ANALYTICAL SOLUTION FOR SLIP FLOW-HEAT TRANSFER IN MICROTUBES INCLUDING VISCOUS DISSIPATION AND AXIAL HEAT CONDUCTION

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ABSTRACT

Heat transfer analysis of two-dimensional, incompressible, constant property, hydrodynamically fully developed, single phase laminar flow in microtube is performed for constant wall temperature thermal boundary condition subject to slip flow regime. The rarefaction effect which is important for slip flow is discussed trough the product $Kn.\beta_v$. The viscous dissipation and the axial conduction within the fluid are included. The energy equation is solved analytically by using finite integral transform technique via Mathematica software. The effects of degree of rarefaction, jump temperature, axial conduction and viscous dissipation on heat transfer are discussed via the parameters ($Kn.\beta_v$, β, Pe, Br). It is found that the heat transfer depends on the competition between the slip velocity and temperature jump. The length of thermal entrance region diminishes with increasing Br and β but increases with decreasing Pe.

Nomenclature

Br	Brinkman number
C_p	specific heat at constant pressure [J kg^{-1} K $^{-1}$]
F_M	momentum accommodation coefficient
F_T	thermal accommodation coefficient
h	convective heat transfer coefficient [W m^{-2} K^{-1}]
k	thermal conductivity [W m^{-1} K^{-1}]
Kn	Knudsen number
Nu	Nusselt number
Pr	Prandlt number
p	slip radius
T	temperature [K]
u	velocity [m s^{-1}]
r	radial coordinate [m]
x	axial coordinate [m]
X	dimensionless axial coordinate
Y	dimensionless radial coordinate

Greek symbols	
α	thermal diffusivity [m^2 s^{-1}]
γ	specific heat ratio
λ	molecular mean free path [m]
μ	dynamic viscosity [Pa s]
ρ	density [kg m^{-3}]
β	jump temperature magnitude
θ	dimensionless temperature
Subscripts and superscripts	
b	bulk
m	mean
max	maximum
s	fluid properties at the surface
w	wall
fd	fully developed
*	temperature integral transform

1. INTRODUCTION

Convection heat transfer in microchannels is encountered in many industrial processes such as biomedical diagnostic technique, biochemical application, microelectromechanical systems (MEMS), cryogenics, thermal control of electronic devices, chemical separation processes, aerospace engineering, vacuum technology, accelerometer flow sensors, micro nozzles, micro valves….etc.

Microchannels are the fundamental part of microfluidic systems. Nominally, microchannels may be defined as channels whose characteristic dimensions are from 1µm to 1mm. Typical applications may involve characteristic dimensions in the range of approximately 10-200µm.

The researchers classified the gas flow in microchannel into four flow regimes: continuum flow regime (Kn<0.001), slip flow regime (0.001<Kn≤0.1), transition flow regime (0.1<Kn≤10) and free molecular flow regime (Kn>10), Knudsen number Kn is defined as the ratio of the molecular mean free path of gas to the characteristic length of the microchannal. In the slip flow regime, the flow and the heat transfer can be described by the continuum governing equations subjected to the slip conditions at the wall.

The slip flow-heat transfer of incompressible gaseous flowing in microtubes was reviewed by [1–7].

[1] studied the convective heat transfer subjected to isothermal boundary condition without viscous dissipation, and discussed the rarefaction effect trough the recalling factor. The energy equation was solved by separation of variables method based on hypergeometric expansion eigenfunctions, the eigenvalues were computed by a new asymptotic approach, it is found that the heat transfer depends both on the degree of rarefaction and on the surface accommodation coefficients. The authors indicate that the rarefaction increases the heat transfer when the jump temperature is neglected or insignificant, and for large jump

temperature at wall, the rarefaction diminishes the heat transfer. [2] examined the microflow heat transfer for laminar rarefied gas flow including viscous dissipation subjected to constant wall temperature, constant wall heat flux and linear variation wall temperature boundary conditions. The energy equation was solved by the finite volume method. The effects of Brinkman and Knudsen numbers on Nusselt number were observed for thermal entrance and fully developed regions. [3] analyzed the convective heat transfer for steady state, laminar fully developed flow, taking into account the viscous dissipation, slip velocity and jump temperature. The energy equation was solved by integral transform technique for isothermal and isoflux conditions. The effects of Knudsen, Brinkman and Prandlt numbers on heat transfer were illustrated. The authors observed that the slip velocity and temperature jump have opposite effects on the Nusselt number.

[4] investigated the extended Graetz problem by considering the rarefaction effect, viscous dissipation and axial heat conduction under uniform wall temperature boundary condition. The energy equation was solved numerically and the solution domain was extended to infinite. The effect of Peclet number on local Nusselt number was discussed. The authors showed that the fully developed Nusselt number and the thermal entrance length increase with the decreasing Peclet number. [5] studied the microscale heat transfer with constant wall heat flux thermal boundary condition. The rarefaction, the viscous dissipation and the axial conduction were included. The energy equation was solved analytically by using general eigenfunctions expansion. The authors have found that the local Nusselt number decreases with increasing Knudsen and Brinkman numbers. The local Nusselt number converges to the same fully-developed value for all values of Peclet and Brinkman numbers. The thermal entrance length increases with decreasing Peclet number. [6] analysed the hydrodynamically and thermally fully developed flow by including viscous dissipation. Isothermal and isoflux thermal boundary conditions have been considered. [7] examined the convective heat transfer in an infinite microtube subjected to mixed boundary conditions, taking into account the axial conduction and the rarefaction. The velocity was considered to be constant (slug flow). The energy equation has been solved by variables separation method. The authors observed that the local Nusselt number increases with increase in Peclet number but decreases with increase in Knudsen number. The fully-developed Nusselt number decreases with increase in Knudsen number but, for a fixed value of Knudsen number, it reaches to a constant value for all Peclet number values.

