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Evaluating the Hydraulic Efficiency of Rectangular, Labyrinth, and Piano Key Weirs Under Different Factors

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ABSTRACT

Weirs are widely used hydraulic structures designed to regulate, measure, and control the flow of water in rivers, canals, and hydraulic systems. Their performance is often assessed in terms of hydraulic efficiency. There are many types of weirs, including rectangular, labyrinth, and piano key. Rectangular weirs are simple but prone to sedimentation; labyrinth and piano key designs maximize discharge under low heads, ideal for environmental variability, but maintenance against sediment buildup is crucial; and temperature effects are generally minor compared to sedimentation and elevation impacts. This study investigates the hydraulic performance of different weirs based on their performance characteristics under different environmental conditions and seeks to analyze the performance of discharge of rectangular, trapezoidal labyrinth, trapezoidal, piano key weirs (PKWs), and stepped labyrinth weirs concerning how various factors, such as height and structure, affect flow dynamics. The analysis describing compositions of various weirs were performed using differing discharge measurements at conditions. The resultant conclusions established that rectangular weirs gave steady discharge irrespective of elevation changes. At the same time, the trapezoidal labyrinth weir showed no consistency concerning discharge, whereas the trapezoidal PKWs was markedly inefficient, with a discharge reduction with respect to an increase of height. It is recommended, based on this study's findings, that rectangular and stepped labyrinth weirs be used that would offer stable discharge in varying height conditions, while trapezoidal labyrinth and PKWs would have to be optimized over specific height ranges. The future work must be directed accordingly, i.e., to refine weir designs using computational modeling and real-time monitoring to help enhance their efficiency in various environmental conditions.

1. INTRODUCTION

A weir is a small overflow-type dam or barrier built across a river, canal, or stream to alter the flow characteristics of water. It raises the water level upstream or regulates its flow. Weirs are widely used in hydraulic engineering, irrigation, and hydrology [1]. The hydraulic efficiencies of weirs, particularly of rectangular, labyrinth, and piano key weirs (PKWs), are greatly dependent on design and operating conditions. Studies have confirmed that labyrinth weirs have proved to be 67% more efficient than rectangular weirs in terms of discharge efficiency, particularly under quite certain geometric constraints. The variation in efficiency is affected by approach velocity, geometry of the weir, and grade of flow conditions [2]. The performance of the weirs is affected by the distribution of lateral velocities and nappe interference, in particular at high headwaters, which can modify the flow pattern and affect the efficiency. In fact, the coefficient of discharge is also reduced by increasing the approach velocity and pool aspect ratio, which shows a flow condition sensitivity. An understanding of these effects is critical to the optimized operability of labyrinth weirs under the diverse flow conditions [3].

PKWs are designed to take advantage of both the labyrinth feature and the overhang so as to improve their discharge capacity, particularly at low head [4]. The provision of high-flow capability across a lower head-age range means that PKWs are preferred for modern hydraulic applications. While labyrinth weirs have been established to be considerably more efficient in ecological terms, the PKWs provide unique advantages in their design framework and flood management [5]. This begs the possibility of a trade-off between hydraulic performance and contexts of their practicality. Some studies find under various submerged conditions, labyrinth weirs may perform less well than rectangular weirs, further highlighting the context in hydraulic assessment [6].

Rectangular weirs are used in open channels to measure the known flow, as they are inexpensive, easy to construct, and accurate. These gradients in hydraulic potential, strengths in mechanical energy, and variances between kinetic and potential energy (attributed to friction with the bottom) govern the control of flow depth and flow velocity, and hence the regulation of transport of these biogeochemical constituents; hydraulic properties (sometimes called hydraulic counterparts) are characterized through experimental and numerical studies of engineering [7]. Modeled flow over linear rectangular and PKWs with subcritical flow using CFD numerical approach. The output showed that discharge through the PKW was 1.3–1.67 times more than that through the linear rectangular side weir, and better in terms of water surface profile, flow and velocity contours. This makes it clear why PKWs far outperform linear rectangular side weirs under certain flows [8].

There remain scant researches that compare the hydraulic performance of various weir types under different environmental conditions, such as elevations, temperatures, and sedimentation. Much of the work up to now has been done in view of the individual weir designs in isolation, with a limited scope to account for the combined effects of environmental factors on discharge performance. Besides, the effects of height and sedimentation on the flow behavior across various weir types have not been fully investigated [9]. There is a growing gap in the massively understood optimal performance ranges of these weir types, especially when it comes to elevation and environmental factors, and how those factors affect their operational efficiency under varying conditions [10].

