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Fractional Mechanism with Power Law (Singular) and Exponential (Non-singular) Kernels and Its Applications in Bio Heat Transfer Model

Muhammad Imran Asjad

Department of Mathematics, University of Management and Technology Lahore, Pakistan

Corresponding Author Email: Imran.asjad@umt.edu.pk

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ABSTRACT

This study deals with breast cancer therapy with fractional derivative in porous medium. The classical model of energy balance equation can be generalized to some fractional derivatives namely, Caputo (C) with singular kernel and Atangana-Baleanu (ABC) with non-singular kernel. We have found the semi exact solutions of initial value problem via Laplace transform. Hyperthermia technique named as a moderate method for the treatment of breast cancer. Graphical illustration is provided for numerical values of embedded parameters with the help of MATHCAD software. As a result, a steady-state time required the therapeutic temperature point to get the death of tumor cell has been computed. A comparison has also been drawn between the solutions of fractional models with fractional derivatives and found that Atangana-Baleanu fractional model is well suited in exhibiting the memory effect of the temperature function. The findings of this research may serve the advantage that it is affordable and important in clinical or medical practice.

1. INTRODUCTION

Hyperthermia is a technique for treatment of malignant and benign tumors by administering heat in1different ways by the Oncologist. On the bases of temperature and time duration hyperthermia can be divided into three categories namely (1) long term low temperature hyperthermia with targeted temperature $(T < 41\,^oC, t = 6-72hrs)$. (2) Moderate temperature hyperthermia with following restriction $(41^oC < T < 46^oC, t = 15-60)$ minutes. (3) High temperature hyperthermia or thermal ablation with $T \geq 46^oC$ for 4-6 minutes. Further this type of therapy depends on the intensity of volumetric heat source. The condition of abnormal cell determines which type of hyperthermia can be used. It further classified as follows (a) Localised in which high temperature needed for smaller area (b) regional in which temperature needed for whole body (c) the therapy applied to the larger area of tissue or in the whole body hyperthermia. There are different external heat sources such as radio-frequency, microwave, magnetically. Excitable thermo-seeds, infra-radiation and further details can be seen in [1-10]. Scientifically, abnormal growth of cells (i.e. malignancy) is known as cancer. Cancer is a1complex life-threatening disease which affects millions of people worldwide. World Health Organization [11] remarked that the disease is a leading cause of death in developed countries and second leading cause of death in developing countries. Cancer is any malignant growth or tumor caused by abnormal and uncontrolled cell division. If not stopped, it may spread to other parts of the body through the lymphatic system or blood stream. Its revelent detail is found in the following references. [12-16].

The technique of fractional calculus has been used to formulate mathematical modeling in various technological development, engineering applications and industrial sciences. Nowadays, it has proved to be an effective method for generalizing the complex dynamics of fluid flow. Particularly, for fluid dynamics, signal processing, tracer in fluent currents, viscoelasticity, bio-engineering, biological science, electrochemistry, and finance. Riemann-Liouville and Caputo fractional derivatives are among commonly applied fractional derivatives. The main advantage of using fractional PID is to control continuous plants, where we can limit the maximum value of the input control to much less value than with conventional PID for nearly the same response. If the fractional PID controller is set designed "perfectly", then such result shall be correct. Better design almost means a good approximation of the FOPID by a rational transfer function. Atangana et al. [17] used the variable order derivative for generalized ground water flow equation. Application of Atangana-Baleanu fractional technique to non-linear system can be found in Ref. [18]. Mirza et al. [19] found the fundamental solutions for advection equation using non-singular fractional calculus derivative. Hristov [20] studied the heat conduction in singular fading found analytical solutions with Caputo-Fabrizio fractional derivative. Baleanu et al. [21] discussed the some new properties of Mittag-Leffler function. Recently, Imran et al. [22] studied the comparison approach between Caputo-Fabrizio and Atangana-Baleanu fractional derivative and found that Atangana-Baleanu fractional derivative is excellent in exhibiting the memory effect in fluid flow problems. Some recent studies related to the applications of modern techniques of fractional derivatives can be seen [23-33]. Due to vast applications of fractional derivatives, applied this technique to multilayered tissues of breast cancer and found semi exact solutions with and found some useful conclusion for experts to treat cancer cells with suitable amount of heat source and have compared the results to see the memory effect of temperature function.