Slip flow-heat transfer of incompressible gaseous in parallel plate microchannel was conducted by [8–10]. [8] studied the effect of shear work at solid boundaries in small scale gaseous flows where slip effects were included. The author illustrated the effect of shear work at the boundary on convective heat transfer subjected to the constant wall heat flux boundary condition. [9] studied the extended Graetz problem including viscous dissipation and axial heat conduction. The energy equation for both isothermal and isoflux conditions was solved by using eigenfunction expansion. The effects of Peclet number, Knudsen number, Brinkman number on heat transfer were showed. The results indicate that The Nusselt number decreases as Knudsen number or Brinkman number increases and as Peclet number decreases. [10] analysed the problem of slip flow-heat transfer inside parallel plate microchannel subject to constant wall temperature and including viscous dissipation. The energy equation was solved by finite integral transform technique. The effects of the parameters $Kn.\beta_v$, β, Br on heat transfer were illustrated. The results indicate that, as $Kn.\beta_v$ increases, the velocity at the wall and local Nusselt number increase but the friction factor diminishes. As β increases the local Nusselt number and the thermal entrance length decrease.

The microscale heat transfer for slip flow regime in rectangular microchannal was analyzed by [11–13]. [11] used the integral transform technique to derive the velocity and the temperature distributions under constant wall temperature boundary condition subjected to the eight possible thermal versions. It is found that with the perfect accommodation for velocity and temperature, the rarefaction effect decreases the heat transfer for the eight thermal versions. [12] numerically solved the Navier-Stokes and energy equations by control-volume method. The effects of Reynolds number, channel aspect ratio and Knudsen number on the simultaneously developing velocity, temperature fields, entrance length, friction coefficient and Nusselt number are examined in detail. The authors have shown that in the entrance region very large reductions were observed in the friction factor and Nusselt number due to rarefaction effects. [13] determined the temperature profile by using the mathematical similarity between the heat conduction and convection under constant wall heat flux boundary condition. Additionally the average Nusselt number was determined for any all eight thermal versions. The authors showed that the rarefaction decreases the heat transfer.

Slip flow heat transfer in annular space formed by two concentric microcylinders have been undertaken by [14-15]. [14] analytically studied the laminar forced convection in micro-annulus for both hydrodynamically and thermally fully developed flows including viscous dissipation. Two different cases of the thermal boundary conditions are considered: uniform heat flux at the outer wall and adiabatic inner wall (case A) and uniform heat flux at the inner wall and adiabatic outer wall (case B). The velocity, the temperature and the Nusselt number were obtained for different values of aspect ratio, Knudsen number and Brinkman number. [15] solved the energy equation with the viscous dissipation term by using an hybrid application of the Laplace transformation technique and the local adaptive differential quadrature method (La-DQM). The constant wall temperature boundary condition was applied. The authors analyzed the effects of the Brinkman number, Knudsen number, and the radius ratio of inner to outer cylinders on the temperature distribution and the Nusselt number. The results indicate that the effect of radius ratio is to reduce the thermal entrance length, the mean fluid temperature and temperature-jump at the surface, but is to increase the fully developed Nusselt number. As the parameter Kn increases, the slip velocity at the surface increases, while the friction factor and the fully developed Nusselt number decrease. The Nusselt number in the thermal entrance region increases as Br increases.

Convective heat transfer with second order slip model was investigated by [16-17]. [16] incorporated the spatial rescaling factor (or slip radius) in velocity and temperature fields, and the energy equation was solved by separation of variables method, the eigenfunctions have been represented by polynomial expression. The effects of degree of rarefaction and jump temperature on heat transfer were discussed. [17] determined the analytical solutions for Nusselt and Poiseuille numbers in terms of the degree of rarefaction, slip flow model parameters, creep flow and Brinkman number. The results from [17] indicate that the second order term and creep velocity effects are significant within the slip flow regime.

[18-19] studied the thermal microflow with using Langumuir slip model which is based on the theory of the gases adsorption into solids. [18] solved the energy equation by finite integral transform technique including axial heat conduction term and discussed the effects of Knudsen and Peclet numbers on heat transfer. [19] used the Lattice-Boltzmann method (LB) to capture the slip velocity and temperature jump in microfluidics. The authors [19] examined the particular problem of flat microchannel with different temperature at the walls, and concluded that the Nusselt number decreases with increasing rarefaction. The decrease of Nusselt number with higher Eckert number is much than that with low Eckert number. The Reynolds analogy is really preserved in the Lattice-Boltzmann scheme.

Recently, [20] reviewed the problem of forced convective heat transfer in microtube and parallel plate microchannel for slip flow regime in presence of the viscous dissipation under isothermal boundary condition. The authors concluded that the heat transfer depends on both degree of rarefaction, measured by $Kn.\beta_v$ (or slip radius p), and jump temperature magnitude measured by β parameter. The slip velocity and the jump temperature have opposite effects on heat transfer.

In the present paper, the integral transform technique is employed to study the forced convection problem for incompressible rarefied gaseous flowing inside microtube subject to constant wall temperature boundary thermal condition under slip flow regime. The viscous dissipation and the axial heat conduction within the fluid are taking into account. The flow is assumed to be steady, laminar and fully developed. The numerical results are performed by using 'Mathematica Softwere'. The present paper could be considered as an extension of the work undertaking by Mecili and Mezaache [20]. The rescaling factor p, (called slip radius) is incorporated in velocity and energy equations. The slip radius characterizes the rarefaction effect and gives for a microconduits energy equation form similar to that of macroconduits. The convenient analysis needs the incorporation of the product ($Kn.\beta_v$) parameter which characterizes the degree of rarefaction, and the β parameter ($\beta=\beta_t/\beta_v$) which characterizes the relative importance of slip velocity and jump temperature.