This work is significant for comparative evaluation of the various weir designs under realistic environmental variables: height, sedimentation, and temperature, to see how hydraulic performance differs. The findings present pertinent information regarding the stability and efficiency of the weir structures in these various environmental conditions, key to managing water resources optimally, especially in areas where variation in elevation and environmental issues occur. The study results assist engineers and hydrologists in selecting the more apt weir for the environmental conditions under occurrence toward the more efficient and sustainable water flow management. It also shows the importance of taking into account other environmental factors like sedimentation in the design of weirs built in various geographical locations. This work is quite novel and is expected to serve as a substantial bench-mark, mainly in comparison to a multitude of weir designs such as rectangular, trapezoidal labyrinth, trapezoidal PKWs, and stepped labyrinth under varied set of environmental conditions. In contrast to many previous works that tend to focus on only isolated variables or particular weir designs, the present study investigates the combined effect of height, temperature, sedimentation, and evaporation on the discharge performance of these weirs.

2. METHODS AND MATERIALS

Rectangular weirs are used of controlling flow of water into streams, canals, rivers and various kinds of water bodies. They assist in water elevation and flow regulation, particularly in times of heavy rain, where weir height can be regulated to reduce flooding [11].

2.1 Rectangular weir

When the flow runs over the weir, the flow changes from subcritical to supercritical status flows freely over a weir, the upper profile is referred to as nappe flow. Special case of flow-over weir is the rectangular weir [12]:

$$Q = \frac{3}{2} \cdot C_d \cdot L_{net} \cdot \sqrt{2g}. H_T^{1.5} \tag{1}$$

To understand the discharge equation regarding rectangular weirs, we must focus on its several integral parameters, which are the gravity 'g', discharge (Q), the head, total head over the weir 'H_T', net crest length 'L_{net}', and the 'C_d', which is the discharge coefficient. But out of all these parameters mentioned, the discharge coefficient is the one that stands out as the most crucial parameter when analyzing the flow characteristics of a rectangular weir. When making comparisons of discrete hydraulic characteristics of varying weirs, the net crest length becomes unimportant relative to the discharge coefficient. This indicates that the performance of the weir is better represented by the discharge coefficient. Since the results of the weir's performance are based on the discharge coefficient, which are not dependent on the height (P) of the weir, these results will be more effective in comparing performance of different types of weirs. To examine the discharge performance of linear T-weirs the following formula, which is derived from rearranging Eq. (1), is employed to compare the discharge coefficients. This procedure entails stating the relationship between the dimensionless total head (HT/P) and the discharge coefficient in the case of rectangular weirs and in particular, Eq. (1). This development allows for comparison across different weirs without changing the parameters running in the weirs [13, 14].

$$C_d = \frac{Q}{\frac{3}{2} \cdot L_{net} \cdot \sqrt{2g}. H_T^{1.5}} \tag{2}$$

Rectangular side weir [15]

$$C_d = \left[1.1308 - 0.1492(F_1)^{0.8292} - 1.5396\left(\frac{P}{L}\right)^{0.0394} + 0.0105\left(\frac{y_1}{L}\right)^{3.6295} + 0.487\left(\frac{B}{L}\right)^{-0.0357}\right]^{0.2322} \tag{3}$$

2.2 Labyrinth weir

A labyrinth weir is a special type of weir designed to increase the effective crest length within a limited channel width, thereby improving its discharge capacity compared to a conventional linear weir. the efficiency of a labyrinth weir is often assessed by the discharge coefficient (C_d) and its variation with head-to-height ratio (H/P).

Trapezoidal labyrinth side weir [16]

$$C_d = \left[-0.001(F_1)^{-1.78} + 0.10\left(\frac{L}{B}\right)^{0.22} - 2.036\left(\frac{y_1 - P}{P}\right)^{0.03} + 0.02\left(\frac{L_{ef}}{L.\sin\alpha}\right)^{0.02}\right]^{5.77} \tag{4}$$

Stepped labyrinth side weir [17]

$$C_d = \left[-3.801 - 0.257\left(\frac{L}{B}\right)^{0.239} + 2158.362\left(\frac{b_s}{L}\right)^{8.933} + 13.264\left(\frac{b_s}{a_s}\right)^{-0.002} - 8.116\left(\frac{L}{y_1}\right)^{-0.001} + 0.583\left(\frac{P}{y_1}\right)^{4.117} - 0.17F_1^{-0.471}\right]^{6.249} \tag{5}$$

2.3 PKWs

It is nonlinear weir designed to increase the hydraulic efficiency of spillways while minimizing footprint and construction costs.

