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Heat and Mass Transfer of Williamson Nanofluid with the Effects of Viscous Dissipation and Chemical Reaction

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ABSTRACT

D A numerical analysis is performed for the mathematical model of boundary layer flow of nanofluids. Heat and mass transfer are analyzed for an incompressible fluid with viscous dissipations and chemical reaction past a stretching surface. An appropriate set of similarity transformations are used to transform the governing partial differential equations (PDEs) into a system of nonlinear ordinary differential equations (ODEs). The resulting system of ODEs is solved numerically by using Adams-Moultan method along with shooting method. Furthermore, we compared our results with the existing results for especial cases which are in an excellent agreement. The numerical values obtained for various non-dimensional physical quantities together with velocity, temperature and concentration profiles are presented through graphs and tables. The effects of different physical parameters on the flow and heat transfer characteristics are discussed in detail.

1. INTRODUCTION

The boundary layer is the region adjacent to the surface of an object around which the fluid is flowing. Flow of the boundary layer play an important role in fluid mechanics and has been extensively studied in the literature. Prandtl [1] was the first who presented the concept of boundary layer. Makinde and Motsumi [2] studied the boundary layer flow of nanofluids over a continuously moving surface. Ibrahim and Makinde [3] investigated the thermal boundary layer flow with effects of double stratification over a vertical sheet. In addition to that in a series of article, Makinde [4, 5] studied the boundary layer flow of nanofluids passing over a at plate. They further analyzed the impacts of viscous dissipation and Newtonian heating for various types of geometry including permeable surface. The main purpose of their study is the computation of mathematical models of nanofluid over steady/unsteady stretching sheet. The boundary layer flow over a moving surface have a number of applications in engineering and industrial fields. Sakiadis et al. [6] presented the concept of the boundary layer flow through a stretching surface.

The small solid particle is known as nanoparticle, these nanoparticles ranges from 1-100 nanometers in size. The nanofluid is defined as the homogenous mixture of the base fluid and nanoparticle. In 1995, Choi [7] in his pioneering work introduced the terminology of nanofluids. Since then an extensive research is carried out on this topic by many researchers due to its potential industrial applications. In the current progress in the field of science and technology, the nanotechnology has a wide range of applications in different fields. In the last couple of decades, the development in nanotechnology is exponentially increasing. The effects of nanoparticle migration on force convection of nanofluid in a channel are studied for alumina [8] critical analysis of thermophysical characteristics of nanofluids are investigated by Khanafer and Vafai [9].

The heat transfer in the boundary layer flow of nanofluid has been an interesting topic for researchers. Masuda et al. [10] found that nanofluids are enhancing thermal conductivity, they further noted the potential applications of nanofluid in nuclear technology. They studied the characteristics of nanofluid by dispersing ultra-_ne particles in base fluid with varying viscosity and thermal conductivity. Buongiorno [11] developed a model with analytic solution for convective heat transfer in a Brownian diffusion of nanofluid. He observed the effects of diffusion and thermophoresis in nanofluid. A cavity ow with heat transfer and entropy generation are analyzed by Z. Mehrez et al. [12].

Viscous dissipation is unavoidable in case of flow field in high gravitational field. Viscous flow past a nonlinearly stretching sheet was deliberated by Vajravelu [13]. For external natural convention flow over a stretching medium, the impact of viscous dissipation was also studied by Mollendro and Gebhart [14], whereas the impact of viscous dissipation and Joule heating on the forced convection flow with thermal radiation was presented by Duwairi [15].

In this section we review the work S. Nadeen et al. [16] and is extended for numerical analysis of heat and mass transfer by taking additional effects of viscous dissipation and chemical reaction parameter.

2. PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION

Let us consider the numerical investigation of MHD boundary layer flow of an incompressible Williamson nanofluid. The flow is two-dimensional past a stretching surface with porous medium. The plate has been stretched with velocity $u = ax$, $(a > 0)$ along x −axis. The temperature at surface is T_w, U_w, C_w represent fluid velocity, nanoparticle concentration at surface respectively. The general equations for nanofluid are given by Buongiorno [11].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\nu\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} + \tau\left(D_B\left(\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right) + \frac{\nu}{(\rho c)_f}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial C}{\partial y}\right)^2 + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - k_0 C, \tag{4}$$

