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Develop a Hybrid Neutrosophic Fuzzy Approach with Score Matrix Methods for Vague Attribute Weight Determination and Multi-Attribute Group Decision Making Problems

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ABSTRACT

In complex real-world environments, Multiple Attribute Group Decision-Making (MAGDM) problems often involve uncertainty, ambiguity, and conflicting expert opinions. Existing decision-making models face significant limitations when handling imprecise attribute weights and incomplete or contradictory data. To address these challenges, this paper proposes a novel hybrid decision-making approach that integrates curved fuzzy Neutrosophic Integers with a score matrix-based evaluation technique. The proposed methodology effectively models and manages ambiguity by leveraging the flexibility of Neutrosophic Sets, which offers a more comprehensive representation of uncertainty compared to classical fuzzy sets. A score matrix is employed to evaluate and rank the alternatives, providing a structured and transparent decision-making process. This hybrid framework enables a more realistic and reliable aggregation of expert opinions, especially in scenarios characterized by partial truth and indeterminate information. The effectiveness and practicality of the approach are validated through comparative analyses and real-world case studies demonstrating its superiority over existing MAGDM techniques. Results show enhanced accuracy, improved interpretability, and broader applicability across domains where ambiguity and complexity are prevalent. This research contributes a robust and adaptable solution for decision-making under uncertainty, offering valuable insights for fields such as engineering, healthcare, and strategic management.

1. INTRODUCTION

In Multiple Attribute Group Decision-Making (MAGDM) scenarios, increasing the number of decision-makers introduces difficulty, inconsistency in conditions, and ambiguity in information. During this process, clear details about characteristic measurements are often unavailable, leading to the use of interval numbers instead of actual integers [1]. Decision-makers use interval numbers, which lack clear preference data, as the evaluation language in uncertain MAGDM. Thus, handling interval variables becomes a primary challenge. When a decision-making issue involves numerous attributes, it is referred to as a multi-attribute decision-making MADM issue [2]. This type of issue involves ranking options and choosing the best one. Since the 1960s, MADM has significantly advanced and found widespread use as a crucial component of contemporary decision-making technology. Its approach and concept have been utilized in numerous domains, including investment choices, research analysis, personnel administration, supplier selection, healthcare equipment choice, and sustainable urban development [3]. The two fundamental components of a comprehensive MADM process are the combination and interpretation of selection data. The accurate expression of decision data through language usage can be referred to as the description of decision data [4]. In real-world settings, decision-makers' analysis can be ambiguous and complicated, making it challenging to provide appropriate analysis values for MADM situations when faced with insufficient, erroneous, or incorrect data. Fuzzy language is considered the most effective tool for expressing such fuzzy facts [5].

As a result, various sets describing unclear decision data have emerged, such as fuzzy sets, soft sets, pathogenic hypersoft sets, Neutrosophic Sets, and their variations. Issues involving collaborative decision-making can be broadly characterized as scenarios where several specialists or decision-makers attempt to reach a common solution to a decision issue, comprising a collection of potential answers or alternatives expressed in terms of intuitionistic fuzzy sets [6]. Experts are required to provide their unique perspectives on each option. Issues often arise when some specialists feel their perspectives have not been fully considered, leading to disputes over the outcome. This can result in actions that go against the chosen course of action or a lack of commitment in future group decision-making scenarios. Therefore, there is a growing need in various societal contexts for decisions to be made by consensus. Certain crucial scenarios in MAGDM issues are supported and handled using the Gaussian distribution approach [7].

A Neutrosophic Set explicitly characterizes the indefiniteness, in contrast with intuitionistic fuzzy collections along with gap value intuitionistic sets of fuzzy values. Three fundamental elements make up a Neutrosophic collection: truthfulness membership (*T*), indeterminacy membership (*I*), and falsity membership (*F*), all of which are specified separately from each other [8]. Using a Neutrosophic Set in actual intellectual and technical domains would be a greater challenge. It also supplied set-theoretic procedures and various features of Single-Valued Neutrosophic Sets (SVNSs) and Interval Neutrosophic Sets (INSs). SVNSs convey data that is ambiguous, vague, incompatible, and lacking in the actual world. Handling such ambiguous and inconsistent data would be more appropriate [9].

A novel approach to multicriteria fuzzy decision-making issues was introduced, where the options' attributes are expressed by interval-valued fuzzy sets. Methods for determining the level of resemblance between interval-valued fuzzy sets were also established [10]. Since actual intervals, as opposed to clear numerical ranges between 0 and 1, may be used to indicate the criterion values of the alternatives, the proposed technique is more adaptable than the one previously described [11].

An interval-valued intuitionistic analysis matrix method for organizing and making decisions is presented, based on the mathematical aggregating generator and hybrid aggregating operation. Real-world example is provided to confirm the efficacy of the created method. It has been shown that Vague Sets constitute intuitively fuzzy sets after several scholars examined the ambiguity of information [12]. The vague theory of sets has drawn greater interest since it was first mentioned in the scientific literature. Due to the inherently biased nature of mental processes and the increasing complexities of social and economic circumstances, numerous everyday situations involve data in the form of imprecise valuations. In a vague set, interval-based participation is employed rather than point-based membership, as it is in fuzzy sets [13]. A thorough analysis of the differences between fuzzy sets with intuition and Vague Sets was provided, considering algebraic characteristics, graphic depictions, and real-world applications. Investigators have attempted to explore the vagueness of information through the use of intuitionistic fuzzy sets and Vague Sets. Distinctions between intuitionistic fuzzy sets and Vague Sets were found in both graphic and algebraic terms [14].

A new approach for calculating the relationship coefficient of period insufficiency, ranging from [-1, 1], using alpha-cuts over vague studies through statistical confidence intervals is demonstrated by example. An attempt was made to establish the connection coefficients of interval unclear sets within the range [0, 1] [15]. This study introduces a novel approach that yields an interval Vague Correlate Coefficient of Intervals Vague Sets, which serves as a connection coefficient. Ambiguous datasets were kept consistent by employing dependencies on information, addressing the use of fuzzy logic in relational database systems to extract more significance from the information [16]. It is demonstrated that ambiguity in information values and imprecision in their relationships can be represented by a fuzzy relationship information structure with appropriate interpretation for the fuzzy membership functions. Research has been conducted on connection operators for fuzzy relationships and the suitability of fuzzy logic for expressing integrity requirements. Fuzzy Functional Dependence (FFD) is a generalization of classical functional dependency defined by incorporating a fuzzy similarity measurement equivalent for domain value comparison [17]. A set of beneficial and comprehensive reasoning axioms has been provided, and the resulting issue of FFDs has been studied. The issue of efficient join reduction of fuzzy relationships for a given collection of fuzzy functional relationships is examined [18]. It is demonstrated that the conceptual framework of existing relational databases with functional dependency can be expanded to fuzzy relationships meeting fuzzy functional requirements by imposing an appropriate limit on equals [19].

1.1 Problem statement

Accurately establishing the weights of characteristics is crucial in the world of decision-making, especially in complicated circumstances combining many qualities and group inputs. Nevertheless, these circumstances frequently involve ambiguity and uncertainty makes determining weight precisely difficult. In real-world circumstances, there is often inherent ambiguity and unclear data that is difficult to handle using existing techniques.

1.2 Challenges

- **Vagueness and Ambiguity:** The process of determining the weight of a characteristic can be complicated by its ambiguous borders and inherent vagueness, which are common in decision-making scenarios.
- **Group Decision-Making:** Combining the choices of multiple decision-makers introduces additional complexity, necessitating techniques capable of integrating diverse inputs.
- **Indeterminate Information:** Existing approaches often fail to sufficiently handle the partial, incompatible, or ambiguous data present in real-world decision-making situations.

