IIETA International Information and Engineering Technology Association
Advancing the World of Information and Engineering

International Journal of Safety and Security Engineering
Vol. 16, No. 6, June, 2026, pp. 1387-1397
Journal homepage: http://iieta.org/journals/ijsse

# Dynamic Modeling of Stakeholder Mobilization Using Probabilistic Graphs: A Framework for HSE Governance and Decision Support

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https://doi.org/10.18280/ijsse.160619

**Received:** 7 May 2026
**Revised:** 18 June 2026
**Accepted:** 26 June 2026
**Available online:** 30 June 2026

***Keywords:***
*dynamic modeling, HSE performance, industrial performance, probabilistic graphs, stakeholder engagement, stakeholder management, transition matrix*

**ABSTRACT**

Stakeholder mobilization plays a central role in improving industrial performance, particularly in relation to Health, Safety, and Environment (HSE) management. However, most existing approaches remain descriptive and static, providing limited capability for predicting the evolution of stakeholder engagement over time. This study proposes a probabilistic graph-based model to analyze and predict the dynamic mobilization of stakeholders within industrial organizations. The proposed approach represents stakeholder categories as interconnected nodes linked through weighted probabilistic relationships that describe information exchange, influence, and mobilization dynamics. A transition matrix and mobilization vectors are used to evaluate stakeholder behavior over time and determine asymptotic mobilization states. The methodology is applied to the case of an Algerian cement plant (SCIMAT) to illustrate the operational use of the model in an HSE governance context. The results show that the proposed model enables the quantification of stakeholder engagement and the prediction of stable mobilization configurations. The findings also demonstrate the usefulness of probabilistic graphs as a decision-support tool for stakeholder mobilization and provide a structured basis for future empirical validation using direct HSE performance indicators. Overall, this work contributes to the development of predictive approaches for stakeholder mobilization and highlights the potential integration of probabilistic modeling into industrial performance management systems.

## 1. INTRODUCTION

Industrial organizations increasingly operate in environments characterized by strong economic competition, stricter environmental regulations, and growing societal expectations regarding Health, Safety, and Environment (HSE) performance. In this context, organizations are required not only to achieve economic efficiency, but also to ensure sustainable and responsible management practices capable of minimizing occupational, environmental, and societal impacts [1-3]. Consequently, stakeholder mobilization has become an important component of industrial governance and HSE performance management, particularly in high-risk industrial sectors.

Stakeholder theory (ST) provides a useful framework for understanding the interactions between organizations and the different actors that may influence or be influenced by organizational activities [4-6]. In industrial systems, stakeholder engagement contributes to improving communication, strengthening safety culture, and supporting operational risk management [7-9]. Effective stakeholder mobilization also enhances organizational resilience and promotes participatory governance practices capable of integrating both organizational and societal expectations into decision-making processes.

Despite the growing importance of stakeholder management, most existing approaches remain essentially descriptive and static, focusing mainly on stakeholder identification and classification without adequately considering the dynamic evolution of stakeholder mobilization over time [10]. Although some dynamic stakeholder management models have been proposed, they often rely on repeated qualitative evaluations and provide limited predictive capability regarding future stakeholder positions and stabilization states [11]. As a result, current approaches still provide limited support for anticipating stakeholder behavior and evaluating the long-term evolution of stakeholder engagement within industrial HSE projects.

In parallel, probabilistic graph theory has demonstrated significant potential for modeling dynamic systems characterized by interconnected interactions and evolving behaviors [12-14]. Probabilistic graphs enable the representation of relational structures and transition dynamics through weighted probabilistic relationships, making them particularly suitable for modeling stakeholder interactions within industrial organizations. Accordingly, the present study proposes a probabilistic graph-based approach for modeling and predicting stakeholder mobilization dynamics within an industrial HSE performance project.

The methodology is applied to the case of the SCIMAT cement plant in Algeria in order to analyze stakeholder interactions, evaluate mobilization levels, and predict asymptotic stakeholder configurations capable of supporting strategic HSE decision-making.

The remainder of this paper is organized as follows. Section 2 presents the theoretical background related to stakeholder management and HSE performance. Section 3 describes the proposed probabilistic graph methodology and its mathematical formalization. Section 4 presents the application of the model to the SCIMAT cement plant and obtained results. Section 5 discusses the main findings. Finally, Section 6 concludes the paper and outlines future research perspectives.

## 2. STAKEHOLDER MANAGEMENT AND HSE PERFORMANCE

### 2.1 Literature review on stakeholders

Over the past decades, ST has evolved considerably, both in its theoretical foundations and practical applications. Initially developed within the field of organizational management, the theory has progressively expanded into several domains, including project management, supply chain management, sustainability assessment, and environmental governance [1-5]. This evolution reflects the growing recognition that organizational performance depends not only on internal resources, but also on the quality of relationships established with the various actors surrounding the organization.

ST is commonly regarded as a framework for understanding the relationships between organizations and the groups or individuals that may influence, or be influenced by, organizational activities [6]. In his seminal work, Freeman defined stakeholders as “any group or individual who can affect or is affected by the achievement of an organization’s objectives” [5]. This definition introduced a broader managerial perspective in which organizational success depends on balancing the expectations and interests of multiple actors rather than focusing exclusively on shareholders.

This perspective highlights the bidirectional nature of the relationship between organizations and stakeholders. On the one hand, stakeholders can influence organizational objectives and performance through their actions, expectations, or pressures. On the other hand, organizational decisions and activities directly affect stakeholder well-being, interests, and perceptions [7, 8]. Consequently, ST has progressively developed along two complementary perspectives: an instrumental perspective, in which stakeholder management contributes to organizational performance, and a normative perspective, which emphasizes the moral responsibility of organizations toward their stakeholders [9, 10].