Trapezoidal PKWs in straight channel [8]

$$C_d = \left[\left(-13.169 + \left(\frac{n}{P}\right)^{0.888}\right)\left(14.012 + \left(\frac{h_1}{P}\right)^{-0.764}\right)\left(-0.687 + \left(\frac{y_1}{P}\right)^{-0.764}\right)\left(0.046 + \left(\frac{b_1}{P}\right)^{1.692}\right)\left(-0.135 + \left(\frac{L}{P}\right)^{1.858}\right)\left(1.322 + (F_1)^{0.421}\left(\frac{b}{P}\right)^{-4.11}\right)\right] \quad (6)$$

P, B, y_1, F_1, L, L_{ef}, θ, α, as, bs, and b are the variables in the equation and are representative of different parameters of the weir and flow system. Such parameters include the P, which is the weir height, B, which is the channel width, y_1, which is the depth of flow at the upstream end of the side weir, F_1, which is the Froude number at the upstream end of the side weir, L for the width of the side weir, L for the total crest length of the side weir, θ for the angle of the oblique side weir, α for the angle of the triangular side weir, as for the step height, bs for the step width, and b for the upstream-downstream length of the PKWs. In addition, ai are constants where i denotes an integer that can be between 1 and 10, inclusive. Another parametric variable is P, which denotes the height of the foundation of the PKWs, n is the number of piano keys, h_1 is the piezometric head at the upstream end of the side weir and b is the over length of the PKW downstream [4]. The bed sediment discharges which flow across the river cross section would go as:

$$Q_q = q_b . b \quad (7)$$

with b = average river width and sediment discharge per meter wide of river:

$$q_b = 0.053\ \frac{T^{2.1}}{D^{0.3}}[(s-1)g]^{0.5} D_{50}^{1.5} \quad (8)$$

Since the gravity is effect by height then we have $g = g_0\left(1 - \frac{2h}{R}\right)$ where $g_0 = 9.8$. For substituting the value of g from in q_b we get $\left(C_d = \frac{Q}{\frac{3}{2}\cdot L_{net}\cdot\sqrt{2g}.H_T^{1.5}}\right)$. In a seasonal river in the northeastern part of Greece, measurements were taken for the water discharge and the amount of transported sediment in the form of bed load. The mean streamflow rate ranged between 0.019 m ³/s and 0.314 m ³/s, whereas the sediment load transport per unit width was between 0.00001 kg/m/s and 0.00213 kg/m/s. The sediment concentration was estimated by other nonlinear regression equations corrected with Yang's formulas [18]. Substituting value of gravity into Eq. (9), a new equation is formulated.

$$q_b = 0.053\ \frac{T^{2.1}}{D^{0.3}}\left[(s-1)^{0.5}\frac{Q}{\frac{3}{2}\cdot C_d\cdot L_{net}\cdot\sqrt{2}.H_T^{1.5}}\right] D_{50}^{1.5} \quad (9)$$

Transport stage parameter:

$$T = \frac{(u_*')^2 - (u_{*,cr})^2}{(u_{*,cr})^2} \quad (10)$$

Grain-shear velocity:

$$u' = \frac{\bar{u}}{C'}\frac{Q}{\frac{3}{2}\cdot C_d\cdot L_{net}\cdot\sqrt{2}.H_T^{1.5}} \quad (11)$$

Chezy coefficient:

$$C' = 18\log\left(\frac{12R_b}{3D_{90}}\right) \quad (12)$$

For wide channel the assumption: R_b = d (depth of flow) [19].

3. RESULT AND DISCUSSION

3.1 Sedimentation behavior with weir length

The decreasing sedimentation discharge with increasing weir length, especially that in paralleled arrangement, as shown in Figure 1, can be explained through a few key hydraulic and sediment transport principles. The length of the weir significantly distributes the net outflow over the weir's surface. In a parallel weir configuration, the total discharge divides among the segments of each of the weirs, collectively passing through each section with part of the total flow. The more extended the weir length, the more significant is the total area of flow along the surface of the weir. This increased surface area assures an even distribution of flow over the weir and, in the case of sediment-laden water, sediment deposition along the flow founder surface, depending on the velocity and turbulence. As its length increases, the mean velocity of flow over the weir surface decreases. It happens because the total discharge is already spread out on a more extended length of the weir. Lower flow velocities lessen the capacity of the water to carry sediment, which results in an increase in deposition. Sedimentation is based on the flow velocity and the dimension of sediment particles. For sediment to remain suspended, the velocity of water needs to be such that the particles are unable to settle down. The greater the length of the weir and the less velocity, the more significant the possibility of larger particles settling on the weir surface because of slower flow. Consequently, with the weir's increasing length, there will be more sediment movement along the weir, and as a corollary, a decrease in sedimentation discharge.