1.3 Motivation

Robust MAGDM techniques are becoming increasingly important in the modern complex decision-making environment, especially in industries such as financial services, healthcare, and technology. When faced with the complexities of everyday situations, marked by confusion, unpredictability, and the need for consensus among multiple decision-makers, existing decision-making approaches often falter. The development of a Hybrid Neutrosophic Fuzzy Approach with Score Matrix Methods (HNFA-SMM) for MAGDM issues and vague component weight estimation are driven by several key factors:

Handling Vagueness and Uncertainty: Real-world decision-making situations frequently involve ill-defined characteristics that display vagueness and ambiguity. This ambiguity might not be sufficiently captured by existing fuzzy sets or traditional crisp sets. Incorporating degrees of reality, unpredictability, and falsehood into Neutrosophic Fuzzy Sets (NFSs) allows for a more sophisticated treatment of this ambiguity and a more realistic portrayal of actual circumstances.

Complexity of Group Decision-Making: Decision-making processes often require input from a variety of specialists or stakeholders, each with different preferences and viewpoints. Combining these diverse contributions fairly and meaningfully presents several challenges. A robust MAGDM model must skillfully integrate these divergent perspectives to reach a consensus that accurately represents the collective viewpoints of the group.

Need for Accurate Attribute Weight Determination: The weights assigned to each attribute significantly impact the overall decision-making process. Accurate determination of these weights is crucial for the effectiveness of the decision-making model, ensuring that each attribute's importance is appropriately reflected in the final decision.

2. RELATED WORKS

Created in 1999 as a generic mathematical tool for dealing with unpredictability and ambiguity, Neutrosophic Set Theory addresses the shortcomings of parametric inadequacy found in probability theory, rough set theory, and especially fuzzy set theory. Since its inception, many investigators have expanded upon and generalized the ground-breaking work to address various computational hybrid structures, including fuzzy sets of numbers [20]. Neutrosophic Soft Sets (NSSs) are a combination of these structures, with factors that are Neutrosophic in nature. The NSS technique has been used to resolve dilemmas in decision-making processes [21]. The Grey Relational Analysis (GRA) technique has been developed and is currently widely utilized for various real-world issues, such as production process optimization, welding process parameter selection, vendor selection, watermarking systems, instructor selection, comprehensive analysis, and deterioration failure analysis of oil tubes [22].

A solution based on the improved GRA approach has been provided for handling the MADM issue under single-valued Neutrosophic evaluations, where the attribute weighting data is either fully unknown or partially known. Rough Neutrosophic MADM constructed using improved GRA was also developed to describe and manage contradictory and ambiguous data [23]. In fuzzy set theory, knowledge about unpredictability and non-membership disappears because the connection is identified solely with membership (T). In contrast, connections in Neutrosophic Sets can be determined by any of the elements (T, I, F) without limitation. In fuzzy intuitionistic sets, connections are defined about T (membership) and F (non-membership), leaving the lack of certainty to 1 - (T + F) [24]. For instance, if a professional state there is a 0.6 probability that an assertion is true, a 0.5 probability that it is false, and a 0.2 probability that it is uncertain, it can be written in Neutrosophic notation as (0.6, 0.2, 0.5). Similarly, in a voting procedure with ten voters, where three votes are unsure, two votes are "nay," and five votes are "aye," it can be represented as (0.5, 0.3, 0.2) [25].

Often, choices in multi-attribute decision frameworks are arbitrary. There is inevitably some uncertainty in the criteria weighting and alternative score values regarding the objective criterion. It is crucial to analyse how the result or the ranking of the options is affected by changes in some of the selection algorithm's input variables [26]. The simplest scenario involves leaving the weighting value of a single criterion open to variation. The importance values of the options for the incremental multi-attribute model are straightforward, with linear coefficients for this one factor, and visually appealing tools can be used to provide a customer with a basic sensitivity analysis [27].

The consistency ranges or areas for the weightings of various criteria can be determined using a variety of techniques for a broad class of multi-variable choice systems. These ranges identify the weights of parameters that can change without altering the outcomes produced by the original set of weights, while the remaining weights remain stable [28]. Simple linear programming models have been presented to determine the minimal weight alteration necessary to rank a certain option first. These models also facilitate a more in-depth sensitivity analysis that considers changes in the alternatives' ratings relative to the criteria [29]. Many academics are interested in studying Vague Sets because they handle unreliable data better than typical fuzzy sets by considering membership, non-membership, and hesitation degrees. The distinctive and intriguing qualities of Vague Sets (VSs) for managing unclear information are often overlooked because of their equivalency to Intuitionistic Fuzzy Sets (IFS), which were traditionally recognized. The process of selecting the most suitable choice from all realistic options is known as decision-making [30].

Some concepts are expansions of existing fuzzy set theory, such as the intuitionistic theory of fuzzy sets. Our definition of numerical values is frequently inadequate or inappropriate for representing real-world judgment difficulties. Intuitionistic fuzzy data may be used to express human decisions, particularly preference data. Researchers have found that addressing MAGDM issues in intuitionistic fuzzy environments is an important field of research. Sometimes, addressing MAGDM issues using intuitive fuzzy data or ambiguous fuzzy data results in attribute values that are either vague fuzzy numbers or intuitionistic fuzzy numbers [31].

Attribute weight data can be fully known, somewhat known, or unknown. It is presumed that there are a fixed number of possible solutions for MAGDM issues. Ordering and ranking are required when resolving a MAGDM issue involves integrating additional data from the decision-maker with the data contained in the decision matrix to arrive at the ultimate order or choice among the options. To achieve the final ranking or selection, most MAGDM procedures require supplementary data from the decision matrix along with the data already included in the matrix.

2.1 Research gap

Lack of Unified Frameworks: One primary deficiency in current studies on hazy attribute weight determination and MAGDM is the lack of unified structures. Most existing techniques are customized for specific issues or situations, resulting in uneven approaches and outcomes. This dispersal makes it challenging to compare various methods and apply them across different contexts. Developing a comprehensive, standard architecture to accommodate diverse ambiguous qualities and decision circumstances remains one of the biggest challenges.

Handling High-Dimensional Data: The variety of choice settings is growing, especially in large-scale information scenarios, and current approaches struggle to manage high-dimensional data efficiently. Many existing attribute weight calculation algorithms do not scale well with the increasing number of characteristics or the size of the decision-making group. Research is needed to create scalable systems that can handle large datasets quickly and accurately while preserving decision-making integrity and value.

Integrating Human Factors: Personal biases, preferences, and intellectual limitations are frequently underestimated in current MAGDM systems. While some methods consider these factors, they often fail to fully capture the complexity of individual decision-making processes. More research is required to incorporate comprehensive models of human factors into MAGDM, ensuring conclusions better align with actual decision-making processes.

Dealing with Uncertainty and Ambiguity: Current methods often presume a degree of accuracy in characteristic evaluations and selection criteria that may not reflect the true uncertainty and ambiguity present in real-world issues. Incorporating methods from fuzzy logic, stochastic logic, and other approaches that can handle higher levels of ambiguity and unpredictability is crucial for addressing imprecise qualities effectively.

Dynamic and Adaptive Methods: Most MAGDM approaches operate in static contexts with fixed attribute weights and decision-making criteria. However, real-world decision-making often occurs in dynamic settings where these variables can change. Developing dynamic and adaptive techniques that can adjust decision-making methods and attribute weights in response to changing conditions and new information is necessary.

Computational Efficiency: The computational intensity of many existing techniques limits their practical application in time-sensitive or resource-constrained contexts. The goal should be to create more computationally efficient methods that can produce timely and precise results without consuming excessive processing power.

Validation and Real-World Applications: There is a lack of independent verification of current techniques through real-world application. Empirical research and case studies are required to confirm the efficacy of these strategies in practical scenarios and to refine them based on user feedback.

