



Estimating Seismic Ground Motion and Assessing Damage Potential in Northern Algeria Using ARMA Models

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ABSTRACT

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This study employs an Autoregressive Moving Average (ARMA) statistical model to generate artificial seismograms, an innovative approach to address the lack of real-world data. The model is driven by white Gaussian noise and modulated by an amplitude envelope function, allowing it to faithfully reproduce the characteristics of seismic motion. This model proved particularly effective in simulating the non-stationary behavior of accelerograms from historical earthquakes in Algeria, including those in Boumerdès, Asnam, and Affroun. A comparative analysis of key parameters such as peak acceleration and shaking duration confirmed the validity of the simulations. Beyond simple reproduction, the generated seismograms made it possible to assess damage potential. The researchers measured crucial structural demand indicators, such as ductility demand and hysteretic energy, to assess potential damage. The response spectra of the simulations match well with those of the real recordings, which reinforces the reliability of the damage predictions. The quasi-linear correlation between the damage index and the initial structural period reinforces this observation. In summary, the ARMA(2,1) approach offers a practical and reliable framework for damage assessment and the design of earthquake-resistant structures.

1. INTRODUCTION

The limited availability and inherent variability of natural accelerogram records often hinder a full analysis of structural response. Consequently, the generation of artificial ground motions has become an essential and widely accepted practice in earthquake engineering. These simulated accelerograms allow for a systematic evaluation of structures under both linear and nonlinear conditions, offering enhanced control over crucial parameters such as amplitude, duration, and frequency content. Response spectra play a central role in seismic design, particularly for single-degree-of-freedom (SDOF) systems [1]. When multiple records are used to develop design spectra, a significant degree of variability emerges [2]. To address this, records are frequently normalized with respect to peak parameters, such as acceleration or velocity [3]. This approach has been widely used, for example, in the seismic design of nuclear power plants [4].

This study aims to develop minimal stochastic models for earthquake ground motions to address these shortcomings. The objective is to create a simple yet descriptive modeling method that captures the damage-relevant characteristics of seismic input. To achieve this, we propose using a time-varying Auto Regressive Moving Average (ARMA) process, specifically a low-order ARMA(2,1) model. This model is

driven by white noise and modulated by an envelope function, which allows it to effectively reproduce the non-stationary behavior of seismic events.

The methodology, applied to Algerian earthquakes recorded on dense soils, confirmed that the model generates ground motions closely matching actual seismic records. The study's results demonstrate that the model can accurately predict the mean values and variances of key spectral response ordinates. The simulations show a good correspondence for key damage indicators, including peak displacement, ductility demand, and hysteretic energy dissipation. These findings confirm the validity of our approach as a valuable tool for assessing damage potential in seismic design applications.

2. FORMULATION OF THE ARMA MODEL

This study analyzes seismic ground motion in the time domain using ARMA models, a method initially introduced by Jebb and Tay [5], and later adapted for structural dynamics by Gao et al. [6]. The ARMA framework was chosen for its ability to create simplified stochastic models that accurately capture ground motion characteristics while maintaining a clear physical interpretation. This interpretability is highly beneficial for structural design, as it allows a direct correlation between model parameters and measurable quantities like

acceleration or displacement.

The ARMA model at any time step, k , is represented by the following equation:

$$A_k - \phi_1 A_{k-1} - \dots - \phi_p A_{k-p} = W_k - \theta_1 W_{k-1} - \dots - \theta_q W_{k-q} \quad (1)$$

where, ϕ_i , θ_j are constant coefficients.

The left-hand side of this equation is the autoregressive (AR) part of order p , while the right-hand side is the moving average (MA) part of order q . In this context, Z_t represents the time series of measured data, and the sequence a_t is a set of independent, identically distributed Gaussian random variables.

The constant coefficients, ϕ_i and θ_j , along with the model orders (p , q), are estimated from the data using a maximum likelihood approach. The seismic accelerograms from Table 1 were selected for this study.