The associated boundary conditions:

$$\left.\begin{aligned} u = U_w(x) = ax, T = T_w, C = C_w \text{ at } y = 0,\\ u = 0, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty \end{aligned}\right\} \tag{5}$$

3. DIMENSIONLESS FORM OF THE GOVERNING EQUATIONS

In order to obtain the solution of the problem, first of all system of Eqns. (1)-(4) together with the boundary conditions (5) is converted into the dimensionless form by using suitable similarity transformation. The following similarity transformation as defined in [16] has been used.

$$\eta = y\sqrt{\frac{a}{\nu}},\ \psi = \sqrt{a\nu}\,xf(\eta), \qquad \theta(\eta) = \frac{T-T_=}{T_w-T_\infty}, \quad \beta(\eta) = \frac{C-C_\infty}{C_w-C_\infty}. \tag{6}$$

The continuity Eq. (1) is identically satisfied for the stream function $\psi(x, y)$.

The velocity components are given by:

$$u = \frac{\partial\psi}{\partial y},\ v = -\frac{\partial\psi}{\partial x} \tag{7}$$

Using the similarity transformation from Eq. (11) in momentum Eq. (2), energy Eq. (3) and concentration Eq. (4) along the boundary conditions (5) we get the following system of ODEs:

$$f''' + ff'' - (f')^2 + \lambda f''f''' = 0 \tag{8}$$

$$\frac{\theta''}{Pr} + f\theta' + \frac{Nc}{Le}\theta'\beta' + \frac{Nc}{Le\times N_{bt}}(\theta')^2 + Ec(f'')^2 = 0 \tag{9}$$

$$\beta'' + Sef\beta' + \frac{1}{N_{bt}}\theta'' - Le\gamma\beta = 0 \tag{10}$$

The transformed BCs in the model problem are:

$$\left.\begin{aligned} f'(0) = 1, f(\theta) = 0, \theta(0) = 1, \beta(0) = 1, \text{at } \eta = 0,\\ f'(\infty) \to 0, \theta(\infty) \to 0, \beta(\infty) \to 0 \text{ as } \eta \to \infty \end{aligned}\right\} \tag{11}$$

The associated parameters appearing in the modeled problem are:

$$\lambda = \Gamma x\sqrt{\frac{2a^3}{\nu}}, Pr = \frac{\nu}{\alpha}, Le = \frac{\alpha}{D_B}, Se = \frac{\nu}{D_B}, N_{bt} = \frac{D_B T_\infty(C_\infty - C_m)}{D_T\nu T_\infty(T_w - T_\infty)},$$

$$Nb = \frac{\tau D_B(C_w - C_\infty)}{\nu}, Ec = \frac{u_w^2}{\rho_f\big((T_w - T_\infty)\big)}, \gamma = \frac{k_0 U(C_\infty - C_w)}{\nu}.$$

where λ the non-Newtonian Williamson parameter, Pr denoted the Prandtl number, Le the Lewis number, Sc the Schmidt number, Nc the heat capacities ratio, i.e., nanoparticles heat capacity/nanofluid, N_{bt} heat capacity, diffusivity ratio, i.e., Brownian diffusivity/thermophoretic diffusivity, Nb the Diffusivity ratio thermophoresis parameter, Ec the Eckert number and γ is the chemical reaction parameter.

4. NUMERICAL PROCEDURE

The numerical solution of the following Eqns. (8)-(10) with corresponding boundary conditions (11) can be obtained by shooting technique. To use the shooting method, first we convert these Eqns. (8)-(10) into a system of first order ODEs. For these purposes, we denote f by y_1, f' by y_2, f'' by y_3, θ by y_4 and β by y_6. The coupled nonlinear momentum, heat and concentration equations are converted into system of seven first order ODEs as into the following form:

$$y_1' = y_2, \qquad y_1(0) = 0 \tag{12}$$

$$y_2' = y_3, \qquad y_2(0) = 1 \tag{13}$$

$$y_3' = \frac{-y_1y_3 + (y_2)^2}{1+\lambda y_3} \qquad y_3(0) = r \tag{14}$$

$$y_4' = y_5 \qquad y_4(0) = 0 \tag{15}$$

$$y_5' = -Pry_1y_5 - \frac{Nc}{Le}y_5y_7 - \frac{Nc}{Le\times N_{bt}}(y_5)^2 + Ecy_3^2 \qquad y_5(0) = s \tag{16}$$

$$y_6' = y_7 \qquad y_6(0) = 1 \tag{17}$$

$$y_7' = -Sey_1y_7 - \frac{1}{N_{bt}}y_5' + Le\gamma y_6 \qquad y_7(0) = t \tag{18}$$