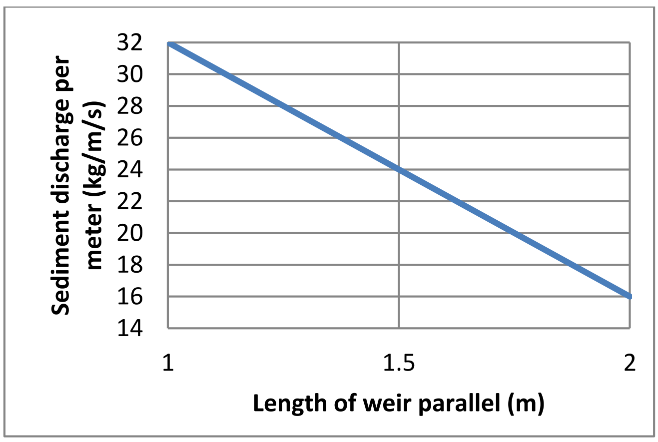

Figure 1. Sedimentation discharge with length of weir

The discharging, controlling, and eroding function of the weir depends on its design and length. The longer the weir length, the wider the area of energy dissipation because a larger volume of water spills over greater surfaces. The more the travel length of the water on the weir, the higher the energy loss due to turbulence and friction, leading to a reduced potential of the flow for sediment transport and to increased deposition, thus reducing sedimentation discharge over the weir. Each weir section generally receives a smaller portion of the total discharge, as arranged by the parallel configuration weirs. Because the parallel weir arrangement allows an even distribution of flow across each weir section, energy is reduced further to transport sediment downstream. Thus, as weir length increases, each parallel weir section loses sediment transport capacity, leading to reduced sedimentation discharge as weir length increases, as shown in Figure 1.

3.2 Sedimentation discharge and transport stage parameters

The exponential increase in sedimentation discharge can be explained in terms of the interaction of sediment transport within the conduit and flow dynamics as influenced by transport stage parameters, such as sediment concentration, size, velocity of flow, and hydraulic conditions. In the sedimentation process, the most commonly dependent flow parameter in the sediment transport stage is the flow velocity, which, in turn, defines the ability of water to carry sediment. Thus, when the flow velocity is increased, shear stress on the riverbed or weir surface is proportionally increased. The shear stress governs the ability of the flow to transport sediment whereby for higher shear stresses, the flow gains the ability to act as a carrier for sediment particles. Large sediment particles can be carried off by flow with high velocity while, in low flow velocity, sediment settling occurs. With increasing flow velocities, since the transport stage shifts into heightened values, shear forces have an exponential increase, where an exaggerated increase in sediment transport capacity would be shown in the given graph in Figure 2.

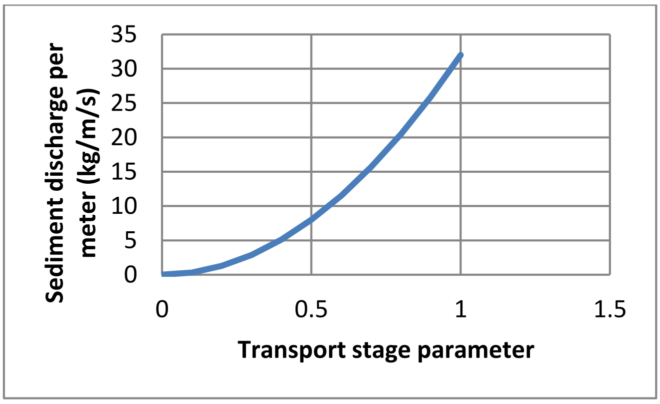

Figure 2. Sedimentation discharge with transport stage parameters

Sediment transport often follows a non-linear relationship with transport stage parameters and in some cases might experience an exponential upturn in sediment transport discharge. This is particularly true if the flow exceeds a threshold, transport capacity, and nuclei flow themselves transport more sediment. As flow transport stage parameters increase, sediment concentration, particle size, etc., the flow has the ability to support even higher transport rates. This exponential increase may be due to an increase in sediment transport itself as the flow continues to interact with sediment particles, and the increased turbulence and shear forces promote the suspension of yet more particles, so that when sediment particles are mobilized by increasing shear stress, their transport through the water column becomes more efficient. In addition, as the flow gains in speed, yet more sediment is added from the weir surface and taken away, thus generating an exponential increase in sedimentation discharge. Another reason for sedimentation discharge growing exponentially is concentration-suspension behavior. With the sediment concentration near saturation the turbidity increases and thus more sediment particles are in motion.

The transport stage progresses to a period in which the sediment concentration is high enough for interference between particles; this increases their ability to stay in suspension. The rate of sediment transport is, therefore, directly proportional to flow velocity and concentration of the suspended particles at the point of motion. At this stage, the sediment transport increases exponentially because more and more particles are entrained in the flow. For that there is a proportionality that is not only the flow velocity but also the concentration of suspended particles, an increase of both parameters ensures exponential increment of the sedimentation discharge. The hydraulic conditions in a weir, like water depth, velocity, and turbulence, gain significance while determining the sediment transport capacity. Once the flow enters the turbulent regime (often involving substantial flow velocity), the sediment transport increases exponentially because turbulence accelerates the mixing of water and sediment, reducing the probability of sediment settling down into the water.

