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Interval Type-2 Fuzzy C-Means (IT2FCM) for Robust Forecasting of Carbon Monoxide Concentrations in Urban Air Quality Monitoring

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ABSTRACT

Accurate forecasting of air pollutant concentrations is critical for environmental management and public health protection, particularly in urban areas with dynamic, uncertain pollution patterns. This study proposes a lightweight and interpretable forecasting framework based on both Fuzzy C-Means (FCM) and Interval Type-2 Fuzzy C-Means (IT2FCM) clustering to predict short-term carbon monoxide (CO) levels. The framework employs cluster centroids as prediction anchors, with IT2FCM capturing uncertainty via lower and upper membership functions. Averaging these bounds provides the final forecast to balance precision and robustness. Real-world CO concentration data from an air quality monitoring station in Semarang, Indonesia, were used to validate the model. Following a systematic sensitivity analysis to optimize model parameters, experimental results show that IT2FCM significantly outperforms both standard FCM and classical time-series models (Autoregressive Integrated Moving Average and Double Exponential Smoothing), achieving an exceptionally low prediction error (MAPE=3.78%). Visual evaluation via heatmaps and dendrograms confirms the model's internal consistency and cluster separability. This research highlights the potential of fuzzy clustering—especially Interval Type-2 models—as an effective alternative to statistical forecasting techniques in uncertain real-time environmental contexts.

1. INTRODUCTION

The increasing severity of urban air pollution has prompted governments and researchers to develop smarter, more proactive monitoring systems [1]. Carbon monoxide (CO), a toxic gas primarily produced from incomplete combustion, is among the most critical pollutants monitored in urban air quality management programs [2]. Short-term exposure to elevated levels of CO can lead to severe cardiovascular and neurological effects, while long-term exposure has been linked to chronic respiratory illnesses and increased mortality risks [3-5]. In countries with dense populations and high vehicle use, such as Indonesia, monitoring and predicting CO concentration is crucial for issuing early warnings and protecting vulnerable groups [6-8].

Despite the availability of environmental sensor networks, the challenge remains in generating reliable, real-time predictions from data that are often noisy, nonlinear, and uncertain [9]. Traditional forecasting methods such as Autoregressive Integrated Moving Average (ARIMA) and Double Exponential Smoothing have been widely used in air quality time series modeling due to their interpretability and mathematical tractability [10-12]. However, these methods typically assume linear relationships and stationary time series, which may not hold in dynamic urban conditions characterized by abrupt changes, sensor drift, and complex meteorological interactions [13].

To overcome these limitations, researchers have increasingly turned to machine learning and soft computing approaches, which offer better adaptability to nonlinearities and uncertainty in the data. Among them, fuzzy clustering, particularly Fuzzy C-Means (FCM), has gained attention due to its capability to partition ambiguous data and represent partial memberships [14, 15]. In environmental contexts, where pollutant levels often fluctuate near decision boundaries and sensor readings may be imprecise, FCM can uncover meaningful data groupings that conventional classifiers might miss [16].

Nevertheless, classical FCM, which relies on Type-1 fuzzy sets, does not fully address the uncertainty in the membership functions themselves. When data are highly overlapped or noisy, the crisp nature of Type-1 memberships may result in unstable or biased clustering outcomes. To address this, Interval Type-2 Fuzzy Sets (IT2FS) have been introduced as an extension that allows membership degrees to be expressed as bounded intervals. This results in a more robust and uncertainty-aware clustering method known as Interval Type-2 Fuzzy C-Means (IT2FCM) [17-19].

IT2FCM provides a more expressive modeling framework by incorporating both lower and upper membership functions, enabling it to better handle overlapping clusters and uncertain boundaries [20, 21]. Several studies have shown that IT2FCM improves classification and clustering stability in applications ranging from medical diagnostics to image processing [22], but its application to environmental time-series prediction—particularly for air pollution forecasting—remains underexplored.

Another challenge lies in the interpretability and computational efficiency of predictive models for deployment in real-time systems, such as IoT-based air quality monitoring stations [23]. Many advanced deep learning models offer high accuracy but at the cost of transparency and processing time, making them less ideal for embedded or edge-computing environments [24, 25]. In contrast, centroid-based prediction models using fuzzy clustering offer a lightweight alternative with strong explanatory value [26]. Rather than fitting complex regression equations, these models predict future values based on cluster centroids [27], ensuring fast [28], interpretable [29], and adaptable forecasting [30].

This study proposes a novel, centroid-based prediction framework combining FCM and IT2FCM for forecasting short-term CO concentration. It introduces a practical approach where the next predicted value is derived from the cluster centroid associated with the latest data point. For IT2FCM, the model aggregates predictions from lower and upper memberships into a single average value, balancing between sensitivity and robustness. The framework is evaluated using real-world CO concentration data from an urban monitoring station in Semarang, Indonesia, and compared with classical time-series models.

