)$ being the relaxation spectrum as introduced by Walters [14, 15]. This idealized model is a valid approximation of Walters liquid (Model B') taking very short memories into account so that terms involving

$$\int_0^\infty t^n N(\tau) d\tau, n \geq 2 \quad (1.5)$$

have been neglected.

Walters [16] reported that the mixture of polymethyl metha crylate and pyridine at 25° C containing 30.5 gm of polymer per litre and having density 0.98 gm/ml fits very nearly to this model. Polymers are used in the manufacture of space crafts, aeroplanes, tyres, belt conveyers, ropes, cushions, seats, foams, plastic, engineering equipments, contact lens etc. Walters liquid (Model B') forms the basis for the manufacture of many such important and useful products.

2. MATHEMATICAL FORMULATION

Consider the steady flow of a visco-elastic electrically conducting fluid past a vertical porous plate with constant heat flux and chemical reaction in presence of a heat source. A uniform magnetic field is assumed to be applied transversely to the direction of the free stream taking into account the induced magnetic field and mass transfer.

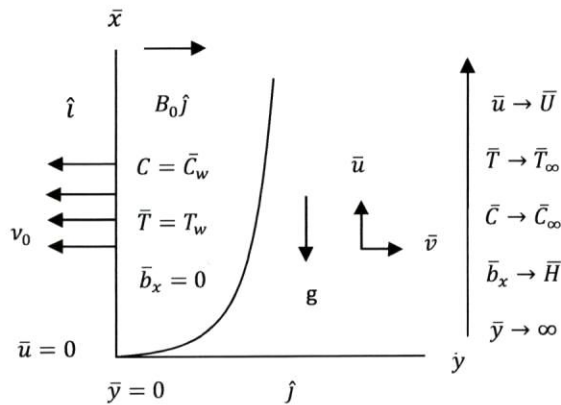


Figure1: The physical model of the problem

Our investigation is restricted to the following assumptions:

- All fluid properties except the density in the buoyancy force term are constant.
- The plate is subjected to constant suction.
- The plate is electrically non conducting.

We introduce a coordinate system $(\bar{x}, \bar{y}, \bar{z})$ with \bar{x} -axis vertical upwards along the plate, \bar{y} -axis perpendicular to it and directed into the fluid region and \bar{z} -axis along the width of the plate. Let $\bar{q} = (\bar{u}, \bar{v}, 0)$ be the fluid velocity and $(\bar{b}_x, \bar{b}_y, 0)$ be the components of magnetic induction vector at a point $(\bar{x}, \bar{y}, \bar{z})$ in the fluid.

With the foregoing assumption, Boussinesq approximation and under usual boundary layer approximations, the governing equations are:

Equation of Continuity:

$$\frac{d\bar{v}}{d\bar{y}} = 0 \quad (2.1)$$

which is satisfied with $v = -v_0$, a constant.

Momentum equation:

$$-v_0 \frac{d\bar{u}}{d\bar{y}} = v \frac{d^2\bar{u}}{d\bar{y}^2} + \frac{k_0}{\rho} v_0 \frac{d^3\bar{u}}{d\bar{y}^3} + g\beta(\bar{T} - \bar{T}_\infty) + g\bar{\beta}(\bar{C} - \bar{C}_\infty) + \frac{\eta\sigma B_0}{\rho} \frac{d\bar{b}_x}{d\bar{y}} \quad (2.2)$$

Energy equation:

$$-v_0 \frac{d\bar{T}}{d\bar{y}} = \frac{k}{\rho C_p} \frac{d^2\bar{T}}{d\bar{y}^2} + \frac{v}{C_p} \left(\frac{d\bar{u}}{d\bar{y}} \right)^2 + \frac{k_0 v_0}{\rho C_p} \frac{d\bar{u}}{d\bar{y}} \frac{d^2\bar{u}}{d\bar{y}^2} + \frac{\sigma \eta^2}{\rho C_p} \left(\frac{d\bar{b}_x}{d\bar{y}} \right)^2 + \frac{Q(\bar{T}_\infty - \bar{T})}{\rho C_p} \quad (2.3)$$

Magnetic induction equation:

$$\eta \frac{d^2\bar{b}_x}{d\bar{y}^2} + B_0 \frac{d\bar{u}}{d\bar{y}} + v_0 \frac{d\bar{b}_x}{d\bar{y}} = 0 \quad (2.4)$$

Species continuity equation:

$$-v_0 \frac{d\bar{C}}{d\bar{y}} = D \frac{d^2\bar{C}}{d\bar{y}^2} - \bar{k}(\bar{C} - \bar{C}_\infty) \quad (2.5)$$

where k_0 is the visco-elastic parameter, σ is the electrical conductivity, k is the thermal conductivity, C_p is the specific heat at constant pressure, η is the magnetic diffusivity, ρ is the density of the fluid, Q is the heat source parameter, \bar{C} is the species concentration, D is the coefficient of chemical molecular diffusivity and the other symbols have their usual meanings.