For solving above system numerically, we replace the domain $(0, \infty]$, by the bounded domain $[0, \eta_\infty]$ where η_∞ is some suitable real number. In the above system of equations, we have $y_3(\eta)$, $y_5(\eta)$ and $y_7(\eta)$ at $\eta = 0$ i.e., r, s and t are missing conditions and are to be chosen such that

$$y_2(\eta_\infty, r, s, t) \approx 0, y_4(\eta_\infty, r, s, t) \text{ and } y_6(\eta_\infty, r, s, t) \approx 0.$$

Finally, the choice of $\eta_{max} = 12$ was more than enough for end condition. The convergence criteria is chosen to be successive value agree up top 3 significant digits.

5. CODE VALIDATION

In Table 1 and 2, comparison of Nusselt number for different values of $\boldsymbol{\lambda}, \boldsymbol{Le}, \boldsymbol{N_{bt}}, \boldsymbol{Nc}, \boldsymbol{Pr}$ is displayed. Furthermore, our findings are compared with the published work of S. Nadeem et al. [16], which show a good agreement of numerical results.

Table 1. Comparison Values of wall temperature gradient $-\theta'(0)$

λ	Le	N_{bt}	Nc	Pr	$-\theta'(0)$ S. Nadeem [16]	$-\theta'(0)$ Present Value
0.0	4.0	2.0	0.5	0.5	0.314	0.313593600
0.2					0.309	0.308631400
0.4					0.302	0.302629700
0.2	4.0	2.0	0.5	0.5	0.309	0.308631400
	10				0.332	0.331970600
	20				0.340	0.340100600
0.2	4.0	0.5	0.5	0.5	0.286	0.285833100
		1.0			0.301	0.300789200
		2.0			0.309	0.308631200
0.2	4.0	2.0	0.5		0.309	0.308631400
			1.0		0.273	0.273043200
			2.0		0.213	0.212952100
				0.2	0.144	0.179527600
				0.6	0.355	0.352547300
				1.2	0.588	0.586684900

Table 2. Comparison Values of wall nano particle volume fraction gradient $-\beta'(0)$

λ	Le	N_{bt}	Sc	$-\beta'(0)$ S. Nadeem [16]	$-\beta'(0)$ Present Value
0.0	4.0	2.0	0.5	0.833	0.834052200
0.2				0.824	0.824271200
0.4				0.811	0.811198500
0.2	4.0	2.0	0.5	0.824	0.824271200
	10			0.815	0.814918800
	20			0.812	0.811866000
0.2	4.0	0.5	0.5	0.620	0.623315500
		1.0		0.751	0.751818400
		2.0		0.823	0.823637800
0.2	4.0	2.0	0.5	0.228	0.240325600
			1.0	0.475	0.473748100
			2.0	0.824	0.823637900

6. RESULTS AND DISCUSSION

This section aims to examine the effect of different parameters λ, Pr, Le, Nc, Nbt, Ec and χ (i.e., Non-Newtonians Williamson parameter, Prandtl number, Lewis number, heat capacities ratio, i.e., nanoparticles heat capacity/nanofluid, heat capacity, diffusivity ratio, i.e., Brownian diffusivity/thermophoretic diffusivity, Eckert number and Chemical reaction parameter) on dimensionless velocity, temperature and concentration in the form of tables and graphs. Here, we include the conversation on numerical results obtained by shooting technique. Also, the velocity, temperature and concentration profile is plotted in which the influence of different parameters is discussed.

To observe the effects of different parameters on dimensionless velocity $f'(\eta)$, dimensionless temperature $\theta(\eta)$ and dimensionless concentration $\beta(\eta)$ graphs are plotted below.

From Figure 1 it is observed that for a nanofluid the velocity as well as the boundary layer thickness decreases with the increase in non-Newtonian parameter λ.