2. SLIP VELOCITY AND TEMPERATURE JUMP

In microscale analysis, the Knudsen number Kn is the most important parameter. In slip flow regime $10^{-3} < Kn < 10^{-1}$, where: $Kn=\lambda/L$, λ is the molecular mean free path, L is characteristic length of micro-conduit.

The fluid particle has a tangential velocity at the wall denoted slip velocity, given by the kinetic theory of gases:

$$u_s = -\frac{2-F_M}{F_M}\lambda\left(\frac{du}{dr}\right)_{r=R} \tag{1}$$

With: F_M is the momentum tangential accommodation coefficient, which is the fraction of tangential momentum lost by a molecule upon collision, and varies from 0 for specular reflections (no accommodation), to 1 for diffuse reflections (full accommodation). $F_M = 1$ for heavy atoms and $F_M \neq 1$ for light atoms [2].

The fluid particle have a finite temperature difference at the solid surface called the temperature jump, it can be expressed as:

$$T_s - T_w = -\frac{2-F_T}{F_T}\frac{2\gamma}{\gamma+1}\frac{\lambda}{Pr}\left(\frac{\partial T}{\partial r}\right)_{r=R} \tag{2}$$

Where: T_s is the temperature of the gas at the wall, T_w the wall temperature, γ the specific heat ratio, F_T is the thermal accommodation coefficient, it is a measure of incomplete energy exchange between gas molecules and system boundaries, and varies from 0 to unity. Pr is the Prandlt number.

3. ANALYSIS AND GOVERNING EQUATIONS

The fully developed velocity profile for gaseous flowing in microtube is given by [1]

$$\frac{u}{u_{max}} = \frac{1-\left(\frac{r}{R}\right)^2 + 4Kn\beta_v}{1+4Kn\beta_v} \tag{3}$$

Where: R is radius of microtube, Kn is the Knudsen number, β_v is dimensionless coefficient, which depends on F_M coefficient. Kn and β_v can be expressed as

$$Kn = \frac{\lambda}{2R}, \beta_v = \frac{2-F_M}{F_M} \tag{4,5}$$

The energy balance equation including viscous dissipation and axial heat conduction can be expressed as

$$u(r)\frac{\partial T}{\partial x} = \alpha\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial x^2}\right] + \frac{\mu}{\rho C_p}\left(\frac{du}{dr}\right)^2 \tag{6}$$

Boundary conditions:

$$T(x=0,r) = T_0 : 0 \le r \le R, \quad \left(\frac{\partial T}{\partial r}\right)_{r=0} = 0 \tag{7-8}$$

Where: T_0 is the inlet temperature.

By introducing the non-dimensional parameters Kn and β_t in equation (2), we can write

$$T_s = T_w - 2KnR\beta_t\left(\frac{\partial T}{\partial r}\right)_{r=R} \tag{9}$$

$$\beta_t = \frac{2-F_T}{F_T}\frac{2\gamma}{\gamma+1}\frac{1}{\mathrm{Pr}} \tag{10}$$

Introducing a dimensionless parameter noted β:

$$\beta = \frac{\beta_t}{\beta_v} \tag{11}$$

Where: β characterizes the effect of gas-surface interaction, and is a relative measure of temperature jump effects.

Thus: the equation (9) becomes

$$T_s = T_w - 2KnR\beta\beta_v\left(\frac{\partial T}{\partial r}\right)_{r=R} \tag{12}$$

By introducing the non-dimensional quantities as follows

$$p^2 = \frac{1}{1+4Kn\beta_v}, Y = \frac{rp}{R}, \theta = \frac{T-T_w}{T_0-T_w} \tag{13}$$

$$X = \frac{xp^2(2-p^2)}{RPe}, Pe = RePr, Pe^* = \frac{Pe}{p(2-p^2)} \tag{14}$$

Where: the parameter p is the dimensionless rescaling factor (called the slip radius), it depends on $Kn\beta_v$ parameter and characterizes the rarefaction effect, Pe is the Peclet number, Re, Pr, Br are the Reynolds, the Prandlt and the Brinkman numbers, defined as

$$Re = \frac{\rho u_m R}{\mu}, Pr = \frac{\mu C_p}{k}, Br = \frac{\mu u_{max}^2}{k(T_0 - T_w)},$$
$$\frac{u_{max}}{u_m} = \frac{2}{2-p^2} \tag{15}$$

The dimensionless energy equation and dimensionless boundary conditions can be given by

$$(1-Y^2)\frac{\partial\theta}{\partial X} = \frac{1}{Y}\frac{\partial}{\partial Y}\left(Y\frac{\partial\theta}{\partial Y}\right) + \frac{1}{Pe^{*2}}\frac{\partial^2\theta}{\partial X^2} + 4BrY^2 \tag{16}$$

$$\theta(X=0,Y) = 1 : 0 \le Y \le p \tag{17}$$

$$\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0} = 0 \tag{18}$$

$$\theta(X, Y=p) + \frac{\beta(1-p^2)}{2p}\left(\frac{\partial\theta}{\partial Y}\right)_{Y=p} = 0 : X > 0 \tag{19}$$

4. ANALYTICAL SOLUTION

To solve the energy equation Eq.(16) with the boundary conditions Eqs.(17-19), we use the integral transform technique based on eigenfunctions and eigenvalues. The eigenfunctions-eigenvalues problem takes the following form

$$\frac{d^2\Omega_i(Y)}{dY^2} + \frac{1}{Y}\frac{d\Omega_i(Y)}{dY} + a_i^2(1-Y^2)\Omega_i(Y) = 0 \tag{20}$$