The relevant boundary conditions are

$$\left. \begin{aligned} \bar{y} = 0: \bar{u} = 0, \frac{d\bar{T}}{d\bar{y}} = -\frac{\bar{q}}{K}, \bar{b}_x = 0, \bar{C} = \bar{C}_\infty \\ \bar{y} \rightarrow \infty: \bar{u} = \bar{U}, \bar{T} \rightarrow \bar{T}_\infty, \bar{b}_x = \bar{H}, \bar{C} \rightarrow \bar{C}_\infty \end{aligned} \right\} \quad (2.6)$$

We introduce the following non-dimensional quantities:

$$\begin{aligned} y = \frac{\bar{y}v_0}{v}, u = \frac{\bar{u}}{v_0}, \theta = \frac{Kv_0(\bar{T} - \bar{T}_\infty)}{\bar{q}}, \\ U = \frac{\bar{U}}{v_0}, b_x = \frac{\bar{b}_x}{B_0}, K = \frac{\bar{k}v}{v_0^2}, Ec = \frac{Kv_0^3}{\bar{q}vC_p}, \\ \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty}, Gr = \frac{g\beta v \bar{q}}{Kv_0^4}, S = \frac{Qv^2}{Kv_0^2}, \\ G_m = \frac{g\bar{\beta}v(\bar{C}_w - \bar{C}_\infty)}{v_0^3}, Sc = \frac{v}{D}, M = \frac{\sigma B_0^2 v}{\rho v_0^2}, \\ P_r = \frac{\mu C_p}{k}, P_m = \frac{v}{\eta} \end{aligned}$$

where Ec is the Eckert number, Gr is the Grashof number for heat transfer, S is the heat source parameter, G_m is the Grashof number for mass transfer, Sc is the Schmidt number,

M is the Hartmann number, Pr is the Prandtl number and Pm is the magnetic Prandtl number.

The non-dimensional governing equations are

$$k_1 \frac{d^3 u}{dy^3} + \frac{d^2 u}{dy^2} + \frac{du}{dy} = -Gr\theta - Gm\phi - \frac{M}{P_m} \frac{db_x}{dy} \quad (2.7)$$

$$\begin{aligned} \frac{d^2 \theta}{dy^2} + Pr \frac{d\theta}{dy} - S\theta \\ = -EcPr \left(\frac{du}{dy} \right)^2 - k_1 EcPr \frac{du}{dy} \frac{d^2 u}{dy^2} \\ - \frac{MEcPr}{Pm^2} \left(\frac{db_x}{dy} \right)^2 \end{aligned} \quad (2.8)$$

$$\frac{d^2 b_x}{dy^2} + Pm \frac{db_x}{dy} = -Pm \frac{du}{dy} \quad (2.9)$$

$$\frac{1}{Sc} \frac{d^2 \phi}{dy^2} + \frac{d\phi}{dy} - k\phi = 0 \quad (2.10)$$

subject to boundary conditions:

$$\left. \begin{aligned} y = 0: u = 0, \frac{d\theta}{dy} = -1, \phi = 1, b_x = 0 \\ y \rightarrow \infty: u = U, \theta = 0, \phi = 0, b_x = H \end{aligned} \right\} \quad (2.11)$$

METHOD OF SOLUTIONS

The solution of the equation (2.10) subject to the boundary conditions (2.11) is

$$\phi = e^{-A_1 y} \quad (3.1)$$

Now, in order to solve the equations (2.7)-(2.9) under the boundary conditions given by (2.11), it is assumed that the solutions of the equations to be of the form

$$u = u_0 + Ecu_1 + Ec^2 u_2 + \dots \quad (3.2)$$

$$\theta = \theta_0 + Ec\theta_1 + Ec^2 \theta_2 + \dots \quad (3.3)$$

$$b_x = \phi_0 + Ec\phi_1 + Ec^2 \phi_2 + \dots \quad (3.4)$$

where $Ec \ll 1$.