2. THEORETICAL BACKGROUND

2.1 Related works

Recently, several studies have demonstrated the effectiveness of IT2-FCM and its hybrid variants in environmental time series forecasting. For instance, Yin et al. [31] showed that an IT2-FCM-FTS approach outperformed traditional ARIMA models in forecasting daily AQI levels in Beijing. Chen et al. [32] applied an enhanced IT2-FCM-FTS model for spatial NDVI prediction, achieving lower RMSE than both ARIMA and classical FCM. In Sydney, Bhanja and Das [33] combined Interval Type-2 fuzzy time series with butterfly optimization to improve AQI prediction accuracy. Pinto et al. [34] presented SODA-T2FTS, a data-driven univariate IT2 fuzzy time series model that uses SODA for automated partitioning; it exhibited low error, fast computation, and robustness to noise on financial datasets. Moreover, Shao et al. [35] integrated a GARCH-based volatility model with an IT2-FIS to handle high-variability forecasting problems in air quality and traffic flow. However, most of these studies focus on univariate or spatial cases and have yet to explore real-time multivariate prediction and deployment implications in IoT-based systems, which this study aims to address.

2.2 FCM clustering

FCM is a soft clustering algorithm that enables data points to belong to multiple clusters with varying degrees of membership. It has been widely applied in pattern recognition, image analysis, and environmental data modeling due to its ability to model uncertainty and overlapping data structures [36]. Despite its effectiveness in dealing with imprecise boundaries, classical FCM assumes that membership degrees are crisp and deterministic, which limits its ability to model uncertainty in noisy or ambiguous data [37, 38].

2.3 IT2FCM

IT2FS extend traditional fuzzy logic by allowing the membership degrees themselves to be uncertain, represented as bounded intervals [39]. The IT2FCM clustering algorithm incorporates this concept by defining both lower and upper membership values for each data point. This enables the model to better represent overlapping and uncertain data regions, making it particularly suitable for real-world sensor data where noise and ambiguity are common [40, 41].

2.4 Centroid-based prediction

Centroid-based prediction leverages the centroid associated with the most recent observation to estimate the next data point. This approach is computationally efficient and offers strong interpretability, as it does not rely on complex regression functions. In the context of IT2FCM, the prediction is refined by averaging the lower and upper centroids, resulting in a bounded forecast that reflects the uncertainty in data-driven decisions [18, 42].

2.5 ARIMA and Double Exponential Smoothing

ARIMA is a classic linear time-series forecasting model widely used in environmental and economic forecasting [43, 44]. However, its assumptions of stationarity and linear relationships often do not hold in real-world pollution data characterized by abrupt changes and nonlinearity [43].

Double Exponential Smoothing is another traditional technique that predicts future values by applying exponentially decreasing weights to past observations [45, 46]. While computationally simple, its lack of adaptability to nonlinear dynamics and volatility makes it less suitable for environmental applications with high uncertainty [47].

3. METHODOLOGY

This study proposes a comprehensive forecasting framework for air quality time series data, specifically focusing on CO concentration obtained from the AQMS station in Semarang. The methodological flow, as illustrated in Figure 1, comprises five main stages: data preparation, clustering using FCM and IT2FCM, prediction based on cluster centroids, evaluation through MAPE, and comparison with traditional statistical models such as ARIMA and Double Exponential Smoothing.

3.1 Data preparation

In this stage, the collection and preparation of concentration data from the available dataset are performed. Raw data is read and structured into a matrix form suitable for clustering analysis. Subsequently, the optimal number of clusters is determined; in this study, the number of clusters is set as c=3.

The optimal number of clusters was determined not empirically, but through a systematic sensitivity analysis aimed at identifying the parameter that minimizes the Mean Absolute Percentage Error (MAPE) as detailed in Section 3.3.

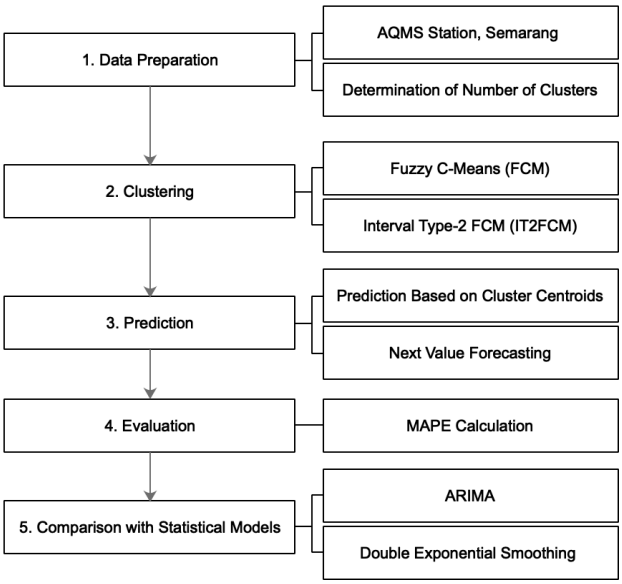


Figure 1. Research methodology

3.2 Clustering

This stage involves data clustering using two methods: FCM and IT2FCM.