The evolution of ST also reflects a transition from traditional economic considerations toward broader sustainability and governance concerns. Early studies mainly focused on the role of stakeholders in supporting organizational continuity and competitive advantage [11-13]. More recent research has increasingly emphasized ethical responsibility, corporate social responsibility, sustainability, and participatory governance [14-19]. In this context, stakeholders are considered active contributors to organizational resilience, and long-term performance.

In industrial environments, stakeholder management has become particularly important due to increasing regulatory requirements, environmental pressures, and HSE-related challenges. Industrial organizations operate within complex socio-technical systems where numerous actors (including employees, suppliers, local communities, public authorities, insurers, and customers) may directly or indirectly influence organizational performance [20, 21]. As a result, effective stakeholder engagement has become an essential component of sustainable industrial management.

Several studies have shown that stakeholder mobilization positively affects organizational and HSE performance [22-25]. Active stakeholder involvement contributes to improving communication, strengthening safety culture, facilitating information sharing, and supporting collective decision-making processes. Furthermore, stakeholder collaboration enables organizations to better anticipate risks, respond to societal expectations, and enhance operational performances [26-29].

Despite these advances, a number of limitations remain in existing stakeholder mobilization approaches. First, many traditional studies remain essentially descriptive and static, focusing primarily on stakeholder identification and classification without considering the dynamic evolution of stakeholder positions over time [5, 13]. Second, although some dynamic stakeholder management models have been proposed, such as the action–reaction framework introduced by Preble [30], these approaches often require repeated qualitative evaluations and remain difficult to operationalize in practice. Third, most existing models lack quantitative and predictive mechanisms capable of modeling stakeholder interactions and anticipating future mobilization states.

Consequently, current stakeholder management approaches still provide limited answers to important practical questions, such as:

- How does stakeholder mobilization evolve over time?
- How long does it take for stakeholders to reach a stable engagement state?
- Which stakeholder groups play a regulatory or stabilizing role within the mobilization process?

Addressing these limitations requires the development of more dynamic and quantitative approaches capable of representing stakeholder interactions and predicting their evolution within industrial systems.

### 2.2 Stakeholder mobilization and HSE performance

Today, stakeholder mobilization has become a strategic necessity for organizations seeking to improve their overall performance and maintain sustainable operations [31-35]. In industrial sectors characterized by significant HSE risks, stakeholder engagement is increasingly considered an important factor in supporting operational safety, environmental protection, and organizational resilience.

Stakeholder management refers to the set of strategies and practices implemented by organizations to identify, involve, and coordinate stakeholders according to organizational objectives and stakeholder expectations [36-38]. However, stakeholders rarely share identical interests, priorities, or levels of HSE awareness. Consequently, organizations must continuously balance potentially divergent interests while maintaining effective governance and operational performance.

In this context, HSE management has become closely linked to stakeholder mobilization. Industrial activities that generate environmental or occupational risks are increasingly exposed to pressures from regulators, local communities, public opinion, and civil society organizations [39, 40]. These pressures encourage companies to adopt more transparent, participatory, and collaborative management practices.

Effective stakeholder mobilization contributes to HSE performance in several ways. At the strategic level, mobilized stakeholders support the formulation and implementation of HSE policies and objectives. At the operational level, they contribute to risk prevention, safety communication, knowledge sharing, and continuous improvement initiatives [41-43]. As a result, stakeholders increasingly act as decision-support agents capable of strengthening organizational value creation and improving industrial sustainability.

Furthermore, the literature highlights that stakeholder mobilization extends beyond traditional contractual stakeholders such as employees, suppliers, and shareholders. Non-contractual stakeholders, including local communities, associations, and public institutions, also play an important role through dialogue, societal expectations, and external influence [44-47]. Consequently, organizations are progressively adopting participatory governance approaches based on collaboration, transparency, and information exchange.

Among the existing stakeholder mobilization frameworks, the model proposed by El Abboubi and Cornet [48] remains one of the most widely used. This model classifies stakeholders according to their organizational and societal engagement into four categories: passive, militant, allied, and engaged stakeholders. The stakeholder mobilization grid provides a useful visual representation of stakeholder positioning and helps organizations identify problematic or weakly mobilized actors (Figure 1).

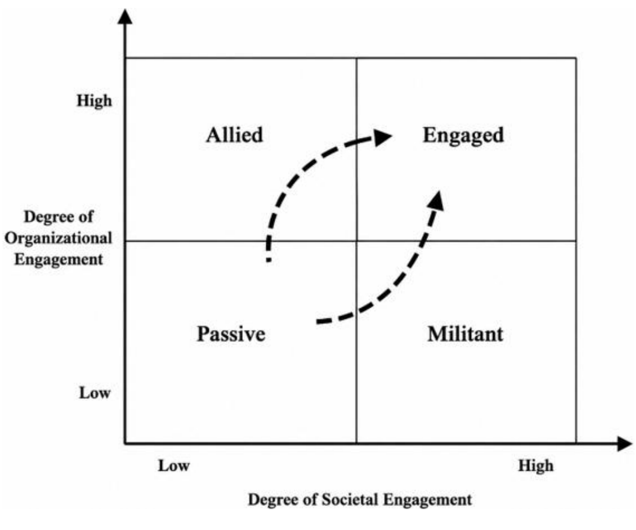


**Figure 1.** Stakeholder mobilization grid

However, this type of grid mainly offers a static representation of stakeholder positioning at a specific moment. In practice, stakeholder mobilization is a dynamic process influenced by communication flows, organizational decisions, stakeholder interactions, and external pressures. Evaluating these changes generally requires repeated qualitative investigations based on interviews, workshops, focus groups, or surveys [49, 50]. To overcome these limitations, more predictive and quantitative approaches are needed.