Where: $\Omega_i(Y)$ are the eigenfunctions, and a_i are the eigenvalues. The eigenfunctions are subject to the following boundary conditions

$$\Omega_i(Y=p) + \frac{\beta(1-p^2)}{2p}\left(\frac{d\Omega_i}{\partial Y}\right)_{Y=p} = 0 \tag{21}$$

$$\left(\frac{d\Omega_i}{dY}\right)_{Y=0} = 0 \tag{22}$$

The solution of the ordinary differential equation (20), gives the eigenfunctions expression. The eigenfunctions may be expressed in term of confluent hypergeometric function 1F1

$$\Omega_i(Y) = exp\left(-\frac{a_iY^2}{2}\right)1F1\left(\frac{2-a_i}{4};1;a_iY^2\right) \tag{23}$$

The application of boundary condition (Eq. 21), gives the following transcendental equation

$$1F1\left(\frac{2-a_i}{4};1;a_ip^2\right) + \frac{\beta}{2}a_i(1-p^2)\begin{bmatrix}\left(\frac{2-a_i}{2}\right)1F1\left(\frac{6-a_i}{4};2;a_ip^2\right) \\ -1F1\left(\frac{2-a_i}{4};1;a_ip^2\right)\end{bmatrix} = 0 \tag{24}$$

Now, we introduce the integral transform of temperature as follows

$$\theta_i^*(X) = \int_0^p Y(1-Y^2)\Omega_i(Y)\theta(X,Y)dY \tag{25}$$

The inverse transform is written as

$$\theta(X,Y) = \sum_{i=1}^{\infty}\left(\frac{1}{N_i}\right)\Omega_i(Y)\theta_i^*(X) \tag{26}$$

The factor N_i is obtained by application of the normalization condition

$$\int_0^p Y(1-Y^2)\Omega(a_i,Y)\Omega(a_j,Y) = \begin{cases}0 : i \ne j \\ N(a_i) : i = j\end{cases} \tag{27}$$

$$N_i = \int_0^p Y(1-Y^2)\Omega_i^2(Y)dY \tag{28}$$

Multiplying the both sides of the energy equation Eq.(16) by (ΩY), and integrated over the domain $0 \le Y \le p$, the energy equation becomes

$$\frac{1}{Pe^{*2}}\sum_{i=1}^{\infty}\left[A_{in}\frac{d^2\theta_n^*(X)}{dX^2}\right] - \frac{d\theta_i^*(X)}{dX} - a_i^2\theta_i^*(X) = -4Br\int_0^p Y^3\Omega_i(Y)dY \tag{29}$$

Where

$$A_{in} = \frac{1}{N_n}\int_0^p Y\Omega_i(Y)\Omega_n(Y)dY \quad (30)$$

By retaining only one term in the series ($i=n$), the equation (29) takes the form

$$\frac{A_{ii}}{Pe^{*2}}\frac{d^2\theta_i^*(X)}{dX^2} - \frac{d\theta_i^*(X)}{dX} - a_i^2\theta_i^*(X) = -4Br\int_0^p Y^3\Omega_i(Y)dY \quad (31)$$

With the boundary condition

$$\theta_i(X=0) = \int_0^p Y(1-Y^2)\Omega_i(Y)dY \quad (32)$$

The solution of the second order ODE problem Eq.(31) subject to the boundary condition Eq.(32) is

$$\theta_i^*(X) = \left(Z_i - \frac{4BrG_i}{a_i^2}\right)exp\left[\frac{XPe^{*2}(1-B_{ii})}{2A_{ii}}\right] + \frac{4BrG_i}{a_i^2} \quad (33)$$

With: $G_i = \int_0^p Y^3\Omega_i(Y)dY$, $Z_i = \int_0^p Y(1-Y^2)\Omega_i(Y)dY$ (34)

$$B_{ii} = \sqrt{1+\frac{4a_i^2A_{ii}}{Pe^{*2}}}\ ,\ A_{ii} = \frac{1}{N_i}\int_0^p Y\Omega_i^2(Y)dY \quad (35)$$

The substitution of Eq.(33) in Eq.(26), gives the temperature field

$$\theta(X,Y) = \sum_{i=1}^{\infty}\frac{\Omega_i(Y)}{N_i}\left\{\begin{matrix}\left(Z_i - \frac{4BrG_i}{a_i^2}\right)exp\left[\frac{XPe^{*2}(1-B_{ii})}{2A_{ii}}\right] \\ +\frac{4BrG_i}{a_i^2}\end{matrix}\right\} \quad (36)$$

5.1. Evaluation of dimensionless bulk temperature and local Nusselt number

$$\theta_b(X) = \frac{\int_0^p Y(1-Y^2)\theta(X,Y)dY}{\int_0^p Y(1-Y^2)dY} = \left(\frac{4}{2p^2-p^4}\right)\int_0^p Y(1-Y^2)\theta(X,Y)dY \quad (37)$$

$$Nu(X) = \frac{h(2R)}{k} = -2p\frac{\left[\frac{\partial\theta}{\partial Y}\right]_{Y=p}}{\theta_b} \quad (38)$$

In the limiting case $p=1$ ($Kn\beta_v=0$), we recover the macrochannel case (continuum flow).