Table 1. Real earthquakes characteristic

Event	Boumerdes	Asnam	Affroun
Magnitude	6.8	5.8	6.8
Duration (second)	25.96	21.7	80
Max Acceleration $\times$ (g)	0.215	0.189	0.03
Number of points	1299	1087	16001

3. MODELING PROCEDURE

In this research, the modeling process begins by normalizing the digitized seismic acceleration data to simplify the fitting of the model. For each time step, k , a moving window of 100 time steps centered at k is used to compute the root mean square S_k value.

This process generates a normalized time series, $Z_k = A_k/S_k$, with a zero mean and unit variance. This series is then modeled as a second-order stationary process using the ARMA formulation Eq. (1).

The comprehensive procedure for fitting an ARMA model to a seismic acceleration record includes the following steps:

Normalization: Compute the experimental envelope function S_k and use it to normalize the raw acceleration data.

Envelope function: Define an analytical expression for S_k and estimate its parameters through least-squares analysis.

Stabilization: Remove any non-stationary trends from the original record using the envelope function to stabilize the signal.

Correlation analysis: Compute the autocorrelation and partial autocorrelation functions of the stabilized, normalized signal.

Order selection: Choose the appropriate order p for the AR component and order q for the MA component of the ARMA model (Eq. (1)).

Coefficient estimation: Estimate the coefficients ϕ_i and θ_j using maximum likelihood analysis.

Model validation: Validate the model by re-computing the autocorrelation and partial autocorrelation functions of the residuals to confirm that the selected orders are adequate and that the process is stationary.

Alternative model evaluation: Use the Akaike Information Criterion (AIC), denoted as $AIC(p, q)$, to evaluate alternative model orders and select the one that minimizes the AIC value [7].

3.1 ARMA model and its parameters

From a parameter count perspective, the ARMA model offers a more efficient representation for simulating earthquake phenomena, particularly when data is affected by noise. A crucial step in evaluating ARMA parameters is analyzing the autocorrelation of the seismic trace. The relationship between autocorrelation and AR parameters highlights the primary importance of the AR part of the model as follows Eq. (4).

$$\begin{bmatrix} 1 & R_1 & \dots & R_{k-1} \\ R_1 & \dots & \dots & R_{k-2} \\ \vdots & \dots & \ddots & \vdots \\ R_{k-1} & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_k \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_k \end{bmatrix} \quad (2)$$

Once the AR component is identified, the MA part can be constructed. For selecting the order of the ARMA(p , q) model, the minimum of the Akaike Information Criterion (AIC) is used, as investigated by Korte et al. [8]. For this purpose, the minimum of the AIC index:

$$AIC(p, q) = N \cdot \ln(\sigma_a^2) + 2(p + q) \quad (3)$$

where, N is the sample size, and σ_a^2 is the estimated variance from the residual maximum likelihood.

A comparison between several models for each earthquake: ARMA(1,1), ARMA(1,2), ARMA(2,1) and ARMA(2,2), using the AIC criterion described in Table 2.

Table 2. Application of AIC criteria to the different models

Parameter	ARMA Model			
	(1,1)	(1,2)	(2,1)	(2,2)
Asnam	-1145.4	-1157.9	-1158.3*	-1155.0
Boumerdes	-1170.3	-1185.5	-1190.2*	-1186.5
Affroun	-3.64068	-4.78432	-5.09023	-5.26325*

* Indicates that the model has the minimum AIC.

The final model parameters for each earthquake are presented in Table 3.

Table 3. Parameters of the selected model

Parameter	AR(1)	AR(2)	MA(1)	σ_a
Asnam	0.359	-0.222	0.181	0.559
Boumerdes	0.359	-0.869	-0.304	0.551
Affroun	1.80244	-0.888599	-0.888599	-0.888599

This research selects an ARMA(2,1) model, which provided the best fit for the input accelerograms. This model can be represented by:

$$A_k - \phi_1 A_{k-1} - \phi_2 A_{k-2} = W_k - \theta_1 W_{k-1} \quad (4)$$

For the simulation, the envelope function is given by the expression:

$$S_k = a \cdot t^b \cdot e^{-ck} \quad (5)$$

The parameters a , b , and c are estimated through nonlinear regression analysis (Table 4).