3.3 Impact of discharge coefficient on sediment transport

To emphasize reduction in sedimentation discharge (or entrapment of the same) with the help of the discharge coefficient of a particular weir as shown in Figure 3, we need to study the relationships among discharge coefficient, fluid dynamics, sediment transportation pathways, and weir shape. Technically, the discharge coefficient is the term showing the effectiveness of the weir in transferring water in a particular area. It is a dimensionless number that shows the capacity of the weir with such factors as the geometry of the weir, the dimensions, deepness of the flow over the weir, and the flow condition that is present. It represents the ratio of the discharge (or water volume) passing through the weir to the theoretical flow corresponding to a given weir height. Thus, a weir having a far less discharge coefficient will control the flow of very little water for a similar flow depth, allowing the fluid to flow at the surface. On the other hand, a weir with a low discharge coefficient may let only a smaller portion of a certain depth be passed depending on the flow conditions. The most important thing with the discharge coefficient is that it directly controls the flow speed and the flow disturbance which, in turn, affects the sediment transport as well as the discharge & entrapment of solids.

A high velocity (which is due to a high discharge coefficient) causes more sediment to be lifted and transported downstream. However, if the discharge coefficient is reduced, the speed of the flow also drops and the flow will not be able to hold the sediments in suspension anymore. Consequently, the sedimentation discharge is prone to decrease. The discharge coefficient and hydraulic geometry are the main factors that are connected to the flow. A greater discharge coefficient usually means a greater flow velocity, which indicates the water possesses higher energy to move and transport sediments. However, if the flow decreases (e.g., by the weir section or the flow conditions changing), the velocity also diminishes. This ranking of speed decreases the water's ability to carry off the sediments. As the discharge coefficient decreases, the ability of the weir to hold sediments in the suspension diminishes, and correspondingly, the sedimentation discharge lessens.

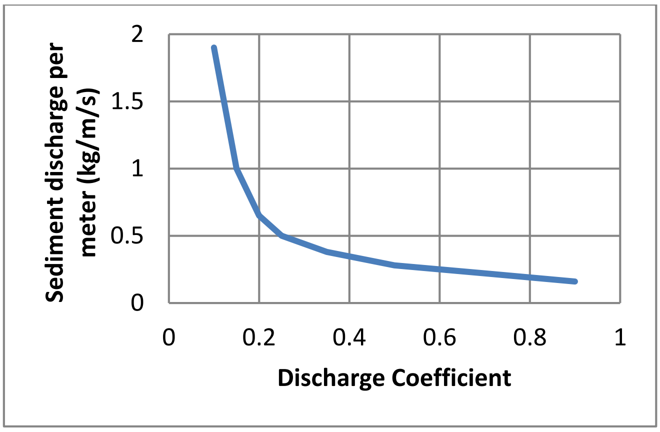

Figure 3. Sedimentation discharge with discharge coefficient

3.4 Influence of density ratio

The link between the density ratio of sediment particles and the size of sedimentation discharge is an important thing. The density ratio that simulates the sediment particle to water density ratio is crucial in the study of the sediment deposition and transport processes in a hydraulic structure, such as a weir. The density ratio interacts the behavior of sediment particles within the flow, altering their transport processes, settling velocities, and thus also matter in the sedimentation discharge as seen in Figure 4. Since the density of the particles is lower than water they do not settle easily but the sedimentation discharge remains relatively constant. We too note that the suspension and transport of these particles don't have to do with the density ratio but rather with the flow conditions (for example, velocity, or turbulence). As the density ratio exceeds 1, the sediment particles become denser than water that will increase their coupling. The higher the sedimentation discharge, the quicker a particle will fall out of suspension, thus increasing the sedimentation discharge. This is due to the fact that the particles that are denser fall faster.

- **Settling Velocity Increase:** There is an upward trend of the sedimentation rate as the density of the particle becomes denser than the water, hence, the settling velocity becomes more rapid. The increased settling velocity creates a higher speed of the deposition process, which results in a higher rate of sedimentation discharge. The particles become too heavy to float in the flow and they are no longer conveyed by the flow because their higher density causes them to fall faster than the flow can transport them.
- **Increased Sedimentation Efficiency:** When the density of the particles is greater than the water, the hydraulic structure's ability to trap and retain sediment is strengthened. Upon the increment in the density ratio, particles move faster and hence are less likely to be disturbed by the surge. The entire deposition and sedimentation discharge are enhanced by the more rapid sedimentation of the particles.
- **Flow Conditions and Turbulence Impact:** While the flow conditions can still be the reason, the density ratio causes a more powerful effect when it gets to the level of 1. As the density ratio increases, especially particles can be less likely to be dragged by the flow, although the flow conditions are not as turbulent. This is because the flow velocity effect is smaller on the high-density particles comparing to the increase of the particle density, and so, the sedimentation rate also increases.