Checking through the literature, it is worth remarking that the effects of variations in blood thermal conductivity, porosity, thermal radiation, heat source and blood perfusion on temperature distribution during microwave heating of hyperthermia therapy has not been studied with new kinds of fractional derivatives namely Caputo and Atangana-Baleanu.

2. STATEMENT OF THE PROBLEM

A combination of thermal conduction, convection, production of metabolic heat and perfusion of blood is known as heat transfer mechanism that occur in the living tissues. At the beginning, constant temperature $T_o = 37^o C$ of body tissu is heated by some external heat source by assuming

- At the skin surface radiation was assumed to be almost negligible.
- Blood and tissue are assumed to be in local thermal equilibrium [15].
- Both the temperatures (Venous blood, local tissue) are equal and arterial blood temperature assumed uniform 1throughout the 1tissue [16].

The one-dimensional governing equation is [16]:

$$\rho_b C_{pb} T_t = K_b T_{yy} + \omega_b \rho_b C_{pb}(T_b - T) + Q_0(T)|E|^2, \quad (1)$$

Porous media theory in modeling of bio heat transfer models are stressed by [8, 9].

$$\rho_b C_{pb} T_t = K_b T_{yy} - \frac{\partial q}{\partial y} + \frac{\epsilon}{k} + \omega_b \rho_b C_{pb}(T_b - T) + Q_m(T - T_0) \quad (2)$$

Subject to the following boundary conditions:

$$T(y,0) = T(0,t) = T_0 + (T_b - T_0)37^0 C,$$
$$T(d,t) = T_0 + (T_b - T_0)45^0 C. \quad (3)$$

Using Rossland diffusion approximation for radiation [8, 33]. Hence, Eq. (2)

$$\rho_b C_{pb} T_t = K_b\left(1 + \frac{4}{3}R\right)T_{yy} + \frac{\epsilon}{k} + \omega_b \rho_b C_{pb}(T_b - T) + Q_m(T - T_0) \quad (4)$$

The following dimensionless parameters are introduced

$$y^* = \frac{y}{d}, \quad t^* = \frac{t}{d^2}, \quad \theta^* = \frac{(T - T_0)}{(T_b - T)}, \quad \alpha = \frac{K_b}{\rho_b C_{pb}},$$
$$\beta = \frac{d^2\theta}{K\,\rho_b C_{pb}(T_b - T)}, \gamma = d^2\omega_b,\ \lambda = \frac{d^2 Q_m}{\rho_b C_{pb}},\ and\ R = \frac{4\sigma T_b^3}{\delta K_b}. \quad (5)$$

Substitute the dimensionless quantities (5) into Eq. (4) and dropping out ⋆ notation we have

$$\frac{\partial\theta(y,t)}{\partial t} = \alpha\left(1 + \frac{4}{3}R\right)\frac{\partial^2\theta(y,t)}{\partial y^2} + (\beta + \gamma) - (\gamma - \lambda)\theta(y,t). \quad (6)$$

The corresponding conditions becomes

$$\theta(y,0) = 37^0 C,\ \theta(0,t) = 37^0 C,\ \theta(1,t) = 45^0 C. \quad (7)$$

α =Thermal conductivity of blood β =Porosity parameter γ =Perfusion rate of blood λ =Heat source.

3. BASIC DEFINITIONS AND LAPLACE TRANSFORMS OF FRACTIONAL OPERATORS

3.1 Fractional derivative of Caputo with singular kernel [24]

$$^{C}D_t^{\alpha}\theta(x,t) = \begin{cases} h_n(t) * \dfrac{\partial^n\theta(x,t)}{\partial t^n}, & n\,\varepsilon\, N, n > 0, \quad n-1 < \alpha < n \\ \dfrac{\partial^n\theta(x,t)}{\partial t^n}, & n\,\varepsilon\, N \end{cases} \quad (8)$$

with Laplace transform

$$L^C D_t^{\alpha}\theta(x,t) = L\{h_n(t)\}L\left\{\frac{\partial^n\theta(x,t)}{\partial t_n}\right\}$$
$$= \frac{1}{s^{n-\alpha}}\left[s^n L\{\theta(x,t)\} - \sum_{k=1}^{n} s^{n-k}\theta^{(k-1)}(0)\right] \text{and}$$
$$= s^{\alpha}L\{\theta(x,t)\} - \sum_{k=1}^{n} s^{\alpha-k}\theta^{(k-1)}(0)$$
$$(\theta_1 * \theta_2)(t) = \theta_1(t) * \theta_2(t) = \int_0^t \theta_1(t-\tau)\theta_2(\tau)d\tau, \quad (9)$$

∗ *notation represents the convolution product*

$$h_n(t) = \frac{t^{n-\alpha-1}}{\Gamma(n-\alpha)} \quad (10)$$

where, Eq. (10) is the power law kernel and Γ is the Euler's function of second kind (Gamma-function).