Substituting (3.2)-(3.4) in the equations (2.7)-(2.9) and equating the coefficient of the same degree terms and neglecting terms of $O(Ec^2)$, the following differential equations are obtained:

Zeroth order equations:

$$k_1 u_0''' + u_0'' + u_0' = -Gr\theta_0 - Gme^{-A_1 y} - \frac{M}{P_m} b_{x0}' \quad (3.5)$$

$$\theta_0'' + Pr\theta_0' - \alpha\theta_0 = 0 \quad (3.6)$$

$$b_{x0}'' + Pmb_{x0}' = -Pmu_0' \quad (3.7)$$

First order equations:

$$k_1 u_1''' + u_1'' + u_1' = -Gr\theta_1 - \frac{M}{P_m} b_{x1}' \quad (3.8)$$

$$\begin{aligned} \theta_1'' + Pr\theta_1' - S\theta_1 \\ = -Pr u_0'^2 - k_1 Pr u_0' u_0'' - \frac{MPr}{Pm^2} b_{x0}'^2 \end{aligned} \quad (3.9)$$

$$b_{x1}'' + Pmb_{x1}' = -Pmu_1' \quad (3.10)$$

corresponding to the boundary conditions

$$y = 0: u_0 = 0, u_1 = 0, \theta_0' = -1, \theta_1' = 0, b_{x0} = 0, b_{x1} = 0$$

$$y \rightarrow \infty: u_0 = U, u_1 = 0, \theta_0 = 0, \theta_1 = 0, \phi = 0, b_{x0} = H,$$

$$b_{x1} = 0 \quad (3.11)$$

Using multi-parameter perturbation technique and taking $k_1 \ll 1$ (as for small shear rate k_1 is very small), we assume

$$u_0 = u_{00} + k_1 u_{01} \quad (3.12)$$

$$u_1 = u_{10} + k_1 u_{11} \quad (3.13)$$

$$b_{x0} = b_{x00} + k_1 b_{x01} \quad (3.14)$$

$$b_{x1} = b_{x10} + k_1 b_{x11} \quad (3.15)$$

$$\theta_1 = \theta_{10} + k_1 \theta_{11} \quad (3.16)$$

Now, using equations (3.12)-(3.16) in equations (3.5), (3.7), (3.8), (3.9) and (3.10) and equating the coefficients of like powers of k_1 and neglecting the higher power of k_1 , we get the following set of differential equations:

Zeroth-order equations:

$$u_{00}'' + u_{00}' = -GrB_1 e^{-A_2 y} - Gme^{-A_1 y} - \frac{M}{P_m} b_{x00}' \quad (3.17)$$

$$b_{x00}'' + Pmb_{x00}' = -Pmu_{00}' \quad (3.18)$$

$$u_{10}'' + u_{10}' = -Gr\theta_{10} - \frac{M}{P_m} b_{x10}' \quad (3.19)$$

$$\theta_{10}'' + Pr\theta_{10}' - S\theta_{10} = -Pr u_{00}'^2 - \frac{MPr}{Pm^2} b_{x00}'^2 \quad (3.20)$$

$$b_{x10}'' + Pmb_{x10}' = -Pmu_{10}' \quad (3.21)$$

First order equations:

$$u_{00}''' + u_{00}'' + u_{00}' = -\frac{M}{P_m} b_{x01}' \quad (3.22)$$

$$b_{x01}'' + Pmb_{x01}' = -Pmu_{01}' \quad (3.23)$$

$$u_{10}''' + u_{10}'' + u_{10}' = -Gr\theta_{11} - \frac{M}{P_m} b_{x11}' \quad (3.24)$$

$$\begin{aligned} \theta_{11}'' + Pr\theta_{11}' - S\theta_{11} \\ = -2Pr u_{00}' u_{01}' - Pr u_{00}' u_{00}'' \\ - 2 \frac{MPr}{Pm^2} b_{x00}' b_{x01}' \end{aligned} \quad (3.25)$$

$$b_{x11}'' + Pmb_{x11}' = -Pmu_{11}' \quad (3.26)$$

subject to boundary conditions

$$y = 0: u_{00} = 0, u_{01} = 0, u_{10} = 0, u_{11} = 0, \theta_{00}' = -1, \theta_{01}' = 0,$$

$$\theta_{10}' = 0, \theta_{11}' = 0, b_{x00} = 0, b_{x01} = 0, b_{x10} = 0, b_{x11} = 0$$

$$y \rightarrow \infty: u_{00} = U, u_{01} = 0, u_{10} = 0, u_{11} = 0, \theta_{00}' = 0, \theta_{01}' = 0,$$

$$\theta_{10}' = 0, \theta_{11}' = 0, b_{x00} = H, b_{x01} = 0, b_{x10} = 0, b_{x11} = 0 \quad (3.27)$$

The solutions of the equations (3.17)-(3.26) subject to the boundary conditions (3.27) and constants are obtained but not given here due to brevity.

