

## Spherical Storage Tank Development Through Mathematical Modeling of Constituent Sections

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### ABSTRACT

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Spherical shapes are known to be the ideal theoretical profile for a vessel that resists internal pressure mainly because the tensile stresses developed at the wall of a pressurized spherical vessel is uniform in all directions. This makes a sphere very strong structure and the preferred shape for storing high pressure fluids since it is the only known shape that has no weak points. The major reason why spherical tanks are not commonly used is because of the complexity of the procedure for their development and the associated high cost of production. This work has successfully developed a relatively simplified framework for the design of the constituent sections of a spherical storage tank thereby providing a way out of the difficulty normally associated with its development. The framework developed was validated by using it to design and develop a 225 liter capacity spherical tank. Two models of same capacity were developed and evaluated. The first model was developed through 10-section members while the second model used 30-section members.

## 1. INTRODUCTION

A sphere is the optimal geometry for a closed pressure vessel in the sense of being the most structurally efficient shape. A cylindrical vessel is somewhat less efficient for two reasons: (1) the wall stresses vary with direction, (2) closure by end caps can alter significantly the ideal membrane state, requiring additional local reinforcements. However the cylindrical shape may be more convenient to fabricate and transport.

Spherical storage tanks are preferred for storage of high pressure fluids. A spherical tank is considered stronger than its counterparts such as the common fixed roof tank, open top tank, and floating roof tank [1-3]. The even distribution of stresses on the sphere's surfaces, both internally and externally, generally means that there are no weak points. Pressure inside a true spherical tank is known to be identical on every axis. Common storage tanks are comprised of numerous components or pieces of metal that are welded or bolted together, in the field or in the shop. Welds and seams are generally accepted as weak points in high pressure scenarios. The spherical shape creates great strength in resisting these pressures and offers the least amount of exterior surface, which reduces the transfer of warmer ambient temperatures on the overall volume. Spherical storage tanks are more expensive to fabricate than the other common types, and become more economically feasible as the tank design gets larger. Vertical cylindrical tanks resting on the ground are however sometimes used in lieu of spherical tanks on account of their low manufacturing cost and higher capacities [1].

Spherical tank finds great use in applications involving pressure vessels and pressure vessels have been in wide use for many years in petroleum, chemical, military industries and also in nuclear power plants [4, 5]. Other recorded uses of spherical tanks include storing of various liquids ranging from

non-flammable liquids to dangerous flammable or toxic chemicals with explosive nature and are installed almost in each sector of contemporary industry like nuclear, energy, chemical etc. [3]. During ordinary operation, the liquid storage. They are known to be usually subjected to high pressures and temperatures which may be constant or cycling.

## 2. CONVENTIONAL METHODS OF FABRICATING SPHERICAL SHAPES

The conventional method of manufacturing spherical tanks/vessels involves forming of spherical petal blanks after which the petal blanks are fabricated and welded to make spherical vessel. An approach previously developed involves integral forming after fabrication and welding, in which the sheet metal will be first cut into predesigned shapes of flat blanks and thereafter fabricated and welded into a closed single-curvature shell [6]. The shell will then filled be with a liquid after which it will be hydro bulged where resulting internal pressure from the pressure pump will cause plastic deformation in the shell under the effect of the hastening-circle moment which promotes the tendency of an internally pressurized shell to be spherical.

There were two other different procedures developed whereby pressure vessels can be manufactured. The first method was described as a process of spin forming and Tungsten Inert Gas (TIG) welding as shown in Figure 1 while the second method involved blow forming together with solid-state diffusion bonding process [7-9].

Another method for the design and manufacturing process of a spherical tank as described involved manufacturing by the dieless hydra-bulging technology involving cutting, roll-bending, assembling, welding, hydro-bulging, final inspecting and tempering [9].

$$\sigma = \frac{Pr}{2t} \quad (1)$$

where  $\sigma$  represents the tensile stresses in the wall of the vessel, the fluid pressure is  $p$  and  $t$  is the wall thickness

Any section that passes through the center of the sphere yields the same result.