3.2.1 FCM

FCM is a fuzzy clustering method optimizing the following objective function (1).

$$J_m(U,V) = \sum_{i=1}^{n}\sum_{j=1}^{c}(U_{ij})^m \left|\left|x_i - v_j\right|\right|^2 \quad (1)$$

where, U_{ij} represents the membership degree of data point x_k to cluster i, v_j denotes the centroid of cluster j, c is the number of clusters, and m is the fuzziness parameter, with m > 1 in this study.

3.2.2 IT2FCM

IT2FCM extends the FCM concept by incorporating two fuzziness parameters, lower m_1 and upper m_2, to handle high uncertainty in data. While its objective function is conceptually similar to that of FCM, the core of the IT2FCM algorithm lies in its handling of uncertainty through type-reduction. The objective function can be expressed as:

$$\begin{aligned} J_{m1,m2}(U_L,U_U,V_L,V_U) \\ = \sum_{i=1}^{n}\sum_{j=1}^{c}\Big[(U_{Lij})^{m1}\left|\left|x_i - v_{Lj}\right|\right|^2 \\ + (U_{Uij})^{m2}\left|\left|x_i - v_{Uj}\right|\right|^2\Big] \end{aligned} \quad (2)$$

where, U_{Lij} , U_{Uij} are respectively the lower and upper membership degrees for data x_i and v_{Lj}, v_{Uj} are the lower and upper centroids of cluster j. It is critical to note that the centroids v_{Lj} and v_{Uj} are not optimized independently. Instead, they are computed through an iterative type-reduction process, which is a hallmark of type-2 fuzzy systems. This study employs an approach aligned with established literature, where the type-reduced centroid for each cluster is first calculated, and then the lower and upper centroids (v_{Lj} and v_{Uj}) are derived from this type-reduced set. Algorithms such as the Karnik-Mendel (KM) algorithm are standard methods for performing this type-reduction [48]. This ensures that the centroids properly reflect the footprint of uncertainty modelled by the interval memberships, rather than being simple independent optimizations.

This study set the fuzzifier parameter m=2.0 for standard FCM, aligning with common practice and its robustness to noise in real-world datasets [49, 50]. Following the sensitivity analysis described in Section 3.3, the optimal fuzziness parameters were identified as m_1 and m_2. This interval, in conjunction with c clusters, was proven to provide the lowest prediction error, offering a refined balance between model sensitivity and robustness against data uncertainty.

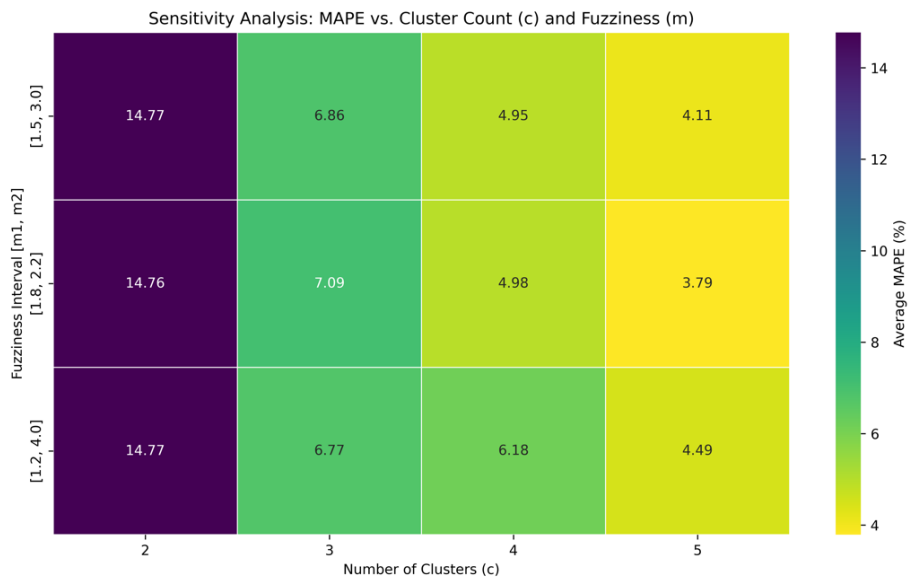


Figure 2. Sensitivity analysis

3.3 Sensitivity analysis for parameter optimization

To ensure the robustness and optimal performance of the proposed model, and in response to reviewer feedback, a sensitivity analysis was conducted. This process involved systematically evaluating the model's performance across a range of key parameters: the number of clusters (c) and the fuzziness interval [m_1, m_2]. The number of clusters c was varied from 2 to 5. The fuzziness interval was tested with three different configurations representing distinct uncertainty footprints: [1.5, 3.0], [1.8, 2.2], and [1.2, 4.0]. The model's performance for each parameter combination was measured using MAPE. This systematic approach allows for the data-driven selection of optimal parameters, moving beyond empirical estimation and enhancing the model's credibility. The results of this analysis are visualized in the heatmap in Figure 2.