3. RESULTS AND DISCUSSION

The velocity profile is given by

$$u = (u_{00} + k_1 u_{01}) + Ec(u_{10} + k_1 u_{11}) \quad (4.1)$$

The non-dimensional skin friction σ at the plate $y=0$ is given by

$$\sigma = \left[\frac{\partial u}{\partial y} + k_1 \frac{\partial^2 u}{\partial y^2} \right]_{y=0} \quad (4.2)$$

The non-dimensional heat flux σ at the plate $y=0$ in terms of Nusselt number is given by

$$Nu = \left(\frac{\partial \theta}{\partial y} \right)_{y=0} \quad (4.3)$$

The non-dimensional mass flux σ at the plate $y=0$ in terms of Sherwood number is given by

$$Sh = \left(\frac{\partial \phi}{\partial y} \right)_{y=0} \quad (4.4)$$

The purpose of the present study is to bring out the effects of elasto-viscous parameter on mixed convective MHD flow of a visco-elastic fluid past an infinite vertical porous surface by imposing the effect of induced magnetic field in the governing fluid flow system. The elasto-viscous effect is exhibited through the non-dimensional parameter k_1 . The non zero values of the parameter k_1 characterize the visco-elastic fluid and $k_1=0$ represents the Newtonian fluid flow phenomenon.

In order to get physical insight into the problem the fluid velocity u is depicted against y in the figures 2-8. The variation of skin friction σ against various flow parameters viz. Pm, Pr, Gm, S, Sc, M is illustrated in the figures 9-14. Figure 15-17 reveal the variation of Nusselt number against the flow parameter Gr, Pr and Sc. The numerical calculations are to be carried out for $U=1$, $Ec=0.001$, $k=0.5$ throughout the discussion. The various combinations of flow parameters are given in the table 1.

figures 2 to 8 represent the pattern of velocity profile u against the distance y for various values of other flow parameters. The graphs show that the velocity profile boosts up considerably in the neighbourhood of the plate and then it starts to converge to free stream velocity for both Newtonian and visco-elastic. The elasticity factor of Walters liquid (Model B') diminishes the speed of the fluid in comparison with a Newtonian fluid.

Figures 2 and 3 represent the variation of fluid velocity against Grashof number for heat transfer (Gr) and Grashof number for mass transfer (Gm). Grashof number studies the behaviour of free convection and it is defined as the ratio of buoyancy force to viscous force. It plays an important role in both heat and mass transfer mechanisms. Gr characterizes the free convection parameter for heat transfer and Gm characterizes the free convection parameter for mass transfer. In both the cases, it is observed that velocity profile boost up to a considerable amount and then follows a steady path.

The effect of Pr on the fluid flow is illustrated in Figure 4. With the rising value of Prandtl number fluid velocity experiences a decelerating trend. This phenomenon is observed in both Newtonian and elasto-viscous fluid.

The magnetic Prandtl number (Pm) signifies the relative importance of momentum diffusion and magnetic diffusion as it is defined as the ratio of momentum diffusivity to magnetic diffusivity. The effects of Pm on Newtonian and

non-Newtonian fluids have been observed in figure 5. It shows that the rising value of Pm accelerates the fluid flow but due to the presence of elasticity, the visco-elastic fluid flow experiences a declined trend during the enhancement of magnetic Prandtl number.

The effects of heat source parameter and Hartmann number on fluid velocity have been cited in figure 6 and 7. An inclined trend is observed for both kind of velocity profile with the growing nature of magnetic parameter and heat source parameter. The maximum effect of both the parameters on visco-elastic fluid and Newtonian fluid is seen in the neighbourhood of the plate.

Schmidt number signifies the ratio of momentum diffusivity to concentration diffusivity. The role of Schmidt number on the fluid velocity is illustrated in figure 8. Increasing value of Schmidt number increases the velocity of the Newtonian fluid as well as visco-elastic fluid. Also, the velocity of the visco-elastic fluid subdues with the enhancement of Schmidt number in comparison with simple Newtonian fluid.

Figures 9 to 14 exhibit the variation of skin friction σ against various flow parameters. In addition to this, the visco-elastic effects of skin frictions are also measured in these graphs. The elasticity factor present in the visco-elastic fluid subdues the shearing stress at the plate in comparison with Newtonian fluid.

Figures 9 and 10, characterize the variations of shearing stress against Pm and Pr. The positive values of Prandtl number signify the dominant effect of viscosity. It is noticed that the shearing stress formed by the visco-elastic fluid flow is negative, which interprets that the viscous drag experiences a reverse direction.

Figure 11 and 12 shows that the magnitude of skin friction reduced along with the amplified values of Grashof number for mass transfer Gm and heat source parameter S for Newtonian as well as non-Newtonian cases.

Figure 13 shows the effects of Schmidt number on shearing stress. It is noticed that the magnitude of skin friction increases with the increasing value of Sc for visco-elastic fluid but an opposite behaviour is observed in case of Newtonian fluid. The intensity of transverse magnetic field is shown in figure 14. Hartmann number subdues the shearing stress of visco-elastic fluid in comparison with the Newtonian fluid.