In this regard, probabilistic graph theory offers promising opportunities for modeling dynamic stakeholder interactions. Unlike static stakeholder matrices, probabilistic graphs enable the representation of evolving relationships, information flows, and transition probabilities between stakeholder states. Such approaches provide not only visual representations of stakeholder dynamics, but also quantitative tools capable of predicting asymptotic mobilization states and identifying key regulatory actors within the system.

Accordingly, the present study proposes a probabilistic graph-based framework to model and predict stakeholder mobilization dynamics within an industrial HSE performance project. The objective is to provide a more operational and predictive decision-support tool capable of improving stakeholder management and strengthening HSE governance in industrial organizations.

## 3. METHODOLOGY

### 3.1 Overview of graph theory

Graph theory is widely used to represent and analyze complex systems characterized by interconnected components and evolving interactions. Initially developed to solve mathematical problems such as shortest-path optimization, graph theory has progressively expanded into several scientific and engineering fields, including transportation systems, communication networks, biology, industrial risk assessment, and reliability analysis [51]. Its ability to model relationships among system components makes it suitable for studying organizational interactions and stakeholder dynamics.

In general, a graph $G$ is defined by two finite sets: a set of vertices $V$ and a set of edges $E$:

$$G = (V, E) \tag{1}$$

where, $V$ represents the set of vertices or nodes, and $E$ represents the set of edges connecting the vertices.

Vertices are used to represent entities within a system, whereas edges describe the relationships or interactions between these entities. When the interactions are directional, the graph is referred to as a directed graph. In weighted graphs, numerical values are associated with the edges in order to quantify the strength, intensity, or probability of interactions between vertices.

Graph theory provides both a visual and mathematical framework for representing complex interaction systems. In industrial engineering and safety management, graph-based approaches have been successfully applied to accident propagation analysis, domino effect modeling, reliability assessment, and decision-support systems [52, 53].

### 3.2 Probabilistic graphs as an appropriate tool for stakeholder modeling

The present study proposes the use of probabilistic graphs to model and predict stakeholder mobilization dynamics within industrial HSE projects. In this framework, stakeholder categories are represented as interconnected nodes, while the relationships among them are represented by weighted directed edges associated with transition probabilities.

The probabilistic graph approach offers several advantages for stakeholder modeling:

- Representation of interaction and information flows;
- Quantification of stakeholder mobilization dynamics;
- Prediction of stakeholder evolution over time;
- Identification of key regulatory actors;
- Estimation of asymptotic mobilization states.

Unlike conventional stakeholder matrices, probabilistic graphs provide both qualitative visualization and quantitative modeling capabilities. Therefore, the proposed methodology can be considered analogous to a discrete-time Markov process in which stakeholder mobilization evolves through probabilistic transitions between stakeholder categories.

Furthermore, probabilistic graphs have already demonstrated their effectiveness in several engineering applications [52-54]. Their capacity to model dynamic interactions and uncertain processes makes them suitable for analyzing stakeholder mobilization in industrial HSE systems.

### 3.3 Steps of the proposed method

Step 1. Defining the stakeholder typology

The first step of the proposed methodology consists of identifying and classifying the stakeholders involved in the industrial HSE project. It should be noted that several typologies for classifying stakeholders coexist, depending on the objective pursued. These include, for example, classification based on the relationship with the firm, which distinguishes between internal and external stakeholders [5], and classification based on the salience model, which rests on the attributes of power, legitimacy, and urgency [55]. In the present work, a classification was proposed according to the degree of importance of stakeholders in decision-making and according to their functional roles in HSE governance.

Based on the organizational structure and interaction mechanisms observed in industrial systems, stakeholders are grouped into three major categories:

- Decision-Making Stakeholders (DMS);
- Operational Stakeholders (OS);
- Civil Society Stakeholders (CSS).

DMS include internal and external actors responsible for strategic orientations, governance, and organizational decision-making. OS correspond to actors directly involved in operational and production activities. CSS include external actors such as local communities, associations, and public institutions that may influence or be influenced by organizational activities.

This classification enables the simplification of stakeholder interactions while preserving the principal organizational relationships influencing HSE performance.

Step 2. Graphical representation of stakeholders and edge weighting

After identifying stakeholder categories, the stakeholder mobilization system is represented as a directed probabilistic graph (Figure 2). In this graph:

- Vertices represent stakeholder categories;
- Directed edges represent stakeholder relationships and interaction flows;
- Edge weights represent stakeholder mobilization probabilities.

The directed nature of the graph reflects the fact that stakeholder interactions are not always symmetrical. Some relationships involve one-way information transmission, while others involve feedback and bidirectional exchanges.

The represented interactions may include:

- Information exchange;
- Operational instructions;
- Organizational decisions;
- Complaints and societal pressure;
- Knowledge sharing and feedback processes.

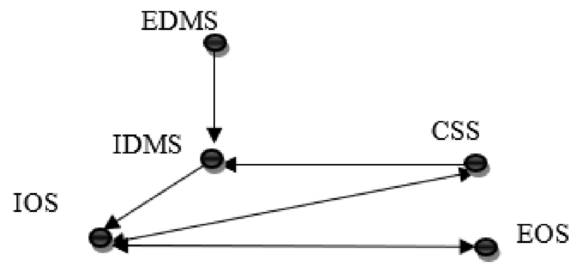


**Figure 2.** Directed hierarchical representation of stakeholder mobilization dynamics

Therefore, the graph structure provides a simplified but operational representation of stakeholder interaction dynamics within the industrial system.