The foundational assumption of this analysis is that an earthquake can be considered a single realization from a population of earthquakes characterized by an ARMA process.

Table 4. The parameters of the envelope function

Event	Envelope Function Parameters		
	A	B	C
Boumerdes	1.2198	1.1077	0.671
Asnam	1.1567	0.8810	0.689
Affroun	9.61e-9	10.91	0.6055

To simulate the acceleration time series using this approach, we first generate a white noise signal with a zero mean and a variance equal to the estimated variance. This white noise is then used as input to the ARMA model to produce a stationary time series. Finally, this stationary series is multiplied by the envelope function, $S(k)$, to obtain the non-stationary simulated acceleration series.

Since the ARMA model is a linear combination of previous values and Gaussian random variables, simulated earthquakes can be generated inductively. For the ARMA(2,1) model, the recursive equation is used. To initiate the calculations, the first p values of A_k (in this case, A_0 and A_{-1}) are assumed to be zero. Gaussian random variables (W_k) with variance σ_a^2 are generated as the white noise input.

Early simulations indicated that the Fourier spectra and responses derived from the simulated earthquakes had large values at low frequencies, which the original filtered earthquakes did not exhibit. This common issue is addressed by applying a low-pass filter to the simulated data to eliminate these low-frequency components.

The ARMA coefficients and the white noise standard deviation (σ_w) are detailed in Table 5.

Table 5. ARMA coefficients and white noise standard deviation

Event	ARMA Parameters			
	ϕ_1	ϕ_2	ϕ_3	σ_w
Boumerdes	0.6626	-0.0738	-0.521	0.555
Asnam	0.4244	0.2411	-0.613	0.560
Affroun	1.8635	-0.922845	-1.28836	0.0407843

For the estimator used, we provide the SSE, R-squared, adjusted R-squared, and RMSE in Table 6.

Table 6. Summary of fit metrics for the envelope function

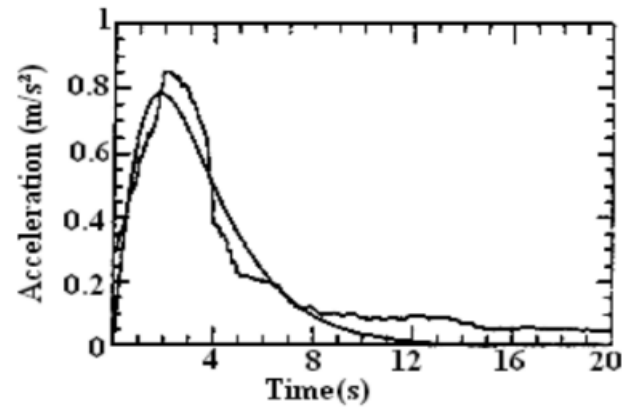
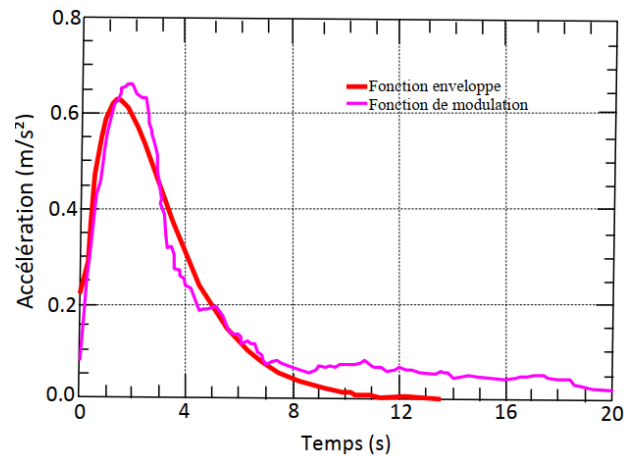
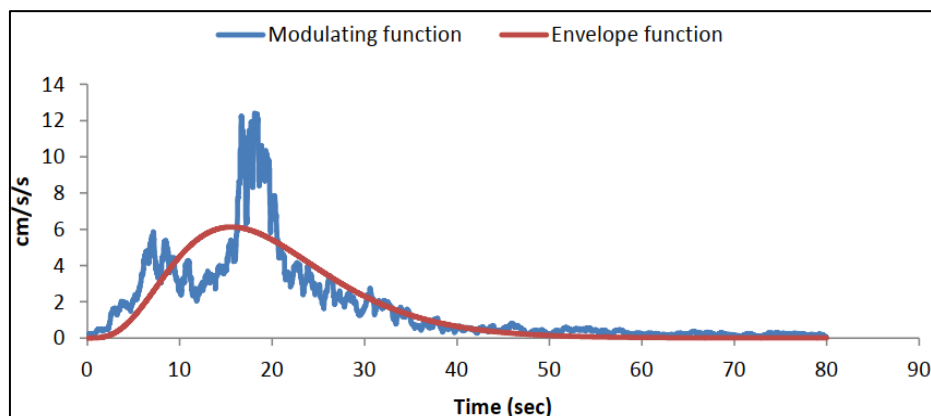
Event	SSE	R ²	Adjusted R ²	RMSE
Affroun	2.674e+04	0.6978	0.6978	1.293