Nusselt number studies the rate of heat transfer through the fluid system. Here we have investigated the nature of Nusselt number on the flat plate. The graphical presentations of rate of heat transfer are given in figures 15 to 17. Visco-elasticity factor present in the complex fluid flow system subdues the rate of heat transfer in comparison with the Newtonian fluid flow system.

Figure 15 characterizes the pattern of rate of heat transfer against Gr. It shows that increasing values of Gr modify the rate of heat transfer of Newtonian fluid in comparison with visco-elastic fluids. Figures 16 and 17 analyze the effects of heat source parameter S and Schmidt number Sc on variation of Nusselt number. In the both the cases, it is observed that with the increasing values of flow parameters Nusselt number for non-Newtonian fluid increases.

Table 2 demonstrates the variation of induced magnetic field b_x for various combinations of flow parameters (Table 1). From the table 2, it is observed that b_x enhances with the increasing values of the visco-elastic parameter k_1 in comparison with the Newtonian fluid for different cases. Also, the induced magnetic field b_x increases with the increase of Grashof number Gr (cases I & II) for visco-elastic fluid, but remains same for viscous fluid. Due to the increase of all other flow parameters (cases I & III, IV, V, VI, VII, VIII) the values of induced magnetic field slightly diminishes for non-Newtonian cases but modifies in case of Newtonian cases.

4. CONCLUSIONS

The study concludes the following results:

- The velocity profile shows an enhancement trend in the neighbourhood of the plate and then follows a steady path.
- The visco-elasticity factor decelerates the speed of fluid flow in comparison with the Newtonian fluid.
- The shearing stress formed at the plate is subdued with the growing trend of visco-elastic parameter.
- The increasing values of Gm, S and M lessen the shearing stress formed by visco-elastic fluid.
- Skin friction profile modifies with the increasing values of Schmidt number Sc.
- Rate of heat transfer enhances with the enhancement of S and Sc.
- The rate of mass transfer is not significantly affected by visco-elastic parameter.

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Table 1: Various combinations of flow parameters

| Cases | Gr | Gm | Pr | Pm | S | M | Sc |
|-------|----|----|----|----|---|---|----|
| I | 8 | 7 | 5 | 4 | 2 | 5 | 5 |
| II | 12 | 7 | 5 | 4 | 2 | 5 | 5 |
| III | 8 | 10 | 5 | 4 | 2 | 5 | 5 |
| IV | 8 | 7 | 8 | 4 | 2 | 5 | 5 |
| V | 8 | 7 | 5 | 6 | 2 | 5 | 5 |
| VI | 8 | 7 | 5 | 4 | 4 | 5 | 5 |
| VII | 8 | 7 | 5 | 4 | 2 | 8 | 5 |
| VIII | 8 | 7 | 5 | 4 | 2 | 5 | 6 |

Table 2: The values of induced magnetic field b_x .

| Cases | $k_I=0$ | | | $k_I=0.05$ | | | $k_I=0.1$ | | |
|-------|---------|--------|-------|------------|--------|--------|-----------|--------|--------|
| | $y=0$ | $y=2$ | $y=4$ | $y=0$ | $y=2$ | $y=4$ | $y=0$ | $y=2$ | $y=4$ |
| I | 0 | 0.9998 | 1 | 0 | 1.0123 | 1.0017 | 0 | 1.0186 | 1.0025 |
| II | 0 | 0.9998 | 1 | 0 | 1.0125 | 1.0017 | 0 | 1.0188 | 1.0025 |
| III | 0 | 0.9998 | 1 | 0 | 1.0123 | 1.0016 | 0 | 1.0185 | 1.0025 |
| IV | 0 | 0.9999 | 1 | 0 | 1.0119 | 1.0016 | 0 | 1.0179 | 1.0024 |
| V | 0 | 1 | 1 | 0 | 1.0099 | 1.0013 | 0 | 1.0148 | 1.0020 |
| VI | 0 | 1 | 1 | 0 | 1.0117 | 1.0016 | 0 | 1.0176 | 1.0024 |
| VII | 0 | 1.0001 | 1 | 0 | 1.0027 | 1.0004 | 0 | 1.0041 | 1.0005 |
| VIII | 0 | 0.9999 | 1 | 0 | 1.0107 | 1.0014 | 0 | 1.0161 | 1.0022 |

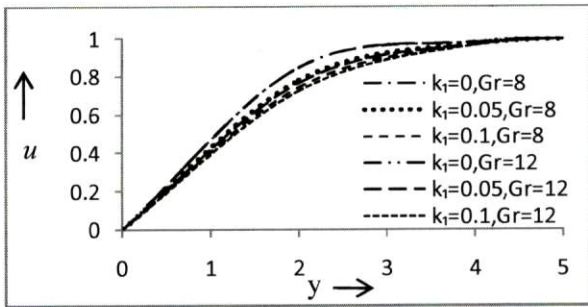


Figure2. Variation of transient velocity u against y for $Pr=5$, $Pm=4$, $Gm=7$, $S=2$, $Sc=5$, $M=5$.