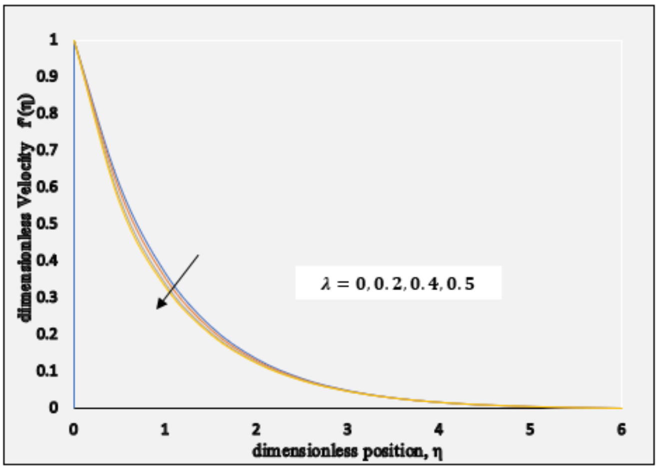

Figure 1. Dimensionless velocity vs λ when $Pr = 0.5, Nc = 0.5, Le = 4, Nbt = 2, Sc = 2, Ec = 0.5$ and $\chi = 0.2$

Figure 2 are the graphical representation which depicts the influence of Prandtl number Pr on dimensionless temperature and concentration. It is seen that temperature profile reduces for the rising estimations of Prandtl number. Greater the Prandtl number outcomes the lower thermal diffusivity. In this way increment in Prandtl number decreases conduction and henceforth the variation in thermal characteristics increases.

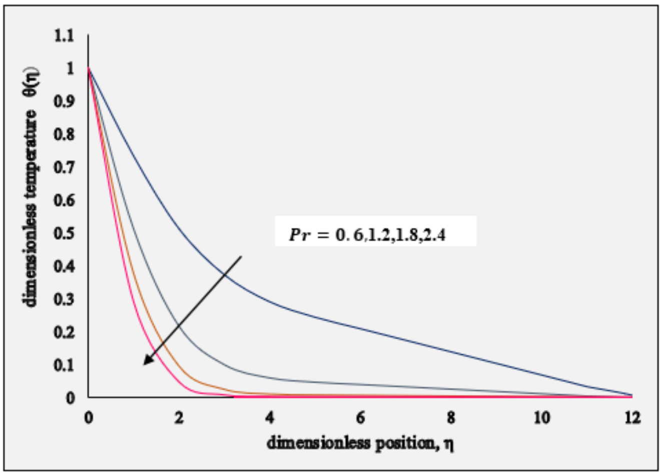

Figure 2. Dimensionless temperature vs Pr when $\beta = 0.2, Nc = 0.5, Le = 4, Nbt = 2, Sc = 2, Ec = 0.5$ and $\chi = 0.2$

The impact of Le on dimensionless temperature profile $\theta(\eta)$ and concentration profile $\beta(\eta)$ can be seen as in Figures 3 and 4. From the figures, it is observed that by increasing values of Lewis number temperature decreases and concentration as well as the thickness of concentration increases. This is due to the fact that Le physically expresses the respective contribution of rate of thermal diffusion to the rate of species diffusion in the boundary layer regime. As increasing values of Lewis number reduce the thickness of thermal boundary layer and temperature decrease. It also reveals that the concentration gradient at surface of the plate increases.

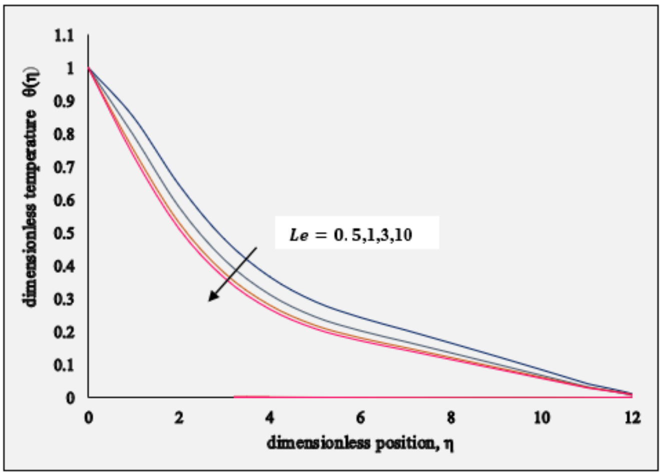

Figure 3. Dimensionless temperature vs Le when $\beta = 0.2, Nc = 0.5, Pr = 0.5, Nbt = 2, Sc = 2, Ec = 0.5$ and $\chi = 0.2$