### 3.2 Sections development

Mathematical modeling of spherical tank as previously developed was derived based on the structure and the output transfer function where the spherical tank system was identified as a nonlinear complex structure [12]. The framework developed in this work is however simplified and has been successfully used to develop some prototypes. Another previous Mathematical model of spherical screw lines developed also involved the use of relatively complex single-blade helicoid equation together with the equation of a curve of crossing of surfaces of the sphere and helicoid [13, 14].

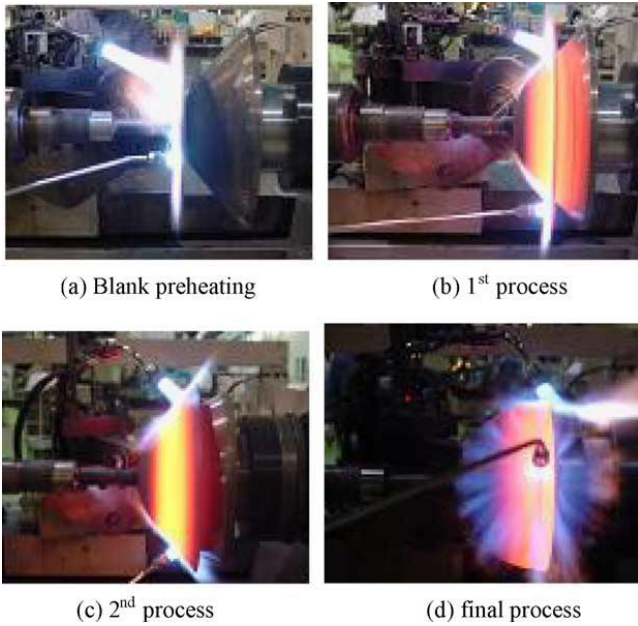
In this work however, the approach employed in developing the spherical tank involved dividing the sphere into two hemispheres and subsequent division of each of the hemispheres into sections. The material used is mild steel and a simplified procedure was established for the section/profile development and the number of sections needed to obtain a near-perfect spherical shape established. Each of the sections developed is as represented by the profile PCD in Figures 3 and 4.

The procedure for the sectional development (Figures 3 and 4) involves the following

- i. Divide the sphere into two hemispheres
- ii. If the number of sections/profiles each hemisphere will be divided into is represented by  $n_1$ , then the arc length CD in Figure 3 will be the ratio of the circumference of one of the great circles by  $n_1$ . That is, length of each of the base of the divided spherical section is

$$X_0 = 2\pi R/n_1 \quad (2)$$

- iii. If the angle subtended by the arc CD at the centre E of the sphere is  $\alpha$ , then  $\alpha = \frac{360}{n_1}$ ; (See Figure 6)
- iv. The length of arc PQ (Figures 3c and 4) is  $L = \frac{90}{360} 2\pi R = \frac{\pi R}{2}$
- v. The development of the sectional profile PCD in Figure 3 with the arc length PQ transforming into the height of the bisecting line PQ of magnitude  $L = \frac{\pi R}{2}$  is shown in Figure 5
- vi. In contrast to the basic mass-balance equation used by others [1, 4, 5, 16] to analyze the minor circles in a hemispherical shape, the method employed here is a simple geometrical analysis of the shape. The arc length AB of one of the minor circles of the sphere in Figure 3a is represented by  $x_5$  in the development shown in Figure 5 with a corresponding minor radius of  $r_5$  and a vertical height  $h_5$  from the sphere horizontal diametral plane as illustrated in Figure 4
- vii. The height PE in Figure 3a represents the radius R of the sphere and can as well be divided into  $n_2$  number of sections with  $n_2$  representing the number of minor circles in each hemisphere as illustrated in Figures 4 and 5
- viii. The angle each arc of the minor circles such as arc AB