3.2 Fractional derivative of Atangana-Baleneau with non-singular kernel

The Atangana-Baleneau time fractional derivative of order α ε [0, 1) is defined [23]

$$^{ABC}D_t^{\alpha}\theta(y,t) = \frac{B(\alpha)}{1-\alpha}\int_0^t E_{\alpha}\left[\frac{-\alpha}{1-\alpha}(T-\tau)^{\alpha}\right]\frac{\partial\theta(y,\tau)}{\partial\tau}d\tau, \quad (11)$$

The Laplace transform of Atangana-Baleanu time derivative is

$$L\left\{^{ABC}D_t^{\alpha}\theta(y,t)\right\} = \frac{B(\alpha)}{1-\alpha}\frac{p^{\alpha}}{p^{\alpha} + \dfrac{\alpha}{1-\alpha}}\left[L\{\theta(y,t)\} - p^{1-\alpha}\theta(0)\right] \quad (12)$$

where, The normalization function B(α) can be any function satisfying the conditions B(1) = B(0) = 1 and 1 Eα is one parameter1Mittag-Leffler function defined as [25]

$$E_{\alpha} = \sum_{0}^{\infty}\frac{z^k}{\Gamma(\alpha k + 1)} \quad (13)$$

The Mittag-Leffler function is an entire function of z and the series of Eq. (13) are converging locally uniformly in the whole complex plane.

4. FRACTIONAL MODEL WITH CAPUTO DERIVATIVE AND ITS SOLUTIONS

A fractional model is developed by generalizing the classical constitutive equation of thermal flux equation by generalized Fouriers law proposed by Hristov [12] and Povstenko [13], Ahmed et al. [14].

$$Q(y,t)=b_{1-\xi}\,{}^{C}D^{1-\xi}{}_{t}\left(\frac{\partial\theta}{\partial y}\right),\ 0<\xi\le 1, \tag{14}$$

where, ${}^{C}D^{\xi}{}_{t}$ is the Caputo fractional operator and $b_{1-\xi}$ is the generalized coefficient of the material and also called quasi properties of materials. So, Eq. (6) becomes

$$\frac{\partial\theta(y,t)}{\partial t}=\alpha\left(1+\frac{4}{3}R\right)b_{1-\xi}\left[{}^{C}D^{1-\xi}{}_{t}\left(\frac{\partial\theta(y,t)}{\partial y}\right)\right] -(\gamma-\lambda)\theta(y,t)+(\beta+\gamma) \tag{15}$$

The time fractional integral operator which is the left inverse operator of [23] is defined as

$$I^{\xi}{}_{t}\theta(y,t)=\left(h_{1-\xi}*\theta\right)(t)=\frac{1}{\Gamma(\alpha)}\int_{0}^{t}\theta(y,\tau)(t-\tau)^{\xi-1}d\tau, \tag{16}$$

Using Eq. (8) and Eq. (16) we obtain that

$$\left(I_{t}^{\xi}.D_{t}^{\xi}\right)\theta(y,t)=I_{t}^{\xi}\left(D_{t}^{\xi}\theta(y,t)\right)=\left[h_{1-\xi}*(h_{\xi}*\dot{\theta}\right](t)= \left[(h_{1-\xi}*h_{\xi})*\dot{\theta}\right](t)=\left[1*\dot{\theta}\right](t)=\theta(y,t)-\theta(y,0). \tag{17}$$

if $\theta(y,0)=0$. Also, using the following property

$$\left(I_{t}^{1-\zeta}\dot{\theta}(y,t)\right)=\left(h_{\zeta}\star\dot{\theta}\right)(t)=D_{t}^{\zeta}\theta(y,t) \tag{18}$$