A comparison of the key characteristics of real and

simulated earthquakes (Table 7) shows that the events are well-characterized by the ARMA(2,1) process and the defined envelope function parameters.

Table 7. Characteristics earthquakes and simulated ones

Header	Characteristics	Earthquakes		
		Boumerdes	Asnam	Affroun
Real	Maximal acceleration $\times$ (g)	0.152	0.189	0.030
	Duration (sec)	25.96	21.7	80
Simulated	Maximal acceleration $\times$ (g)	0.141	0.107	0.022
	Duration (sec)	20	20	80

**Figure 1.** Modulation functions and envelope Boumerdes**Figure 2.** Modulation functions and envelope Asnam**Figure 3.** Modulation functions and envelope Affroun

Figures 1-3 show the appearance of the modulation functions and envelope functions corresponding to the Boumerdes, Asnam and Affroun earthquakes.

4. EVALUATION OF SEISMIC DAMAGE INDICES

A range of damage measures has been developed to quantify the effects of seismic excitation on both linear and nonlinear structural systems [9-13]. Researchers have explored diverse approaches, as highlighted by Grigoriu's comparative evaluation [14]. While frequency spectra are a common way to present structural response data, any single measure is acknowledged to be an imperfect predictor of damage due to the variety of potential failure modes. Consequently, this study evaluates damage potential using a set of general and composite criteria, referred to as damage indices.

One such index, proposed by Park [15] is based on the ratio of structural demand to capacity. This index is a linear combination of the maximum relative deformation and the relative energy dissipation. The general form of the Park-Ang damage index is the frequency, damping and the yield force of a structure.

$$DI = \frac{U_{max}(t)}{U_{max}} + \frac{\beta \cdot [E_H]}{R_Y \cdot U_{max}} \quad (6)$$

where, $U_{max}(t)$ is the maximum displacement under seismic loading. U_u is the ultimate deformation capacity under monotonic loading. R_Y is the yield force. E_H is the hysteretic energy dissipated through inelastic deformation. β is a non-negative parameter that represents the structure's energy absorption capacity.

To calculate a spectrum of damage indices, it is necessary to specify the value of β and the structural parameters, including natural frequency, damping ratio, and yield force.

The structural responses ($U_{max}(t)$) were calculated through numerical integration of the equations of motion, assuming a linear acceleration within each time step [16-21]. To accurately model the energy absorption of structures, a bilinear hysteretic model was adopted.

This model is defined by three key parameters:

u_y : initial yield displacement,

k_I : initial elastic stiffness,

k_y : post-yield stiffness.