5.2. Determination of fully developed temperature, bulk temperature and Nusselt number

For $X\rightarrow\infty$, the energy equation takes the following form

$$\frac{1}{Y}\frac{d}{dY}\left(Y\frac{d\theta_{fd}}{dY}\right) = -4BrY^2 \quad (39)$$

By integrating the above equation and using the boundary conditions (Eqs. 18, 19), we obtain the fully developed temperature profile

$$\theta_{fd}(Y) = -\frac{BrY^4}{4} + \frac{Brp^4(1-2\beta)}{4} + \frac{\beta Brp^2}{2} \quad (40)$$

Then: For $Br\neq 0$, the fully developed bulk temperature and Nusselt number are expressed as

$$\theta_{b\,fd} = \left(\frac{4}{2p^2-p^4}\right)\int_0^p Y(1-Y^2)\theta_{fd}(Y)dY = \frac{Br\left(\beta p^4 + \frac{p^6}{3} - \frac{3\beta p^6}{2} - \frac{p^8}{8} + \frac{\beta p^8}{2}\right)}{2p^2-p^4} \quad (41)$$

$$Nu_{fd} = -2p\frac{\left[\frac{d\theta_{fd}}{dY}\right]_{Y=p}}{\theta_{b\,fd}} = \frac{2p^2-p^4}{\frac{\beta}{2}+\frac{p^2}{6}-\frac{3\beta p^2}{4}-\frac{p^4}{16}+\frac{\beta p^4}{4}} \quad (42)$$

6. RESULTS AND DISCUSSION

The slip flow-forced convective heat transfer in microtube is analytically investigated by the finite integral transform technique via Mathematica software. The combined effects of rarefaction, jump temperature, viscous dissipation and axial conduction on heat transfer are discussed trough the dimensionless parameters $Kn\beta_v$, β, Br, Pe. In the calculations, we take $Pe=10^6$ which corresponds to infinite (or neglected) axial conduction, $Br=0$ corresponds to no viscous dissipation case. $Kn\beta_v$ includes the rarefaction effect and it is varied between 0 (continuum regime) and 0.1 (limiting slip regime). The parameter β includes the gas-surface interaction and measured the magnitude of jump temperature, it is varied between 0 and 10, $\beta=0$ is a fictitious value, to see the effect of slip velocity without jump temperature, $\beta=10$ corresponds to the large temperature jump at the surface.

The numerical evaluation of bulk temperature and local Nusselt number requires the computation of eigenvalues a_i, eigenfunctions $\Omega_i(Y)$ and eigenquantities ($N_i, G_i, Z_i, A_{ii}, B_{ii}$). The eigenvalues can be computed from transcendental equation (24) by using the built-in function FindRoot. The eigenfunctions can be determined from the equation (23) by using the confluent hypergeometric function 1F1(., ., .) available in Mathematica. The eigenquantities (N_i , G_i , Z_i, A_{ii}

$,B_{ii}$) can be calculated from equations (28, 34, 35) by using the built-in function NIntegrate.

Table 1 presents the fully developed Nusselt number values computed for β_v=1, Br=0, Pe=10^6 and for different Kn and β values. Table 2 presents the fully developed Nusselt number values computed for β_v=1, $Br \neq 0$ and for different Kn and β values. Our results are compared with those obtained by [21], and these comparisons reveal a very good agreement.

The eigenvalues (a_i) and other parameters (N_i, G_i, Z_i, A_{ii}) computed for different values of Kn and for $\beta=0$, are summarized in Table 3.

Fig. 1 illustrates the effect of rarefaction (measured by $Kn\beta_v$) on the local Nusselt number without viscous dissipation and axial conduction. We consider two cases: β=0 (negligible temperature jump, Fig. 1.a) and β=10 (large temperature jump at surface, Fig. 1.b).

In the case when temperature jump is neglected (β=0), the rarefaction measured by $Kn\beta_v$ increases the slip velocity at the surface and leads to decreasing the difference between the wall temperature and the fluid bulk temperature, which tends to amplifies the energy exchange near the wall, consequently the local Nu increases, as seen in (Fig. 1. a). This trends is reversed when the large temperature jump is included (β=10, Fig. 1.b), in other words as $Kn\beta_v$ increases the jump temperature increases and leads to increasing the difference between the wall temperature and the fluid bulk temperature, which tends to decreasing the energy exchange, consequently the local Nu decreases. The jump temperature plays a role like a thermal contact resistance between the wall and the gas, while the slip velocity tends to decrease this thermal contact resistance. We can notice that the slip velocity and the jump temperature have opposite effects on heat transfer.

Fig. 2 illustrates the effects of rarefaction, viscous dissipation and temperature jump on local Nu for fixed $Kn\beta_v$ and β parameters and for different Br values. The viscous dissipation affects the temperature profile by playing a role like an energy source; the effect of viscous dissipation becomes the most significant near the wall due to highest velocity gradient occurring there. Viscous dissipation always contributes to internal heating of the fluid; hence the heat transfer mechanism will differ according to the process taking place (wall heating or cooling). We note also that the magnitude of local Nusselt number increases with Br number increasing; this increase in Br number leads to decreasing in the length of the thermal entrance region. In the fully developed region, we observe that the all values of Nu number computed for a different non-zero values of Br number reaches to the same asymptotic value in case when β and $Kn\beta_v$ are fixed, in other words the fully developed Nu number does not depends on Br number, this result is clearly illustrated by (eq. 42).

Fig. 3 demonstrates the effect of the parameter β (jump temperature) on local Nusselt number for fixed $Kn\beta_v$ parameter ($Kn\beta_v$=0.1). We observe that as β increases the length of thermal entrance region decreases, and for higher β values, the profile of local Nusselt number becomes quasi-flatter.