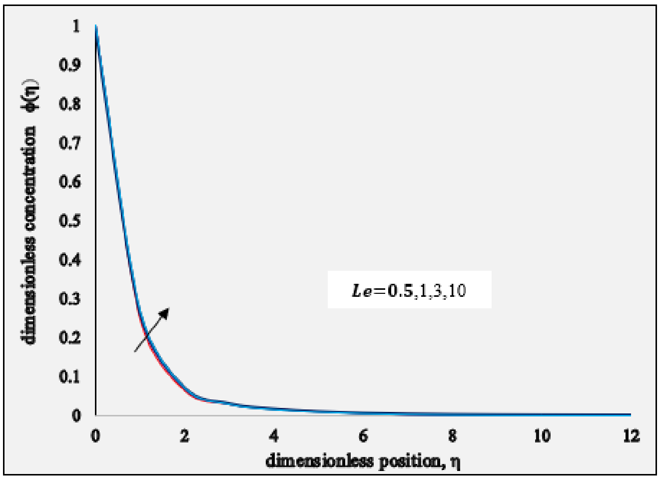

Figure 4. Dimensionless temperature vs Le when $\beta = 0.2, Nc = 0.5, Pr = 0.5, Nbt = 2, Sc = 2, Ec = 0.5$ and $\chi = 0.2$

The variation of temperature verses heat capacity ratio parameter (Nc) is plotted in figure 5. Since the heat capacities ratio is equal to the ratio of nanoparticle heat capacity to the nanofluid heat capacity. It is seen that as Nc increases the temperature of the fluid increases.

The effect of Nbt on temperature and mass volume friction is shown in figures 6 and 7. Nbt is the ratio of Brownian diffusivity to thermophoretic diffusivity. An increase in Nbt the temperature of the fluid is decreases as well as same results were found in mass volume friction.

From figure 8 show that the variation of the Schmidt number (Sc) to the nanoparticle volume friction. Since Schmidt number is the ratio of the momentum diffusivity to Brownian diffusivity. It is seen that nanoparticle volume friction decreases with increases the Schmidt number.

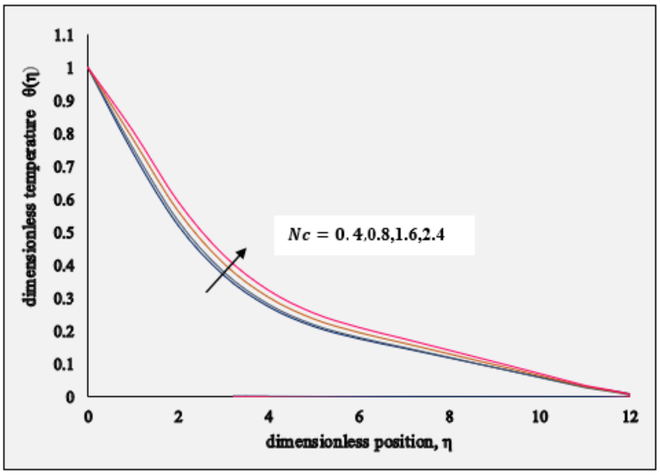

Figure 5. Dimensionless temperature vs Nc when $\beta = 0.2, Le = 4, Pr = 0.5, Nbt = 2, Sc = 2, Ec = 0.5$ and $\chi = 0.2$

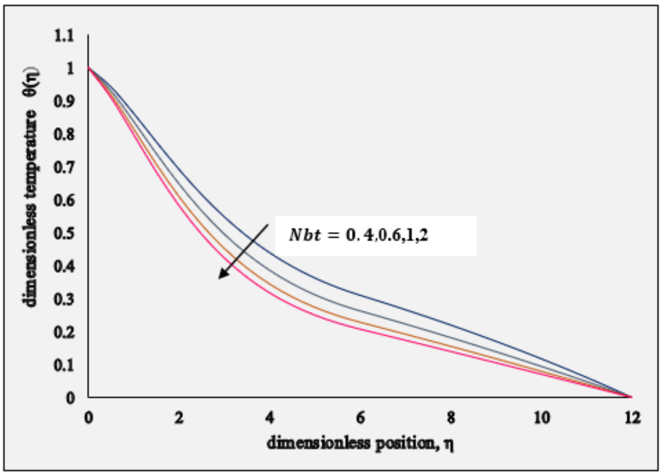

Figure 6. Dimensionless temperature vs Nbt when $\beta = 0.2, Le = 4, Pr = 0.5, Nc = 2, Sc = 2, Ec = 0.5$ and $\chi = 0.2$