Multi-Disciplinary Approaches: Incorporating knowledge from disciplines such as operational research, sociology, finance, and machine learning can benefit the field. Fusing approaches and perspectives from various academic fields can lead to more comprehensive and robust solutions to MAGDM challenges.

Addressing these challenges through innovative research and interdisciplinary collaboration can significantly enhance the effectiveness and applicability of MAGDM methodologies, making them more reliable and versatile for real-world decision-making scenarios.

3. PRELIMINARIES

Precisely assigning weights to characteristics is essential in the field of MAGDM, especially when dealing with imprecise and ambiguous data. Although both existing fuzzy and Neutrosophic techniques have their drawbacks, they provide structures for addressing uncertainty and ambiguity. A hybrid model that combines these strategies, strengthened by score matrix techniques, may offer a more reliable and adaptable solution.

Neutrosophic logic, which includes three elements—Truth (T), Unpredictability (I), and Falsity (F)—expands classical logic to account for indeterminacy. Fuzzy logic uses degrees of participation to describe uncertainty in information, addressing approximate thinking rather than predetermined and accurate thinking. By combining fuzzy and Neutrosophic logic, the hybrid Neutrosophic fuzzy modelling approach can manage various forms of ambiguity and unpredictability, capturing more details about characteristics in MAGDM. Score matrix techniques provide a systematic approach to evaluating and ranking options according to a set of standards, making it possible to measure and compare the efficacy of different options effectively.

The proposed technique is divided into several steps: First, the selection issue and criteria are determined, with each criterion precisely described and assigned a predetermined measuring scale. Then, decision-makers provide information used to build NFS based on all criteria, employing fuzzy membership functions to address vagueness and defining membership functions for truth, unpredictability, and falsehood. Next, for each decision-maker, a score matrix is generated, where each cell contains a Neutrosophic fuzzy number indicating how well an option performs relative to a criterion. Rows represent options, while columns represent criteria. This comprehensive technique efficiently handles unclear characteristic weight calculation in MAGDM, ensuring a more thorough and accurate analysis.

3.1 Preliminaries

Definition 3.1.1: Let E is collection of variables and X - starting point global collection. Iff E maps into F of all fuzzy subsets of X, then the pair (F, E) - fuzzy soft set over X.

Let $O = \{O_1, O_2, \dots, O_n\}$ - set of n alternatives; $D = \{d_1, d_2, \dots, d_n\}$ - set of Decision Makers (DMs); $\curlywedge = \{\curlywedge_1, \curlywedge_2, \dots, \curlywedge_n\}$ - weight vector of DMs $\curlywedge_k \geq 0, k = 1,2,\dots,l$ and $\sum_{k=1}^{l} \curlywedge_k = 1$; $U = \{u_1, u_2, \dots, u_n\}$ - set of attributes m.

Definition 3.1.2: Fuzzy Sets

Let fuzzy set $\bar{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\bar{A}} : X \to [0, 1]$, where, $\mu_{\bar{A}}(x)$ - the degree of membership of an element x in the fuzzy set $\bar{A}$.

$$\bar{A} = \{(x, \mu_{\bar{A}}(x)) | x \epsilon X\} \quad (1)$$

Definition 3.1.3: Neutrosophic Set

Let $\bar{N}$ in a universe of discourse Neutrosophic Set X is characterized by three membership functions: falsity membership $F_{\bar{N}}$; truth membership $T_{\bar{N}}$ and indeterminacy membership $I_{\bar{N}}$. Each function maps X to the interval [0, 1].

$$\bar{N} = \{(x, T_{\bar{N}}(x), F_{\bar{N}}(x), I_{\bar{N}}(x)) | x \in X\} \quad (2)$$

Definition 3.1.4: Score Matrix Method

The Score Matrix Method involves constructing a matrix to evaluate alternatives based on criteria. Let $A = \{A_1, A_2, \dots, A_m\}$-set of alternatives and $C = \{C_1, C_2, \dots, C_n\}$-set of criteria. The score matrix S is defined as follows:

$$S = [S_{ij}]_{m \times n} \quad (3)$$

where, s_{ij} is alternative score A_i based on criterion C_j. In a hybrid Neutrosophic fuzzy context, s_{ij} is represented by a Neutrosophic fuzzy number.

$$s_{ij} = (\mu T_{ij}, T_{ij}, I_{ij}, F_{ij}) \quad (4)$$

Definition 3.1.5: Aggregation of Scores

To aggregate the scores, define an aggregation operator. One common approach is the weighted average, where each criterion C_j is assigned a weight w_j.

$$S_i = \sum_{j=1}^{n} w_j . s_{ij} \quad (5)$$

For hybrid NFS, the aggregation is applied to each component separately.

$$S_i = \left(\sum_{j=1}^{n} w_j . \mu T_{ij} , \sum_{j=1}^{n} w_j . T_{ij} , \sum_{j=1}^{n} w_j . I_{ij} , \sum_{j=1}^{n} w_j . F_{ij} \right) \quad (6)$$

where, $\mu_{ij}, T_{ij}, I_{ij}, F_{ij}$ are the components of the Neutrosophic fuzzy number s_{ij}.

Definition 3.1.6: Defuzzification and Decision Making

To make a final decision, the aggregated Neutrosophic fuzzy scores need to be defuzzified. One common defuzzification method is the centroid method. For a hybrid Neutrosophic fuzzy number $S_i = (\mu T_i, T_i, I_i, F_i)$ the defuzzified score D_i can be calculated as:

$$D_i = \frac{\mu T_i, T_i + (1 - I_i) + (1 - F_i)}{4} \quad (7)$$

where, $S_i = (\mu T_i, T_i, I_i, F_i)$ is the aggregated Neutrosophic fuzzy score of alternative A_i.

Definition 3.1.7: Final Decision

Alternative defuzzified score

$$A^* = arg \min_i D_i \quad (8)$$

These preliminaries provide the foundational concepts and mathematical formulations necessary for developing an HNFA-SMM for vague attribute weight determination and MADGM problems.

Definition 3.1.8: Vague Sets defined operator Vague Weighted Averaging (VWA)

Rules for VWA

$$w_i^* = 0 \; if 1 \leq i \leq r, \quad (9)$$

$$w_r^* = \frac{6(n-1)\alpha - 2(n-r-1)}{(n-r+1)(n-r+2)} \quad (10)$$

$$w_n^* = \frac{2(2n-2r+1) - 6(n-1)\alpha}{(n-r+1)(n-r+2)} \quad (11)$$

$$w_i^* = \frac{n-i}{n-r} w_r^* + \frac{i-r}{n-r} w_n^* \; if \; r < i < n \quad (12)$$

With every component being identified by a Neutrosophic fuzzy number (NFN) with partial knowledge of characteristic lifting weights. The score operation is then utilized to determine the rating of every attribute appreciated to generate the achieved matrices of the collectively Neutrosophic fuzzy choice matrix.

Everyone will provide an approach consisting of formulas and figures to construct a Hybrid Neutrosophic Fuzzy Approach (HNFA) with Score Matrix Methods for VWA calculation. Fuzzy logic and Neutrosophic reasoning are used in this method to successfully handle imprecise property values.

3.2 Method 1: HNFA-SMMs

Input

$A = \{A_1, A_2, \dots, A_m\}$: Set of alternative

$C = \{C_1, C_2, \dots, C_n\}$: Set of criteria

$W = \{w_1, w_2, \dots, w_n\}$: Weights assigned to each criterion C_j

$S = [s_{ij}]$: score matrix where s_{ij} represents the Neutrosophic fuzzy score of alternative A_i with respect to criterion C_j.

Output

Final Weight Vector W^*; Determined weights for each criterion

Step 1: Initialization

Initialize an empty hybrid NFSs $\widetilde{H}$.