The fully-developed Nusselt number as a function of $Kn\beta_v$ and parameterized by β is presented in Fig. 4, it reveals that the fully-developed Nusselt number depends on $Kn\beta_v$ and β parameters and independent of Br and Pe numbers, as seen the rarefaction effect measured by $Kn\beta_v$ increases the fully-developed Nusselt number when the jump temperature is neglected or small (β<1), and reduces it when the jump temperature is significant (β>1). This reduce is due to decreasing in temperature gradient normal to the wall causes by large jump temperature at the surface. Therefore, the fully developed Nusselt number increases, decreases or unchanged depending on competition between slip velocity and jump temperature. In addition, the fully-developed Nusselt number for $Br \neq 0$ is higher than that for Br=0 (see tables 1 and 2).

Fig. 5 illustrates the effect of jump temperature on fluid bulk temperature for $Kn\beta_v$ = 0.01, and without viscous dissipation, the data of this figure indicate that the bulk temperature increases with increasing temperature jump.

Fig. 6 depicts the bulk temperature of the fluid for different values of Br number (Br=-0.1,0.1,-1.0,1.0,0). Positive values of Br correspond to wall heating (T_w>T_0), and negative values correspond to wall cooling (T_w<T_0). It can observe that, the sign of bulk temperature $\theta_b(X)$ changes when Br<0, the same result was reported by [9].

In fig.7 the variation of local Nusselt number is plotted for different $Kn\beta_v$ and Pe values. Pe=1 corresponds to the finite axial heat conduction within the fluid and Pe=10^6 corresponds to a high magnitude of the thermal energy convected to the fluid relative to the thermal energy conducted axially within the fluid (neglected axial heat conduction). This figure shows that Nu_∞=3.66 in the case when Pe=10^6, Br=0 and $Kn\beta_v$=0 (classical Graetz problem) which is exactly value compared with the previous analytical works, additionally it reveals that the local Nusselt number values for Pe=1 are greater than those obtained for Pe=10^6, in other words the axial heat conduction contributes to enhancement of heat transfer. We can note also that the thermal entrance length increases with decreasing Pe but the variation of the degree of rarefaction measured by $Kn\beta_v$ does not affects the thermal entrance length.

In fig.8 the variation of local Nusselt number is plotted for different Br and Pe values and for fixed $Kn\beta_v$ and β ($Kn\beta_v$=0.01, β=1), this figure indicates that in the thermal fully developed region the viscous dissipation dominates heat transfer and all fully developed Nusselt number values converges to the same asymptotic value regardless of the values of Pe and Br numbers. The axial conduction has no influence on fully developed Nusselt number when viscous dissipation is included. It can be concluded that in the presence of the viscous dissipation the axial heat conduction is significant in the thermal entrance region and insignificant in the thermal fully developed region.

Table 1: Fully developed Nusselt number values for β_v=1, Br=0, Pe=10^6

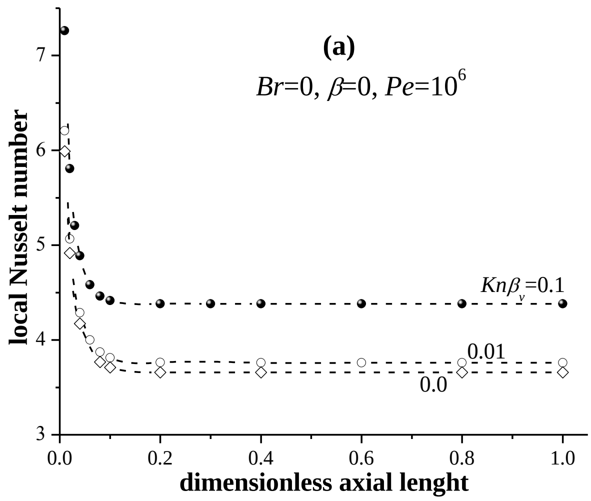
	β=0.0		β=0.5		β=10	
Kn	Present	[21]	present	[21]	present	[21]
0.0	3.656	3.6568	3.6568	3.656	3.6568	3.656
0.02	3.855	3.8556	3.7395	3.739	2.2911	2.291
0.04	4.020	4.0207	3.7783	3.778	1.6237	1.624
0.06	4.160	4.1599	3.7847	3.785	1.2465	1.247
0.08	4.279	4.2789	3.7671	3.767	1.0077	1.008
0.10	4.382	4.3817	3.7318	3.732	0.8440	0.844

Table 2: Fully developed Nusselt number values for β_v=1, $Br \neq 0$, $Pe=10^6$

	β=0.0		β=0.5		β=10	
Kn	Present	[21]	present	[21]	present	[21]
0.0	9.6000	9.598	9.6000	9.598	9.6000	9.598
0.02	9.8723	9.871	8.9853	8.984	3.3190	3.316
0.04	10.0892	10.088	8.3952	8.394	2.0036	1.995
0.06	10.2659	10.264	7.8487	7.848	1.4339	1.423
0.08	10.4127	10.411	7.3500	7.350	1.1160	1.115
0.10	10.5366	10.535	6.9000	6.900	0.9133	0.912

Table 3: Eigenvalues a_i and other parameters (Z_i, G_i, N_i, A_{ii}) for different values of $Kn\beta_v$ and for β=0