The weights associated with graph edges are interpreted as transition probabilities describing the influence or mobilization transfer between stakeholder categories. These probabilities may be estimated through:

- Expert judgment;
- Field observations;
- Stakeholder surveys;
- Communication flow analysis;
- Graph semi-degree analysis.

Step 3. Using the probabilistic graph to predict the dynamic mobilization of stakeholders

To formalize the probabilistic graph mathematically, the graph is transformed into a transition matrix $A$:

$$A = [a_{ij}] \tag{2}$$

where, $a_{ij}$ represents the probability of transition from stakeholder category $i$ toward stakeholder category $j$.

The transition probabilities satisfy the following condition:

$$\sum_{j=1}^{n} a_{ij} = 1 \tag{3}$$

which ensures that the total probability associated with outgoing transitions from a given stakeholder category is equal to unity.

Stakeholder mobilization at a given instant is represented through the mobilization vector:

$$M(t) = \begin{bmatrix} m_1(t) \\ m_2(t) \\ m_3(t) \end{bmatrix} \tag{4}$$

where,

$m_1(t)$ represents the mobilization level of DMS;
$m_2(t)$ represents the mobilization level of OS;
$m_3(t)$ represents the mobilization level of CSS.

The initial stakeholder mobilization state is defined as:

$$M(0) = M_0 \tag{5}$$

Thus, the dynamic evolution of stakeholder mobilization is described by successive applications of the transition matrix, where, $A^T$ denotes the transpose of matrix $A$:

$$M(t+1) = A^T \times M(t) \quad (6)$$

By recurrence, the stakeholder mobilization state after $n$ iterations becomes:

$$M(n) = (A^T)^n \times M_0 \quad (7)$$

This formulation enables the quantitative monitoring of stakeholder mobilization dynamics over time and allows the prediction of stakeholder evolution within the industrial system.

Step 4. Monitoring stakeholder mobilization over time

One of the main advantages of the proposed probabilistic graph model lies in its ability to estimate asymptotic stakeholder mobilization states. The asymptotic mobilization vector corresponds to the stable stakeholder configuration reached after a sufficiently large number of iterations.

Mathematically, the asymptotic state is expressed as:

$$M_\infty = \lim_{n \to \infty} (A^T)^n M_0 \quad (8)$$

The asymptotic mobilization vector provides important managerial information because it:

- Identifies stable stakeholder engagement configurations;
- Highlights weakly mobilized stakeholder groups;
- Reveals regulatory stakeholder categories;
- Supports strategic HSE decision-making.

Consequently, the proposed methodology enables not only the visualization of stakeholder interactions, but also the prediction of stakeholder mobilization trajectories and stabilization mechanisms over time.

Overall, the proposed probabilistic graph framework combines ST with quantitative dynamic modeling in order to provide a predictive and operational decision-support tool for industrial HSE performance management.

## 4. APPLICATION OF THE PROPOSED PROBABILISTIC GRAPH TO AN ALGERIAN CEMENT PLANT

### 4.1 Brief description of the selected cement plant

The cement industry in Algeria faces significant HSE challenges due to the nature of cement manufacturing processes, which generate dust emissions, atmospheric pollutants, noise, and various operational risks. In this context, cement plants are increasingly required to strengthen their HSE management systems while improving their relationships with internal and external stakeholders.

The present study was conducted at the SCIMAT cement plant, located in eastern Algeria. Like many industrial facilities operating in the cement sector, SCIMAT has historically focused on increasing production capacity in order to satisfy the growing demand of the national construction market. However, this production-oriented strategy has also intensified HSE-related challenges.

To address these challenges, SCIMAT has progressively implemented an integrated HSE management strategy aimed at improving its overall industrial performance. As part of this commitment, the company has engaged in several certification processes, including ISO 14001 for environmental management, OHSAS 18001 and later ISO 45001 for occupational health and safety management, as well as ISO 50001 for energy management. These initiatives reflect the company's willingness to adopt continuous improvement practices and strengthen its HSE governance framework.

In parallel, SCIMAT has developed a participatory management approach based on active stakeholder engagement and information sharing. This strategy seeks to improve communication and cooperation among the various actors involved in the company's activities, including management bodies, operational staff, suppliers, customers, local communities, and institutional stakeholders. Through this approach, stakeholder mobilization is considered a key factor for strengthening HSE culture and supporting sustainable industrial performance.

SCIMAT currently produces and markets approximately 1,000,000 tons of cement annually. The cement manufacturing process involves several stages that may generate environmental and occupational risks. Following the extraction of limestone and clay from quarries, the raw materials undergo primary crushing before being transported by conveyor systems to storage areas. The materials are then ground into raw meal, which is preheated in cyclone towers and calcined in rotary kilns to produce clinker. After the addition of gypsum, the clinker undergoes a second grinding phase to produce cement, which is subsequently stored and distributed to the market.

Due to the complexity of these industrial operations and the diversity of the stakeholders involved, SCIMAT provides a relevant case study for applying the proposed probabilistic graph model.

### 4.2 Results

The first stage of the application consisted of identifying and classifying the stakeholders involved in SCIMAT's HSE performance project. Based on the organizational structure and the nature of stakeholder interactions, the identified nine stakeholders were grouped into three principal stakeholder categories according to their functional roles in HSE governance (Table 1).

