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An Approach to Manufacture Small Multiplexer with Dense Field-Effect Transistors

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ABSTRACT

This paper introduces an approach to manufacture a small multiplexer with dense field-effect transistors. First, a framework of two-level current-mode logic gates was designed for the transistors. The logic gates are organized heterogeneously with a substrate, epitaxial layers and buffer layer. Considering the different properties of materials in the framework, the density of transistors was improved by annealing dopant and/or radiation defects, thus reducing the size of the multiplexer. In addition, the mismatch-induced stress was alleviated through ion implantation. Next, an analytical approach was developed to analyze the mass and heat transport in the heterogenous framework during the production of integrated circuits, in the presence of mismatch-induced stress. The proposed method makes it possible to capture both nonlinear variations of parameters in space and time through the production process.

1. INTRODUCTION

In the present time several actual problems of the solid state electronics (such as increasing of performance, reliability and density of elements of integrated circuits: diodes, field-effect and bipolar transistors) are intensively solving. One of them is to decrease dimensions of elements of integrated circuits and increasing their density [1-6]. To increase the performance of these devices it is attracted an interest determination of materials with higher values of charge carriers mobility [7-10]. One way to decrease dimensions of elements of integrated circuits is manufacturing them in thin film heterostructures [3-5, 11]. In this case it is possible to use inhomogeneity of heterostructure and necessary optimization of doping of electronic materials [12] and development of epitaxial technology to improve these materials (including analysis of mismatch induced stress) [13-15]. An alternative approach to increase dimensions of integrated circuits are using of laser and microwave types of annealing [16-18].

Framework the paper we introduce an approach to manufacture field-effect transistors. The approach gives a possibility to decrease their dimensions with increasing their density framework a two-level current-mode logic gates in a multiplexer. We also consider possibility to decrease mismatch-induced stress to decrease quantity of defects, generated due to the stress. In this paper we consider a heterostructure, which consist of a substrate and an epitaxial layer (see Figure 1). We also consider a buffer layer between the substrate and the epitaxial layer. The epitaxial layer includes into itself several sections, which were manufactured by using another material. These sections have been doped by diffusion or ion implantation to manufacture the required types of conductivity (*p* or *n*). These areas became sources, drains and gates (see Figure 1). After this doping it is required annealing of dopant and/or radiation defects. Main aim of the present pa-per is analysis of redistribution of dopant and radiation defects to determine conditions, which correspond to decreasing of elements of the considered voltage reference and at the same time to increase their density. At the same time, we consider a possibility to decrease mismatch-induced stress.

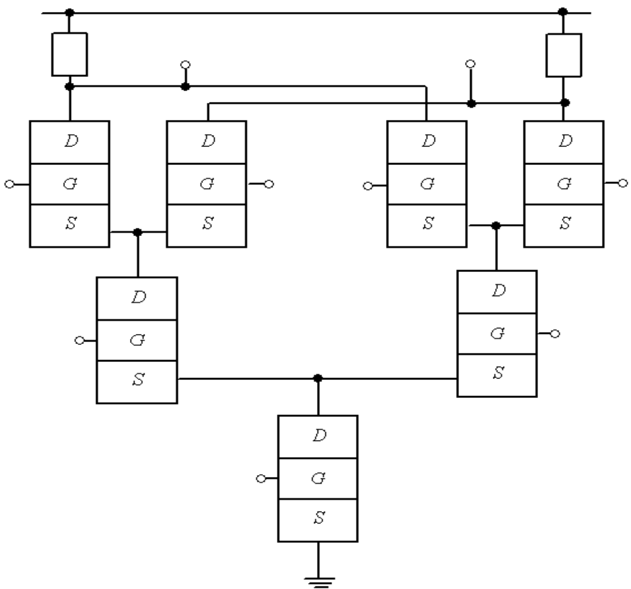

Figure 1*a*. Structure of the considered logic gates [19]

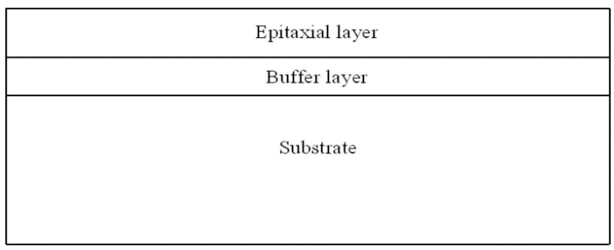

Figure 1*b*. Heterostructure with a substrate, epitaxial layers and buffer layer (view from side)

2. METHOD OF SOLUTION

To solve our aim, we determine and analyzed spatio-temporal distribution of concentration of dopant in the considered heterostructure. We determine the distribution by solving the second Fick's law in the following form [1, 20-23].

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)dW\right]+$$
$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)dW\right]+$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)dW\right]+ \quad (1)$$
$$+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+\frac{\partial}{\partial x}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}\right]+$$
$$+\frac{\partial}{\partial y}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}\right]$$

with boundary and initial conditions

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$
$$\left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{x=L_y}=0, \quad (2)$$
$$\left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{x=L_z}=0,$$
$$C(x,y,z,0)=f_C(x,y,z).$$

Here $C(x,y,z,t)$ is the spatio-temporal distribution of concentration of dopant; Ω is the atomic volume of dopant; ∇_s is the symbol of surficial gradient; $\int_0^{L_z}C(x,y,z,t)dz$ is the surficial concentration of dopant on interface between layers of heterostructure (in this situation we assume, that Z-axis is perpendicular to interface between layers of heterostructure); $\mu_1(x,y,z,t)$ and $\mu_2(x,y,z,t)$ are the chemical potential due to the presence of mismatch-induced stress and porosity of material; D and D_S are the coefficients of volumetric and surficial diffusions. Values of dopant diffusions coefficients depends on properties of materials of heterostructure, speed of heating and cooling of materials during annealing and spatio-temporal distribution of concentration of dopant. Dependences of dopant diffusions coefficients on parameters could be approximated by the following relations [24-26].

$$\begin{cases} D_C=D_L(x,y,z,T)\left[1+\xi\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\times \\ \times\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right] \\ D_S=D_{SL}(x,y,z,T)\left[1+\xi_S\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\times \\ \times\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\end{cases} \quad (3)$$

Here $D_L(x,y,z,T)$ and $D_{LS}(x,y,z,T)$ are the spatial (due to accounting all layers of heterostruicture) and temperature (due to Arrhenius law) dependences of dopant diffusion coefficients; T is the temperature of annealing; $P(x,y,z,T)$ is the limit of solubility of dopant; parameter γ depends on properties of materials and could be integer in the following interval $\gamma\in[1,3]$ [24]; $V(x,y,z,t)$ is the spatio-temporal distribution of concentration of radiation vacancies; V^* is the equilibrium distribution of vacancies. Concentrational dependence of dopant diffusion coefficient has been described in details in Ref. [24]. Spatio-temporal distributions of concentration of point radiation defects have been determined by solving the following system of equations [20-23, 25, 26].

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+$$
$$+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]-k_{I,I}(x,y,z,T)I^2(x,y,z,t)+$$
$$+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]-k_{I,V}(x,y,z,T)I(x,y,z,t)\times$$
$$\times I(x,y,z,t)V(x,y,z,t)+\frac{\partial}{\partial x}\left[\nabla_S\mu(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\times\right.$$
$$\left.\times\frac{D_{IS}}{kT}\right]\Omega+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\right]+$$
$$+\frac{\partial}{\partial x}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}\right] \quad (4)$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+$$
$$+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]-k_{V,V}(x,y,z,T)V^2(x,y,z,t)+$$
$$+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]-k_{I,V}(x,y,z,T)I(x,y,z,t)\times$$
$$\times I(x,y,z,t)V(x,y,z,t)+\frac{\partial}{\partial x}\left[\nabla_S\mu(x,y,z,t)\int_0^{L_z}V(x,y,W,t)dW\times\right.$$
$$\left.\times\frac{D_{VS}}{kT}\right]\Omega+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}V(x,y,W,t)dW\right]+$$
$$+\frac{\partial}{\partial x}\left[\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}\right]$$

with boundary and initial conditions

$$\left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$
$$\left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0, \quad (5)$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$I(x,y,z,0)=f_I(x,y,z),\ V(x,y,z,0)=f_V(x,y,z),$$

$$V(x_1+V_nt,y_1+V_nt,z_1+V_nt,t)=V_\infty\left(1+\frac{2\ell\omega}{kT\sqrt{x_1^2+y_1^2+z_1^2}}\right).$$

Here $I(x,y,z,t)$ is the spatio-temporal distribution of concentration of radiation interstitials; I^* is the equilibrium distribution of interstitials; $D_I(x,y,z,T)$, $D_V(x,y,z,T)$, $D_{IS}(x,y,z,T)$, $D_{VS}(x,y,z,T)$ are the coefficients of volumetric and surficial diffusions of interstitials and vacancies, respectively; terms $V^2(x,y,z,t)$ and $I^2(x,y,z,t)$ correspond to generation of divacancies and diinterstitials, respectively (see, for example, [26] and appropriate references in this book); $k_{I,V}(x,y,z,T)$, $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ are the parameters of recombination of point radiation defects and generation of their complexes; k is the Boltzmann constant; $\omega = a^3$, a is the interatomic distance; ℓ is the specific surface energy. To account porosity of buffer layers we assume, that porous are approximately cylindrical with average values $r=\sqrt{x_1^2+y_1^2}$ and z_1 before annealing [23]. With time small pores decomposing on vacancies. The vacancies absorbing by larger pores [27]. With time large pores became larger due to absorbing the vacancies and became more spherical [27]. Distribution of concentration of vacancies in heterostructure, existing due to porosity, could be determined by summing on all pores, i.e.

$$V(x,y,z,t)=\sum_{i=0}^{l}\sum_{j=0}^{m}\sum_{k=0}^{n}V_p(x+i\alpha,y+j\beta,z+k\chi,t),$$

$$R=\sqrt{x^2+y^2+z^2} \quad (6)$$

Here α, β and χ are the average distances between centers of pores in directions x, y and z; l, m and n are the quantity of pores inappropriate directions.