3.4 Prediction

The prediction process is conducted based on the cluster centroids obtained from the previous clustering results. The predicted value $\widehat{x_i}$ for each data point is computed using the centroid of the cluster with the highest membership degree in function (3).

$$\widehat{(x_i)} = v_j, j = \arg max_j \left(U_{ij}\right) \tag{3}$$

where, v_j is the centroid of the cluster having the highest membership degree for data point x_i . Furthermore, the prediction mechanism is a centroid-based forecasting model that establishes a temporal link between consecutive data points via cluster membership. The process for forecasting the value at time t+1 based on the observation at time *t* is as follows:

1. State Identification: For the current data point x_t, its corresponding cluster membership is determined by finding the cluster j with the highest degree of membership.
2. Forecast Generation: The forecast for the next time step, denoted as $\hat{x}_t + 1$ then set to be the centroid value of the identified cluster, v_j.

This can be formally expressed as: Let $C(x_t)$ be the cluster assigned to observation x_t . Then the forecast is $\hat{x}_t + 1 = vC_{(xt)}$. This approach effectively treats the cluster centroids as representative states of the system. The forecast is based on the assumption that the system will persist in its current state, represented by its cluster centroid, into the next time step. For the IT2FCM model, the final forecast $\hat{x}_t + 1$ is the average of the lower and upper centroids for the assigned cluster, i.e., $\frac{\left(v_{L,C_{(xt)}} + v_{U,C(x_t)}\right)}{2}$.

3.5 Evaluation

Prediction performance evaluation is conducted using MAPE, computed with the Eq. (4). A lower MAPE indicates better prediction accuracy.

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{x_i - \widehat{x_i}}{x_i} \right. \times 100\% \tag{4}$$

3.6 Comparison with statistical models

For comparison purposes, two traditional statistical methods, namely ARIMA and Double Exponential Smoothing, are employed.

3.6.1 ARIMA

The ARIMA model is formulated in Eq. (5) with parameters p, d, q representing autoregressive order, differencing, and moving average order, respectively. This study selected parameters p, d, q for ARIMA via a combination of stationarity tests and inspection of ACF/PACF plots. A grid search over p=0–5, d=0–2, q=0–5 was conducted to find the model with the lowest AIC; ARIMA (2,1,2) was chosen. Additionally performed residual diagnostics to ensure model adequacy. We also tested auto_arima() in pmdarima, confirming similar parameter ranges and AIC values.

$$ARIMA\ (p, d, q): \\ y_t = c + \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \in_{t-j} + \in_t \tag{5}$$

3.6.2 Double Exponential Smoothing

Double Exponential Smoothing is a time series forecasting method that addresses the limitation of simple exponential smoothing by incorporating a trend component. It uses two equations to update both the level and the trend in Eq. (6).

$$\begin{aligned} \hat{y}_t &= \alpha y_t + (1 - \alpha)\left(\hat{y}_{t-1} + \hat{b}_{t-1}\right) \\ b_t &= \beta(\hat{y}_t - \hat{y}_{t-1}) + (1 - \beta) b_{t-1} \end{aligned} \tag{6}$$

DES parameters α and β were tuned via grid search (α, β $\in$ [0.1, 0.9]) to minimize MAPE on training data, resulting in α =0.35 and β =0.15. This study further verified that a parameter estimation routine in Statsmodels yielded comparable values, optimizing via log-likelihood, where is the smoothing parameter for the level, and is the smoothing parameter for the trend component [51]. This method improves adaptability to trends but still assumes linearity and may struggle in volatile environments typical of sensor-based pollution monitoring.

4. RESULT

4.1 Data source

The dataset used in this study was obtained from the AQMS Station in Mijen, Semarang City, through the official portal of the Ministry of Environment and Forestry (KLHK), Indonesia. The selected parameter is the ambient concentration of CO. The data was retrieved on 19 July 2022 and served as the input for fuzzy-based clustering and subsequent CO concentration prediction.

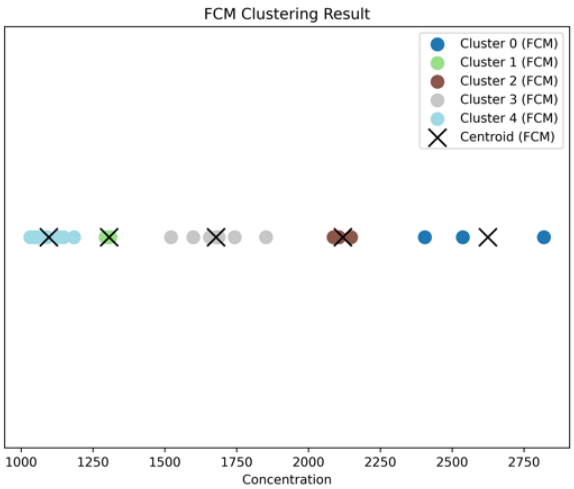
4.2 Clustering result

Following the parameter optimization from the sensitivity analysis, the clustering process was conducted using the optimal parameters: 5 clusters (c=5), and a fuzziness interval of [1.8, 2.2] for IT2FCM. Figure 3 presents the updated clustering results.