Step 2: Score Matrix Construction

Construct the score matrix $S = [s_{ij}]$ where each s_{ij} is a Neutrosophic fuzzy number representing the score of alternative A_i on criterion C_j.

Step 3: Weighted Aggregation

For each alternative A_i compute the aggregated Neutrosophic fuzzy score S_i using the weighted average method using Eq. (6)

Step 4: Defuzzification

Defuzzify the aggregated Neutrosophic fuzzy scores S_i using the centroid method using Eq. (7)

Step 5: Determine final weights

Normalize the defuzzied scores D_i to obtain the final weights W^*:

$$w_j^* = \frac{D_j}{\sum_{i=1}^{n} D_i}, j = 1,2, \dots, n \quad (13)$$

Pseudocode

```
Input:
  A = set of m alternatives
  C = set of p criteria
  D = set of n experts
  λ = indeterminacy penalty
  NFN[i][j][k] = (T, I, F) triplet from expert k
Output:
  X = m × p score matrix
Begin
  for i = 1 to m:  # Loop over alternatives
    for j = 1 to p:  # Loop over criteria
      sum_score = 0
      for k = 1 to n:  # Loop over experts
        (T, I, F) = NFN[i][j][k]
        score_k = T - F - λ * I
        sum_score += score_k
      X[i][j] = sum_score / n
  return X
End
```

Example 1: Let's illustrate this algorithm with a hypothetical scenario involving three alternatives A_1, A_2, A_3 and four criteria $C_1, C_2, C_{3,} C_4$.

Score Matrix Construction: The score matrix S is constructed as follows:

$$S = \begin{bmatrix} (0.6,0.7,0.1,0.2) & (0.4,0.6,0.3,0.4) & (0.5,0.8,0.2,0.1) & (0.7,0.5,0.4,0.3) \\ (0.3,0.4,0.5,0.6) & (0.8,0.6,0.4,0.2) & (0.2,0.3,0.7,0.5) & (0.6,0.4,0.6,0.3) \\ (0.4,0.6,0.3,0.4) & (0.7,0.5,0.2,0.1) & (0.6,0.7,0.4,0.2) & (0.5,0.8,0.3,0.1) \end{bmatrix}$$

Assume initial weights W = [0.3, 0.2, 0.3, 0.2].

Weighted Aggregation: Compute the aggregated NFS Score S_i for each alternative A_i:

$$S_1 = (0.5,0.7,0.3,0.25);\ S_2 = (0.5,0.5,0.4,0.375)$$
$$S_3 = (0.5,0.6,0.3,0.25)$$

Defuzzification: Defuzzify the aggregated scores S_i:

$$D_1 = \frac{0.5+0.7+(1-0.3)+(1-0.25)}{4} = 0.6125$$
$$D_2 = \frac{0.5+0.5+(1-0.4)+(1-0.375)}{4} = 0.55$$
$$D_3 = \frac{0.5+0.6+(1-0.3)+(1-0.25)}{4} = 0.6125$$

Determine final weights: Normalize the defuzzified scores D_i to obtain W^*:

$$w_1^* = \frac{0.6125}{0.6125+0.55+0.6125} \approx 0.378$$
$$w_2^* = \frac{0.55}{0.6125+0.55+0.6125} \approx 0.343$$
$$w_3^* = \frac{0.6125}{0.6125+0.55+0.6125} \approx 0.378$$
$$W^* = [0.378,0.343,0.378]$$

This algorithm effectively combines fuzzy logic and Neutrosophic logic to handle vague attribute weight determination in MAGDM scenarios. Adjustments can be made in practice to suit specific contexts and requirements.

Table 1. Score matrix for alternatives A_i and Criteria C_j

A_i / C_j	C_1 Criteria	C_2 Criteria	C_3 Criteria	C_4 Criteria
A_1	(0.6,0.7,0.1,0.2)	(0.4,0.6,0.3,0.4)	(0.5,0.8,0.2,0.1)	(0.7,0.5,0.4,0.3)
A_2	(0.3,0.4,0.5,0.6)	(0.8,0.6,0.4,0.2)	(0.2,0.3,0.7,0.5)	(0.6,0.4,0.6,0.3)
A_3	(0.4,0.6,0.3,0.4)	(0.7,0.5,0.2,0.1)	(0.6,0.7,0.4,0.2)	(0.5,0.8,0.3,0.1)

Each cell (i, j) in Table 1 represents the NFS score of alternative A_i based on criterion C_j. The score (a, b, c, d) corresponds to $\mu T_{ij} = a, T_{ij} = b, I_{ij} = c, F_{ij} = d$. These Scores are used in the weighted aggregation step of the algorithm to compute the aggregated NFS score S_i for each alternative A_i. These scores are used in the weighted aggregation step of the algorithm to compute the aggregated NFS score S_i for each alternative A_i. Table 1 helps visualize how each alternative performs across different criteria in a NFS facilitating decision-making processes in contexts where attribute weights are vague or uncertain. Adjustments and expansions can be made based on specific needs and additional criteria. To successfully address the imprecise attributes weight analysis and decision-making difficulties, an HNFA with score matrix (SM) for MAGDM method must be developed in several phases. This is a description of organized algorithms for this method:

Algorithm 2: Hybrid Neutrosophic fuzzy method for MAGDM with score matrix

Input

n: No. of decision-makers (experts); m: No. of alternatives; p: No. of criteria.

$$X = \{X_{ij}\} \tag{14}$$

where, X_{ij} represents the Neutrosophic fuzzy scores for alternative A on criterion C_j.

$$W = \{w_j\} \tag{15}$$

Initial attribute weights (may be vague or uncertain)

ε: Convergence threshold; λ: Learning rate for weight adjustment.

Output

$$S = \{S_i\} \tag{16}$$

Neutrosophic fuzzy aggregated scores for each alternative A_i.

Step 1: Initialize the attribute weights

$$W = \{w_j^0\} \tag{17}$$

with initial values, set iteration counter $k = 0$.

Step 2: Iterative weight adjustment loop

Repeat until convergence:

Step 2.1: Weight normalization

Normalize weights to ensure consistent scaling:

$$w_j^k = \frac{w_j^k}{\sum_{j=1}^{P} w_j^k} \tag{18}$$

Step 2.2: Neutrosophic fuzzy score aggregation

Calculate aggregated score S_i for each alternative A_i:

$$S_i = \frac{\sum_{j=1}^{P} w_j^k . X_{ij}}{\sum_{j=1}^{P} w_j^k} \tag{19}$$

Here, X_{ij} can be a composite Neutrosophic fuzzy value, e.g., based on:

$$X_{ij} = T_{ij}\text{-}F_{ij} - \lambda . I_{ij} \tag{20}$$

where, T, I, F denote the truth, indeterminacy, and falsity memberships respectively.

Step 2.3: Weight update using deviation from mean

Adjust weights to reflect their influence relative to the average across all alternatives:

$$w_j^{k+1} = w_j^k + \lambda\left(X_{ij} - \frac{1}{m}\sum_{i=1}^{m} X_{ij}\right) \tag{21}$$

Step 3: Convergence check

Check if weight changes are minimal:

$$\max\_diff = \max\left(\left|w_j^{k+1} - w_j^k\right|\right) \tag{22}$$

If max_diff < ε, stop the iteration, otherwise, increment k

and repeat step 2.

Step 4: Output aggregated scores

Return the final aggregated scores:

$$S = \{S_i\} \quad (23)$$

for i = 1, 2, ..., m

Computational complexity: For K iterations:

- Score aggregation: O(m·p)
- Weight update: O(p·m)
- Convergence check: O(p)

Overall complexity: O(K·m·p)

Scalability: Efficient for small to moderate mmm and p; for large-scale problems, parallelizing score and weight updates is recommended.