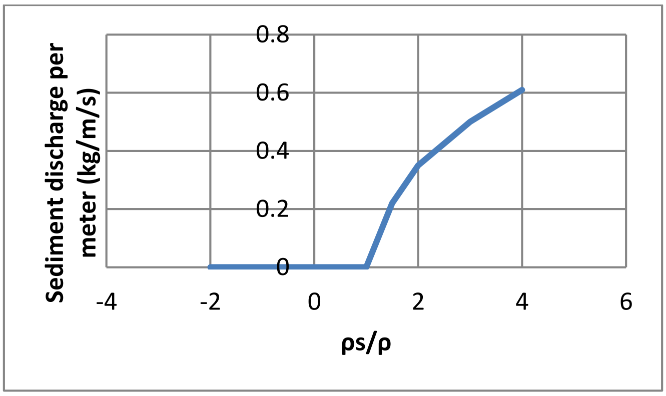

Figure 4. Sedimentation discharge with density ratio

3.5 Discharge vs. Froude number

As the Froude number changes, the inflow in a hydraulic structure follows a nonlinear pattern that starts at the bottom and also rises. This is due to the inflow characteristics set up in the subcritical, transitional, and supercritical administrations. graveness has the topmost influence on inflow in the subcritical inflow governance, where the Froude number is lower than 0.4. The water moves sluggishly, with a moderate depth of inflow. During this governance, the haste of the water is slow and the face area is larger in this terrain. Due to the reduced haste, the water is not distributed unevenly, performing in a reduction in the aggregate inflow. Although deep water has an advanced water content, its movement is slower, performing in lower discharge passing through the structure. thus, the discharge decreases significantly when the Froude number is 0.4. Critical and supercritical processes are affected by the inflow as the Froude number approaches 0.4.

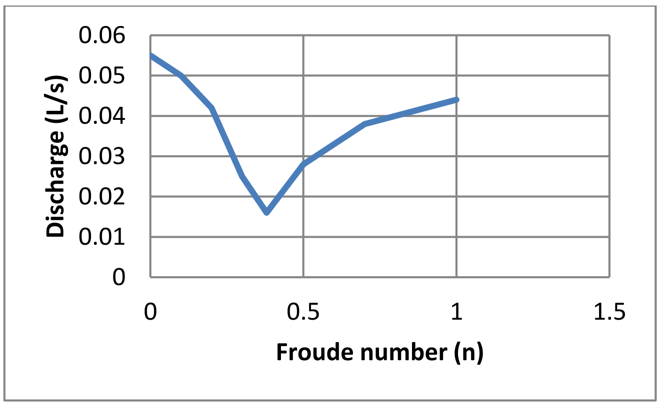

Figure 5. Discharge with Froude number

In the subcritical regime, where flow conditions are deeper and at lower velocities than in Figure 5, the discharge is reduced as depth increases above a Froude number of 0.4. Figure 5 illustrates this phenomenon. Nevertheless, as the

Froude number increases and the flow enters the supercritical range, the discharge is increased due to higher velocities and lower depths in the over supercritical zone, which allows for more water to move through the structure. This non-linear pattern emphasizes the intricate relationship between flow depth, velocity, and gravitational and inertial forces in hydraulic structures.

3.6 Discharge vs. head height

As illustrated in Figure 6, the higher the water level (or hydraulic head) above the crest of a weir, the greater its discharge in hydraulic structures. Weirs are designed to measure the flow of water in rivers, canals, or reservoirs and this behavior is essential to their operation. The amount of water that is released through a weir is directly proportional to the height above its crest, which is indicative of potential energy. This causes an additional barrier when no other source of current exists. Nonlinearities in water flow and discharge are correlated with increases in height. The small escalation in head size results in a more significant increase in discharge, particularly when the head is already high. The potential energy of the water above the crest of H increases with an increase in height. As the potential energy increases, the water flows at a faster rate over the weir. The height raised to the 3/2 power is proportional to resulting discharge. Weirs are utilized for flow regulation, and this aspect is crucial to the understanding. By analyzing the water level above its crest, engineers can determine how much flow will occur.

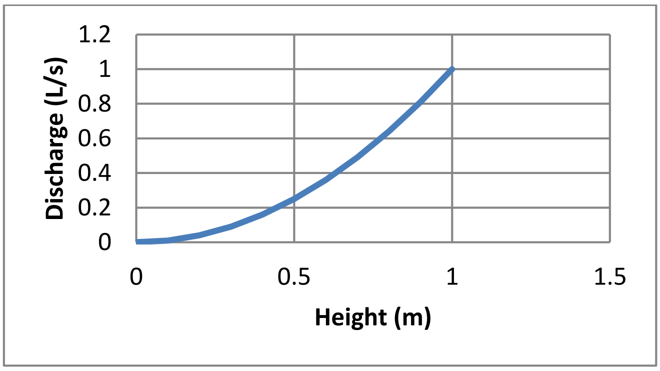

Figure 6. Discharge with height

The weir's height can determine the pressure difference between its upstream and downstream sides, which in turn causes an increase in discharge with height. The upstream side experiences a rise in pressure, which results in more water flowing over the weir. Furthermore, a higher rate of water flow over the weir and an increase in its kinetic energy cause increased discharge. The principle arises from the pressure differential and gravity's interaction, which intensify as the height rises. The curve of height corresponding to discharge is non-linear. Higher water levels result in a greater increase in discharge. An increase in the water level when the head is low may not lead to a significant surge in discharge. This is an example. Nevertheless, as the head height increases, the discharge rate also rises.