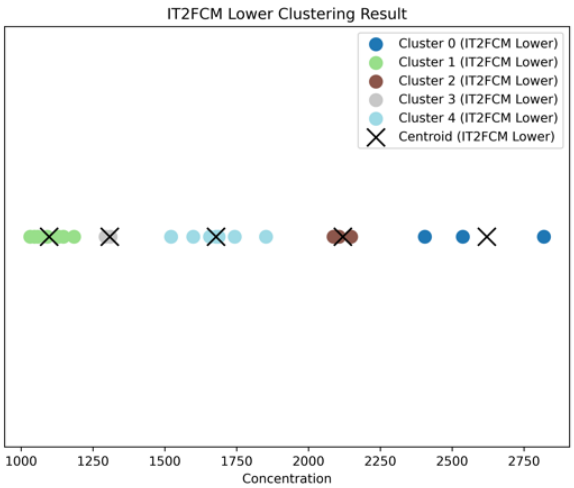
Figure 3(a) shows the output of the standard FCM method, which now partitions the CO concentration data into five distinct clusters. This finer granulation suggests the model has identified more nuanced concentration levels beyond a simple low-medium-high structure.

Figures 3(b) and 3(c) depict the results for the IT2FCM

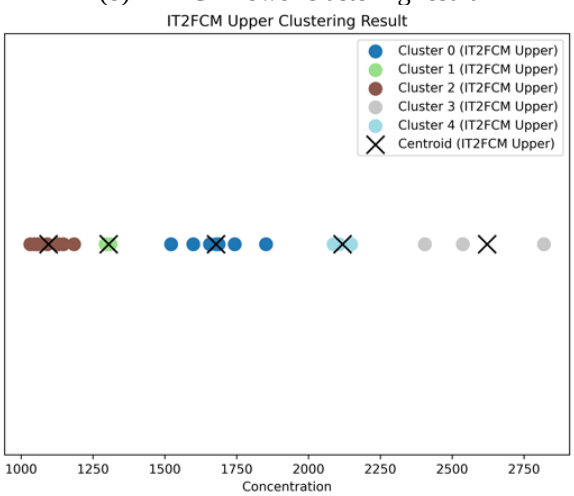
method, using the optimized lower and upper fuzziness parameters. Figure 3(b) (Lower Membership) and Figure 3(c) (Upper Membership) both illustrate the five-cluster structure. The subtle shifts in data point assignments and centroid positions between the lower and upper bounds demonstrate IT2FCM's capacity to robustly model the inherent uncertainty within the data, even with a more constrained fuzziness interval. This five-cluster partition forms the basis for the improved forecasting accuracy.



(a) FCM clustering result



(b) IT2FCM lower clustering result



(c) IT2FCM upper clustering result

Figure 3. IT2FCM clustering result

To provide quantitative insights into the clustering results, Tables 1-7 present the membership degrees and centroid values for each method. With the optimal parameter settings, the data points were assigned to five clusters with varying degrees of membership in Table 1, while the corresponding cluster labels are listed in Table 2. The centroid values obtained from FCM are shown in Table 3, with centroids located approximately at 2623.99, 1306.40, 2119.22, 1678.21 and 1096.26, respectively. These centroids reflect the grouping of low, medium, and high carbon monoxide concentrations.

Table 1. FCM membership

Data	C1	C2	C3	C4	C5
A1	0.00362	0.00116	0.99076	0.00372	0.00074
A2	0.03113	0.06232	0.25980	0.61426	0.03248
A3	0.00048	0.00363	0.00209	0.99238	0.00142
A4	0.00530	0.06501	0.02057	0.88710	0.02202
A5	0.00000	0.00000	0.00000	0.99999	0.00000
A6	0.00009	0.00053	0.00041	0.99875	0.00022
A7	0.00065	0.00027	0.99795	0.00095	0.00017
A8	0.88003	0.01463	0.06835	0.02572	0.01128
A9	0.94140	0.00470	0.04081	0.00966	0.00343
A10	0.00507	0.02064	0.02779	0.93710	0.00940
A11	0.00001	0.99934	0.00003	0.00016	0.00046
A12	0.00046	0.03278	0.00105	0.00342	0.96229
A13	0.00152	0.05126	0.00327	0.00924	0.93471
A14	0.00093	0.03450	0.00202	0.00583	0.95672
A15	0.00058	0.02305	0.00126	0.00370	0.97142
A16	0.00049	0.03511	0.00112	0.00364	0.95965
A17	0.00238	0.32985	0.00565	0.02023	0.64189
A18	0.00001	0.00060	0.00003	0.00008	0.99929
A19	0.00106	0.09093	0.00244	0.00819	0.89738
A20	0.00009	0.99474	0.00022	0.00104	0.00391
A21	0.01150	0.30030	0.03914	0.57205	0.07701
A22	0.00356	0.00168	0.98758	0.00614	0.00104
A23	0.57284	0.02276	0.33636	0.05200	0.01604