By applying the operator $I_{t}^{1-\zeta}(.)\, to\ Eq.(15)$ we get

$$D_{t}^{\zeta}\theta(y,t)=\ a_{1}\left(1+\frac{4}{3}R\right)b_{1-\zeta}\frac{\partial^{2}\theta(y,t)}{\partial y^{2}}-(\gamma-\lambda)I_{t}^{1-\zeta}\theta(y,t)+I_{t}^{1-\zeta}(\beta+\gamma). \tag{19}$$

The general solution of Eq. (19) subject to the conditions given in Eq. (7) by applying the Laplace transform technique

$$\bar{\theta}(y,s)=\left[\frac{37}{s}-\frac{\beta+\gamma}{s+\gamma-\lambda}-\frac{\frac{37}{s}e^{\sqrt{\frac{s+\gamma-\lambda}{as^{1-\iota}(1+\frac{4}{3}R)}}}-\frac{45}{s}-\frac{(\beta+\gamma)\left(e^{\sqrt{\frac{s+\gamma-\lambda}{abs^{1-\iota}(1+\frac{4}{3}R)}}}-1\right)}{s+\gamma-\lambda}}{2\sinh\left(\frac{s+\gamma-\lambda}{abs^{1-\iota}(1+\frac{4}{3}R)}\right)}\right]\times e^{y\sqrt{\frac{s+\gamma-\lambda}{as^{1-\iota}(1+\frac{4}{3}R)}}}$$

$$+\left[\frac{\left(\frac{37}{s}e^{\sqrt{\frac{s+\gamma-\lambda}{\alpha bs^{1-\zeta}(1+\frac{4}{3}R)}}}-\frac{45}{s}\right)-\frac{(\beta+\gamma)\left(e^{\sqrt{\frac{s+\gamma-\lambda}{abs^{1-\zeta}(1+\frac{4}{3}R)}}}-1\right)}{s+\gamma-\lambda}}{2\sinh\left(\sqrt{\frac{s+\gamma-\lambda}{abs^{1-\zeta}(1+\frac{4}{3}R)}}\right)}\right]\times e^{-y\sqrt{\frac{s+\gamma-\lambda}{\alpha bs^{1-\zeta}(1+\frac{4}{3}R)}}}+\frac{\beta+\gamma}{s+\gamma-\lambda} \tag{20}$$

4.1 Solution of temperature field with Atangana-Baleanu derivative

The general solution of temperature modeled with Atangana-Baleanu following the Ref. [11] by applying the Laplace transform technique.

$$\bar{\theta}(y,s)=\left[\frac{37}{s}-\frac{\beta+\gamma}{X(\zeta)s+\gamma-\lambda}-\frac{\left(\frac{37}{s}e^{\frac{1}{\sqrt{abs^{1-\zeta}(1+\frac{4}{3}R)}}\sqrt{X(\zeta)+\gamma-\lambda}}-\frac{45}{s}\right)-\frac{(\beta+\gamma)\left(e^{\sqrt{\frac{s+\gamma-\lambda}{abs^{1-\zeta}(1+\frac{4}{3}R)}}}-1\right)}{X(\zeta)+\gamma-\lambda}}{2\sinh\left(\sqrt{\frac{X(\zeta)+\gamma-\lambda}{abs^{1-\zeta}(1+\frac{4}{3}R)}}\right)}\right]\times e^{\frac{y}{\sqrt{abs^{1-\zeta}(1+\frac{4}{3}R)}}\sqrt{X(\zeta)+\gamma-\lambda}}$$

$$+\left[\frac{\left(\frac{37}{s}e^{\frac{1}{\sqrt{abs^{1-\zeta}(1+\frac{4}{3}R)}}\sqrt{X(\zeta)+\gamma-\lambda}}-\frac{45}{s}\right)-\frac{(\beta+\gamma)\left(e^{\sqrt{\frac{X(\zeta)+\gamma-\lambda}{abs^{1-\zeta}(1+\frac{4}{3}R)}}}-1\right)}{X(\zeta)+\gamma-\lambda}}{2\sinh\left(\sqrt{\frac{X(\zeta)+\gamma-\lambda}{abs^{1-\zeta}(1+\frac{4}{3}R)}}\right)}\right]\times e^{\frac{-y}{\sqrt{abs^{1-\zeta}(1+\frac{4}{3}R)}}\sqrt{X(\zeta)+\gamma-\lambda}}+\frac{\beta+\gamma}{X(\zeta)+\gamma-\lambda}. \tag{21}$$

where, $X(\zeta)=\frac{s^{\zeta}}{s^{\zeta}(1-\zeta)+\zeta}$.