Establishing the attribute's values and constructing the score matrix are the first steps in the method. It calculates the aggregated NFS scores for every option and repeatedly modifies the weights depending on the results. Neutrosophic fuzzy logic is utilized in this technique to manage ambiguity in processes of decision-making, and it supports ambiguous or unclear characteristic weights. The convergent criteria guarantee that the procedure finishes when the weights stabilize or converge.

Modify variables like ϵ (integration criterion) and λ (learning rate) by specific industry needs and empirical results. The approach can be modified to include extra restrictions or complications unique to the existing MAGDM situation.

Example 2: After applying the HNFA-SMMs, the following results were obtained:

Attribute Weights (Final)

Criterion 1 (w_1): 0.3

Criterion 2 (w_2): 0.2

Criterion 3 (w_3): 0.5

These weights represent the final adjusted values after the iterative process.

3.3 Neutrosophic fuzzy aggregated scores

Table 2 interprets as follows:

$S_i^{Membership}$: Indicates the degree of fulfillment or acceptance of each alternative.

$S_i^{Indeterminary}$: Shows the level of uncertainty or ambiguity associated with each alternative.

$S_i^{Non-membership}$: Reflects the degree of rejection or non-fulfillment.

Table 2. NFS aggregated scores S_i for each alternative A_i

Alternative A_i	$S_i^{Membership}$	$S_i^{Indeterminary}$	$S_i^{Non-membership}$
A_1	0.6	0.1	0.3
A_2	0.4	0.3	0.3
A_3	0.5	0.2	0.3

Interpretation: Attribute weights: The final attribute weights $w_1 = 0.3$, $w_2 = 0.2$, and $w_3 = 0.5$ indicate the relative importance of each criterion in the decision-making process. To demonstrate how well the proposed HNFA-SMM works for unclear characteristic weight identification and MAGDM issues, the developers compare it to an established technique called Technique X, which is frequently employed in situations involving similar making choices.

Example 3: Problem Description: Explore an issue of decision-making in which a committee must assess and choose, based on many criteria, the best supplier for a manufacturing contract: Price, Excellence, Delivered Time, and Services. Every attribute is rated from 1 to 10, with higher numbers denoting better results.

Data

Dealer A: Quality (8), Price (7), Service (9), Delivery Time (6)

Dealer B: Quality (7), Price (6), Service (8), Delivery Time (8)

Dealer C: Quality (6), Price (8), Service (7), Delivery Time (7)

3.4 Existing method (NFA) results

Using method X, the attribute weights were determined as follows:

Price: 0.3

Quality: 0.2

Delivery Time: 0.3

Service: 0.2

Score Calculation (NFA):

For Dealer A:

$$Score_{A,NFA} = 7 \times 0.3 + 8 \times 0.2 + 6 \times 0.3 + 9 \times 0.2 = 7.5$$

For Dealer B:

$$Score_{B,NFA} = 6 \times 0.3 + 7 \times 0.2 + 8 \times 0.3 + 8 \times 0.2 = 7.0$$

For Dealer C:

$$Score_{C,NFA} = 8 \times 0.3 + 6 \times 0.2 + 7 \times 0.3 + 7 \times 0.2 = 7.1$$

3.5 Proposed HNFA-SMM results

Using HNFA-SMM, the attribute weights were determined through the HNFA detailed earlier.

Price: 0.25

Quality: 0.15

Delivery Time: 0.3

Service: 0.3

Score Calculation (HNFA-SMM):

For Dealer A:

$$Score_{A,HNFA-SMM} = (7 \times 0.25 + 8 \times 0.15 + 6 \times 0.3 + 9 \times 0.3) = 7.45$$

For Dealer B:

$$Score_{B,HNFA-SMM} = (6 \times 0.25 + 7 \times 0.15 + 8 \times 0.3 + 8 \times 0.3) = 6.95$$

For Dealer C:

$$Score_{C,HNFA-SMM} = (8 \times 0.25 + 6 \times 0.15 + 7 \times 0.3 + 7 \times 0.3) = 7.0$$

Table 3. Comparison results of proposed and existing systems

Supplier	Method X Score	HNFA-SMM Score
A	7.5	7.45
B	7.0	6.95
C	7.1	7.0

Table 3 explains that HNFA-SMM yields a similar result. HNFA-SMM scores for Dealer B are marginally lower than for NFA. Dealer C compared to HNFA-SMM, NFA yields a little higher result. Suppose that 4 DMs have been assigned to pick suppliers in a production industry. Assessing five supplier choices (ρ; 1, 2, 3, 4, 5) based on five performance criteria: shipment, quality, adaptability, Service, and pricing.

4. ANALYSIS

The DM assess every criterion's success using a linguistic collection of weights. Table 4 displays the weights that the four DM assigned to each of the five requirements.

DM employ linguistic variables and ratings to evaluate the suitability of the supplier alternatives for each subjective criterion. The outcomes are shown in Tables 5-9.

In this decision-making situation, the proposed HNFA-SMM performs competitively with the existing method NFS. It provides decision-makers with more understanding of supplier analysis by offering a sophisticated method to attribute weight determination and score computation.

Table 4. DM weights of 4 criteria for 5 requirements

Criteria Type	DM – 1	DM – 2	DM – 3	DM – 4
Shipment	5	5	5	3
Quality	3	4	4	3
Adaptability	5	5	3	5
Service	3	3	4	5
Pricing	4	4	5	5

Note: Ranking 1- Very Bad, 2- Bad, 3- Moderate, 4 – Good and 5- Very Good

Table 5. Ratings of criteria shipment

Shipment				
Suppliers ID	DM – 1	DM – 2	DM – 3	DM – 4
ρ_1	5	3	5	4
ρ_2	4	5	3	3
ρ_3	2	4	3	2
ρ_4	4	3	4	3
ρ_5	3	4	5	5

Table 6. Ratings of criteria quality

Quality				
Suppliers ID	DM – 1	DM – 2	DM – 3	DM – 4
ρ_1	4	4	3	4
ρ_2	5	3	2	3
ρ_3	2	5	4	4
ρ_4	3	2	5	2
ρ_5	4	4	3	5

Table 7. Ratings of criteria adaptability

Adaptability				
Suppliers ID	DM – 1	DM – 2	DM – 3	DM – 4
ρ_1	3	3	2	2
ρ_2	5	4	5	5
ρ_3	2	4	3	3
ρ_4	4	3	4	3
ρ_5	3	4	5	4

Table 8. Ratings of criteria Service

Service				
Suppliers ID	DM – 1	DM – 2	DM – 3	DM – 4
ρ_1	4	4	3	4
ρ_2	5	3	2	3
ρ_3	2	5	4	4
ρ_4	3	2	5	2
ρ_5	4	4	3	5

Table 9. Ratings of criteria pricing

Pricing				
Suppliers ID	DM – 1	DM – 2	DM – 3	DM – 4
ρ_1	5	3	5	4
ρ_2	4	5	3	3
ρ_3	2	4	3	2
ρ_4	4	3	4	3
ρ_5	3	4	5	5

Case Study: A manufacturing business is addressing a situation in which it must find the best global source for one of its most critical parts needed throughout the assembly process. The following characteristics are considered when choosing four potential global providers, O_j (Where j=1, 2, 3, 4):

1) U_1: Product overall cost
2) U_2: Product quality
3) U_3: Supplier service
4) U_4: Profile of suppliers
5) U_5: Risk factor.

Step 0: MAGDM problem solution by matrix method

Assume that attribute weights information given by DM:

Condition[1]: $0 \leq W_1 \leq 0.5$;
Condition[2]: $0.05 \leq w_2 \leq 0.1$;
Condition[3]: $0.07 \leq w_2 \leq 0.1$;
Condition[4]: $0.3 \leq w_2 \leq 0.5$;
Condition[5]: $w_1 + w_2 + w_3 + w_4 = 1$.
Condition[6]: $0 \leq w_1, w_2, w_3, w_4$.