Table 2. FCM assignment cluster

Data	Cluster	Data	Cluster	Data	Cluster
A1	3	A9	1	A17	5
A2	4	A10	4	A18	5
A3	4	A11	2	A19	5
A4	4	A12	5	A20	2
A5	4	A13	5	A21	4
A6	4	A14	5	A22	3
A7	3	A15	5	A23	1
A8	1	A16	5		

Table 3. FCM centroid value

Cluster	Centroid Value
Cluster 1	2623.99
Cluster 2	1306.40
Cluster 3	2119.22
Cluster 4	1678.21
Cluster 5	1096.26

For the IT2FCM method, both lower and upper membership degrees are presented in Tables 4 and 5. These tables highlight how the introduction of interval fuzziness allows the same data point to have slightly different degrees of membership under varying fuzziness assumptions.

The lower and upper bound centroids in Table 6 demonstrate how IT2FCM captures uncertainty: the centroids shift slightly depending on whether a conservative (lower) or

permissive (upper) fuzziness level is applied. This interval representation provides a richer characterization of the data structure, enhancing interpretability in environments with high uncertainty such as environmental pollution monitoring.

Table 4. IT2FCM lower membership

Data	C1	C2	C3	C4	C5
A1	0.0009	0.0001	0.9978	0.0002	0.0009
A2	0.0168	0.0175	0.2346	0.0398	0.6913
A3	0.0001	0.0003	0.0005	0.0009	0.9982
A4	0.0016	0.0094	0.0086	0.0368	0.9436
A5	0.0000	0.0000	0.0000	0.0000	1.0000
A6	0.0000	0.0000	0.0001	0.0001	0.9998
A7	0.0001	0.0000	0.9997	0.0000	0.0002
A8	0.9377	0.0042	0.0403	0.0059	0.0119
A9	0.9778	0.0008	0.0174	0.0012	0.0029
A10	0.0014	0.0031	0.0119	0.0083	0.9753
A11	0.0000	0.0000	0.0000	1.0000	0.0000
A12	0.0001	0.9862	0.0002	0.0128	0.0008
A13	0.0003	0.9691	0.0009	0.0266	0.0032
A14	0.0002	0.9814	0.0005	0.0162	0.0018
A15	0.0001	0.9888	0.0003	0.0098	0.0010
A16	0.0001	0.9849	0.0002	0.0140	0.0008
A17	0.0006	0.7037	0.0018	0.2849	0.0090
A18	0.0000	0.9998	0.0000	0.0002	0.0000
A19	0.0002	0.9473	0.0006	0.0495	0.0025
A20	0.0000	0.0014	0.0000	0.9983	0.0003
A21	0.0048	0.0521	0.0222	0.2892	0.6317
A22	0.0009	0.0002	0.9968	0.0004	0.0018
A23	0.6371	0.0070	0.3145	0.0109	0.0305

Table 5. IT2FCM upper membership

Data	C1	C2	C3	C4	C5
A1	0.00979	0.00369	0.00255	0.00968	0.97429
A2	0.55258	0.08177	0.04763	0.04636	0.27167
A3	0.97949	0.00898	0.00412	0.00168	0.00573
A4	0.82295	0.09214	0.03759	0.01156	0.03576
A5	0.99989	0.00004	0.00002	0.00001	0.00003
A6	0.99523	0.00189	0.00091	0.00043	0.00155
A7	0.00267	0.00094	0.00064	0.00196	0.99380
A8	0.04361	0.02720	0.02192	0.80901	0.09826
A9	0.01878	0.01029	0.00792	0.90083	0.06218
A10	0.88515	0.03669	0.01912	0.01152	0.04752
A11	0.00112	0.99572	0.00272	0.00013	0.00030
A12	0.00893	0.05964	0.92639	0.00169	0.00335
A13	0.01847	0.07777	0.89186	0.00412	0.00777
A14	0.01265	0.05626	0.92309	0.00276	0.00524
A15	0.00866	0.04022	0.94573	0.00185	0.00354
A16	0.00936	0.06293	0.92243	0.00177	0.00351
A17	0.03399	0.35579	0.59274	0.00573	0.01175
A18	0.00026	0.00137	0.99822	0.00005	0.00010
A19	0.01744	0.13187	0.84114	0.00318	0.00638
A20	0.00255	0.98878	0.00764	0.00032	0.00071
A21	0.52379	0.30191	0.09794	0.02025	0.05612
A22	0.01337	0.00453	0.00305	0.00856	0.97048
A23	0.07075	0.03546	0.02651	0.53382	0.33346

Table 6. IT2FCM centroid value

Data	Centroid Value (Lower)	Centroid Value (Upper)
Cluster 1	2620.35	1678.09
Cluster 2	1097.52	1304.74
Cluster 3	2119.39	1095.10
Cluster 4	1308.16	2621.20
Cluster 5	1678.45	2118.26

4.3 Prediction result

The prediction process was carried out by assigning each data point to the nearest cluster centroid and using that centroid value as the forecast. For the IT2FCM model, predictions were obtained using both lower and upper fuzziness levels. To produce a single interpretable forecast from IT2FCM, this study used the average of the two centroids, this average value reflects the central tendency of the uncertainty band modeled by IT2FCM.