Note: The inverse Laplace transform of Eq. (20) and (21) can be found numerically.

5. RESULTS AND DISCUSSIN

It is presented to see the impact of fractional parameter on temperature for fixing other parameters. It is found that for greater values of non-integer parameter, temperature increases for both fractional models. The steady-state time required is small to reach the targeted temperature of (C) is less than (ABC) while in comparison sense the temperature of (ABC) is smaller than (C). Since the kernel of Caputo fractional derivative is a power law singular kernel at the end point and has bad memory while the newly introduced fractional derivative operator named as Atangana-Baleanu has non-local and non-singular kernel has excellent memory effect. The reason is that the Mittag-Leffler used in Atangana-Baleanu differentiation, appears naturally in several physical problems as generalized exponential decay and as power law asymptotic for a very large time. The Mittag-Leffler is essential for description of long-time behavior. As a whole,the Mittag-Leffler displayed the exponential decay memory and displayed power-law memory for different values of fractional parameter. Precisely, Mittag-Leffler memory is the excellent memory. As a result we can say that ABC is well suiting in exiting the memory effect of temperature function.

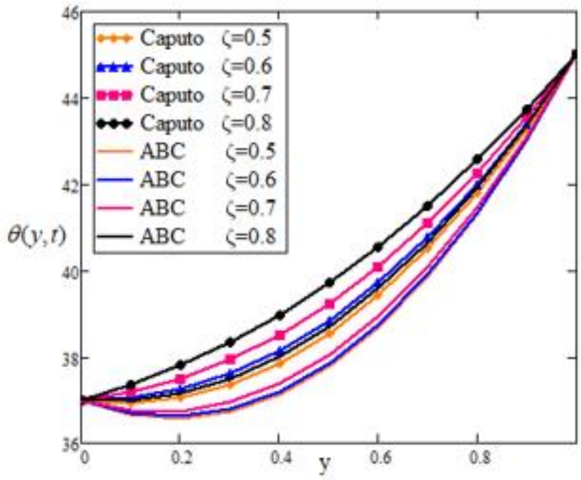

Figure 1. Effect of fractional parameter ζ on temperature profile during hyperthermia treatment of tumor when t = 0.6, α = 0.6, β = 0.4, λ = 0.0001, R = 0.8 and γ = 0.001

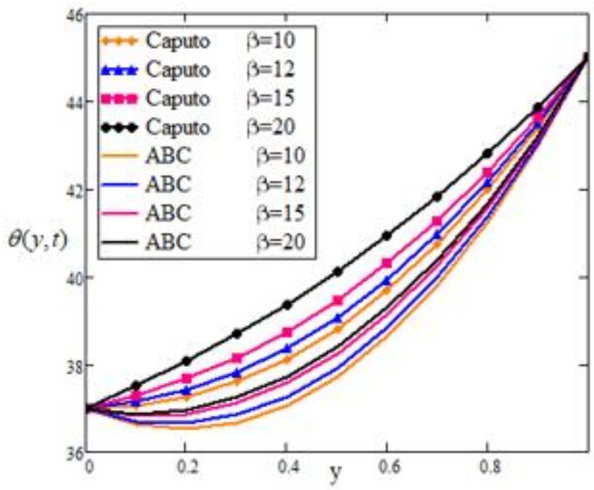

Figure 2. Effect of porosity terms β on temperature profile when t = 0.3, ζ = 0.6, α = 1.1, λ = 0.6, R = 0.1 and γ = 0.001

It depicts the response of porosity parameters β on the temperature profiles. It is observed that the temperature profiles rises as the porosity parameter increases at λ = 0.6 minimum. The reason for this behaviour is that the porosity term provides an additional support to the blood flow mechanism through the porous tissue which causes the blood cell to move at an accelerating rate.