During monotonic loading, the restoring force $R(u, t)$ follows:

$$R(u, t) = K_I \cdot u(t); u(t) \leq u_y \quad (7)$$

$$R(u, t) = K_y \cdot (u(t) - u_y); u(t) > u_y \quad (8)$$

During unloading, the stiffness reverts to the initial elastic stiffness until the displacement again intersects the yield envelope. This elastoplastic behavior is captured by setting $K_y = 0$ and $R(u, t) = K_y \cdot u_y$ for $u(t) > u_y$.

This simplified yet effective model is widely used in seismic analysis to estimate damage potential.

5. NUMERICAL RESULTS

The dataset for this study includes two recorded earthquake ground motions:

- The 2003 Boumerdes (NS) earthquake, with a moment magnitude $M = 6.8$, yielding 1299 data points at 0.02-second intervals.

- The 1980 Asnam (NE) earthquake, with a moment magnitude $M = 5.8$, providing 1087 data points at 0.02-second intervals.

- The 1988 Affroun earthquake, with a moment magnitude $M = 6.8$, yielding 1601 points.

Various ARMA models were fitted to these records. For each earthquake, 20 synthetic acceleration records were generated using the identified ARMA models. These synthetic records were then used as input for elastoplastic SDOF systems, which were modeled with a bilinear hysteresis law (Figure 4).

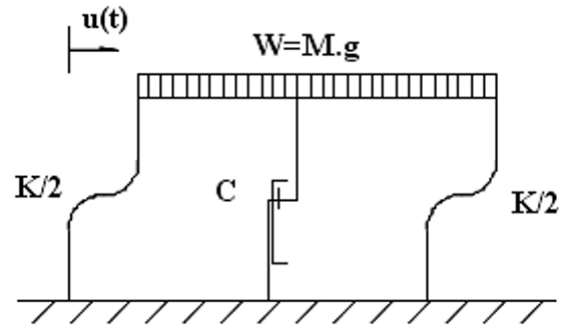


Figure 4. Response of SDOF system

The analyses were performed with damping ratios of 0.02 and 0.05, and yield strength ratios ($Y = R_Y/M \cdot g$) of 0.05, 0.10, and 0.15 to represent different levels of structural ductility. For the damage index calculations, the β parameter in the Park-Ang index was set to 0.05, which is a value reflecting the average energy dissipation capacity of typical steel structures.

The resulting damage index spectra (Figures 5-7) were obtained by evaluating the Park-Ang index across a range of natural frequencies. These spectra are a combination of two fundamental response measures: the ductility demand spectrum and the hysteretic energy demand spectrum.

As shown in Figure 2, the damage index spectra for individual earthquakes display notable irregularities due to the inherent randomness of ground motion characteristics. This highlights the importance of using ensemble averages or statistically representative simulations for damage assessment.