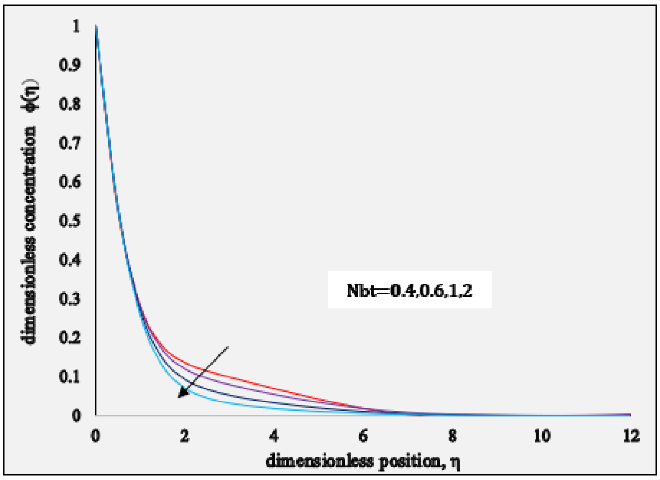

Figure 7. Dimensionless concentration vs Nc when $\beta = 0.2, Le = 4, Pr = 0.5, Nbt = 2, Sc = 2, Ec = 0.5$ and $\chi = 0.2$

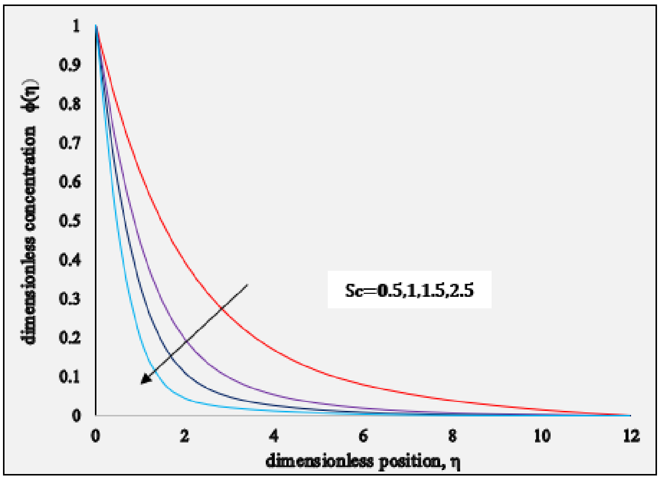

Figure 8. Dimensionless concentration vs Nbt when $\beta = 0.2, Le = 4, Pr = 0.5, Nc = 2, Nbt = 2, Ec = 0.5$ and $\chi = 0.2$

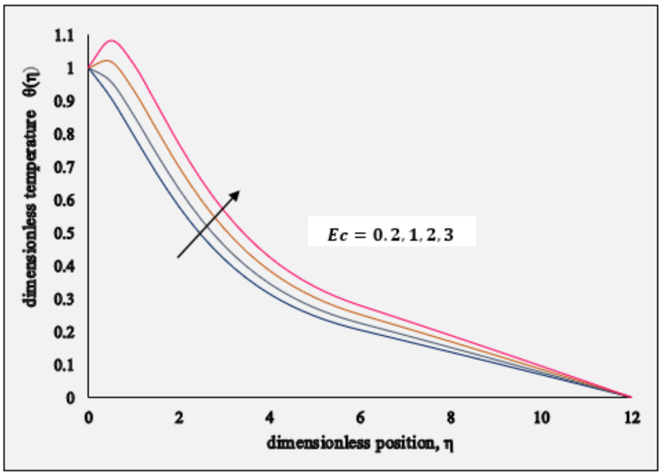

Figure 9. Dimensionless temperature vs Ec when $\beta = 0.2, Le = 4, Pr = 0.5, Nc = 2, Nbt = 2, Nbt = 2$ and $\chi = 0.2$