$Kn\beta_v$	a_1	a_2	a_3	a_4	a_5
0.0	2.70436	6.67903	10.67338	14.67108	18.66987
0.04	2.86309	6.9756	11.0908	15.2017	19.309
0.08	3.0153	7.27672	11.533	15.7824	20.0273
0.1	3.08906	7.42679	11.7573	16.0806	20.3994
$Kn\beta_v$	G_1	G_2	G_3	G_4	G_5
0.0	0.05811	-0.052726	0.038753	-0.030353	0.024913
0.04	0.0441512	-0.039871	0.0283328	-0.02146	0.0170636
0.08	0.0343168	-0.030592	0.0208623	-0.015144	0.0115335
0.1	0.0309966	-0.027679	0.018857	-0.013747	0.0105692
$Kn\beta_v$	Z_1	Z_2	Z_3	Z_4	Z_5
0.0	0.138687	-0.030246	0.0138018	-0.008112	0.0054247
0.04	0.128049	-0.029892	0.0142201	-0.008627	0.005922
0.08	0.118325	-0.028829	0.0140116	-0.008619	0.005976
0.1	0.11386	-0.028181	0.0137921	-0.008517	0.005921
$Kn\beta_v$	N_1	N_2	N_3	N_4	N_5
0.0	0.093934	0.0375198	0.0234421	0.0170471	0.0133936
0.04	0.0853702	0.0348183	0.0219223	0.0160097	0.0126134
0.08	0.07804	0.0322543	0.0203906	0.0149198	0.0117674
0.1	0.0747807	0.0310577	0.0196596	0.0143928	0.0113548
$Kn\beta_v$	A_{11}	A_{22}	A_{33}	A_{44}	A_{55}
0.0	1.25123	1.45617	1.53714	1.58458	1.61708
0.04	1.21279	1.37564	1.4327	1.46286	1.48171
0.08	1.18442	1.31782	1.35967	1.37985	1.39147
0.1	1.17286	1.29481	1.33124	1.34813	1.35755



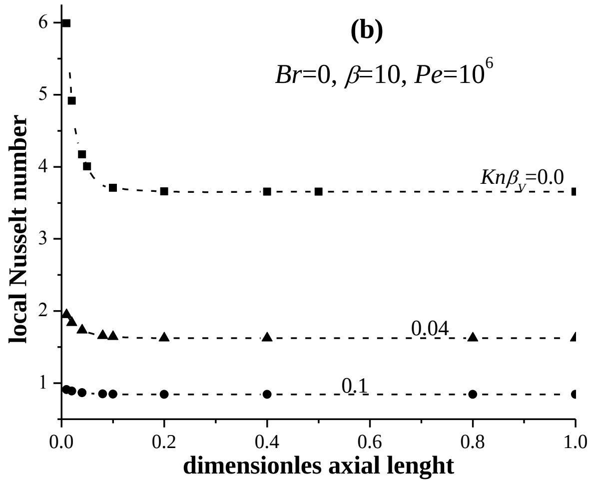


Figure 1: Local Nusselt number parameterized by $Kn\beta_v$ without viscous dissipation (Br=0) and axial conduction ($Pe=10^6$), (Fig. 1a) without jump temperature, (Fig. 1b) in presence of jump temperature.

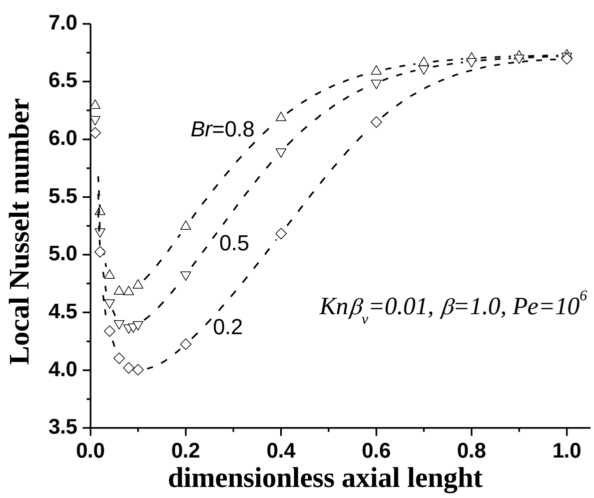


Figure 2: Effects of combined viscous dissipation, rarefaction and jump temperature on local Nusselt number for neglected axial conduction.

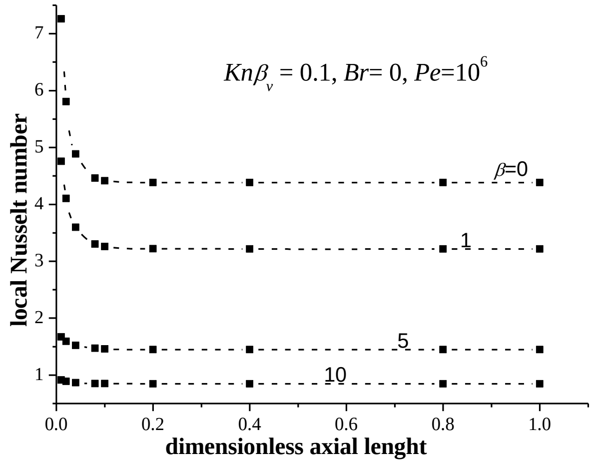


Figure 3: Local Nusselt number parameterized by β and for fixed $Kn.\beta_v$, without viscous dissipation and axial conduction.

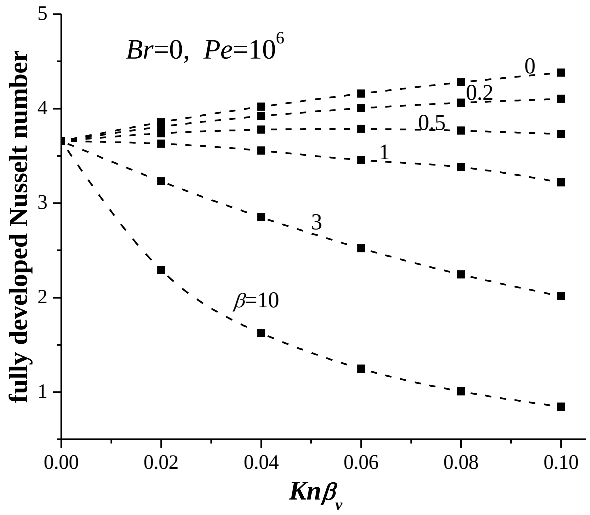


Figure 4: Variation of fully developed Nusselt number with $Kn\beta_v$ for various values of β and without viscous dissipation and axial conduction.