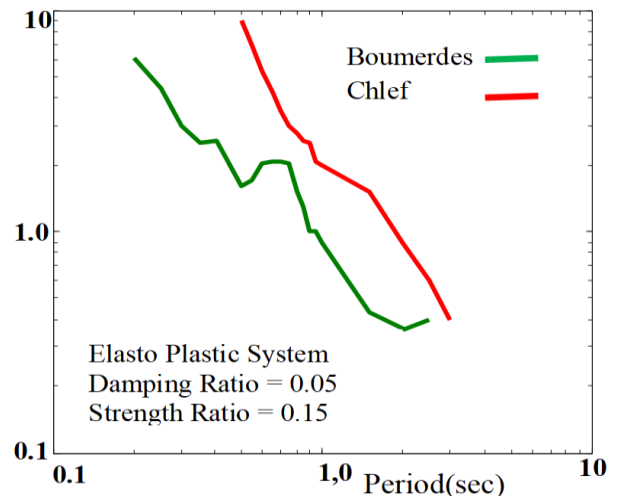


Figure 5. Seismic damage spectra for the 1980 Asnam and 2003 Boumerdes events

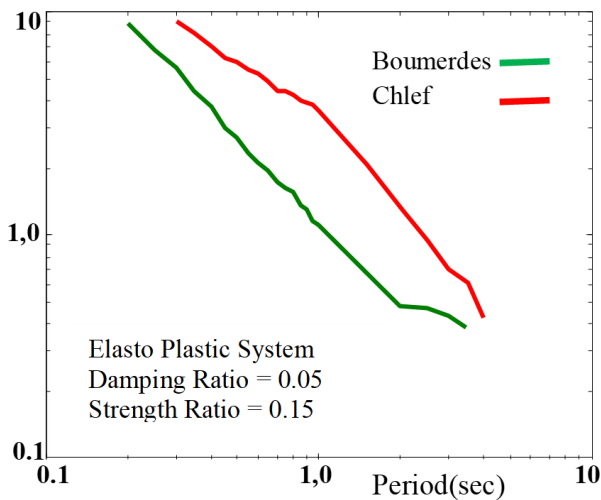


Figure 6. Average spectral damage from synthetic motions for the 1980 Asnam and 2003 Boumerdes events

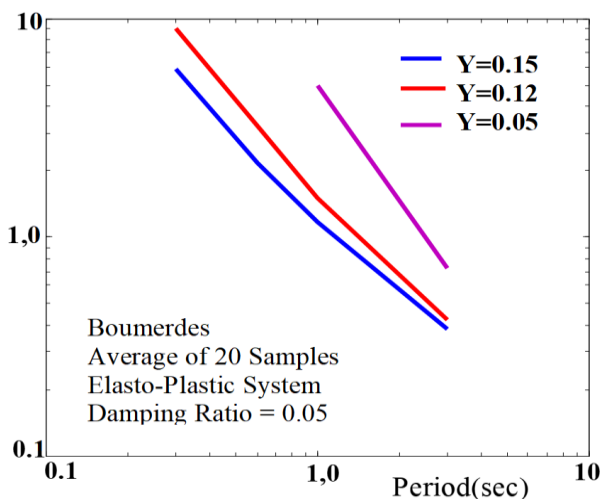


Figure 7. Average spectral damage vs strength reduction factor for the 2003 Boumerdes events

6. CONCLUSION

This study confirms that ARMA(2,1) time-domain models are a compact and effective tool for simulating earthquake ground motions. Despite the model's simplicity, the simulated response spectra show good agreement with real records. However, some discrepancies were noted for long-period motions, likely due to residual low-frequency components from filtering. The envelope function successfully controls the amplitude and duration of the synthetic motions, which are key characteristics of seismic inputs.

The results also emphasize the significant influence of the soil behavior model on seismic response, underscoring the need to incorporate realistic, and potentially nonlinear, site effects into simulations. The ARMA(2,1) based approach therefore provides a practical framework for generating site-specific synthetic records for performance-based seismic design.

Furthermore, the irregularities in the damage index spectra of individual records appear to be the result of stochastic variability rather than intrinsic features of the seismic source. A nearly linear correlation was observed between the logarithm of the average damage index and the logarithm of

the initial structural period. If validated with larger datasets, this trend could greatly improve the predictive power of structural damage models and lead to more accurate seismic hazard assessments. The observed decrease in the average damage index as the structural period increases supports the established understanding that long-period structures are generally less vulnerable.

In summary, these findings advocate for a combined approach using ARMA-based modeling and period-dependent damage assessment to develop next-generation, performance-oriented methodologies for earthquake-resistant design.

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NOMENCLATURE

Z_t	time series of measured data
a_t, W_k	Gaussian random variables
S_k	root mean square value
Z_k	normalized time series
AIC	Akaike Information Criterion
N	sample size
a, b, c	parameters of the envelope function
$S(k)$	envelope function
DI	damage index
u_y	initial yield displacement
k_1	initial elastic stiffness
k_y	post-yield stiffness
SDOF	single-degree-of-freedom
ϕ, Θ	ARMA constant coefficients
σ_a^2	estimated variance
σ_w	standard deviation
(p, q)	model orders