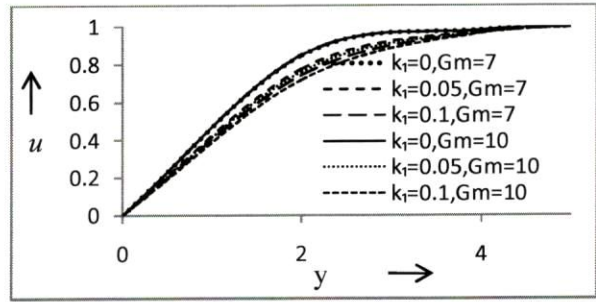


Figure3. Variation of transient velocity u against y for $Pr=5$, $Pm=4$, $Gr=8$, $S=2Sc=5$, $M=5$.

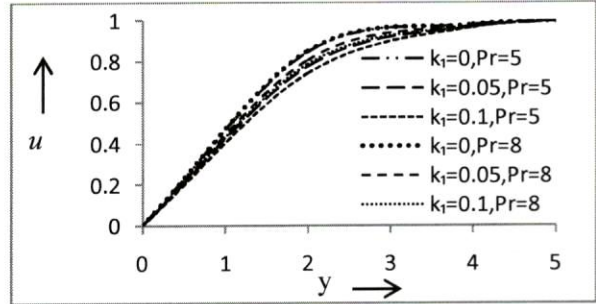


Figure4. Variation of transient velocity u against y for $Gr=8$, $Pm=4$, $Gm=7$, $S=2$, $Sc=5$, $M=5$.

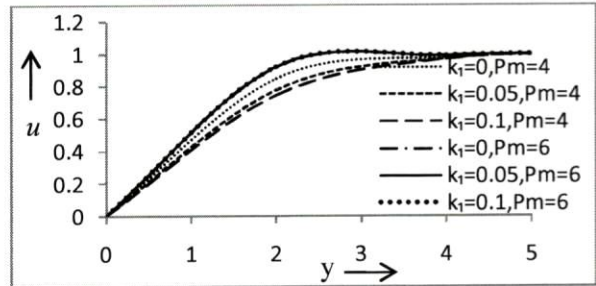


Figure5. Variation of transient velocity u against y for $Pr=5$, $Gr=8$, $Gm=7$, $S=2$, $Sc=5$, $M=5$.

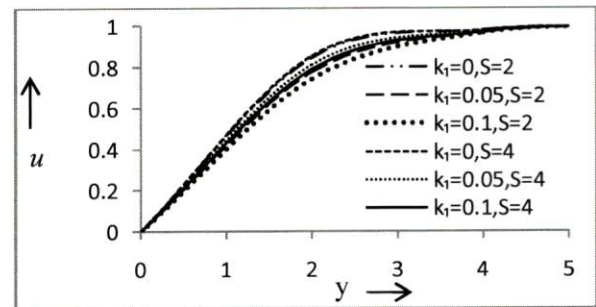


Figure6. Variation of transient velocity u against y for $Pr=5$, $Pm=4$, $Gm=7$, $Gr=8$, $Sc=5$, $M=5$.

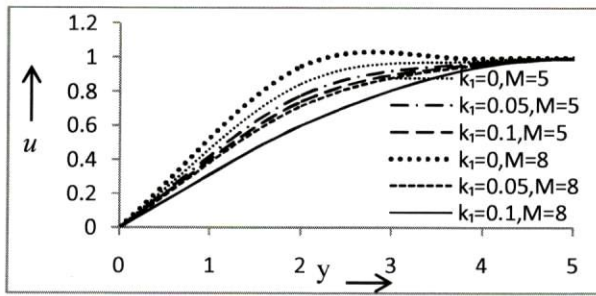


Figure7. Variation of transient velocity u against y for $Pr=5$, $Pm=4$, $Gm=7$, $S=2$, $Sc=5$, $Gr=8$.

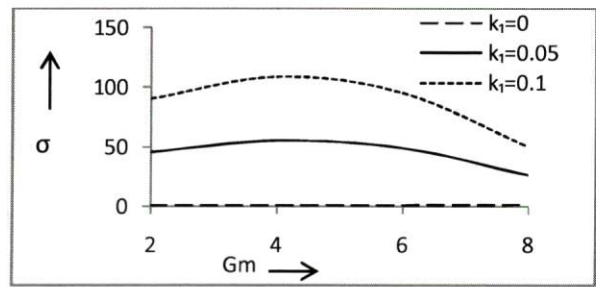


Figure11. Variation of skin friction σ at the plate $y=0$ against Gm for $Pr=5$, $Gr=8$, $Pm=4$, $S=2$, $Sc=5$, $M=5$.