**Table 1.** Typology of SCIMAT stakeholders

| Category | Stakeholder | Number |
|---|---|---|
| Decision-making Stakeholders (DMS) | Internal – SCIMAT Management | 15 |
| | External – GICA Group | 2 |
| | External – Partners | 2 |
| Operational Stakeholders (OS) | Internal – SCIMAT Staff | 567 |
| | External – Potential Clients | 500 |
| | External – Suppliers | 80 |
| | External – Others (insurers, banks, local authorities) | 112 |
| Civil Society Stakeholders (CSS) | Local residents | 4 |
| | External service providers | 20 |

It should be noted that the aggregation of the nine stakeholder groups into three functional categories is justified by the objective of preserving the major structural relationships relevant to HSE governance while reducing the analytical complexity of the network. Specifically, the actors were grouped according to their primary function within the

HSE system and their position within the decision-making, operational, and societal-influence flows.

It should be acknowledged that this aggregation has certain limitations, as it simplifies the diversity of interests, levels of influence, and HSE roles specific to each stakeholder subgroup. Consequently, it may attenuate some important nuances, particularly regarding power, priorities, and modes of interaction with the organization. Nonetheless, this methodological choice was adopted in order to preserve the readability of the model and to highlight the major structural relationships of the network. Thus, the resulting simplification constitutes a deliberate modeling decision intended to keep the model operational, while leaving the possibility of more detailed analyses in future work.

Based on this classification, the stakeholder interaction system was represented using the probabilistic graph shown in Figure 3. The graph illustrates the principal interaction flows among stakeholder categories, including organizational decisions, operational feedback, stakeholder actions, societal expectations, pressure and complaints, internal coordination, and knowledge sharing.

In this regard, DMS mainly influence OS through strategic orientations and organizational decisions, OS interact continuously with CSS through operational activities and information exchange, and CSS in turn exert indirect pressure on the organization through societal expectations, while the self-loops on the DMS and OS nodes in Figure 3 indicate that, over time, flows of information and knowledge also circulate within the decision-making and operational groups themselves. The connectivity of this network is characterized by the semi-degrees of each vertex namely $d^+(DMS) = 2$; $d^-(DMS) = 3$; $d^+(OS) = 3$; $d^-(OS) = 3$; $d^+(CSS) = 2$; and $d^-(CSS) = 1$. It should be noted that these values were established from an analysis of the actual information flows at SCIMAT and corroborated by semi-structured interviews with seven HSE managers and an internal documentary analysis.

These counts describe the topology of the interaction network and identify OS, for which $d^+ = d^- = 3$, as the regulatory vertex mediating between the other categories. To quantify the intensity of these interactions, a weight $w_{ij}$ was assigned to each directed arc according to the relative frequency and importance of the corresponding relationship, yielding the weighted adjacency matrix $W$ and the weighted probabilistic graph presented in Figure 4.

$$W = \begin{pmatrix} 5 & 5 & 0 \\ 3 & 2 & 5 \\ 4 & 6 & 0 \end{pmatrix} \tag{9}$$

The transition probabilities were obtained by normalizing the outgoing weights of each vertex, $a_{ij} = w_{ij} / \sum_k w_{ij}$, in accordance with stochastic graph theory and the axiom of total probability, where the denominator $\sum_k w_{ij}$ is the total outgoing weight (out-strength) of vertex $i$ (equal to 10 for DMS, OS and CSS).

It should be noted that transition probabilities were estimated from the structure of stakeholder interaction flows represented in the graph. The semi-degree values were used to normalize the outgoing and incoming relationships associated with each stakeholder category.

This approach provides a simplified probabilistic representation of stakeholder influence and mobilization transfer. Although the probabilities are structurally derived, their interpretation remains linked to the observed communication and interaction mechanisms within SCIMAT.