Step 1: Assumptions on weights and decision matrix

d_k represented as experts; Oj represented global suppliers; NFS as $\left(r_{ij}^{(k)}\right)$, where, i, j, k range from 1 to 4.

$$R^1 = \begin{bmatrix} <0.25,0.54,0.8> & <0.3,0.4,0.9> & <0.7,0.35,0.5> & <0.9,0.2,0.8> \\ <0.6,0.5,0.5> & <0.6,0.2,0.3> & <0.2,0.4,0.9> & <0.6,0.23,0.7> \\ <0.3,0.45,0.9> & <0.7,0.1,0.4> & <0.6,0.5,0.5> & <0.4,0.2,0.9> \\ <0.45,0.38,0.27> & <0.37,0.68,0.16> & <0.6,0.25,0.3> & <0.1,0.4,0.8> \end{bmatrix}$$

$$R^2 = \begin{bmatrix} <0.1,0.3,0.7> & <0.6,0.6,0.6> & <0.4,0.2,0.1> & <0.3,0.7,0.6> \\ <0.3,0.55,0.37> & <0.75,0.42,0.1> & <0.32,0.67,0.56> & <0.35,0.56,0.72> \\ <0.5,0.4,0.32> & <0.65,0.25,0.32> & <0.6,0.3,0.1> & <0.75,0.25,0.55> \\ <0.27,0.9,0.81> & <0.31,0.4,0.6> & <0.75,0.65,0.55> & <0.3,0.7,0.9> \end{bmatrix}$$

$$R^3 = \begin{bmatrix} <0.32,0.47,0.6> & <0.9,0.1,0.3> & <0.6,0.4,0.5> & <0.3,0.5,0.7> \\ <0.12,0.32,0.52> & <0.17,0.81,0.9> & <0.5,0.3,0.1> & <0.45,0.65,0.27> \\ <0.50,0.6,0.23> & <0.56,0.52,0.23> & <0.3,0.6,0.1> & <0.57,0.52,0.55> \\ <0.54,0.83,0.72> & <0.73,0.86,0.61> & <0.5,0.52,0.4> & <0.6,0.4,0.2> \end{bmatrix}$$

$$R^4 = \begin{bmatrix} <0.7,0.3,0.1> & <0.5,0.4,0.4> & <0.2,0.1,0.6> & <0.7,0.9,0.6> \\ <0.3,0.56,0.73> & <0.57,0.24,0.1> & <0.23,0.76,0.65> & <0.53,0.65,0.27> \\ <0.32,0.32,0.6> & <0.56,0.52,0.32> & <0.1,0.3,0.9> & <0.57,0.52,0.55> \\ <0.72,0.5,0.18> & <0.13,0.6,0.4> & <0.55,0.56,0.78> & <0.7,0.1,0.6> \end{bmatrix}$$

$$R^5 = \begin{bmatrix} <0.52,0.45,0.1> & <0.57,0.37,0.1> & <0.76,0.65,0.23> & <0.57,0.52,0.55> \\ <0.3,0.6,0.7> & <0.7,0.4,0.1> & <0.3,0.7,0.6> & <0.5,0.4,0.6> \\ <0.2,0.3,0.2> & <0.6,0.2,0.5> & <0.1,0.6,0.65> & <0.3,0.9,0.7> \\ <0.27,0.5,0.81> & <0.75,0.25,0.32> & <0.32,0.67,0.56> & <0.35,0.56,0.72> \end{bmatrix}$$

$$R^6 = \begin{bmatrix} <0.4631,0.5867,0.5341> & <0.6300,0.6173,0.5355> & <0.5819,0.5981,0.7088> & <0.6121,0.3316,0.3580> \\ <0.3132,0.4644,0.3843> & <0.5825,0.5302,0.5712> & <0.3204,0.3718,0.3846> & <0.4919,0.4686,0.4716> \\ <0.3061,0.5908,0.4434> & <0.6069,0.6461,0.6276> & <0.3197,0.5128,0.3791> & <0.5260,0.3510,0.3301> \\ <0.4863,0.3247,0.3350> & <0.5503,0.4052,0.5611> & <0.5358,0.4258,0.4225> & <0.4795,0.5419,0.3037> \end{bmatrix}$$

Step 2: Utilize the HNFA-SMM operator (consider $\alpha = (0.1289, 0.1567, 0.1922, 0.2359, 0.2863)^T$ and NFS fuzzy decision matrix $R = (r_{ij})_{m\times n}$.

This is the score matrix $S_0 = (S_{ij})_{5\times 4}$ of the collective Neutrosophic fuzzy decision matrix.

Step 3: The matrix S_{00} is

$$\begin{bmatrix} <0.4631,0.4737> & <0.6300,0.4591> & <0.5819,0.5554> & <0.6121,0.5132> \\ <0.3132,0.4393> & <0.5825,0.5205> & <0.3204,0.5064> & <0.4919,0.5015> \\ <0.3061,0.4263> & <0.6069,0.4908> & <0.3197,0.4332> & <0.5260,0.4900> \\ <0.4863,0.5052> & <0.5503,0.5780> & <0.5358,0.4984> & <0.4795,0.3809> \end{bmatrix}$$

Step 4: The matrix S is $\begin{bmatrix} 0.4943 & 0.5785 & 0.5117 & 0.5440 \\ 0.4162 & 0.5281 & 0.3876 & 0.4952 \\ 0.4179 & 0.5529 & 0.4246 & 0.5177 \\ 0.4905 & 0.4877 & 0.5181 & 0.5573 \end{bmatrix}$

Step 5: Optimal weight vectors $w^{(j)} = (w_1^{(j)}, w_2^{(j)}, w_3^{(j)}, w_4^{(j)})$

$$w^{(1)} = (0.42,0.09,0.09,0.40)$$

$$w^{(2)} = (0.46,0.08,0.08,0.38)$$

$$w^{(3)} = (0.43,0.07,0.09,0.41)$$

$$w^{(4)} = (0.45,0.07,0.08,0.40)$$

Weight matrix Construction

$$w = \begin{bmatrix} 0.42 & 0.09 & 0.09 & 0.40 \\ 0.46 & 0.08 & 0.08 & 0.38 \\ 0.43 & 0.07 & 0.09 & 0.41 \\ 0.45 & 0.07 & 0.08 & 0.40 \end{bmatrix}$$

Then it observes that

$$S^T = \begin{bmatrix} 0.4943 & 0.4162 & 0.4179 & 0.4905 \\ 0.5785 & 0.5281 & 0.5529 & 0.4877 \\ 0.5117 & 0.3876 & 0.4246 & 0.5181 \\ 0.5440 & 0.4952 & 0.5177 & 0.5573 \end{bmatrix}$$

$$S^T w = \begin{bmatrix} 0.7995 & 0.1414 & 0.1546 & 0.7234 \\ 0.9431 & 0.1672 & 0.1831 & 0.8538 \\ 0.8089 & 0.1431 & 0.1567 & 0.7333 \\ 0.9332 & 0.1659 & 0.1822 & 0.8530 \end{bmatrix}$$

$$(S^T w)^T = \begin{bmatrix} 0.7995 & 0.9431 & 0.8089 & 0.9332 \\ 0.1414 & 0.1672 & 0.1431 & 0.1659 \\ 0.1546 & 0.1831 & 0.1567 & 0.1822 \\ 0.7234 & 0.8538 & 0.7333 & 0.8530 \end{bmatrix} \begin{bmatrix} 0.7995 & 0.1414 & 0.1546 & 0.7234 \\ 0.9431 & 0.1672 & 0.1831 & 0.8538 \\ 0.8089 & 0.1431 & 0.1567 & 0.7333 \\ 0.9332 & 0.1659 & 0.1822 & 0.8530 \end{bmatrix}$$

Step 6: Compute normalized Eigen vector ω matrix is $(S^T w)(S^T w)$, $\omega = (0.4818, 0.0440, 0.1051, 0.3691)^T$.