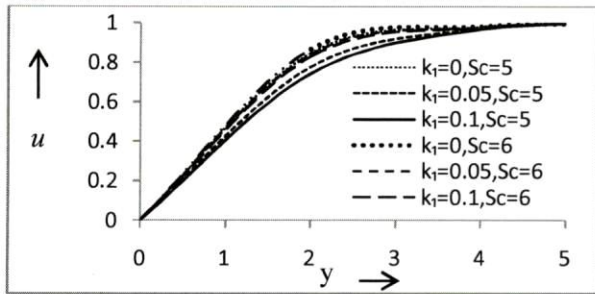


Figure8. Variation of transient velocity u against y for $Pr=5$, $Pm=4$, $Gm=7$, $S=2$, $Gr=8$, $M=5$.

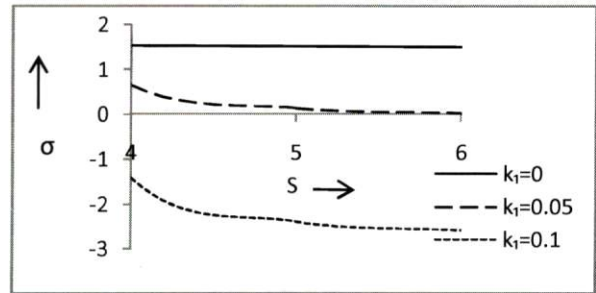


Figure12. Variation of skin friction σ at the plate $y=0$ against S for $Pr=5$, $Gr=8$, $Gm=7$, $Pm=4$, $Sc=5$, $M=5$.

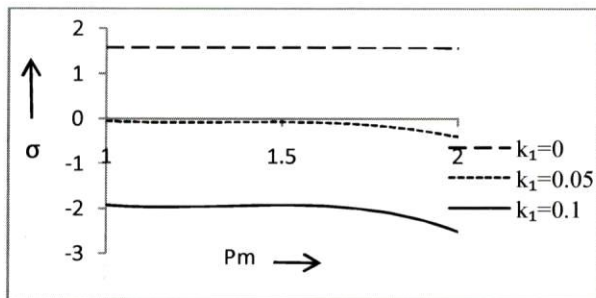


Figure9. Variation of skin friction σ at the plate $y=0$ against Pm for $Pr=5$, $Gr=8$, $Gm=7$, $S=2$, $Sc=5$, $M=5$.

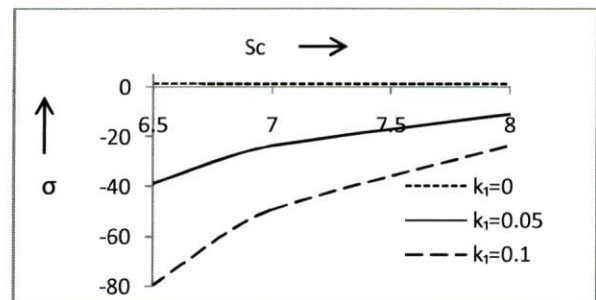


Figure13. Variation of skin friction σ at the plate $y=0$ against Sc for $Pr=5$, $Gr=8$, $Gm=7$, $S=2$, $Pm=4$, $M=5$.

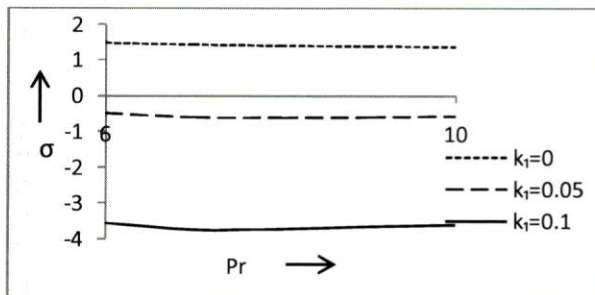


Figure10. Variation of skin friction σ at the plate $y=0$ against Pr for $Pm=4$, $Gr=8$, $Gm=7$, $S=2$, $Sc=5$, $M=5$.

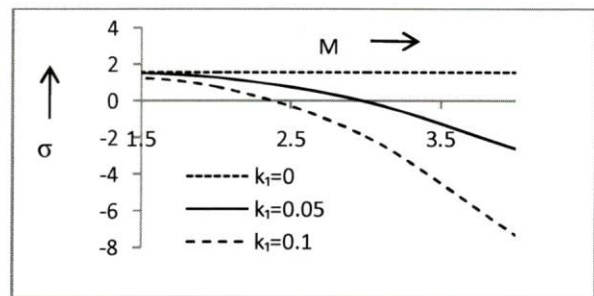


Figure14. Variation of skin friction σ at the plate $y=0$ against M for $Pr=5$, $Gr=8$, $Gm=7$, $S=2$, $Sc=5$, $Pm=4$.