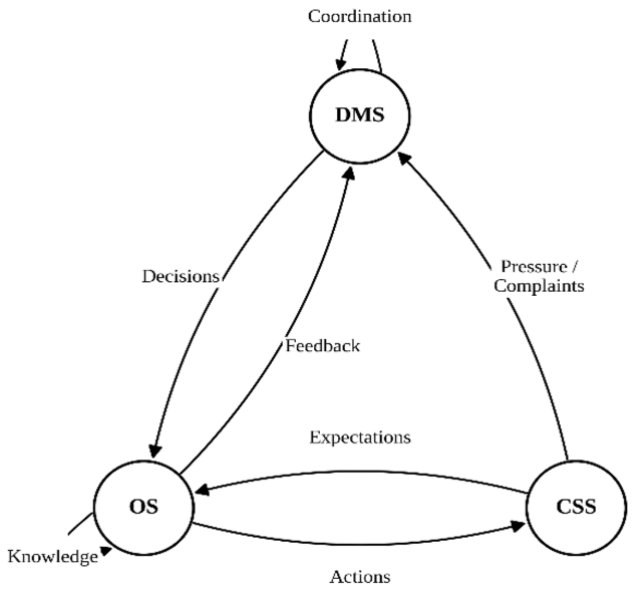


**Figure 3.** Probabilistic graph of stakeholder interaction flows within SCIMAT

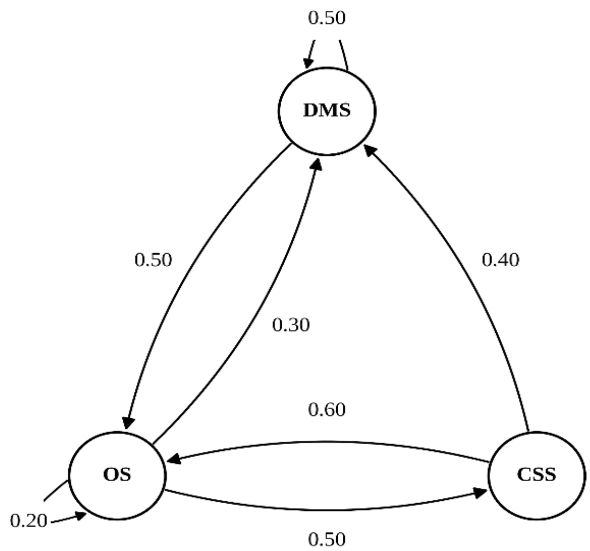


**Figure 4.** Weighted probabilistic graph of stakeholder mobilization within SCIMAT

The graph was subsequently transformed into a probabilistic transition matrix $A$, representing the transition probabilities among stakeholder categories. The corresponding transition matrix is presented below:

$$A = \begin{bmatrix} 0.50 & 0.50 & 0.00 \\ 0.30 & 0.20 & 0.50 \\ 0.40 & 0.60 & 0.00 \end{bmatrix} \tag{10}$$

It can be verified that each row of A sums to unity (0.50 + 0.50 + 0.00 = 1; 0.30 + 0.20 + 0.50 = 1; 0.40 + 0.60 + 0.00 = 1), confirming that $A$ is a valid row-stochastic transition matrix.

The initial mobilization vector $M_0$ was obtained through the same field assessment used to characterize the interaction structure (i.e., semi-structured interviews with seven SCIMAT's HSE managers, complemented by an internal documentary analysis). Each expert independently rated the initial level of involvement of the three stakeholder categories in HSE activities using a normalized 0-1 engagement scale. The individual scores were then aggregated by calculating the arithmetic mean for each stakeholder category, and the resulting values were normalized so that their sum equals unity, yielding the initial mobilization vector $M_0$, which represents the relative distribution of mobilization across the three stakeholder categories.

As these are expert-based estimates rather than direct measurements, their influence on the model's conclusions is examined through the sensitivity analysis, which confirms that the asymptotic state is independent of the specific initial vector.

$$M_0 = \begin{bmatrix} 0.40 \\ 0.45 \\ 0.15 \end{bmatrix} \quad (11)$$

The initial assessment revealed heterogeneous mobilization levels among stakeholders. OS exhibited the highest mobilization level due to their direct involvement in production activities and daily HSE practices. DMS showed a moderate level of mobilization associated with strategic management and governance activities.

In contrast, CSS presented the lowest mobilization level, reflecting weaker involvement in the HSE performance process during the initial evaluation stage.

The evolution of stakeholder mobilization was then evaluated through successive iterations of the transition matrix (Table 2).

**Table 2.** Evolution of stakeholder mobilization over time

| Iteration | DMS | OS | CSS |
|---|---|---|---|
| Initial state $M_0$ | 0.40 | 0.45 | 0.15 |
| $M_1$ | 0.40 | 0.38 | 0.22 |
| $M_2$ | 0.40 | 0.41 | 0.19 |
| $M_3$ | 0.40 | 0.40 | 0.20 |
| Asymptotic state $M_\infty$ | 0.40 | 0.40 | 0.20 |

Note: Decision-Making Stakeholders (DMS); Operational Stakeholders (OS); Civil Society Stakeholders (CSS).

The obtained results demonstrate that stakeholder mobilization evolves progressively toward a stable asymptotic configuration. The asymptotic stakeholder mobilization vector obtained for SCIMAT indicates that:

- 40% of the final mobilization concerns DMS;
- 40% concerns OS;
- 20% concerns CSS.

These results suggest that OS and DMS play dominant roles in sustaining the company's HSE performance dynamics, while CSS maintain a more moderate but still significant influence within the overall mobilization process.

The analysis also highlights the role of regulatory stakeholder groups in stabilizing the mobilization process. In probabilistic graph terms, a vertex is said to be regulatory when its positive semi-degree ($d^+$) and negative semi-degree ($d^-$) are equal, reflecting a balance between the influence it exerts and the influence it receives.

Figure 5 illustrates the evolution of stakeholder positioning within the stakeholder mobilization grid. The initial stakeholder positions (i.e. CSS, OS, DMS) reveal relatively weak mobilization levels which are initially located within the "passive" area of the grid.

However, the dynamic analysis shows that stakeholder mobilization evolves progressively over time. DMS remains relatively stable, OS decreases slightly from 45% to 40%, while CSS increases from 15% to 20% due to strengthened interaction and communication flows.

To assess the robustness of the model with respect to the initial mobilization estimate, we performed several simulations using alternative initial vectors, including deliberately extreme configurations (Table 3). In all tested cases, the model converged to the same asymptotic distribution, $M_\infty$ = [0.40, 0.40, 0.20], independently of the chosen initial state.