**Figure 1.** Hot spinning process of hemisphere for altitude control tank [7]

## 3. MATERIALS AND METHODS

### 3.1 Stress assumptions in spherical vessels

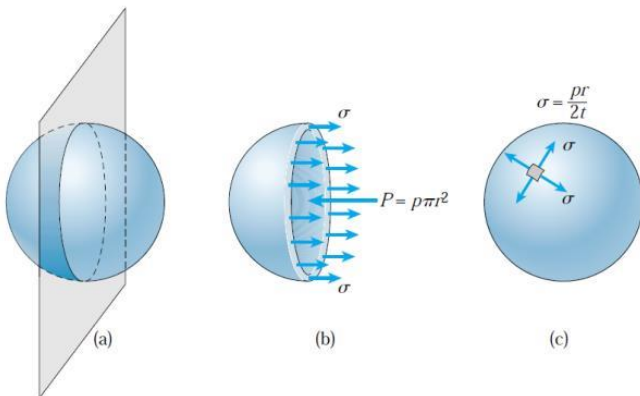
The following assumptions are made when analyzing the stresses in Spherical Vessels

1. All shear stresses are zero:

$$\tau_{r\theta} = \tau_{\theta r} = 0, \tau_{r\phi} = \tau_{\phi r} = 0 \text{ and } \tau_{\theta\phi} = \tau_{\phi\theta} = 0.$$

2. The normal stress  $\sigma_{rr}$  varies from zero on the outside free surface to the negative of the pressure  $p$  on the inside surface. Again there is the need to neglect this value when compared to the other normal stresses and justify this assumption a posteriori.

The force generated on the shell (the surface of revolution) due to axially symmetrical load can be represented by meridional force ( $N_\phi$ ) and circumferential force ( $N_\theta$ ) [6].



**Figure 2.** Tensile stresses  $\sigma$  in a spherical pressure vessel [11]

A more general approach at finding the Tensile Stress  $\sigma$  is by cutting the sphere into two hemispheres [10, 11] as shown in Figure 2 which can be used to show that

in Figure 3a subtends at a distance of radius R from the centre E of the sphere is represented by  $\beta$  as shown in Figure 4.

- ix. Analyses of such sectors of the minor circles described in (viii) above are as shown in Figures 7 to 9 with the accompanying Mathematical formulations.