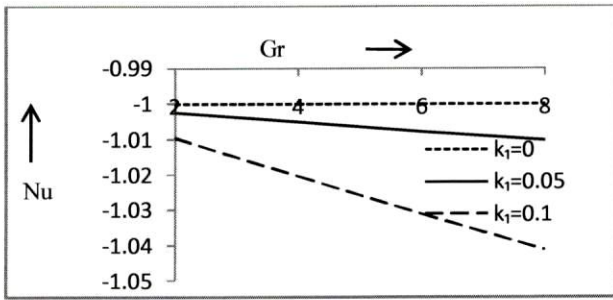


Figure15. Nusselt number against Gr for Pr=5, Sc=5, Gm=7, S=2, M=5, Pm=4.

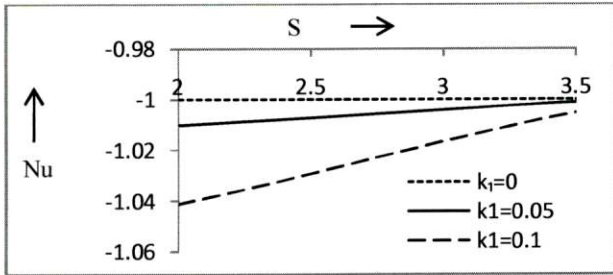


Figure16. Nusselt number against S for Pr=5, Gr=8, Gm=7, Sc=5, M=5, Pm=4.

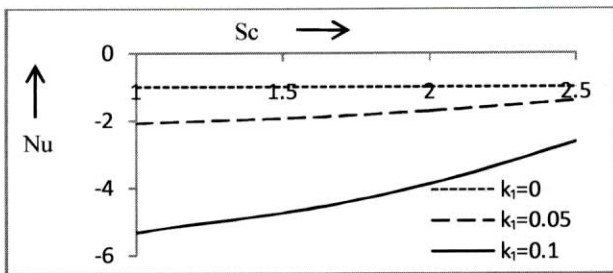


Figure17. Nusselt number against Sc for Pr=5, Gr=8, Gm=7, S=2, M=5, Pm=4.

NOMENCLATURE

| | |
|----|----------------------------------|
| Ec | Eckert number |
| Gm | Grashof number for mass transfer |
| Gr | Grashof number for heat transfer |
| k | Thermal conductivity |
| M | Hartmann number |

| | |
|-------------------------------|--|
| p | Isotropic pressure |
| Pm | Magnetic Prandtl number |
| Pr | Prandtl number |
| S | Heat source parameter |
| Sc | Schmidt number |
| v_0 | Suction velocity (ms^{-1}) |
| β | Volumetric coefficient of thermal expansion (K^{-1}) |
| $\bar{\beta}$ | Volumetric coefficient of mass expansion (K^{-1}) |
| σ^{ik} | Stress tensor |
| g_{ik} | Metric tensor of a fixed co-ordinate system |
| η_0 | Dynamic viscosity ($\text{kg m}^{-1} \text{s}^{-1}$) |
| $N(\tau)$ | Relaxation spectrum |
| $(\bar{x}, \bar{y}, \bar{z})$ | Cartesians coordinates |
| (x, y, z) | Dimensionless Cartesian coordinates |
| \vec{q} | Fluid velocity at a point $(\bar{x}, \bar{y}, \bar{z})$ in the fluid. |
| $(\bar{b}_x, \bar{b}_y, 0)$ | Components of magnetic induction vector at a point $(\bar{x}, \bar{y}, \bar{z})$ in the fluid. |
| k_1 | Visco-elastic parameter |
| G | Acceleration due to gravity (ms^{-2}) |
| \bar{T} | Temperature (K) |
| \bar{T}_∞ | Ambient temperature (K) |
| B_0 | Strength of the magnetic field |
| σ | Electrical conductivity ($(\text{ohmm})^{-1}$) |
| C_p | Specific heat at constant pressure |
| η | Magnetic diffusivity |
| ρ | Density (Kgm^{-3}) |
| ν | Kinematic viscosity ($\text{m}^2 \text{s}^{-1}$) |
| Q | Heat source parameter |
| \bar{C} | Species concentration (kgm^{-3}) |
| \bar{C}_∞ | Free stream concentration |
| C_w | Concentration at the plate |
| D | Coefficient of chemical molecular diffusivity |