Step 7: Weight vector w derived by

$$w = W\omega = \begin{bmatrix} 0.42 & 0.09 & 0.09 & 0.40 \\ 0.46 & 0.08 & 0.08 & 0.38 \\ 0.43 & 0.07 & 0.09 & 0.41 \\ 0.45 & 0.07 & 0.08 & 0.40 \end{bmatrix} \begin{bmatrix} 0.4818 \\ 0.0440 \\ 0.1051 \\ 0.3691 \end{bmatrix}$$

$$((S^T w)(S^T w)) = \begin{bmatrix} 0.7995 & 0.9431 & 0.8089 & 0.9332 \\ 0.1414 & 0.1672 & 0.1431 & 0.1659 \\ 0.1546 & 0.1831 & 0.1567 & 0.1822 \\ 0.7234 & 0.8538 & 0.7333 & 0.8530 \end{bmatrix}$$

$$((S^T w)(S^T w)) == \begin{bmatrix} 0.9726 & 0.5429 & 0.5949 & 0.7813 \\ 0.5429 & 0.0959 & 0.1051 & 0.4914 \\ 0.5949 & 0.1051 & 0.1152 & 0.5385 \\ 0.7813 & 0.4974 & 0.5385 & 0.5176 \end{bmatrix}$$

Thus $w = (0.3634, 0.2416, 0.2034, 0.1916)^T [<0.4036,0.4842,0.5547><0.5981,0.4324,0.4324><0.4691,0.5097,0.4656><0.5422,0.5857,0.6286>]$.

Step 8: $r^* = (1,0,0) = (T_A*, I_A*, F_A*)$, find $d(r*, r_j) = \sqrt{(T_A* - T_{jA})^2 + (I_A* - I_{jA})^2 + (F_A* - F_{jA})^2}$ used to compute the distance between NFS information value:

$d(r^*, r1) = 0.947541392; d(r^*, r2) = 0.731753462;$
$d(r^*, r3) = 0.870880164; d(r^*, r4) = 0.973533500;$

Step 9: Utilize the obtained distance coefficients to rank using O_j.

The arrangement in alternatives Q_i due to rank values is $O_4 > O_1 > O_3 > O_2$.

Step 10: Desirable global supplier identified is O_4.

The MAGDM difficulties are examined in Neutrosophic fuzzy surroundings, and a method for resolving scenarios in which the values of attributes are described by HNFA-SMM and the characteristic weight data is entirely unknown is proposed. Using the HNFA-SMM manager, the proposed method first fuses every individual Neutrosophic fuzzy analysis matrix into a single Neutrosophic fuzzy choice matrix. Total NFS of the possibilities are produced by using the HNFA-SMM operation using the acquired characteristic weights.

Table 10. Comparison of performance measures

Method	Handling of Uncertainty	Accuracy of Ranking	Interpretability	Computational Complexity	Adaptability to Incomplete Data
Classical Fuzzy TOPSIS	Moderate	Moderate	High	Low	Low
Intuitionistic Fuzzy AHP	Good	Moderate-High	Moderate	High	Moderate
Neutrosophic Decision Matrix (NDMM)	High	High	Moderate	Moderate	High
Pythagorean Fuzzy VIKOR	Moderate-High	High	Moderate	Moderate	Moderate
Proposed Hybrid Neutrosophic Fuzzy Method	Very High	Very High	High	Moderate	Very High

As shown in the Table 10, the proposed hybrid method outperforms existing models in handling uncertainty and adaptability to incomplete or contradictory data are critical in real-world decision environments. Unlike classical fuzzy or intuitionistic methods that struggle with conflicting inputs or high indeterminacy, the proposed method uses curved fuzzy Neutrosophic logic to explicitly model truth, falsity, and indeterminacy. This results in more accurate and interpretable rankings of alternatives. Although the method has moderate computational complexity, it remains scalable and tractable for typical MAGDM problems. These advantages make it particularly well-suited for dynamic, uncertain environments such as supply chain decisions, research evaluations, and policy planning.

5. CONCLUSIONS

The hybrid Neutrosophic fuzzy method combined with Score Matrix Methods provides a robust framework for addressing challenges in MAGDM such as VWA calculation. Extensive testing and comparative research have demonstrated that this method effectively manages ambiguous and insufficient information, offering enhanced adaptability and flexibility in decision-making. Fuzzy logic contributes to faster computation and handles membership degrees, while Neutrosophic Sets enable the representation of ambiguous and uncertain qualities. Score Matrix Methods aid in organizing decision factors and aggregating opinions from multiple decision-makers facilitating a comprehensive review process. This hybrid approach offers insights into potential future advancements and applications across various decision-making domains, thereby enhancing decision reliability and precision.

5.1 Future directions

Future investigations into the HNFA combined with Score Matrix Methods for research and advancement in computer intelligence and decision science. It involves enhancing the computational methods underlying the approach to improve algorithm effectiveness and flexibility particularly in managing large-scale MAGDM scenarios. Incorporating machine learning techniques, such as neural networks or reinforcement learning could further enhance the predictive capabilities and adaptability of the strategy leading to more precise decision outcomes. Validating the method's practicality and robustness through application in real-world contexts such as environmental governance, healthcare, and banking will be crucial. Future research efforts may focus on refining data handling processes and adapting the methodology for real-time decision settings and streaming data using internet-based instruction and adaptive techniques. Advancements in decision support systems based on this method could facilitate the development of interactive tools and user-friendly interfaces, thereby improving decision-making accessibility and transparency.

To address confidentiality and integrity concerns in sensitive decision-making contexts, future research should also explore issues related to privacy and security within decision-making frameworks. Comparative studies with alternative decision-making approaches would provide valuable insights into the relative advantages of this strategy and its optimal applications across diverse decision scenarios. These new avenues of research aim to enhance the HNFA-SMMs into a more adaptable and powerful tool for addressing complex decision-making challenges in various domains.

REFERENCES

[1] Ibrahim, H.Z., Al-shami, T.M., Arar, M., Hosny, M. (2024). k_m^n-Rung picture fuzzy information in a modern approach to multi-attribute group decision-making. Complex & Intelligent Systems, 10(2): 2605-2625. https://doi.org/10.1007/s40747-023-01277-z

[2] Naz, S., Saeed, M.R., Butt, S.A. (2024). Multi-attribute group decision-making based on 2-tuple linguistic cubic q-rung orthopair fuzzy DEMATEL analysis. Granular Computing, 9(1): 12. https://doi.org/10.1007/s41066-023-00433-7

[3] Paraskevas, A., Madas, M. (2024). Selection of academic staff based on a hybrid multi-criteria decision method under Neutrosophic environment. Operations Research Forum, 5(1): 23. https://doi.org/10.1007/s43069-024-00309-9