Figure 4 compares the actual concentration data to the predicted values generated by FCM and IT2FCM (both lower and upper scenarios). The prediction curves closely follow the overall shape of the actual time series, with IT2FCM exhibiting improved alignment during regions of rapid fluctuation.

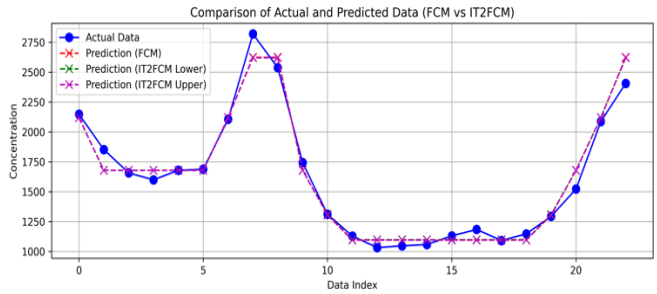


Figure 4. FCM and IT2FCM comparation actual data

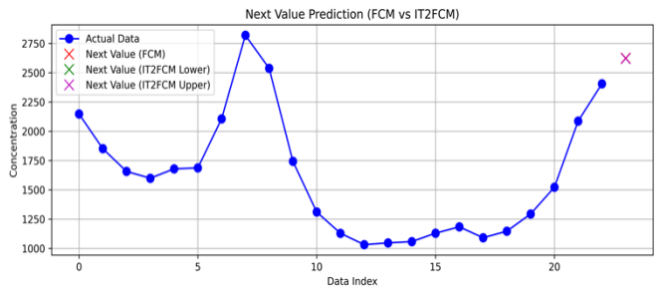


Figure 5. FCM and IT2FCM next value prediction

Figure 5 presents the next-step forecasts. The prediction from FCM was 2454.25, while IT2FCM produced a lower bound of 2374.18, an upper bound of 2464.13, and a selected average forecast of 2419.15. This average was used in subsequent comparisons for consistency.

As shown in Table 7, the use of IT2FCM-Average results in a value that is well-centered between the uncertainty bounds, providing a more stable and realistic forecast than either bound alone. This approach is particularly useful in environmental applications where short-term fluctuations and sensor noise are expected.

Table 7. MAPE and next value prediction

Method	MAPE (%)	Next Value Prediction
FCM	3.790	2623
IT2FCM (Lower)	3.785	2620
IT2FCM (Upper)	3.781	2621
IT2FCM (Average)	3.783	2620

The quantitative evaluation using Mean Absolute Percentage Error (MAPE) supports this strategy. IT2FCM

(Average) achieved a MAPE of 3.783%, which was lower than FCM (3.790%) and comparable to IT2FCM Higher (3.785%) and Upper (3.781%), as seen in the MAPE summary. These findings reinforce the use of the average value as a pragmatic balance between precision and stability.

4.4 Performance comparison with statistical models

To benchmark the effectiveness of the proposed fuzzy clustering-based prediction models, two widely-used statistical forecasting methods—ARIMA and Double Exponential Smoothing—were also evaluated. Their performance was compared with FCM and IT2FCM, (average) in terms of both prediction accuracy and next value estimation. Figure 6 illustrates the actual vs. predicted concentration data for all methods.

Both FCM and IT2FCM closely follow the true data trajectory, with smoother and more stable prediction lines. In contrast, ARIMA and Double Exponential Smoothing tend to produce more erratic results, especially during abrupt changes in concentration levels. These statistical models either overreact to recent fluctuations or underrepresent rapid rises.

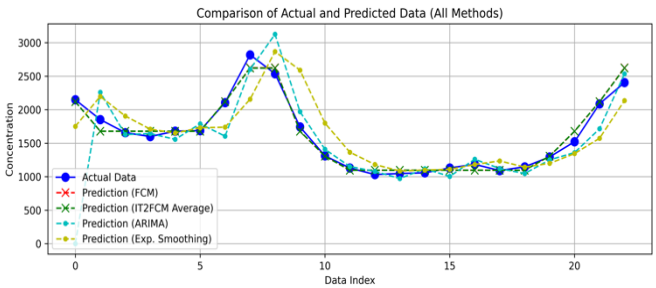


Figure 6. Performance comparison with statistical models

The performance metrics are summarized in the accompanying Table 8. The result is evident that IT2FCM (Average) outperforms all other methods in terms of accuracy, with a MAPE of 3.78%, significantly lower than ARIMA (13.07%) and Double Exponential Smoothing (13.75%). While FCM performs reasonably well, the added flexibility and uncertainty modeling of IT2FCM yield superior and more reliable predictions.

Table 8. Performance comparison with statistical models

Method	MAPE (%)	Next Value Prediction
IT2FCM (Average)	3.78	2620
ARIMA	13.07	2333
Double Exp, Smoothing	13.75	2455

5. DISCUSSION

The results presented in the previous sections demonstrate that the proposed fuzzy clustering-based forecasting approach—especially the IT2FCM—offers notable advantages over conventional statistical models for short-term prediction of CO concentration levels.