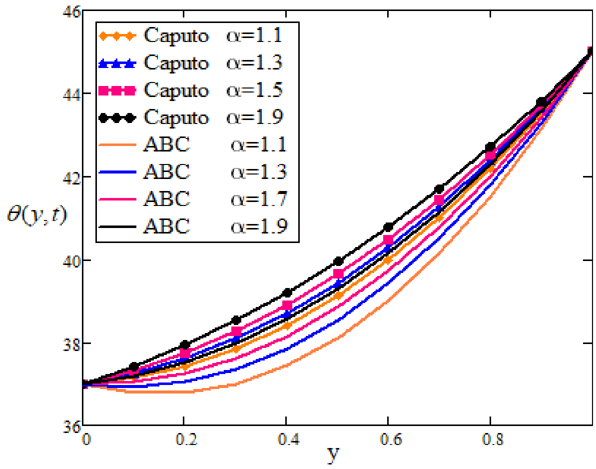

Figure 3. Effect of thermal conductivity α on temperature when t = 0.6, ζ = 0.6, β = 0.5, λ = 0.5, R = 0.5 and γ = 0.4

It shows the influence of variation in values of the thermal conductivity on the temperature distributions. It is observed that an increase in the thermal conductivity value increases the temperature profiles when λ = 0.5. This is because an increase in α causes a rise in the tissue layers thickness when λ = 0.5 and thereby increase the average temperature within the tissue layers. For $\lambda \geq 2$, the tissue layers get thinner and heat is able to diffuse away from the heated surface.

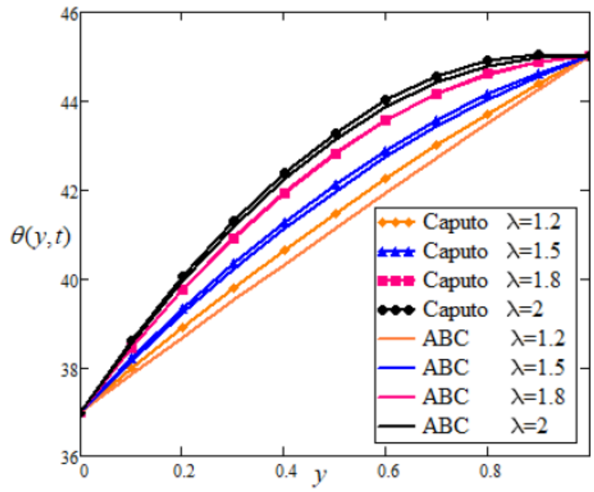

Figure 4. Effect of larger value of heat source on temperature profile

Almost all metallic energy used in these processes is converted into heat within the biological tissue. Metabolic heat generation in the tissue is used for active transport via membrane pumps, energy requiring for chemical reaction, such as muscular work, formation of the glycogen from glucose and proteins from amino acid. Physically, heat source used as active transport agent, energy requiring for the chemical reaction. It's clear from the figure that by taking small values of metabolic temperature is linear in nature. By increasing the values of λ increases the temperature profiles at α = 2 and γ = 0.1 which results in the death of cancerous cells.

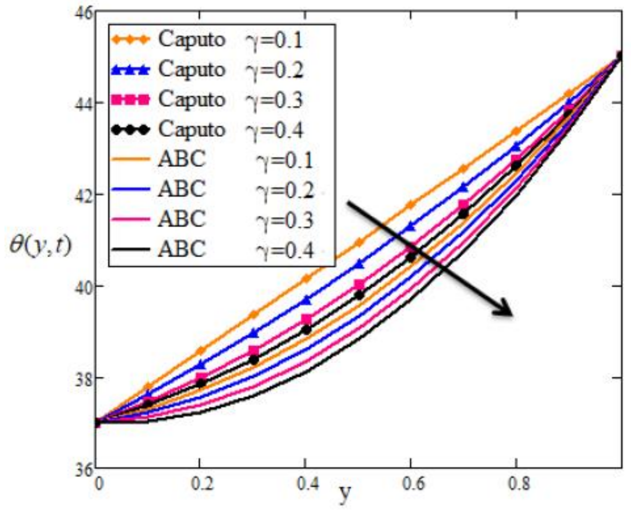

Figure 5. Effect of blood perfusion rate parameter γ on temperature profile when t = 2, ζ = 0.2, β = 0.1, λ = 0.5, R = 0.1 and α = 1.4

It depicts the variation in the temperature profile in relation to the blood perfusion parameter. It is linear in nature especially when λ = 0.5 and maximum temperature is obtained as increases. However, it simply means increase in the blood perfusion rate causes the decrease in the temperature of the biological tissue at λ = 0.5. It is further shown that in areas where convection by blood perfusion is dominant, tissue temperatures are almost uniform except for narrow regions near the boundaries.