Spatio-temporal distributions of divacancies $\Phi_V(x,y,z,t)$ and diinterstitials $\Phi_I(x,y,z,t)$ could be determined by solving the following system of equations [25, 26].

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+$$

$$+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\times\right.$$

$$\left.\times\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)dW\right]+$$

$$+\frac{\partial}{\partial x}\left[\frac{D_{\Phi_IS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_IS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_IS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+$$

$$+k_I(x,y,z,T)I(x,y,z,t) \quad (7)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+$$

$$+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\times\right.$$

$$\left.\times\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_VS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_VS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)dW\right]+$$

$$+\frac{\partial}{\partial x}\left[\frac{D_{\Phi_VS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_VS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_VS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+$$

$$+k_V(x,y,z,T)V(x,y,z,t)$$

with boundary and initial conditions

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$

$$\left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,$$

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ \left.\frac{\partial V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0, \quad (8)$$

$$\left.\frac{\partial V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\Phi_I(x,y,z,0)=f_{\Phi I}(x,y,z),\ \Phi_V(x,y,z,0)=f_{\Phi V}(x,y,z).$$

Here $D_{\Phi I}(x,y,z,T)$, $D_{\Phi V}(x,y,z,T)$, $D_{\Phi IS}(x,y,z,T)$ and $D_{\Phi VS}(x,y,z,T)$ are the coefficients of volumetric and surficial diffusions of complexes of radiation defects; $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ are the parameters of decay of complexes of radiation defects.

Chemical potential μ_1 in Eq. (1) could be determine by the following relation [20].

$$\mu_1=E(z)\Omega\sigma_{ij}[u_{ij}(x,y,z,t)+u_{ji}(x,y,z,t)]/2, \quad (9)$$

where, $E(z)$ is the Young modulus, σ_{ij} is the stress tensor; $u_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ is the deformation tensor; u_i, u_j are the components $u_x(x,y,z,t)$, $u_y(x,y,z,t)$ and $u_z(x,y,z,t)$ of the displacement vector $\vec{u}(x,y,z,t)$; x_i, x_j are the coordinate x, y, z. The Eq. (3) could be transform to the following form

$$\mu(x,y,z,t)=E(z)\frac{\Omega}{2}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]\times$$
$$\times\left\{\frac{1}{2}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]-\varepsilon_0\delta_{ij}+\frac{\sigma(z)\delta_{ij}}{1-2\sigma(z)}\times\right.$$
$$\left.\times\left[\frac{\partial u_k(x,y,z,t)}{\partial x_k}-3\varepsilon_0\right]-K(z)\beta(z)\left[T(x,y,z,t)-T_0\right]\delta_{ij}\right\} \tag{10}$$

where, σ is Poisson coefficient; $\varepsilon_0=(a_s-a_{EL})/a_{EL}$ is the mismatch parameter; a_s, a_{EL} are lattice distances of the substrate and the epitaxial layer; K is the modulus of uniform compression; β is the coefficient of thermal expansion; T_r is the equilibrium temperature, which coincide (for our case) with room temperature. Components of displacement vector could be obtained by solution of the following equations [21].

$$\begin{cases}\rho(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2}= \\ =\frac{\partial\sigma_{xx}(x,y,z,t)}{\partial x}+\frac{\partial\sigma_{xy}(x,y,z,t)}{\partial y}+\frac{\partial\sigma_{xz}(x,y,z,t)}{\partial z} \\ \rho(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}= \\ =\frac{\partial\sigma_{yx}(x,y,z,t)}{\partial x}+\frac{\partial\sigma_{yy}(x,y,z,t)}{\partial y}+\frac{\partial\sigma_{yz}(x,y,z,t)}{\partial z} \\ \rho(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}= \\ =\frac{\partial\sigma_{zx}(x,y,z,t)}{\partial x}+\frac{\partial\sigma_{zy}(x,y,z,t)}{\partial y}+\frac{\partial\sigma_{zz}(x,y,z,t)}{\partial z},\end{cases} \tag{11}$$

where, $\sigma_{ij}=\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}-\frac{\delta_{ij}}{3}\frac{\partial u_k(x,y,z,t)}{\partial x_k}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]\times$
$\times\frac{E(z)}{2[1+\sigma(z)]}+K(z)\,\delta_{ij}\frac{\partial u_k(x,y,z,t)}{\partial x_k}-\left[T(x,y,z,t)-T_r\right]\times$
$\times\beta(z)K(z)$, $\rho(z)$ is the density of materials of heterostructure, δ_{ij} Is the Kronecker symbol. With account the relation for σ_{ij} last system of equation could be written as

$$\rho(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_x(x,y,z,t)}{\partial x^2}+$$
$$+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y}+\frac{E(z)}{2[1+\sigma(z)]}\times$$
$$\times\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial z^2}\right]+\left[K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right]\times$$
$$\times\frac{\partial^2 u_z(x,y,z,t)}{\partial x\partial z}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x}$$
$$\rho(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2}=\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial y}\right]\times$$
$$\times\frac{E(z)}{2[1+\sigma(z)]}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y}+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\times\right.$$
$$\left.\times\left[\frac{\partial u_y(x,y,z,t)}{\partial z}+\frac{\partial u_z(x,y,z,t)}{\partial y}\right]\right\}+\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}\times$$
$$+\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}+\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}\times$$
$$\times\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}+K(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial x\partial y} \tag{12}$$

$$\rho(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_z(x,y,z,t)}{\partial x^2}+\right.$$
$$\left.+\frac{\partial^2 u_z(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial z}+\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}\right]+$$
$$+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_x(x,y,z,t)}{\partial x}+\frac{\partial u_y(x,y,z,t)}{\partial y}+\frac{\partial u_x(x,y,z,t)}{\partial z}\right]\right\}+$$
$$+\frac{1}{6}\frac{\partial}{\partial z}\left\{\left[6\frac{\partial u_z(x,y,z,t)}{\partial z}-\frac{\partial u_x(x,y,z,t)}{\partial x}-\frac{\partial u_y(x,y,z,t)}{\partial y}-\right.\right.$$
$$\left.\left.-\frac{\partial u_z(x,y,z,t)}{\partial z}\right]\frac{E(z)}{1+\sigma(z)}\right\}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Conditions for the system of Eq. (8) could be written in the form

$$\frac{\partial\vec{u}(0,y,z,t)}{\partial x}=0;\ \frac{\partial\vec{u}(L_x,y,z,t)}{\partial x}=0;\ \frac{\partial\vec{u}(x,0,z,t)}{\partial y}=0;$$
$$\frac{\partial\vec{u}(x,L_y,z,t)}{\partial y}=0;\ \frac{\partial\vec{u}(x,y,0,t)}{\partial z}=0;\ \frac{\partial\vec{u}(x,y,L_z,t)}{\partial z}=0;$$
$$\vec{u}(x,y,z,0)=\vec{u}_0;\ \vec{u}(x,y,z,\infty)=\vec{u}_0 \tag{13}$$

We determine spatio-temporal distributions of concentrations of dopant and radiation defects by solving the Eqns. (1), (4) and (7) framework standard method of averaging of function corrections [28]. Previously we transform the Eqns. (1), (4) and (7) to the following form with account initial distributions of the considered concentrations.

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+\frac{\partial}{\partial x}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}\right]+$$
$$+\frac{\partial}{\partial y}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}C\left(x,y,W,t\right)dW\right]+$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}C\left(x,y,W,t\right)dW\right] \qquad (1a)$$

$$\frac{\partial\,I\left(x,y,z,t\right)}{\partial\,t}=\frac{\partial}{\partial\,x}\left[D_I\left(x,y,z,T\right)\frac{\partial I\left(x,y,z,t\right)}{\partial x}\right]+$$
$$+\frac{\partial}{\partial\,y}\left[D_I\left(x,y,z,T\right)\frac{\partial I\left(x,y,z,t\right)}{\partial y}\right]+\frac{\partial}{\partial\,z}\left[D_I\left(x,y,z,T\right)\times\right.$$
$$\left.\times\frac{\partial I\left(x,y,z,t\right)}{\partial\,z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\int\limits_0^{L_z}I\left(x,y,W,t\right)dW\right]+$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\int\limits_0^{L_z}I\left(x,y,W,t\right)d\,W\right]-I^2\left(x,y,z,t\right)\times$$
$$\times\,k_{I,I}\left(x,y,z,T\right)-k_{I,V}\left(x,y,z,T\right)I\left(x,y,z,t\right)V\left(x,y,z,t\right)+$$
$$+f_I\left(x,y,z\right)\delta\left(t\right) \qquad (4a)$$

$$\frac{\partial\,V\left(x,y,z,t\right)}{\partial\,t}=\frac{\partial}{\partial\,x}\left[D_V\left(x,y,z,T\right)\frac{\partial V\left(x,y,z,t\right)}{\partial x}\right]+$$
$$+\frac{\partial}{\partial\,y}\left[D_V\left(x,y,z,T\right)\frac{\partial V\left(x,y,z,t\right)}{\partial y}\right]+\frac{\partial}{\partial\,z}\left[D_V\left(x,y,z,T\right)\times\right.$$
$$\left.\times\frac{\partial V\left(x,y,z,t\right)}{\partial\,z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\int\limits_0^{L_z}V\left(x,y,W,t\right)dW\right]+$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\int\limits_0^{L_z}I\left(x,y,W,t\right)dW\right]-I^2\left(x,y,z,t\right)\times$$
$$\times\,k_{I,I}\left(x,y,z,T\right)-k_{I,V}\left(x,y,z,T\right)I\left(x,y,z,t\right)V\left(x,y,z,t\right)+$$
$$+f_V\left(x,y,z\right)\,\delta\left(t\right)$$