The first key finding is that IT2FCM consistently outperformed both FCM and baseline statistical methods (ARIMA, Exponential Smoothing) in terms of accuracy. This is evident from the MAPE, where the optimized IT2FCM (Average) model achieved a significantly lower MAPE of 3.79%. This result is not just a marginal improvement but a substantial leap in accuracy, with a prediction error less than one-third of that produced by traditional models like ARIMA (13.07%) and Exponential Smoothing (13.75%). These improvements validate the hypothesis that fuzzy clustering methods can more effectively model environmental data, which is often noisy, overlapping, and uncertain [52, 53].

Secondly, the use of interval modeling in IT2FCM introduces flexibility by capturing a range of potential centroid rather than a single point estimate. This design aligns well with real-world air quality monitoring, where variability is common, due to sensor noise, atmospheric conditions, and temporal patterns. By averaging the lower and upper bounds of IT2FCM centroids, the model yields a prediction that is both stable and interpretable, bridging precision and caution—essential in applications involving public health or environmental alerts [54].

A crucial contribution of this revised study is the inclusion of a sensitivity analysis, which addresses the parameter selection limitations of the initial approach. This analysis revealed that increasing the number of clusters from 3 to 5 allows the model to capture more granular patterns in the CO concentration data, likely corresponding to more nuanced states of air quality (e.g., very low, low, medium, high, very high). Similarly, the optimal fuzziness interval of [1.8, 2.2] demonstrates that a more constrained, moderate uncertainty footprint yields better results for this specific dataset compared to wider intervals. This data-driven parameter tuning not only validates the chosen parameters but also enhances the robustness and credibility of the proposed forecasting model.

A particularly insightful contribution is the use of the centroid-based prediction mechanism, which differs from traditional regression-based forecasting [55]. This lightweight mechanism avoids overfitting [56], ensures fast computation [57], and enables deployment in edge-computing scenarios such as IoT-based air quality monitoring stations [58].

A key limitation in the initial version of this study—the empirical selection of parameters—has now been directly addressed. By incorporating a systematic sensitivity analysis to optimize the number of clusters and fuzziness levels, the model's parameters are no longer based on heuristic choices but on a data-driven optimization process. This enhancement significantly strengthens the methodology's rigor. However, a remaining limitation is that the current study focuses solely on univariate prediction. Incorporating multivariate inputs (e.g., temperature, humidity, wind) could further enhance the model's forecasting capabilities.

In summary, the IT2FCM-based approach introduces a novel direction in environmental time-series prediction by integrating fuzzy uncertainty modeling, interpretability, and computational simplicity. It represents a promising tool for smart environmental monitoring systems and offers potential integration into early warning platforms.

Future enhancements of the proposed IT2FCM model could include the use of multivariate input features, such as incorporating auxiliary environmental parameters (e.g., temperature, wind speed, humidity) to better capture the dynamic dependencies influencing pollutant concentration. Additionally, integrating adaptive clustering mechanisms—such as using validity indices or data-driven cluster evolution—could allow the model to adjust the number of clusters in response to non-stationary changes in environmental patterns. These improvements could significantly increase the model's predictive robustness and operational flexibility in dynamic IoT deployments.

6. CONCLUSION

This study proposed a centroid-based forecasting approach for predicting ambient CO concentrations using FCM and IT2FCM clustering methods. The model was designed to handle uncertainty and overlapping patterns commonly found in environmental sensor data by leveraging fuzzy logic and interval modeling.

After performing a sensitivity analysis to identify optimal parameters (c=5, m=[1.8, 2.2]), the experimental results demonstrated the profound superiority of the proposed approach. The optimized IT2FCM (Average) model achieved the lowest MAPE at just 3.78%. This result is not only a significant improvement over the standard FCM model (3.79%) but represents a different class of accuracy compared to the traditional statistical models, ARIMA (13.07%) and Double Exponential Smoothing (13.75%). The findings strongly confirm that a data-driven, optimized IT2FCM model offers a far more accurate and reliable method for forecasting in complex environmental scenarios.

In future work to implement IT2-FCM in microcontroller devices, measure execution time, memory usage, and power draw, and compare performance to ARIMA and DES models under identical conditions. Additionally, we will explore implementation optimizations — including memory-efficient representation and simplified fuzzification — to enable real-time uncertainty-aware forecasting on resource-constrained hardware. In future work to implement IT2-FCM in microcontroller devices, measure execution time, memory usage, and power draw, and compare performance to ARIMA and DES models under identical conditions. Additionally, we will explore implementation optimizations—including memory-efficient representation and simplified fuzzification—to enable real-time uncertainty-aware forecasting on resource-constrained hardware.

Overall, the integration of IT2FCM clustering with a centroid-based prediction mechanism presents a promising, interpretable, and accurate approach to real-time environmental forecasting.

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