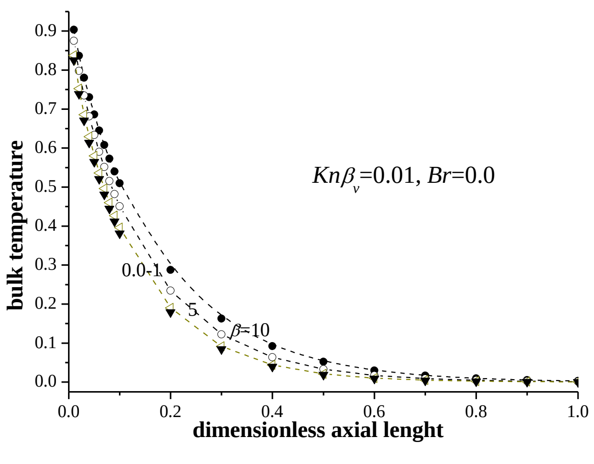


Figure 5: Variation of bulk temperature parameterized by β for $Kn\beta_v$=0.01.

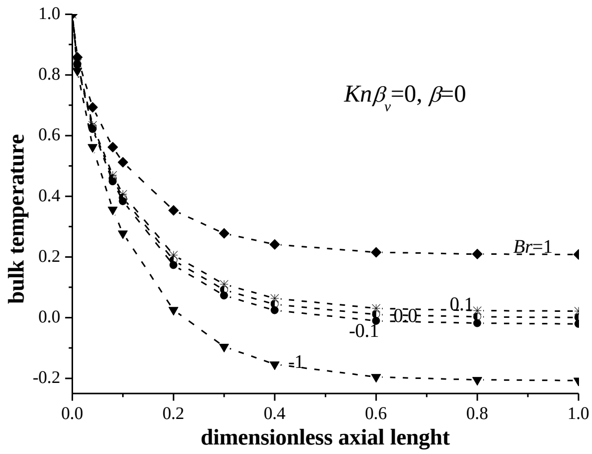


Figure 6: Variation of bulk temperature parameterized by Br for continuum flow case.

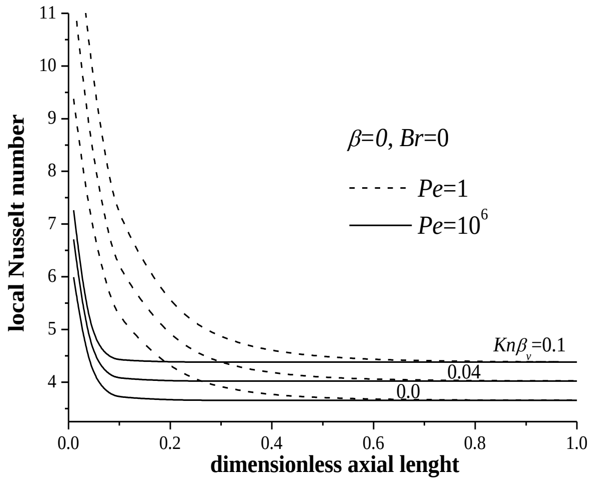


Figure 7: Effects of rarefaction and axial heat conduction on local Nusselt number.

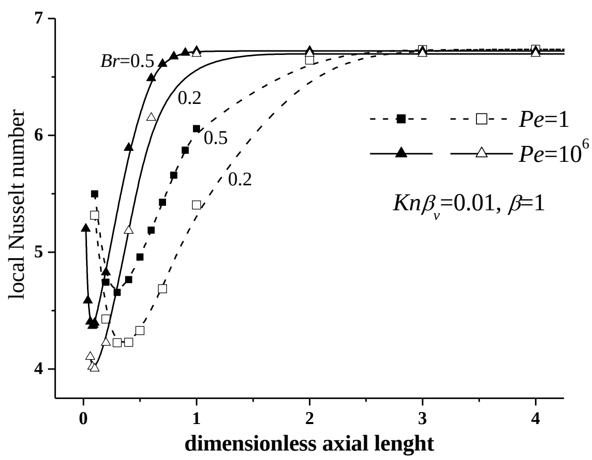


Figure 8**:** Effects of rarefaction, jump temperature, viscous dissipation and axial heat conduction on local Nusselt number

6. CONCLUSION

Slip flow-convective heat transfer inside microtube including viscous dissipation and axial conduction within the fluid subject to isothermal boundary condition was investigated. The energy equation was solved by finite integral transform technique by the help of Mathematica. The effects of rarefaction, jump temperature, viscous dissipation and axial conduction on heat transfer are discussed trough the dimensionless parameters, $Kn\beta v$, β, Br, Pe.

We concluded that the heat transfer depends on both degree of rarefaction and surface accommodation coefficients. The slip velocity and the jump temperature have opposite effects on the heat transfer. The axial heat conduction within the fluid contributes to increasing of local Nusselt Number. The length of thermal entrance region diminishes with increasing Br and β but increases with decreasing Pe. In the thermal fully developed region the viscous dissipation dominates the axial conduction and all fully developed Nusselt number values converges to the same asymptotic value regardless of the values of Pe and Br numbers. The axial conduction has no influence on the thermal fully developed region when viscous dissipation is included, but it is significant in the thermal entrance region.

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