$$\frac{\partial\,\Phi_I\left(x,y,z,t\right)}{\partial\,t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}\left(x,y,z,T\right)\frac{\partial\Phi_I\left(x,y,z,t\right)}{\partial x}\right]+$$
$$+\frac{\partial}{\partial y}\left[D_{\Phi_I}\left(x,y,z,T\right)\frac{\partial\Phi_I\left(x,y,z,t\right)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}\left(x,y,z,T\right)\frac{\partial\Phi_I\left(x,y,z,t\right)}{\partial z}\right]+$$
$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\int\limits_0^{L_z}\Phi_I\left(x,y,W,t\right)dW\right]+$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\int\limits_0^{L_z}\Phi_I\left(x,y,W,t\right)dW\right]+$$
$$+\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial z}\right]+k_I\left(x,y,z,T\right)\,I\left(x,y,z,t\right)+$$
$$+k_{I,I}\left(x,y,z,T\right)I^2\left(x,y,z,t\right)+f_{\Phi_I}\left(x,y,z\right)\delta\left(t\right) \qquad (7a)$$

$$\frac{\partial\,\Phi_V\left(x,y,z,t\right)}{\partial\,t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}\left(x,y,z,T\right)\frac{\partial\Phi_V\left(x,y,z,t\right)}{\partial x}\right]+$$

$$\frac{\partial\,\Phi_V\left(x,y,z,t\right)}{\partial\,t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}\left(x,y,z,T\right)\frac{\partial\Phi_V\left(x,y,z,t\right)}{\partial x}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}\left(x,y,z,T\right)\frac{\partial\Phi_V\left(x,y,z,t\right)}{\partial z}\right]+$$
$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\int\limits_0^{L_z}\Phi_V\left(x,y,W,t\right)dW\right]+$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\int\limits_0^{L_z}\Phi_I\left(x,y,W,t\right)dW\right]+$$
$$+\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial z}\right]+k_I\left(x,y,z,T\right)I\left(x,y,z,t\right)+$$
$$+k_{V,V}\left(x,y,z,T\right)V^2\left(x,y,z,t\right)+f_{\Phi_V}\left(x,y,z\right)\delta\left(t\right).$$

Farther we replace concentrations of dopant and radiation defects in right sides of Eqns. (1a), (4a) and (7a) on their not yet known average values $\alpha_{1\rho}$. In this situation we obtain equations for the first-order approximations of the required concentrations in the following form

$$\frac{\partial C_1\left(x,y,z,t\right)}{\partial t}=\alpha_{1C}\Omega\frac{\partial}{\partial x}\left[z\frac{D_S}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\right]+$$
$$+\alpha_{1C}\Omega\frac{\partial}{\partial y}\left[z\frac{D_S}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\right]+f_C\left(x,y,z\right)\delta\left(t\right)+$$
$$+\frac{\partial}{\partial\,x}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\,\mu_2\left(x,y,z,t\right)}{\partial\,x}\right]+\frac{\partial}{\partial\,y}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\,\mu_2\left(x,y,z,t\right)}{\partial\,y}\right]+$$
$$+\frac{\partial}{\partial\,z}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\,\mu_2\left(x,y,z,t\right)}{\partial\,z}\right] \qquad (1b)$$

$$\frac{\partial\,I_1\left(x,y,z,t\right)}{\partial\,t}=\alpha_{1I}\,z\,\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu\left(x,y,z,t\right)\right]+$$
$$+\alpha_{1I}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{IS}}{kT}\nabla_S\mu\left(x,y,z,t\right)\right]+\frac{\partial}{\partial x}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial x}\right]+$$
$$+\frac{\partial}{\partial y}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial z}\right]+$$
$$+f_I\left(x,y,z\right)\delta\left(t\right)-\alpha_{1I}^2k_{I,I}\left(x,y,z,T\right)-\alpha_{1I}\alpha_{1V}k_{I,V}\left(x,y,z,T\right)$$
$$\frac{\partial\,V_1\left(x,y,z,t\right)}{\partial\,t}=\alpha_{1V}\,z\,\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\right]+$$
$$+\alpha_{1V}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{VS}}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\right]+\frac{\partial}{\partial x}\left[\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial x}\right]+$$
$$+\frac{\partial}{\partial y}\left[\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial z}\right]+$$
$$+f_V\left(x,y,z\right)\delta\left(t\right)-\alpha_{1V}^2k_{V,V}\left(x,y,z,T\right)-\alpha_{1I}\alpha_{1V}k_{I,V}\left(x,y,z,T\right)$$
(1b)

$$\frac{\partial\,\Phi_{1I}\left(x,y,z,t\right)}{\partial\,t}=\alpha_{1\Phi_I}\,z\,\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1\left(x,y,z,t\right)\right]+$$

$$+\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}\right]+\alpha_{1\Phi_I}\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]\times$$
$$\times z\Omega+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}\right]+$$
$$+k_I(x,y,z,T)\,I(x,y,z,t)+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+$$
$$+f_{\Phi_I}(x,y,z)\delta(t) \qquad (7b)$$

$$\frac{\partial\,\Phi_{1V}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+$$
$$+\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial x}\right]+z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]\times$$
$$\times\alpha_{1\Phi_V}+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial \mu_2(x,y,z,t)}{\partial z}\right]+$$
$$+k_V(x,y,z,T)V(x,y,z,t)+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+$$
$$+f_{\Phi_V}(x,y,z)\delta(t)\,.$$

Integration of the left and right sides of the Eqns. (1*b*), (4*b*) and (7*b*) on time gives us possibility to obtain relations for above approximation in the final form

$$C_1(x,y,z,t)=\alpha_{1C}\Omega\frac{\partial}{\partial x}\int_0^t D_{SL}(x,y,z,T)\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\right.$$
$$\left.+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\nabla_S\mu_1(x,y,z,\tau)\left[1+\frac{\xi_S\alpha_{1C}^{\gamma}}{P^{\gamma}(x,y,z,T)}\right]\times$$
$$\times\frac{z}{kT}d\tau+\alpha_{1C}\Omega\frac{\partial}{\partial y}\int_0^t D_{SL}(x,y,z,T)\left[1+\frac{\xi_S\alpha_{1C}^{\gamma}}{P^{\gamma}(x,y,z,T)}\right]\times$$
$$\times\nabla_S\mu_1(x,y,z,\tau)\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$
$$\times\frac{z}{kT}d\tau+\frac{\partial}{\partial x}\int_0^t\frac{D_{CS}}{\bar{V}kT}\frac{\partial\,\mu_2(x,y,z,\tau)}{\partial\,x}d\tau+\frac{\partial}{\partial\,y}\int_0^t\frac{D_{CS}}{\bar{V}kT}\times$$
$$\times\frac{\partial\,\mu_2(x,y,z,\tau)}{\partial\,y}d\tau+\frac{\partial}{\partial\,z}\int_0^t\frac{D_{CS}}{\bar{V}kT}\frac{\partial\,\mu_2(x,y,z,\tau)}{\partial\,z}d\tau+$$
$$+f_C(x,y,z) \qquad (1c)$$

$$I_1(x,y,z,t)=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau+\alpha_{1I}z\Omega\times$$
$$\times\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}d\tau+$$
$$+\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}d\tau-$$
$$-\alpha_{1I}^2\int_0^t k_{I,I}(x,y,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau+$$
$$+f_I(x,y,z) \qquad (1c)$$

$$V_1(x,y,z,t)=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)d\tau+\alpha_{1V}z\Omega\times$$
$$\times\frac{\partial}{\partial y}\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+\frac{\partial}{\partial x}\int_0^t\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}d\tau+$$
$$+\frac{\partial}{\partial y}\int_0^t\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}d\tau-$$
$$-\alpha_{1V}^2\int_0^t k_{V,V}(x,y,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau$$
$$+f_V(x,y,z)$$

$$\Phi_{1I}(x,y,z,t)=\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+$$
$$+\alpha_{1\Phi_I}z\,\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+f_{\Phi_I}(x,y,z)+$$
$$+\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial y}d\tau+$$
$$+\frac{\partial}{\partial z}\int_0^t\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial z}d\tau+\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)d\tau+$$
$$+\int_0^t k_{I,I}(x,y,z,T)I^2(x,y,z,\tau)d\tau \qquad (7c)$$

$$\Phi_{1V}(x,y,z,t)=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+\alpha_{1\Phi_V}\Omega\times$$
$$\times z\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,\tau)\,d\tau+f_{\Phi_V}(x,y,z)+\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{\bar{V}kT}\times$$
$$\times\frac{\partial\mu_2(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial y}d\tau+$$
$$+\frac{\partial}{\partial z}\int_0^t\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial z}d\tau+\int_0^t k_V(x,y,z,T)\,V(x,y,z,\tau)\,d\tau+$$
$$+\int_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,\tau)d\tau\,.$$

We determine average values of the first-order approximations of concentrations of dopant and radiation defects by the following standard relation [28].

$$\alpha_{1\rho}=\frac{1}{\Theta L_xL_yL_z}\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\rho_1(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t\,. \qquad (14)$$

Substitution of the relations (1*c*), (4*c*) and (7*c*) into relation (9) gives us possibility to obtain required average values in the following form

$$\alpha_{1C}=\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_C(x,y,z)\,d\,z\,d\,y\,d\,x\,,\ \alpha_{1V}=\frac{1}{S_{IV00}}\times$$
$$\times\left[\frac{\Theta}{\alpha_{1I}}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}f_I(x,y,z)\,d\,z\,d\,y\,d\,x-\alpha_{1I}S_{II00}-\Theta L_xL_yL_z\right], \qquad (15)$$

$$\alpha_{1I}=\sqrt{\frac{(a_3+A)^2}{4a_4^2}-4\left(B+\frac{\Theta a_3B+\Theta^2L_xL_yL_za_1}{a_4}\right)}-\frac{a_3+A}{4a_4}\,,$$

where,

$$S_{\rho\rho\,ij}=\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}k_{\rho,\rho}(x,y,z,T)V_1^j(x,y,z,t)I_1^i(x,y,z,t)\,d\,z\,d\,y\,d\,x\times$$
$$\times(\Theta-t)\,d\,t\,,\ a_4=S_{II00}\left(S_{IV00}^2-S_{II00}S_{VV00}\right),\ a_2=S_{IV00}\Theta L_x^2\times$$

$$\times L_y^2 L_z^2 + S_{IV00} S_{IV00}^2 \int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_V(x,y,z)\,dz\,dy\,dx + 2S_{VV00}S_{II00}\times$$

$$\times\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx - \Theta L_x^2 L_y^2 L_z^2 S_{VV00} -$$