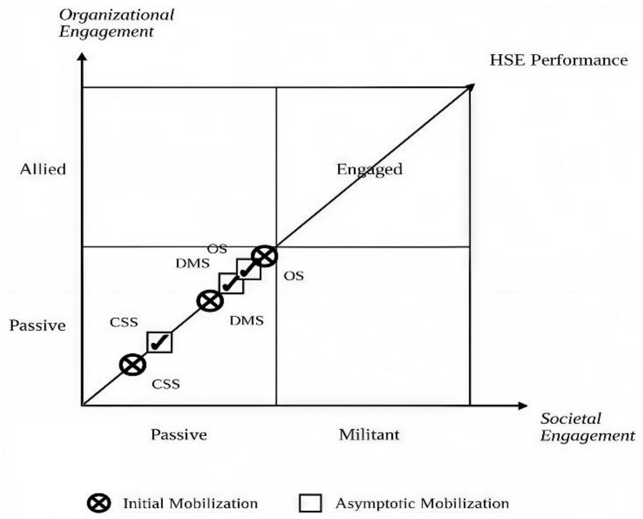


**Figure 5.** Evolution of stakeholder mobilization from initial to asymptotic states

**Table 3.** Tested initial vectors and resulting asymptotic state

| Initial Vector $M_0$ Tested | DMS | OS | CSS |
|---|---|---|---|
| [0.40, 0.45, 0.15] (OS ≈ DMS) | 0.40 | 0.40 | 0.20 |
| [0.70, 0.20, 0.10] (DMS dominant) | 0.40 | 0.40 | 0.20 |
| [0.10, 0.10, 0.80] (CSS dominant) | 0.40 | 0.40 | 0.20 |
| [0.33, 0.33, 0.33] (Uniform) | 0.40 | 0.40 | 0.20 |

Note: Decision-Making Stakeholders (DMS); Operational Stakeholders (OS); Civil Society Stakeholders (CSS).

This behavior is consistent with the Perron–Frobenius theorem [56], which characterizes the properties of irreducible and aperiodic stochastic matrices, for which the long-term dynamics depend essentially on the structure of the interactions encoded in the transition matrix $A$ rather than on the initialization of the system. Therefore, this sensitivity analysis shows that the conclusions of the model are robust. An imperfect estimate of $M_0$ may influence the transient trajectory and the speed of convergence, but it does not call into question the predicted asymptotic state.

## 5. DISCUSSION

The objective of this study is to propose a tool for modeling and predicting stakeholder engagement, rather than to evaluate HSE performance directly through numerical indicators.

However, the proposed approach draws on prior research demonstrating the link between stakeholder mobilization and improved HSE performance. Moreover, SCIMAT has recorded a progressive improvement in its internal safety indicators (e.g., a reduction in reported accidents) since its engagement in the ISO 9001, ISO 14001, ISO 45001, and ISO 50001 certification processes. This may be regarded as indirect, contextual evidence consistent with a favorable role of stakeholder engagement; however, no direct causal link to HSE performance is claimed.

It should nonetheless be emphasized that, in the absence of quantitative before-and-after HSE indicators, the present results must be interpreted with caution and do not establish a direct causal relationship between stakeholder mobilization and HSE performance. The proposed model therefore constitutes a dynamic modeling and decision-support framework rather than empirical proof of a measured impact on HSE outcomes; the empirical validation of this relationship using measurable HSE metrics is identified as a direction for future work.

Therefore, the proposed model should be understood as a decision-support tool that facilitates the prediction of how stakeholder engagement evolves over time.

Consequently, the obtained results focus on the representation of stakeholder interactions, the estimation of transition probabilities, and the evaluation of stakeholder mobilization over time. Particular attention is given to the asymptotic evolution of stakeholder positions and their implications for HSE governance and strategic decision-making.

In this regard, Figure 3 illustrates the flows of data, information, and knowledge exchanged among the different vertices of the probabilistic graph. These flows include organizational decisions, operational instructions, stakeholder actions, complaints, feedback, and various forms of pressure exerted on SCIMAT's stakeholders. Such informational and interaction flows constitute the basis for evaluating stakeholder mobilization within the HSE performance project under study.

Table 1 presents the detailed stakeholder typology identified within SCIMAT. For modeling purposes, the nine stakeholder groups were aggregated into three principal stakeholder categories: DMS, OS, and CSS. These three categories constitute the vertices of the probabilistic graph represented in Figure 4.

The weights associated with the graph arcs in Figure 4 represent the transition probabilities between the three stakeholder categories. These probabilities quantify the intensity of stakeholder interactions and the dynamics of mobilization within the organization. These were determined such that, for each vertex of the probabilistic graph, the outgoing transition probabilities (i.e., The entries of each row of the transition matrix) sum to unity, in accordance with the defining property of a row-stochastic matrix [57]. This normalization condition guarantees the coherence and stability of the probabilistic model.

The analysis of the probabilistic graph also reveals several important characteristics of stakeholder mobilization dynamics within SCIMAT:

- The largest negative semi-degree ($d^-$) indicates insufficient consideration of the stakeholder category represented by the corresponding graph vertex.
- Stakeholder categories for which the positive semi-degree ($d^+$) and negative semi-degree ($d^-$) are approximately equal play a regulatory role within the mobilization process. These stakeholders are mobilized at nearly 40% within SCIMAT's HSE performance project.
- In practical managerial terms, the fact that OS exhibits balanced positive and negative semi-degrees $d^+ = d^- = 3$ indicates that this stakeholder category receives as much influence as it transmits, occupying a pivotal bridging position between strategic decisions (DMS) and societal expectations (CSS). This structural property has direct implications for HSE governance. First, HSE communication strategies should be channeled primarily through OS representatives rather than relying solely on top-down directives from DMS. Second, OS should be granted a formal and visible voice within HSE governance structures (e.g., joint health and safety committees and HSE steering groups) in view of its intermediary role. Third, HSE monitoring dashboards should track the specific engagement of OS and the quality of OS-CSS communication as priority indicators, since a disruption at this regulatory node would propagate toward both DMS and CSS. Fourth, training and resource allocation should prioritize equipping OS with the dual capacity to relay strategic instructions downward and to convey field- and society-level feedback upward.
- The stakeholder categories characterized by the highest semi-degree values ($d^+$) and ($d^-$) represent the largest proportion of the overall stakeholder population.