3.7 Elevation effects on discharge

In practice, sluices are designed to accommodate a range of water situations and give an accurate dimension of inflow over varying conditions. As the height of the water rises, the discharge increases, and it's important for masterminds to consider the maximum design head to ensure the block can handle the anticipated inflow rates without being overwhelmed. also, if the water height exceeds certain thresholds, the inflow may transition from laminar to turbulent, which could affect the delicacy of the discharge dimension and the effectiveness of inflow control. The rise in water volume above the crest of a weir leads to an increase in discharge over that area. The flow rate is nonlinear with height, as shown by the equation $Q=C_d \cdot L \cdot H^{3/2}$. The pressure differential and kinetic energy increase together with the rise in water level result in greater discharge. Understanding this connection is essential for the creation and operation of weirs in multiple hydraulic applications, such as flood control measures, water measurements, and irrigation management. However, engineers must also consider flow instability or inefficiency when limiting head height. The nature of Figure is reminiscent of that of the study [20].

Whether above or below sea level can cause the discharge in hydraulic structures, like weirs, is subject to variation. These structures are designed to control water flow, and their effectiveness is greatly affected by atmospheric pressure, water density, hydraulic head height, etc. The hydraulic head is the vertical distance of water from a specific point, typically the base of the structure. This height is known as the hydraulic angle. In weirs, the hydraulic head is directly linked to the discharge. As the weir's height changes with rising sea level, it affects the head's available area. Upon reaching a higher altitude, the water level above the crest of the weir decreases, as illustrated in Figure 7. By reducing the potential energy of water, the discharge is reduced. The water flow through the weir has less pressure and therefore flows at a slower rate. A weir situated beneath sea level results in a higher hydraulic head due to the elevated water level above its crest. The potential energy for flow is raised by the rise in water level above the weir, resulting in higher discharge rates.

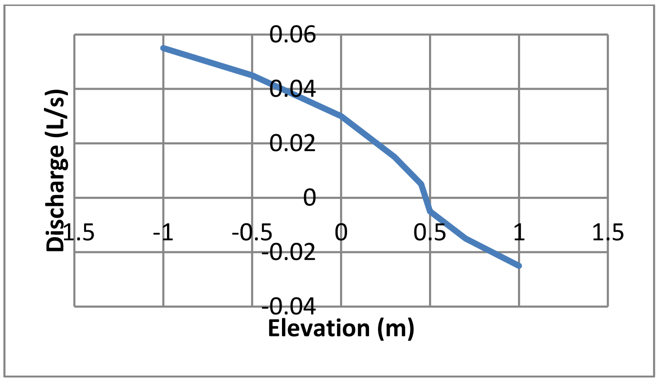

Figure 7. Discharge above and below sea level

Elevation causes a decrease in atmospheric pressure. The density of water is slightly decreased. This alteration is usually minor but can have a significant impact on flow characteristics, particularly in systems with high levels of sensitivity or low-flow conditions. However, Areas with lower atmospheric pressure are also higher. With increased pressure, the water density can increase and, if desired, discharge (especially in pressurized weirs) may slightly increase due to reduced air density at higher elevations, which will also reduce the resistance of water flowing overhanging on this feature.

The discharge for the trapezoidal PKWs weir shows a more intricate pattern: it falls slightly, then rises to a maximum at the surface; after which, it falls again as the height increases and finally becomes a minimum just above the surface. Such variations can be typical for the PKWs weir having features that may include but not be limited to multiple notches or varying paths of flow. Consequently, these may be the causes due to which discharge will vary with height. With a discharge of maximum value at the surface, this gives an indication that some optimal flow conditions are attained either at the Earth's surface itself or quite near, whereby the gravitational influence on the flow is most favorable to discharge work efficiently.

So, having looked at how these weirs let water flow, we see that the rectangular weir lets the least water through. After this comes the trapezoidal labyrinth weir, the trapezoidal PKWs weir, and the stepped labyrinth weir, which is the most permeable. The order of this one indicates that the trickier the craft of weir is the more it can adjust with the different weights of the flows, and handled different amount of water but simultaneously might not show that much stability in some cases.

The trapezoidal labyrinth and stepped labyrinth weirs, with their more intricate designs, appear to handle water flow at various heights more than the basic rectangular weir. The trapezoidal PKWs weir, which has a more complicated structure, displays fluctuations in its water discharge. This suggests it might cope with changing flow conditions more under specific height-related circumstances. Variations in discharge among these weir types demonstrate that structural complexity really does matter when it comes to driving flow dynamics. Although a rectangular weir will provide stability and simplicity but, it is worse suited to variable height conditions than the more complex forms of trapezoidal labyrinth and stepped labyrinth weirs. Variation in discharge of the trapezoidal PKWs weir pattern might indicate that harmonics are possibly more efficient at heights, and less so others. This implies that structural design and height differences require more attention when choosing weir type for specific hydraulic applications in future. Further studies would look at the mechanisms behind discharge increase beyond these critical heights, particularly for the trapezoidal PKWs and stepped labyrinth weir types to make optimal use of them in diverse operating conditions.