$$-S_{IV00}^2 \int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx\,,\; a_3 = S_{IV00}S_{II00} + S_{IV00}^2 -$$

$$-S_{II00}S_{VV00}\,,\; a_1 = S_{IV00}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx\,,$$

$$a_0 = S_{VV00}\times$$

$$\times\left[\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx\right]^2,\; A=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}}\,, \tag{15}$$

$$B=\frac{\Theta a_2}{6a_4}+\sqrt[3]{\sqrt{q^2+p^3}-q}-\sqrt[3]{\sqrt{q^2+p^3}+q}\,,\; q=\frac{\Theta^3 a_2}{24a_4^2}\times$$

$$\times\left(4a_0-\Theta L_xL_yL_z\frac{a_1a_3}{a_4}\right)-\Theta^2\frac{a_0}{8a_4^2}\left(4\Theta a_2-\Theta^2\frac{a_3^2}{a_4}\right)-\frac{\Theta^3a_2^3}{54a_4^3}-$$

$$-L_x^2L_y^2L_z^2\frac{\Theta^4a_1^2}{8a_4^2}\,,\; p=\Theta^2\frac{4a_0a_4-\Theta L_xL_yL_za_1a_3}{12a_4^2}-\frac{\Theta a_2}{18a_4}\,,$$

$$\begin{cases}\alpha_{1\Phi_I}=\dfrac{R_{I1}}{\Theta L_xL_yL_z}+\dfrac{S_{II20}}{\Theta L_xL_yL_z}+\\ \qquad+\dfrac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_{\Phi_I}(x,y,z)\,dz\,dy\,dx\\ \alpha_{1\Phi_V}=\dfrac{R_{V1}}{\Theta L_xL_yL_z}+\dfrac{S_{VV20}}{\Theta L_xL_yL_z}+\\ \qquad+\dfrac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} f_{\Phi_V}(x,y,z)\,dz\,dy\,dx,\end{cases} \tag{16}$$

where,

$$R_{\rho i}=\int\limits_0^{\Theta}(\Theta-t)\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z} k_I(x,y,z,T)\,I_1^i(x,y,z,t)\,dz\,dy\,dx\,dt\,.$$

We determine approximations of the second and higher orders of concentrations of dopant and radiation defects framework standard iterative procedure of method of averaging of function corrections [28]. Framework this procedure to determine approximations of the n-th order of concentrations of dopant and radiation defects we replace the required concentrations in the Eqns. (1c), (4c), (7c) on the following sum $\alpha_{n\rho}+\rho_{n-1}(x,y,z,t)$. The replacement leads to the following transformation of the appropriate equations

$$\frac{\partial C_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left(\left\{1+\xi\frac{\left[\alpha_{2C}+C_1(x,y,z,t)\right]^\gamma}{P^\gamma(x,y,z,T)}\right\}\times\right.$$

$$\times D_L(x,y,z,T)\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times$$

$$\left.\times\frac{\partial C_1(x,y,z,t)}{\partial x}\right)+\frac{\partial}{\partial y}\left(\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times\right.$$

$$\left.\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial y}\left\{1+\xi\frac{\left[\alpha_{2C}+C_1(x,y,z,t)\right]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right)+$$

$$+\frac{\partial}{\partial z}\left(D_L(x,y,z,T)\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times\right.$$

$$\left.\times\frac{\partial C_1(x,y,z,t)}{\partial z}\left\{1+\xi\frac{\left[\alpha_{2C}+C_1(x,y,z,t)\right]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right)+$$

$$+f_C(x,y,z)\delta(t)+\frac{\partial}{\partial x}\left[\frac{D_{CS}}{\bar V kT}\frac{\partial\,\mu_2(x,y,z,t)}{\partial x}\right]+$$

$$+\frac{\partial}{\partial y}\left[\frac{D_{CS}}{\bar V kT}\frac{\partial\,\mu_2(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[\frac{D_{CS}}{\bar V kT}\frac{\partial\,\mu_2(x,y,z,t)}{\partial z}\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2C}+C(x,y,W,t)\right]dW\right\}+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2C}+C(x,y,W,t)\right]dW\right\} \tag{1d}$$

$$\frac{\partial I_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial x}\right]+$$

$$+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial y}\right]-k_{I,I}(x,y,z,T)\times$$

$$\times\left[\alpha_{1I}+I_1(x,y,z,t)\right]^2+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial z}\right]-$$

$$-k_{I,V}(x,y,z,T)\left[\alpha_{1I}+I_1(x,y,z,t)\right]\left[\alpha_{1V}+V_1(x,y,z,t)\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,t)\right]dW\right\}+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,t)\right]dW\right\}+$$

$$+\frac{\partial}{\partial x}\int\limits_0^t\frac{D_{IS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int\limits_0^t\frac{D_{IS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial y}d\tau+$$

$$+\frac{\partial}{\partial z}\int\limits_0^t\frac{D_{IS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial z}d\tau \tag{4d}$$

$$\frac{\partial V_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial x}\right]+$$

$$+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial y}\right]-k_{V,V}(x,y,z,T)\times$$

$$\times\left[\alpha_{1V}+V_1(x,y,z,t)\right]^2+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial z}\right]-$$

$$-k_{I,V}(x,y,z,T)\left[\alpha_{1I}+I_1(x,y,z,t)\right]\left[\alpha_{1V}+V_1(x,y,z,t)\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,t)\right]dW\right\}+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{VS}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\alpha_{2V}+V_1\left(x,y,W,t\right)\right]dW\right\}+$$

$$+\frac{\partial}{\partial x}\int\limits_0^t\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,\tau\right)}{\partial x}d\tau+\frac{\partial}{\partial y}\int\limits_0^t\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,\tau\right)}{\partial y}d\tau+$$

$$+\frac{\partial}{\partial z}\int\limits_0^t\frac{D_{VS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,\tau\right)}{\partial z}d\tau$$

$$\frac{\partial\Phi_{2I}\left(x,y,z,t\right)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}\left(x,y,z,T\right)\frac{\partial\Phi_{1I}\left(x,y,z,t\right)}{\partial x}\right]+$$

$$+\frac{\partial}{\partial y}\left[D_{\Phi_I}\left(x,y,z,T\right)\frac{\partial\Phi_{1I}\left(x,y,z,t\right)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}\left(x,y,z,T\right)\frac{\partial\Phi_{1I}\left(x,y,z,t\right)}{\partial z}\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}\left(x,y,W,t\right)\right]dW\right\}+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}\left(x,y,W,t\right)\right]dW\right\}+$$

$$+k_{I,I}\left(x,y,z,T\right)I^2\left(x,y,z,t\right)+k_I\left(x,y,z,T\right)I\left(x,y,z,t\right)+$$

$$+\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_I S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial z}\right]+f_{\Phi_I}\left(x,y,z\right)\delta\left(t\right)\qquad(7d)$$

$$\frac{\partial\Phi_{2V}\left(x,y,z,t\right)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}\left(x,y,z,T\right)\frac{\partial\Phi_{1V}\left(x,y,z,t\right)}{\partial x}\right]+$$

$$+\frac{\partial}{\partial y}\left[D_{\Phi_V}\left(x,y,z,T\right)\frac{\partial\Phi_{1V}\left(x,y,z,t\right)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}\left(x,y,z,T\right)\frac{\partial\Phi_{1V}\left(x,y,z,t\right)}{\partial z}\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}\left(x,y,W,t\right)\right]dW\right\}+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int\limits_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}\left(x,y,W,t\right)\right]dW\right\}+$$

$$+k_{V,V}\left(x,y,z,T\right)V^2\left(x,y,z,t\right)+k_V\left(x,y,z,T\right)V\left(x,y,z,t\right)+$$

$$+\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[\frac{D_{\Phi_V S}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial z}\right]+f_{\Phi_V}\left(x,y,z\right)\delta\left(t\right).$$

Integration of the left and the right sides of Eqns. (1*d*), (4*d*) and (7*d*) gives us possibility to obtain relations for the required concentrations in the final form

$$C_2\left(x,y,z,t\right)=\frac{\partial}{\partial x}\int\limits_0^t\left[1+\varsigma_1\frac{V\left(x,y,z,\tau\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,\tau\right)}{\left(V^*\right)^2}\right]\times$$

$$\times D_L\left(x,y,z,T\right)\left\{1+\xi\frac{\left[\alpha_{2C}+C_1\left(x,y,z,\tau\right)\right]^\gamma}{P^\gamma\left(x,y,z,T\right)}\right\}\frac{\partial C_1\left(x,y,z,\tau\right)}{\partial x}d\tau+$$

$$+\frac{\partial}{\partial y}\int\limits_0^t D_L\left(x,y,z,T\right)\left[1+\varsigma_1\frac{V\left(x,y,z,\tau\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,\tau\right)}{\left(V^*\right)^2}\right]\times$$

$$\times\frac{\partial C_1\left(x,y,z,\tau\right)}{\partial y}\left\{1+\xi\frac{\left[\alpha_{2C}+C_1\left(x,y,z,t\right)\right]^\gamma}{P^\gamma\left(x,y,z,T\right)}\right\}+$$

$$+\frac{\partial}{\partial z}\int\limits_0^t\frac{\partial C_1\left(x,y,z,\tau\right)}{\partial z}\left[1+\varsigma_1\frac{V\left(x,y,z,\tau\right)}{V^*}+\varsigma_2\frac{V^2\left(x,y,z,\tau\right)}{\left(V^*\right)^2}\right]\times$$

$$\times D_L\left(x,y,z,T\right)\left\{1+\xi\frac{\left[\alpha_{2C}+C_1\left(x,y,z,\tau\right)\right]^\gamma}{P^\gamma\left(x,y,z,T\right)}\right\}d\tau+f_C\left(x,y,z\right)+$$

$$+\Omega\frac{\partial}{\partial x}\int\limits_0^t\frac{D_S}{kT}\nabla_S\mu\left(x,y,z,\tau\right)\int\limits_0^{L_z}\left[\alpha_{2C}+C_1\left(x,y,W,\tau\right)\right]dW\,d\tau+$$