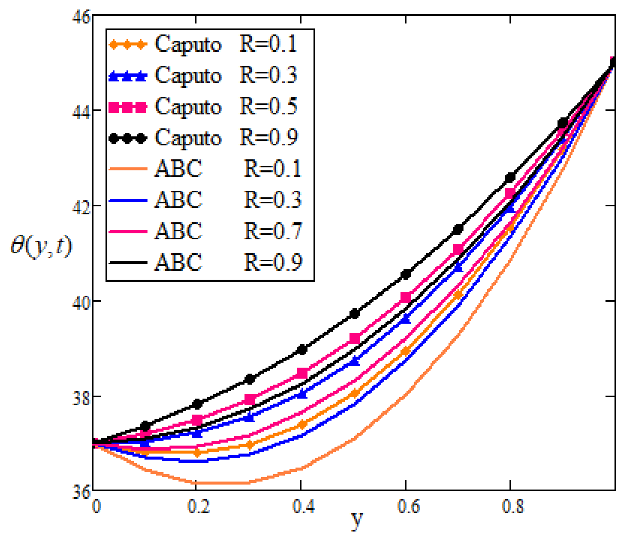

Figure 6. The effect of radiation parameter R on the temperature profiles

As radiation may affect the skin or change the structure of the cell that's why we have assumed that radiation was considered only at the skin surface and is negligible everywhere else. It is noticed that the figure has minimum values, whenever there is increase in heat source, the rate at which temperature increases become smaller. It is seen that as the value of R increases, there is corresponding increase in the temperature profiles at and this due to an increase in the rate at which heat penetrate tumour cell as a result.

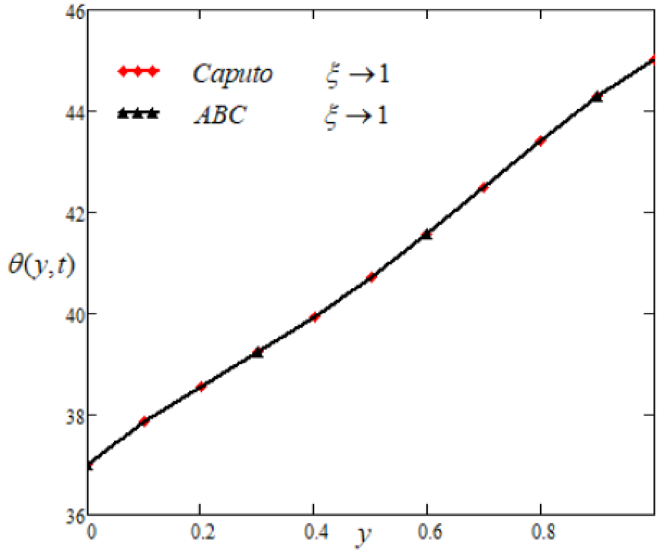

Figure 7. Validation and comparison between the solutions of the temperatures when fractional parameter $\xi \to 1$

Table 1. Values of physical parameters of biological tissue in finite domain

Parameter	Symbol	Value
Blood specific heat at constant pressure	C_p	3500
Blood density	ρ_b	1060
Blood perfusion rate	ω_b	0.00125
Blood thermal conductivity	K_b	0.24
Local tissue temperature	T_o	37^o
Arterial temperature	T_b	45^o

6. CONCLUSIONS

This paper deals with the Pennes bioheat model used for cancer treatment without damaging the surrounding tissues by moderate hyperthermia (Radiative microwave heating). Experts should embrace it due to the advantage that it is affordable and important in clinical or medical practice. The main outcomes are of the present study as follows

- For larger values fractional parameter ζ, heat source λ and radiative heat flux significantly affected the temperature profile at the target area of breast cancer during treatment in the control and prediction of temperature at targeted region in treatment process of breast cancer cells.
- Increasing the values of metabolic heat source parameter, temperature also increases.
- The high temperature at the outer surface that is being controlled by cooling pad due to the effect of boundary conditions.
- The temperature modeled with Caputo is higher and

linear than Atangana-Baleanu but in the other sense that Atanagan-Baleanu model of energy equation exhibits better memory of the function.

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NOMENCLATURE

To	reference temperature
Qm	Heat source due to metabolic heat generation in tissue (Wm^{-3})
q	Radiative heat flux (Wm^{-2})
y	Space coordinate (m)
t	Time (s)

Greek symbols

α	Blood thermal diffusivity, m^2. s^{-1}
β	Prosity parmeter
γ	Boold perfusion rate term
λ	Heat source term
ξ	Fractional parameter