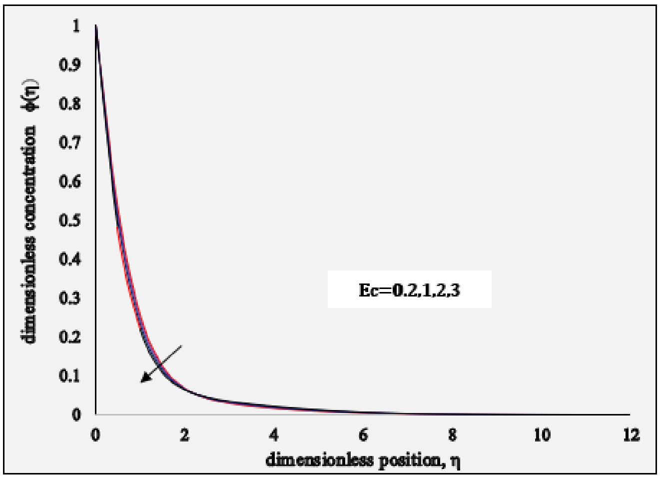

Figure 10. Dimensionless concentration vs Ec when $\beta = 0.2, Le = 4, Pr = 0.5, Nc = 2, Nbt = 2, Nbt = 2$ and $\chi = 0.2$

Figure 9 and 10 shows the effect of viscous dissipation on temperature and concentration profile. It is observed that increase in dissipation also increases temperature whereas concentration profile decreasing.

Figure 11 and 12 explains the influence of the chemical reaction parameter on the profile of concentration. It is noted that increasing values of chemical reaction parameter concentration as well as the thickness of concentration decrease. It is because of the fact that the chemical reaction in this system results in chemical dissipation and therefore results in decrease in the profile of concentration. The most significant influence is that chemical reaction tends to decrease the overshoot in the concentration profiles and their associated boundary layer.

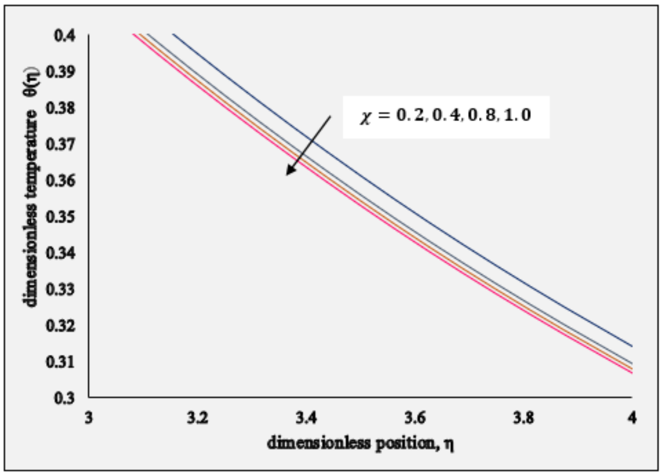

Figure 11. Dimensionless temperature vs Ec when $\beta = 0.2, Le = 4, Pr = 0.5, Nc = 2, Nbt = 2, Nbt = 2$ and $Ec = 0.2$

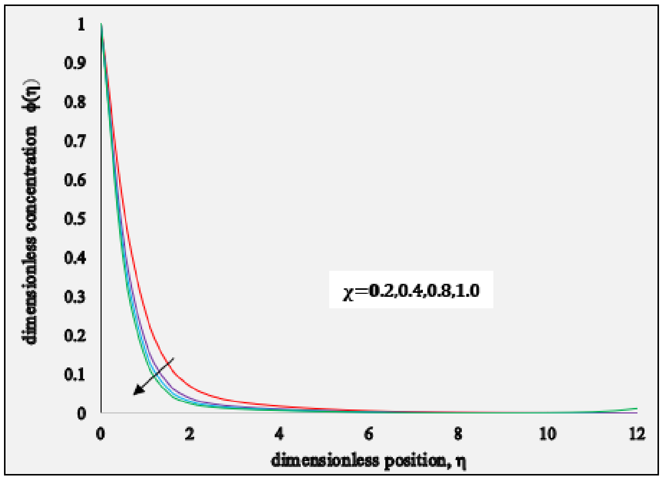

Figure 12. Dimensionless concentration vs Ec when $\beta = 0.2, Le = 4, Pr = 0.5, Nc = 2, Nbt = 2, Nbt = 2$ and $Ec = 0.2$

7. CONCLUSION

After a thorough investigation, we have reached the following concluding observation.

- Fluid velocity decreases with an increase in the non-Newtonian Williamson parameter.
- Due to an increase in the chemical reaction parameter, the temperature field as well as concentration profile decreases.

- ❖ The influence of diffusivity ratio parameter or heat capacities ratio parameter reduces the heat transfer coefficient.
- ❖ Energy profile θ increases by enlarging Ec.

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