The initial stakeholder mobilization assessment indicates heterogeneous mobilization levels among the three stakeholder categories. The values of the initial mobilization vector $M_0$ show low mobilization for CSS (15%), relatively low mobilization for DMS (40%), and moderate mobilization for OS (45%). In the stakeholder mobilization grid presented in Figure 5, these values correspond to the "passive" area of the grid for DMS, OS, and CSS.

These initial stakeholder positions reveal insufficient overall mobilization within SCIMAT's HSE project and therefore highlight the need for strengthening stakeholder engagement strategies. However, when considering both the dynamic evolution of stakeholder positions over time and the probabilistic reference data of the model (transition matrix $A$ and initial mobilization vector $M_0$ , the asymptotic mobilization vector $\widetilde{M}$ reveals the following trends (Figure 5):

- A relative stagnation in the mobilization level of DMS;
- A slight decrease in the mobilization of OS, estimated at approximately 5% compared with the initial state;
- An increase of approximately 5% in the mobilization of CSS relative to their initial mobilization level.

Although the increase in CSS mobilization reflects improved interaction with external stakeholders, the asymptotic evolution of stakeholder positions may still negatively influence the achievement of the targeted HSE performance objectives if stakeholder engagement efforts are not continuously reinforced.

Consequently, sustained efforts to strengthen stakeholder mobilization must be maintained over time if SCIMAT aims to progressively position all stakeholder categories along the principal diagonal of the mobilization grid, particularly within the "engaged" area corresponding to high organizational and societal engagement levels.

In this respect, DMS should adopt a more collaborative and participatory management strategy capable of strengthening the involvement of both OS and CSS within the HSE project.

Overall, the proposed probabilistic graph approach overcomes several limitations associated with conventional stakeholder management models.

First, the approach enables the dynamic modeling of stakeholder interactions and information flows. By representing stakeholders as graph vertices and their relationships as weighted probabilistic arcs, the methodology provides a structured representation of stakeholder mobilization dynamics.

Second, the approach introduces a quantitative dimension into stakeholder mobilization analysis. Through the construction of transition matrices and mobilization vectors, it becomes possible to estimate stakeholder engagement levels at different stages of the mobilization process and to predict their asymptotic stabilization states.

Third, the probabilistic framework provides rapid answers to several strategic management questions, including:

- The speed at which a stakeholder category may evolve toward a desired mobilization state;
- The identification of regulatory stakeholder groups characterized by balanced positive and negative influence;
- The prediction of asymptotic stakeholder mobilization scenarios capable of supporting strategic HSE decision-making.

Finally, the proposed mathematical formalization provides an operational decision-support and simulation tool that could eventually be extended toward more advanced probabilistic frameworks such as Bayesian networks. Such developments may contribute to strengthening strategic management dashboards dedicated to industrial safety, environmental sustainability, and organizational performance.

In summary, probabilistic graphs introduce a dynamic, predictive, and operational dimension into stakeholder mobilization analysis. By overcoming the limitations of conventional descriptive and qualitative approaches, the proposed methodology offers an innovative framework for quantifying, monitoring, and managing stakeholder mobilization within industrial HSE performance projects.

## 6. CONCLUSION

This study proposed a probabilistic graph-based approach for modeling and predicting stakeholder mobilization within an industrial HSE performance project. The proposed methodology combines ST with probabilistic graph modeling to represent stakeholder interactions, evaluate mobilization levels, and predict their evolution over time. The application to the SCIMAT cement plant demonstrated the usefulness of the approach for identifying stakeholder dynamics, regulatory actors, and asymptotic mobilization states. The results highlight the importance of strengthening collaboration and stakeholder engagement in order to improve HSE performance and organizational governance.

Overall, the proposed framework provides a dynamic and quantitative decision-support tool that overcomes the limitations of conventional descriptive approaches. Nevertheless, the model remains dependent on the quality of stakeholder classification and probability estimation, which should be further validated in future empirical applications. Future research may focus on integrating Bayesian networks and applying the methodology to other high-risk industrial sectors to further improve its predictive capabilities and operational applicability. In particular, the integration of quantitative before-and-after HSE indicators (e.g. incident rates, frequency and severity indices, and environmental compliance measures) would make it possible to validate empirically the impact of stakeholder mobilization on HSE performance.

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## NOMENCLATURE

| | |
|---|---|
| $G$ | Probabilistic graph |
| $V$ | Set of vertices (nodes) |
| $E$ | Set of edges (arcs) |
| $A$ | Transition probability matrix |
| $a_{ij}$ | Transition probability from stakeholder $i$ to stakeholder $j$ |
| $M_0$ | Initial stakeholder mobilization vector |
| $M(t)$ | Stakeholder mobilization vector at time $t$ |
| $M(n)$ | Stakeholder mobilization vector after $n$ iterations |
| $M_\infty$ | Asymptotic stakeholder mobilization vector |
| $d^+$ | Positive semi-degree of a graph vertex |
| $d^-$ | Negative semi-degree of a graph vertex |
| DMS | Decision-Making Stakeholders |
| OS | Operational Stakeholders |
| CSS | Civil Society Stakeholders |
| HSE | Health, Safety, and Environment |
| ISO | International Organization for Standardization |
| SCIMAT | Société des Ciments de Aïn Touta |
| $P_{ij}$ | Probability associated with stakeholder transition |
| $n$ | Number of iterations |
| $t$ | Time step or iteration index |