[4] Kirişci, M. (2024). Multiple criteria group decision-making using Fermatean fuzzy soft matrices and their medical applications. Journal of Uncertain Systems, 17(2): 2440007. https://doi.org/10.1142/S1752890924400075
[5] Kamran, M., Ashraf, S., Kalim, K.S., Hussain, K.A., Zardi, H., Mehmood, S. (2024). Integrated decision-making framework for hospital development: A single-valued Neutrosophic probabilistic hesitant fuzzy approach with innovative aggregation operators. Yugoslav Journal of Operations Research, 34(3): 515-550. https://doi.org/10.2298/YJOR230915034K
[6] Rong, Y., Yu, L., Liu, Y., Peng, X., Garg, H. (2024). An integrated group decision-making framework for assessing S3PRLPs based on MULTIMOORA-WASPAS with q-rung orthopair fuzzy information. Artificial Intelligence Review, 57(6): 163. https://doi.org/10.1007/s10462-024-10782-7
[7] Farman, S., Khan, F.M., Bibi, N. (2024). T-Spherical fuzzy soft rough aggregation operators and their applications in multi-criteria group decision-making. Granular Computing, 9(1): 6. https://doi.org/10.1007/s41066-023-00437-3
[8] Regragui, H., Sefiani, N., Azzouzi, H., Cheikhrouhou, N. (2024). A hybrid multi-criteria decision-making approach for hospitals' sustainability performance evaluation under fuzzy environment. International Journal of Productivity and Performance Management, 73(3): 855-888. https://doi.org/10.1108/IJPPM-10-2022-0538
[9] Goyal, S., Rani, P. (2024). Multi-criteria decision-making based on Einstein operators, WASPAS method and quadripartitioned Single-Valued Neutrosophic Sets. Granular Computing, 9(3): 54. https://doi.org/10.1007/s41066-024-00482-6
[10] Al-Mashhadani, A.K., Salman, M.S., Al-Yasiri, F.A.H.N. (2025). Quality of service attributes based hybrid decision-making framework for ranking cloud service providers under Fermatean fuzzy environment. Yugoslav Journal of Operations Research, 35(1): 85-112. https://doi.org/10.2298/YJOR231215010A
[11] Pang, K., Martínez, L., Li, N., Liu, J., Zou, L., Lu, M. (2024). A concept lattice-based expert opinion aggregation method for multi-attribute group decision-making with linguistic information. Expert Systems with Applications, 237: 121485. https://doi.org/10.1016/j.eswa.2023.121485
[12] Xian, S., Liu, M., Xian, Z., Chai, J., Lu, S., Qing, K. (2024). χ-linguistic sets and its application for the linguistic Multi-Attribute Group Decision Making. Artificial Intelligence Review, 57(4): 92. https://doi.org/10.1007/s10462-023-10695-x
[13] Lei, S., Ma, X., Qin, H., Wang, Y., Zain, J.M. (2024). A new multi-attribute group decision-making method based on Einstein Bonferroni operators under interval-valued Fermatean hesitant fuzzy environment. Scientific Reports, 14(1): 12370. https://doi.org/10.1038/s41598-024-62762-0
[14] Feng, X., Liu, S., Xu, W. (2024). A new multi-attribute group decision-making method based on probabilistic multi-valued linguistic spherical fuzzy sets for the site selection of charging piles. Archives of Control Sciences, 171-210.
[15] Qiu, J., Jiang, L., Fan, H., Li, P., You, C. (2024). Dynamic nonlinear simplified Neutrosophic Sets for multiple-attribute group decision making. Heliyon, 10(5): e27493. https://doi.org/10.1016/j.heliyon.2024.e27493
[16] Ferreira, M.A.D.D.O., Ribeiro, L.C., Schuffner, H.S., Libório, M.P., Ekel, P.I. (2024). Fuzzy-set-based multi-attribute decision-making, its computing implementation, and applications. Axioms, 13(3): 142. https://doi.org/10.3390/axioms13030142
[17] Wang, H., Xu, T., Feng, L., Ullah, K. (2024). An improved ARAS approach with T-spherical fuzzy information and its application in multi-attribute group decision-making. International Journal of Fuzzy Systems, 26(7): 2132-2156. https://doi.org/10.1007/s40815-024-01718-y
[18] Zheng, Y., Ma, C. (2024). A hybrid multi-attribute decision-making approach considering correlations among indicators. International Journal of Computers Communications & Control, 19(3): 6515. https://doi.org/10.15837/ijccc.2024.3.6515
[19] Zhang, H., Liu, S., Zhang, L. (2024). A multi-attribute decision making approach considering economic psychology. International Journal of Knowledge-Based and Intelligent Engineering Systems, 28(4): 700-717. https://doi.org/10.3233/KES-230015
[20] Palanikumar, M., Kausar, N., Pamucar, D., Kadry, S. (2024). Novelty of different distance approach for multi-criteria decision-making challenges using q-rung vague sets. Computer Modeling in Engineering & Sciences (CMES), 139(3): 3353-3385. https://doi.org/10.32604/cmes.2024.031439
[21] Luo, M., Zhang, G. (2024). Divergence-based distance for picture fuzzy sets and its application to multi-attribute decision-making. Soft Computing-A Fusion of Foundations, Methodologies & Applications, 28(1): 253-269. https://doi.org/10.1007/s00500-023-09205-6
[22] Liu, P., Shen, J., Zhang, P. (2024). Multi-attribute group decision-making method using single-valued Neutrosophic credibility numbers with the Dombi extended power aggregation operator and its application in intelligent transportation system data collection scheme selection. Engineering Applications of Artificial Intelligence, 133: 108639. https://doi.org/10.1016/j.engappai.2024.108639
[23] Nie, J. (2024). A novel IFN-CSM-CoCoSo approach for multiple-attribute group decision-making with intuitionistic fuzzy sets: An application in assessing corporate social responsibility performance. Heliyon, 10(7): e29207. https://doi.org/10.1016/j.heliyon.2024.e29207
[24] Javeed, S., Javed, M., Jameel, A., Senapati, T. (2024). A picture fuzzy multi-attribute decision-making approach based on Hamacher Muirhead mean operators. Granular Computing, 9(3): 64. https://doi.org/10.1007/s41066-024-00486-2
[25] Sarfraz, M. (2024). Multi-attribute decision-making for T-spherical fuzzy information utilizing Schweizer-Sklar prioritized aggregation operators for recycled water. Decision Making Advances, 2(1): 105-128. https://doi.org/10.31181/dma21202425
[26] Chu, Y.M., Garg, H., Rahim, M., Amin, F., Asiri, A., Ameer, E. (2024). Some p, q-cubic quasi-rung orthopair fuzzy operators for multi-attribute decision-making. Complex & Intelligent Systems, 10(1): 87-110.

https://doi.org/10.1007/s40747-023-01092-6
[27] Iqbal, Q., Khan, Z.U. (2024). Efficacy of induced complex aggregation operators in multi-attribute decision-making with confidence levels. Acadlore Transactions on Applied Mathematics and Statistics, 2(2): 64-71. https://doi.org/10.56578/atams020201
[28] Garg, H., Limboo, B., Dutta, P. (2024). Multi-criteria group decision-making process using convex combination of q-rung orthopair basic probability assignment with application to medical diagnosis. Engineering Applications of Artificial Intelligence, 133: 108421. https://doi.org/10.1016/j.engappai.2024.108421
[29] Palanikumar, M., Kausar, N., Garg, H., Nasseredine, H., Pamucar, D. (2024). Selection process based on new building construction work using square root vague sets and their aggregated operators. Engineering Applications of Artificial Intelligence, 131: 107794. https://doi.org/10.1016/j.engappai.2023.107794
[30] Abbas, F., Ali, J., Mashwani, W.K., Syam, M.I. (2024). An integrated group decision-making method under q-rung orthopair fuzzy 2-tuple linguistic context with partial weight information. PloS One, 19(5): e0297462. https://doi.org/10.1371/journal.pone.0297462
[31] Hussain, A., Amjad, A., Ullah, K., Pamucar, D., Ali, Z., Al-Quran, A. (2024). Multi-attribute group decision-making for supplier selection based on Dombi aggregation operators under the system of spherical fuzzy Hamy mean. Journal of Intelligent & Fuzzy Systems, 46(4): 9639-9662. https://doi.org/10.3233/JIFS-23