$$+\Omega\frac{\partial}{\partial y}\int\limits_0^t\frac{D_S}{kT}\nabla_S\mu\left(x,y,z,\tau\right)\int\limits_0^{L_z}\left[\alpha_{2C}+C_1\left(x,y,W,\tau\right)\right]dW\,d\tau+$$

$$+\frac{\partial}{\partial x}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[\frac{D_{CS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial z}\right]\qquad(1e)$$

$$I_2\left(x,y,z,t\right)=\frac{\partial}{\partial x}\int\limits_0^t D_I\left(x,y,z,T\right)\frac{\partial I_1\left(x,y,z,\tau\right)}{\partial x}d\tau+$$

$$+\frac{\partial}{\partial y}\int\limits_0^t D_I\left(x,y,z,T\right)\frac{\partial I_1\left(x,y,z,\tau\right)}{\partial y}d\tau+\frac{\partial}{\partial y}\int\limits_0^t D_I\left(x,y,z,T\right)\times$$

$$\times\frac{\partial I_1\left(x,y,z,\tau\right)}{\partial z}d\tau-\int\limits_0^t k_{I,I}\left(x,y,z,T\right)\left[\alpha_{2I}+I_1\left(x,y,z,\tau\right)\right]^2d\tau-$$

$$-\int\limits_0^t k_{I,V}\left(x,y,z,T\right)\left[\alpha_{2I}+I_1\left(x,y,z,\tau\right)\right]\left[\alpha_{2V}+V_1\left(x,y,z,\tau\right)\right]d\tau+$$

$$+\Omega\frac{\partial}{\partial x}\int\limits_0^t\frac{D_{IS}}{kT}\nabla_S\mu\left(x,y,z,\tau\right)\int\limits_0^{L_z}\left[\alpha_{2I}+I_1\left(x,y,W,\tau\right)\right]dW\,d\tau+$$

$$+\Omega\frac{\partial}{\partial y}\int\limits_0^t\frac{D_{IS}}{kT}\nabla_S\mu\left(x,y,z,\tau\right)\int\limits_0^{L_z}\left[\alpha_{2I}+I_1\left(x,y,W,\tau\right)\right]dW\,d\tau+$$

$$+\frac{\partial}{\partial x}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[\frac{D_{IS}}{\bar{V}kT}\frac{\partial\mu_2\left(x,y,z,t\right)}{\partial z}\right]+f_I\left(x,y,z\right)\qquad(4e)$$

$$V_2\left(x,y,z,t\right)=\frac{\partial}{\partial x}\int\limits_0^t D_V\left(x,y,z,T\right)\frac{\partial V_1\left(x,y,z,\tau\right)}{\partial x}d\tau+$$

$$+\frac{\partial}{\partial y}\int\limits_0^t D_V\left(x,y,z,T\right)\frac{\partial V_1\left(x,y,z,\tau\right)}{\partial y}d\tau+\frac{\partial}{\partial y}\int\limits_0^t D_V\left(x,y,z,T\right)\times$$

$$\times\frac{\partial V_1\left(x,y,z,\tau\right)}{\partial z}d\tau-\int\limits_0^t k_{V,V}\left(x,y,z,T\right)\left[\alpha_{2V}+V_1\left(x,y,z,\tau\right)\right]^2d\tau-$$

$$-\int\limits_0^t k_{I,V}\left(x,y,z,T\right)\left[\alpha_{2I}+I_1\left(x,y,z,\tau\right)\right]\left[\alpha_{2V}+V_1\left(x,y,z,\tau\right)\right]d\tau+$$

$$+\Omega\frac{\partial}{\partial x}\int_0^t \frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]dW\,d\tau+$$

$$+\Omega\frac{\partial}{\partial y}\int_0^t \frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,\tau)\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]dW\,d\tau+$$

$$+\frac{\partial}{\partial x}\left[\frac{D_{VS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[\frac{D_{VS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[\frac{D_{VS}}{\bar V kT}\frac{\partial\mu_2(x,y,z,t)}{\partial z}\right]+f_V(x,y,z)$$

$$\Phi_{2I}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial x}d\tau+$$

$$+\frac{\partial}{\partial y}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t D_{\Phi_I}(x,y,z,T)\times$$

$$\times\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial y}d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{\Phi_I S}}{kT}\times$$

$$\times\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\,d\tau+\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(x,y,z,\tau)\times$$

$$\times\Omega\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\,d\tau+\int_0^t k_{I,I}(x,y,z,T)\times$$

$$\times I^2(x,y,z,\tau)\,d\tau+\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_I S}}{\bar V kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial x}d\tau+$$

$$+\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_I S}}{\bar V kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t\frac{D_{\Phi_I S}}{\bar V kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial z}d\tau+$$

$$+f_{\Phi_I}(x,y,z)+\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)\,d\tau \qquad (7e)$$

$$\Phi_{2V}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial x}d\tau+$$

$$+\frac{\partial}{\partial y}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t D_{\Phi_V}(x,y,z,T)\times$$

$$\times\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial y}d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\frac{D_{\Phi_V S}}{kT}\times$$

$$\times\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]dW\,d\tau+\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(x,y,z,\tau)\times$$

$$\times\Omega\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]dW\,d\tau+\int_0^t k_{V,V}(x,y,z,T)\times$$

$$\times V^2(x,y,z,\tau)\,d\tau+\frac{\partial}{\partial x}\int_0^t\frac{D_{\Phi_V S}}{\bar V kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial x}d\tau+$$

$$+\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_V S}}{\bar V kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial y}d\tau+\frac{\partial}{\partial z}\int_0^t\frac{D_{\Phi_V S}}{\bar V kT}\frac{\partial\mu_2(x,y,z,\tau)}{\partial z}d\tau+$$

$$+f_{\Phi_V}(x,y,z)+\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)\,d\tau\,.$$

Average values of the second-order approximations of required approximations by using the following standard relation [28].

$$\alpha_{2\rho}=\frac{1}{\Theta L_xL_yL_z}\times$$

$$\times\int_0^{\Theta}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[\rho_2(x,y,z,t)-\rho_1(x,y,z,t)\right]dz\,dy\,dx\,dt\,. \quad (17)$$

Substitution of the relations (1e), (4e), (7e) into relation (10) gives us possibility to obtain relations for required average values $\alpha_{2\rho}$.

$$\alpha_{2C}=0,\ \alpha_{2\Phi I}=0,\ \alpha_{2\Phi V}=0,$$

$$\alpha_{2V}=\sqrt{\frac{(b_3+E)^2}{4b_4^2}-4\left(F+\frac{\Theta a_3F+\Theta^2L_xL_yL_zb_1}{b_4}\right)}-\frac{b_3+E}{4b_4},$$

$$\alpha_{2I}=\left[C_V-\alpha_{2V}^2S_{VV00}-\alpha_{2V}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)-S_{VV02}-\right.$$

$$\left.-S_{IV11}\right]/\left(S_{IV01}+\alpha_{2V}S_{IV00}\right), \qquad (18)$$

where,

$$b_4=\frac{1}{\Theta L_xL_yL_z}S_{IV00}^2S_{VV00}-\frac{1}{\Theta L_xL_yL_z}S_{VV00}^2S_{II00},\ b_3=-\left(\Theta L_x\times\right.$$

$$\times L_yL_z+2S_{VV01}+S_{IV10}\Big)\frac{S_{II00}S_{VV00}}{\Theta L_xL_yL_z}+\frac{S_{IV00}S_{VV00}}{\Theta L_xL_yL_z}\Big(S_{IV01}+L_xL_yL_z\times$$

$$\times\Theta+S_{IV01}+2S_{II10}\Big)+\frac{S_{IV00}^2}{\Theta L_xL_yL_z}\left(2S_{VV01}+\Theta L_xL_yL_z+S_{IV10}\right)-$$

$$-\frac{S_{IV00}^2S_{IV10}}{\Theta^3L_x^3L_y^3L_z^3},\ b_2=\frac{S_{II00}S_{VV00}}{\Theta L_xL_yL_z}\left(S_{VV02}+S_{IV11}+C_V\right)- \qquad (19)$$

$$-\left(S_{IV10}-2S_{VV01}+\Theta L_xL_yL_z\right)^2+\frac{S_{IV01}S_{VV00}}{\Theta L_xL_yL_z}\Big(2S_{II10}+S_{IV01}+$$

$$+\Theta L_xL_yL_z\Big)+\frac{S_{IV00}}{\Theta L_xL_yL_z}\left(S_{IV01}+2S_{II10}+2S_{IV01}+\Theta L_xL_yL_z\right)\Big(\Theta\times$$

$$\times L_xL_yL_z+2S_{VV01}+S_{IV10}\Big)-S_{IV00}^2\frac{C_V-S_{VV02}-S_{IV11}}{\Theta L_xL_yL_z}-2\frac{S_{IV10}}{\Theta L_x}\times$$

$$\times\frac{S_{IV00}}{L_yL_z}S_{IV01}+\frac{C_IS_{IV00}^2}{\Theta^2L_x^2L_y^2L_z^2},\ b_1=S_{II00}\frac{S_{IV11}+S_{VV02}+C_V}{\Theta L_xL_yL_z}\Big(\Theta L_x\times$$

$$\times L_yL_z+2S_{VV01}+S_{IV10}\Big)+S_{IV01}\frac{\Theta L_xL_yL_z+2S_{II10}+S_{IV01}}{\Theta L_xL_yL_z}\Big(S_{IV10}+$$

$$+2S_{VV01}+\Theta L_xL_yL_z\Big)-\frac{S_{IV10}S_{IV01}^2}{\Theta L_xL_yL_z}-\frac{S_{IV00}}{\Theta L_xL_yL_z}\Big(3S_{IV01}+2S_{II10}+$$

$$+\Theta L_xL_yL_z\Big)\left(C_V-S_{VV02}-S_{IV11}\right)+2C_IS_{IV00}S_{IV01},\ b_0=S_{II00}\times$$

$$\times\frac{\left(S_{IV00}+S_{VV02}\right)^2}{\Theta L_xL_yL_z}-\frac{C_V-S_{VV02}-S_{IV11}}{L_xL_yL_z\Theta}\left(\Theta L_xL_yL_z+2S_{II10}+S_{IV01}\right)\times$$