Figure 8 shows the discharge of different weir types: rectangular, stepped labyrinth and trapezoidal PKWs nearly constant with height but decreases in the case of trapezoidal labyrinth; For various weir designs by differing height conditions. With increasing height, the discharge of the rectangular weir stays almost stable. The reason for this observation is the simplicity of the rectangular weir, with flow being primarily controlled by its width and the depth of water that is sitting behind it. This is indicative as discharge is similar made the flow does not operate significantly over its principal range at a particular range operational height. The limited expansion of flow area as water height increases could be partly offset by the increased head. The low, unobstructed flow path of the square weir means that variations in height have little effect on discharge so it is convenient for stable regulation of flow.

Also, the stepped labyrinth weir has nearly flat discharge with height. The stepped labyrinth weir which is having more involved geometry than rectangular type has nearly constant pattern of discharge for varying heights. The constancy is somewhat counter-intuitive though, as one would expect that the higher geometry would more certainly impact the underlying flow more so than in this case. At least that is why this weir behaved according to the rule of constant discharge, because of the steps which shape adaptive flow paths. The requirement is that the steps should provide uniform control and pattern of flow such that even though heights differ, significant variations in discharge do not arise. Since the discharge is steady, we can be assured that for all manner of flow it has been carefully designed to cope with this not only at lower but also progressively so over the steps to rise head. Hence, the design of the stepped labyrinth weir can accommodate a variation in height without having undue effect on total flow. The trapezoidal PKWs weir, as for the rectangular and stepped labyrinth weirs as well nearly exhibits uniform discharge with increase in height. The trapezoidal PKWs weir not the be a constant discharge with rising height, it should be suspected that this design nature of it come from the fact the trapezoid one is more tolerant to a range of flow conditions thus keep discharge stable. Being weir with some kind of weir-head configuration (presumably having a larger area for the water to spill out over) keeps discharge from getting all willy-nilly due to change in height. The fact that discharge is extremely constant suggests that the weir is not very height-sensitive and must have been designed to perform adequately under different heights of water.

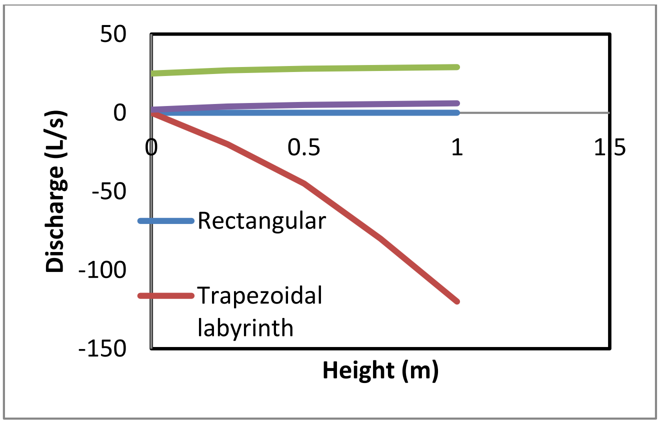

Figure 8. Discharge with height for different weir

4. CONCLUSION

A comprehensive analysis of the hydraulic conditions for several weir types and designs including evaluations of their effectiveness for controlling discharges for a range of flow conditions is provided in this paper. With this, it provides several different isocline representations for different dimensions with the nonlinear nature of deformation in discharge to head representation which leads to different distinct behaviors of hydraulic heads under varying flow rate conditions which can highlight rectangular, trapezoidal, labyrinth and PKWs weirs behavior. The rectangular weirs had a similar performing trend and the labyrinth and stepped labyrinth weirs show a better adaptive response with variable flow conditions. Within this context, trapezoidal PKWs weirs have been encouraging despite their dependency on particular elevations and flow changes. It is recommended, based on this study's findings, that rectangular and stepped labyrinth weirs be used that would offer stable discharge in varying height conditions, while trapezoidal labyrinth and PKWs weirs would have to be optimized over specific height ranges. The future work must be directed accordingly, i.e., to refine weir designs using computational modeling and real-time monitoring to help enhance their efficiency in various environmental conditions.

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NOMENCLATURE

Q	discharge
C_d	discharge coefficient
L_{net}	net crest length
g	gravity
H	total head over the weir
P	Height of weir
B	channel width
y_1	Depth of flow
F_1	Froude number
L	width of the side weir
θ	angle of the oblique side weir
α	angle of the triangular side weir
h_1	piezometric head
b	average river width
R_b	Depth of flow
PKW	Piano Key Weir