$$\times S_{IV01}+2C_IS_{IV01}^2-S_{IV01}\frac{C_V-S_{VV02}-S_{IV11}}{\Theta L_xL_yL_z}\Big(\Theta L_xL_yL_z+2S_{II10}+$$

$$+S_{IV01}\Big),\ C_I=\frac{\alpha_{1I}\alpha_{1V}}{\Theta L_xL_yL_z}S_{IV00}+\frac{\alpha_{1I}^2S_{II00}}{\Theta L_xL_yL_z}-\frac{S_{II20}S_{II20}}{\Theta L_xL_yL_z}-\frac{S_{IV11}}{\Theta L_x}\times$$

$$\times\frac{1}{L_yL_z},\ C_V=\alpha_{1I}\alpha_{1V}S_{IV00}+\alpha_{1V}^2S_{VV00}-S_{VV02}-S_{IV11},$$

$$F=\frac{\Theta a_2}{6a_4}+$$

$$+\sqrt[3]{\sqrt{r^2+s^3}-r}-\sqrt[3]{\sqrt{r^2+s^3}+r},\ E=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}},$$

$$r=\frac{\Theta^3b_2}{24b_4^2}\left(4b_0-\Theta L_xL_yL_z\frac{b_1b_3}{b_4}\right)-b_0\frac{\Theta^2}{8b_4^2}\left(4\Theta b_2-\Theta^2\frac{b_3^2}{b_4}\right)-$$

$$-\frac{\Theta^3b_2^3}{54b_4^3}-L_x^2L_y^2L_z^2\frac{\Theta^4b_1^2}{8b_4^2},\ s=\Theta^2\frac{4b_0b_4-\Theta L_xL_yL_zb_1b_3}{12b_4^2}-\frac{\Theta b_2}{18b_4}.$$

Farther we determine solutions of Eq. (8), i.e. components of displacement vector. To determine the first-order approximations of the considered components framework method of averaging of function corrections we replace the required functions in the right sides of the equations by their not yet known average values α_i. The substitution leads to the following result.

$$\rho(z)\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x},$$
$$\rho(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y},$$
$$\rho(z)\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Integration of the left and the right sides of the above relations on time t leads to the following result.

$$u_{1x}(x,y,z,t)=u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-$$
$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta,$$
$$u_{1y}(x,y,z,t)=u_{0y}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-$$
$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta, \qquad (19)$$
$$u_{1z}(x,y,z,t)=u_{0z}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta-$$
$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta.$$

Approximations of the second and higher orders of components of displacement vector could be determined by using standard replacement of the required components on the following sums $\alpha_i+u_i(x,y,z,t)$ [28]. The replacement leads to the following result.

$$\rho(z)\frac{\partial^2 u_{2x}(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x^2}+$$
$$+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}+\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}+\right.$$
$$\left.+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial z^2}\right]\frac{E(z)}{2[1+\sigma(z)]}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x}+$$
$$+\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x\partial z}$$

$$\rho(z)\frac{\partial^2 u_{2y}(x,y,z,t)}{\partial t^2}=\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial y}\right]\times$$
$$\times\frac{E(z)}{2[1+\sigma(z)]}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y}+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\times\right.$$
$$\left.\times\left[\frac{\partial u_{1y}(x,y,z,t)}{\partial z}+\frac{\partial u_{1z}(x,y,z,t)}{\partial y}\right]\right\}+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}\times$$
$$\times\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}+\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}+$$
$$+K(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y} \qquad (20)$$

$$\rho(z)\frac{\partial^2 u_{2z}(x,y,z,t)}{\partial t^2}=\left[\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial y^2}+\right.$$
$$\left.+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial z}+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}\right]\frac{E(z)}{2[1+\sigma(z)]}+\frac{\partial}{\partial z}\left\{K(z)\times\right.$$
$$\left.\times\left[\frac{\partial u_{1x}(x,y,z,t)}{\partial x}+\frac{\partial u_{1y}(x,y,z,t)}{\partial y}+\frac{\partial u_{1x}(x,y,z,t)}{\partial z}\right]\right\}+$$
$$+\frac{\partial}{\partial z}\left[6\frac{\partial u_{1z}(x,y,z,t)}{\partial z}-\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\right.$$
$$\left.-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\right]\frac{E(z)}{6[1+\sigma(z)]}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Integration of the left and right sides of the above relations on time t leads to the following result.

$$u_{2x}(x,y,z,t)=\frac{1}{\rho(z)}\frac{\partial^2}{\partial x^2}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\left\{\frac{5E(z)}{6[1+\sigma(z)]}+\right.$$
$$\left.+K(z)\right\}-\frac{1}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\left\{\frac{E(z)}{3[1+\sigma(z)]}-\right.$$
$$\left.-K(z)\right\}+\frac{E(z)}{2\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial y^2}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$
$$\left.+\frac{\partial^2}{\partial z^2}\int_0^t\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]+\frac{\partial^2}{\partial x\partial z}\int_0^t\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\times$$
$$\times\frac{1}{\rho(z)}\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}-K(z)\frac{\partial}{\partial x}\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\times$$
$$\times\frac{\beta(z)}{\rho(z)}-\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\times$$
$$\times\frac{1}{\rho(z)}-\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\partial y}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\times$$
$$\times\frac{1}{\rho(z)}-\frac{E(z)}{2\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$
$$\left.+\frac{\partial^2}{\partial z^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]-\frac{\partial^2}{\partial x\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\times$$
$$\times\frac{1}{\rho(z)}\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}+\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\times$$
$$\times K(z)\beta(z)/\rho(z)+u_{0x} \qquad (21)$$

$$u_{2y}(x,y,z,t)=\frac{E(z)}{2\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial x^2}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$
$$\left.+\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\right]+\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\times$$
$$\times\frac{K(z)}{\rho(z)}+\frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int_0^t\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\left\{\frac{5E(z)}{12[1+\sigma(z)]}+\right.$$

$$+K(z)\Bigg\}+\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.\right.$$

$$\left.\left.+\frac{\partial}{\partial y}\int_0^t\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}-K(z)\int_0^t\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\times$$

$$\times\frac{\beta(z)}{\rho(z)}-\frac{\partial^2}{\partial y\partial z}\int_0^t\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\left\{\frac{E(z)}{6\left[1+\sigma(z)\right]}-K(z)\right\}\times$$

$$\times\frac{1}{\rho(z)}-\frac{E(z)}{2\rho(z)\left[1+\sigma(z)\right]}\left[\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$

$$\left.+\frac{\partial^2}{\partial x\partial y}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\right]-K(z)\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta\times$$

$$\times\frac{\beta(z)}{\rho(z)}-\frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+\right.$$

$$\left.+K(z)\right\}-\frac{K(z)}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\times\right.$$

$$\left.\times\left[\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}\times$$

$$\times\frac{1}{2\rho(z)}+\frac{1}{\rho(z)}\frac{\partial^2}{\partial y\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\left\{\frac{E(z)}{6\left[1+\sigma(z)\right]}-\right.$$

$$\left.-K(z)\right\}+u_{0y}$$

$$u_z(x,y,z,t)=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$

$$+\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial x\partial z}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+$$

$$\left.+\frac{\partial^2}{\partial y\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\frac{1}{\rho(z)}+\frac{1}{\rho(z)}\frac{\partial}{\partial z}\left\{K(z)\times\right.$$

$$\times\left[\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$

$$\left.\left.+\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}+\frac{1}{6\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\times\right.$$

$$\times\left[6\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta-\right.$$

$$\left.\left.-\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}-$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0z}\,.$$

Framework this paper we determine concentration of dopant, concentrations of radiation defects and components of displacement vector by using the second-order approximation framework method of averaging of function corrections. This approximation is usually enough good approximation to make qualitative analysis and to obtain some quantitative results. All obtained results have been checked by comparison with results of numerical simulations.

3. DISCUSSION

In this section we analyzed dynamics of redistributions of dopant and radiation defects during annealing and under influence of mismatch-induced stress and modification of porosity. Typical distributions of concentrations of dopant in heterostructures are presented on Figures 2 and 3 in direction, which is perpendicular to interface between epitaxial layer substrate. Figure 2 corresponds to diffusion type of doping. Figure 3 corresponds to diffusion type of doping. Increasing of number of curve corresponds to increasing of difference between values of dopant diffusion coefficient in layers of heterostructure under condition, when value of dopant diffusion coefficient in epitaxial layer is larger, than value of dopant diffusion coefficient in substrate. These distributions have been calculated for the case, when value of dopant diffusion coefficient in doped area is larger, than in nearest areas. Curves 1 and 3 on Figure 3 corresponds to annealing time $\Theta=0.0048(L_x^2+L_y^2+L_z^2)/D_0$. Curves 2 and 4 on Figure 3 corresponds to annealing time $\Theta=0.0057(L_x^2+L_y^2+L_z^2)/D_0$. Curves 1 and 2 corresponds to homogenous sample. Curves 3 and 4 corresponds to heterostructure under condition, when value of dopant diffusion coefficient in epitaxial layer is larger, than value of dopant diffusion coefficient in substrate. Annealing time of dopant, which corresponds to Figure 2, is equal to $\Theta=0.005(L_x^2+L_y^2+L_z^2)/D_0$. The figures show, that inhomogeneity of heterostructure gives us possibility to increase compactness of concentrations of dopants and at the same time to increase homogeneity of dopant distribution in doped part of epitaxial layer. However, this manufacturing approach of bioplar transistor cannot effectively anneal the dopants and/or radiation defects. The annealing process should be optimized for the following reasons. If annealing time is small, the dopant did not achieve any interfaces between materials of heterostructure. In this situation one cannot find any modifications of distribution of concentration of dopant. If annealing time is large, distribution of concentration of dopant is too homogenous. We optimize annealing time framework recently introduces approach [29-37]. Framework this criterion we approximate real distribution of concentration of dopant by step-wise function (see Figures 4 and 5). These figures show spatial distributions of dopant in heterostructure after infusion (for Figure 4) or implantation of dopant (for Figure 5). Curve 1 is idealized distribution of dopant. Curves 2-4 are real distributions of dopant for different values of annealing time. Increasing of number of curve corresponds to increasing of annealing time. Farther we determine optimal values of annealing time by minimization of the following mean-squared error.

$$U=\frac{1}{L_xL_yL_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[C(x,y,z,\Theta)-\psi(x,y,z)\right]\,d z\,d y\,d x \quad (22)$$

where, $\psi(x,y,z)$ is the approximation function. Dependences of optimal values of annealing time on parameters are presented on Figures 6 and 7 for diffusion and ion types of doping, respectively. Curve 1 is the dependence of dimensionless optimal annealing time on the relation a/L and $\xi=\gamma=0$ for equal to each other values of dopant diffusion coefficient in all parts of heterostructure. Curve 2 is the dependence of dimensionless optimal annealing time on value of parameter ε for a/L=1/2 and $\xi=\gamma=0$. Curve 3 is the dependence of dimensionless optimal annealing time on value of parameter ξ for a/L=1/2 and $\varepsilon=\gamma=0$. Curve 4 is the dependence of dimensionless optimal annealing time on value of parameter γ for a/L=1/2 and $\varepsilon=\xi=0$. It should be noted, that it is necessary to anneal radiation defects after ion implantation. One could find spreading of concentration of distribution of dopant during this annealing. In the ideal case distribution of dopant achieves appropriate interfaces between materials of heterostructure during annealing of radiation defects. If dopant did not achieve any interfaces during annealing of radiation defects, it is practicably to additionally anneal the dopant. In this situation optimal value of additional annealing time of implanted dopant is smaller, than annealing time of infused dopant.

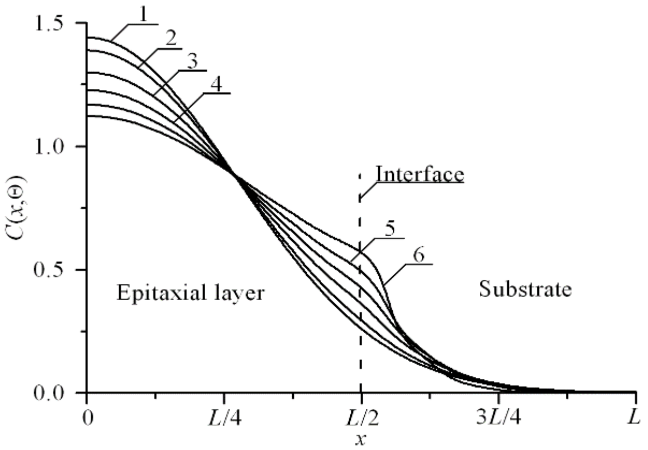

Figure 2. Distributions of concentration of infused dopant

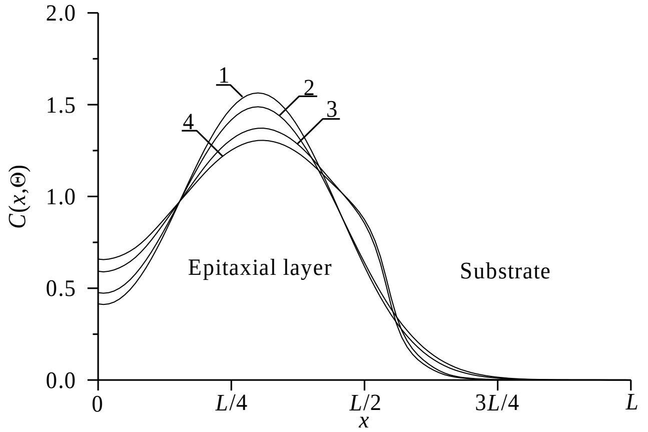

Figure 3. Distributions of concentration of implanted dopant

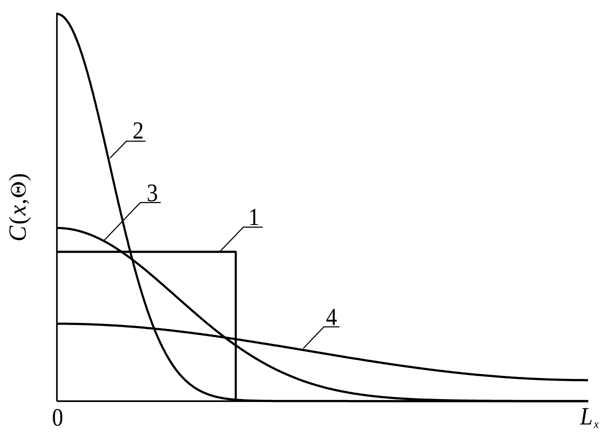

Figure 4. Typical spatial distributions of dopant in heterostructure after dopant infusion

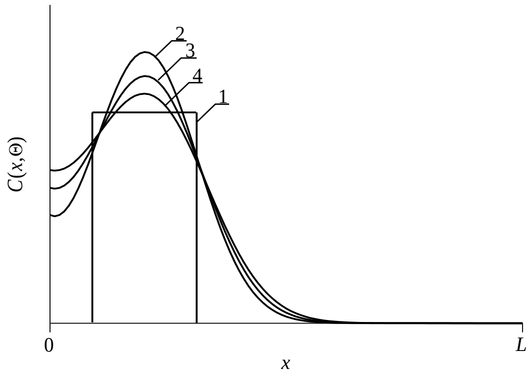

Figure 5. Typical spatial distributions of dopant in heterostructure after ion implantation

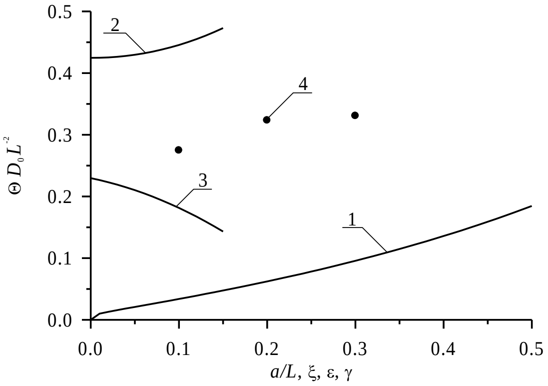

Figure 6. Dependences of dimensionless optimal annealing time for doping by diffusion

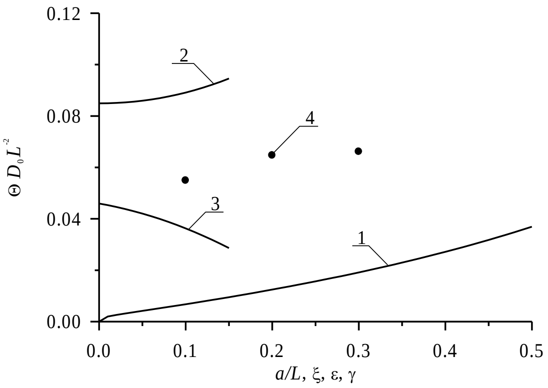

Figure 7. Dependences of dimensionless optimal annealing time for doping by ion implantation

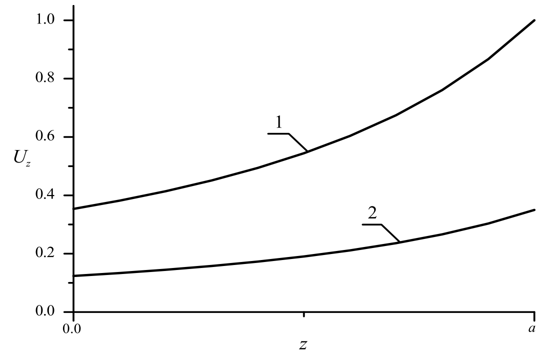

Figure 8. Normalized dependences of component u_z of displacement vector on coordinate z for nonporous (curve 1) and porous (curve 2) epitaxial layers

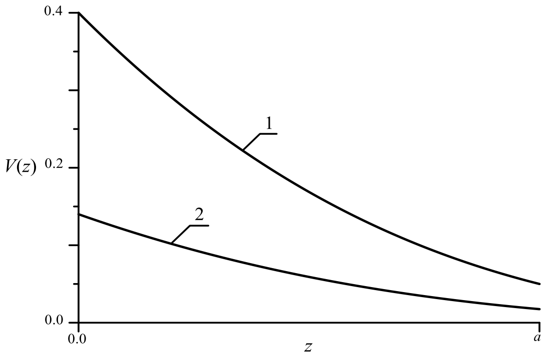

Figure 9. Normalized dependences of vacancy concentrations on coordinate *z* in unstressed (curve 1) and stressed (curve 2) epitaxial layers

Farther we analyzed influence of relaxation of mechanical stress on distribution of dopant in doped areas of heterostructure. Under following condition $\varepsilon_0 < 0$ one can find compression of distribution of concentration of dopant near interface between materials of heterostructure. Contrary (at $\varepsilon_0 > 0$) one can find spreading of distribution of concentration of dopant in this area. This changing of distribution of concentration of dopant could be at least partially compensated by using laser annealing [37]. This type of annealing gives us possibility to accelerate diffusion of dopant and other processes in annealed area due to inhomogenous distribution of temperature and Arrhenius law. Accounting relaxation of mismatch-induced stress in heterostructure could leads to changing of optimal values of annealing time. At the same time modification of porosity gives us possibility to decrease value of mechanical stress. On the one hand mismatch-induced stress could be used to increase density of elements of integrated circuits. On the other hand, it could leads to generation dislocations of the discrepancy. Figures 8 and 9 show distributions of concentration of vacancies in porous materials and component of displacement vector, which is perpendicular to interface between layers of heterostructure.

4. CONCLUSION

In this paper we model redistribution of infused and implanted dopants with account relaxation mismatch-induced stress during manufacturing field-effect heterotransistors framework a two-level current-mode logic gates in a multiplexer. We obtain, that using difference between materials of heterostructure and optimization of annealing of dopant and/or radiation defects gives a possibility to decrease dimensions of transistors and to increase their density. We obtain, that using ion implantation gives a possibility to decrease mismatch-induced stress. At the same time, it is necessary to choose materials with higher charge carrier motility and minimal mismatch-induced stress. Minimization of mismatch-induced stress gives a possibility to use diffusion type of doping without radiation damage of materials of heterostructure. Increasing of charge carrier motility gives a possibility to accelerate of transport of charge carriers. We also introduce an analytical approach to model diffusion and ion types of doping with account concurrent changing of parameters in space and time. At the same time the approach gives us possibility to take into account nonlinearity of considered processes.

REFERENCES

[1] Lachin, V.I., Savelov, N.S. (2001). Electronics. Phoenix. Rostov-on-Don.

[2] Polishscuk, A. (2004). Anadigm Programmable analog integrated circuits: The entire spectrum of analog electronics on a single chip. First meeting. Modern Electronics, 12: 8-11.

[3] Volovich, G. (2006). Modern chips UM3Ch class D manufactured by firm MPS. Modern Electronics, 2: 10-17.

[4] Kerentsev, A., Lanin, V. (2008). Constructive-technological features of MOSFET-transistors. Power Electronics, 1: 34-38.

[5] Ageev, A.O., Belyaev, A.E., Boltovets, N.S., Ivanov, V.N., Konakova, R.V., Kudrik, Ya.Ya., Litvin, P.M., Milenin, V.V., Sachenko, A.V. (2009). Au-TiB$_x$-n-6H-SiC Schottky barrier diodes: the features of current flow in rectifying and nonrectifying contacts. Semiconductors, 43(7): 865-871. https://doi.org/10.1134/S1063782609070070

[6] Tsai, J.H., Chiu, S.Y., Lour, W.S. Guo, D.F. (2009). High-performance InGaP/GaAs PNP δ-doped heterojunction bipolar transistor. Semiconductors, 43(7): 939-942. https://doi.org/10.1134/S1063782609070227

[7] Alexandrov, O.V., Zakhar'in, A.O., Sobolev, N.A., Shek, E.I., Makoviychuk, M.M., Parshin, E.O. (1998). Formation of donor centers upon annealing of dysprosium-and holmium-implanted silicon. Semiconductors, 32(9): 921-923. https://doi.org/10.1134/1.1187515

[8] Ermolovich, I.B., Milenin, V.V., Red'ko, R.A., Red'ko, S.M. (2009). Specific features of recombination processes in CdTe films produced in different temperature conditions of growth and subsequent annealing. Semiconductors, 43(8): 980-984. https://doi.org/10.1134/S10 6378260908003X

[9] Sinsermsuksakul, P., Hartman, K., Kim, S.B., Heo, J., Sun, L., Park, H.H., Chakraborty, R., Buonassisi, T., Gordon, R.G. (2013). Enhancing the efficiency of SnS solar cells via band-offset engineering with a zinc oxysulfide buffer layer. Applied Physics Letters, 102(5): 053901-053905. https://doi.org/10.1063/1.4789855

[10] Reynolds, J.G., Reynolds, C.L., Mohanta, Jr.A., Muth, J.F., Rowe, J.E., Everitt, H.O., Aspnes, D.E. (2013). Shallow acceptor complexes in *p*-type ZnO. Applied Physics Letters, 102(15): 152114-152118. https://doi.org/10.1063/1.4802753

[11] Volokobinskaya, N.I., Komarov, I.N., Matyukhina, T.V., Reshetnikov, V.I., Rush, A.A., Falina, I.V., Yastrebov, A.S. (2001). A study of technological processes in the production of high-power high-voltage bipolar transistors incorporating an array of inclusions in the collector region. Semiconductors, 35(8): 974-978. https://doi.org/10.1134/1.1393038

[12] Pankratov, E.L., Bulaeva, E.A. (2013). Doping of materials during manufacture *p-n*-junctions and bipolar transistors. Analytical approaches to model technological approaches and ways of optimization of distributions of dopants. Reviews in Theoretical Science, 1(1): 58-82. https://doi.org/10.1166/rits.2013.1004

[13] Kukushkin, S.A., Osipov, A.V., Romanychev, A.I. (2016). Epitaxial growth of zinc oxide by the method of atomic layer deposition on SiC/Si substrates. Physics of

the Solid State, 58(7): 1448-1452. https://doi.org/10.1134/S1063783416070246

[14] Trukhanov, E.M., Kolesnikov, A.V., Loshkarev, I.D. (2015). Long-range stresses generated by misfit dislocations in epitaxial films. Russian Microelectronics, 44(8): 552-558. https://doi.org/10.1134/S1063739715080 119

[15] Pankratov, E.L., Bulaeva, E.A. (2015). On optimization of regimes of epitaxy from gas phase. Some analytical approaches to model physical processes in reactors for epitaxy from gas phase during growth films. Reviews in Theoretical Science, 3(4): 365-398. https://doi.org/10.1166/rits.2015.1041

[16] Ong, K.K., Pey, K.L., Lee, P.S., Wee, A.T.S., Wang, X.C., Chong, Y.F. (2006). Dopant distribution in the recrystallization transient at the maximum melt depth induced by laser annealing. Applied Physics Letters, 89 (17): 172111-172114. https://doi.org/10.1063/1.2364834

[17] Wang, H.T., Tan, L.S., Chor, E.F. (2005). Pulsed laser annealing of Be-implanted GaN. Journal of Applied Physics, 98(9): 094901-094905. https://doi.org/10.1063/1.2120893

[18] Bykov, Y.V., Yeremeev, A.G., Zharova, N.A., Plotnikov, I.V., Rybakov, K.I., Drozdov, M.N., Drozdov, Y.N., Skupov, V.D. (2003). Diffusion processes in semiconductor structures during microwave annealing. RadioPhysics and Quantum Electronics, 43(9): 749-755. https://doi.org/10.1023/B:RAQE.0000025008.97954.1c

[19] Jang, I., Kim, J., Kim, S. (2015). Accurate delay models of CMOS CML circuits for design optimization. Analog Integrated Circuits and Signal Processing, 82(1): 297-307. https://doi. org/10.1007/s10470-014-0460-4

[20] Zhang, Y.W., Bower, A.F. (1999). Numerical simulations of island formation in a coherent strained epitaxial thin film system. Journal of the Mechanics and Physics of Solids, 47(11): 2273-2297. https://doi.org/10.1016/S0022-5096(99)00026-5

[21] Landau, L.D., Lefshits, E.M., Kosevich, A.M., Pitaevskii, L.P. (2001). Theoretical Physics. 7 (Theory of elasticity). Physmatlit, Moscow.

[22] Kitayama, M., Narushima, T., Carter, W.C., Cannon, R.M., Glaeser, A.M. (2000). The Wulff shape of alumina: I, modeling the kinetics of morphological evolution. Journal of the American Ceramic Society, 83(10): 2561-2531. https://doi.org/10.1111/j.1151-2916.2000.tb01591.x

[23] Cheremskoy, P.G., Slesov, V.V., Betekhtin, V.I. (1990). Pore in solid bodies. Energoatomizdat, Moscow.

[24] Gotra, Z.Y. (1991). Technology of microelectronic devices. Radio and Communication, Moscow.

[25] Fahey, P.M., Griffin, P.B., Plummer, J.D. (1989). Point defects and dopant diffusion in silicon. Reviews of Modern Physics, 61(2): 289-388. https://doi.org/10.1103/RevModPhys.61. 289

[26] Vinetskiy, V.L., Kholodar', G.A. (1979). Radiative physics of semiconductors. Naukova Dumka, Kiev.

[27] Mynbaeva, M.G., Mokhov, E.N., Lavrent'ev, A.A., Mynbaev, K.D. (2008). High-temperature diffusion doping of porous silicon carbide. Technical Physics Letters, 34(9): 13. https://doi.org/10.1134/S106378500809 0034

[28] Sokolov, Y.D. (1955). About the definition of dynamic forces in the mine lifting. Applied Mechanics, 1(1): 23-35.

[29] Pankratov, E.L. (2007). Dopant diffusion dynamics and optimal diffusion time as influenced by diffusion-coefficient nonuniformity. Russian Microelectronics, 36(1): 33-39. https://doi.org/10.1134/S1063739707010040

[30] Pankratov, E.L. (2008). Redistribution of a dopant during annealing of radiation defects in a multilayer structure by laser scans for production of an implanted-junction rectifier. International Journal of Nanoscience, 7(4-5): 187-197. https://doi.org/10.1142/S0219581X08 005328

[31] Pankratov, E.L., Bulaeva, E.A. (2013). Doping of materials during manufacture *p-n*-junctions and bipolar transistors. Analytical approaches to model technological approaches and ways of optimization of distributions of dopants. Reviews in Theoretical Science, 1(1): 58-82. https://doi.org/10.1166/rits.2013.1004

[32] Pankratov, E.L., Bulaeva, E.A. (2012). Decreasing of quantity of radiation defects in an implanted-junction rectifiers by using overlayers. Int. J. Micro-Nano Scale Transp., 3(3): 119-130.

[33] Pankratov, E.L., Bulaeva, E.A. (2015). Optimization of manufacturing of emitter-coupled logic to decrease surface of chip. International Journal of Modern Physics B, 29(5): 1-12. https://doi.org/10.1142/S021797921550023X

[34] Pankratov, E.L. (2017). On approach to optimize manufacturing of bipolar heterotransistors framework circuit of an operational. Amplifier to increase their integration rate. influence mismatch-induced stress. J. Comp. Theor. Nanoscience, 14(10): 4885-4899. https://doi.org/10.1166/jctn.2017.6899

[35] Pankratov, E.L., Bulaeva, E.A. (2015). An approach to increase the integration rate of planar drift heterobipolar transistors. Materials Science in Semiconductor Processing, 34: 260-268. https://doi.org/10.1016/j.mssp.2015.02. 054

[36] Pankratov, E.L., Bulaeva, E.A. (2014). An approach to manufacture of bipolar transistors in thin film structures. On the method of optimization. International Journal of Micro-Nano Scale Transport, 4(1): 17-31.

[37] Pankratov, E.L., Bulaeva, E.A. (2016). An analytical approach for analysis and optimization of formation of field-effect heterotransistors. Multidiscipline Modeling in Materials and Structures, 12(4): 578-604. https://doi.org/10.1108/MMMS-09-2015-0057