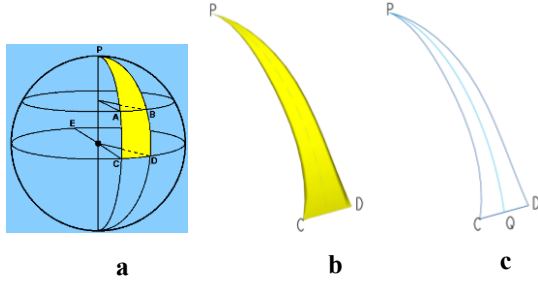


Figure 3. Sectional development

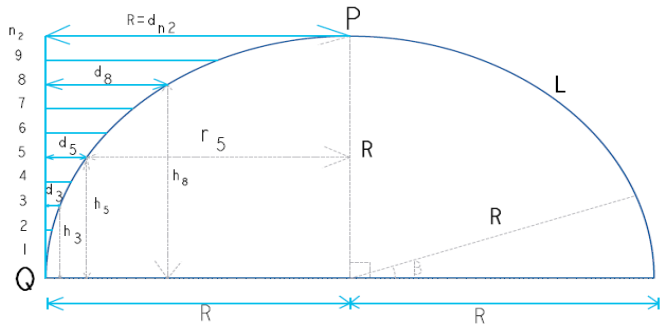


Figure 4. Hemisphere used for mathematical analysis

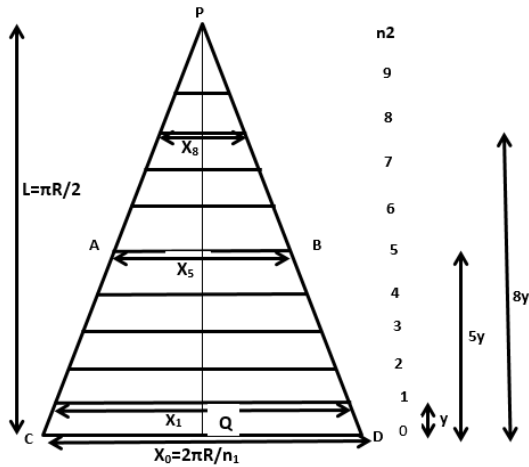


Figure 5. Sectional divisions along minor circles

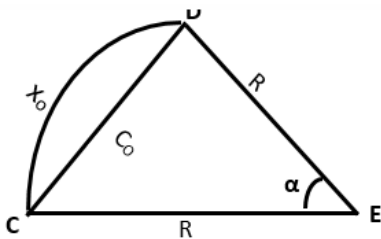


Figure 6. Major circle sectional analysis

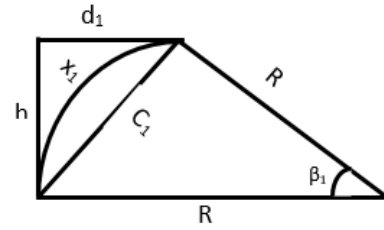


Figure 7. First minor circle sectional analysis

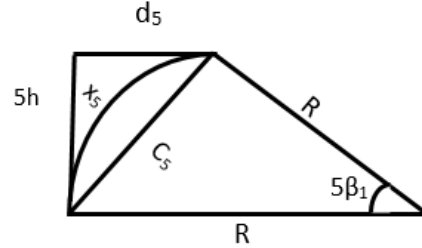


Figure 8. Fifth minor circle sectional analysis

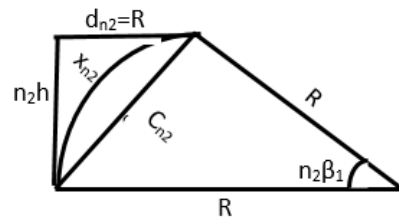


Figure 9.  $n_2$ th minor circle sectional analysis

From Figure 6 and using the Cosine Rule for the cord length  $C_0$  is

$$C_0^2 = R^2 + R^2 - 2.R.R.Cos\alpha$$

$$C_0 = \sqrt{2R^2(1 - Cos\alpha)}$$

$$C_0 = \sqrt{2R^2(1 - Cos(360/n_1))} \quad (3)$$

Length of the arc CD from Figures 5 and 6 is

$$x_0 = \frac{2\pi R}{n_1} = \frac{\alpha}{360} 2\pi R \quad (4)$$

Using Figures 7 to 9;

From the Cosine Rule, Cord length  $C_1$  corresponding to arc length  $x_1$  is

$$C_1^2 = R^2 + R^2 - 2.R.R.Cos\beta_1$$

$$C_1 = \sqrt{2R^2(1 - Cos\beta_1)} \quad (5)$$

where  $\beta_1 = 90/n_2$  (see Figure 4)

From Figure 7, using Pythagoras theorem and taking  $h_1 = h = R/n_2$ ;  $d_1 = \sqrt{C_1^2 - h^2}$ ;  $r_1 = R - d_1$  (see Figure 8).

Similarly from Figure 8,

$$C_5 = \sqrt{2R^2(1 - Cos\beta_5)}$$

where  $\beta_5 = 5\beta_1 = 5 * 90/n_2$  and

$$d_5 = \sqrt{C_5^2 - h_5^2} = \sqrt{C_5^2 - (5h)^2}; \quad r_5 = R - d_5$$

#### 4. RESULTS AND DISCUSSIONS

For  $n_2$  number of divisions which is equivalent to the number of minor circles obtained for each of the two hemispheres of the spherical shape of interest,

$$d_{n_2} = \sqrt{C_{n_2}^2 - h_{n_2}^2} = \sqrt{C_{n_2}^2 - (n_2 \cdot h)^2}; \quad (6)$$

$$r_{n_2} = R - d_{n_2} = R - R = 0$$

$$\text{since } d_{n_2} = R \quad (7)$$

Eq. (7) implies that there is no minor circle at the tip of the sphere.

With reference to Eq. (6);

Since  $d_{n_2} = R$ ; together with  $n_2$  and  $h$  are known

Eq. (6) becomes

$$R = \sqrt{C_{n_2}^2 - (n_2 \cdot h)^2} \quad \text{or}$$

$$\text{At } x_o = \frac{2\pi R}{n_1}; d_o = 0; h_o = 0$$

For our spherical shape to be well formed, the cord length  $C_0$  must be approximately equal to the arc length  $x_0$ .

Using Eqns. (3) and (4) and tabulating Cord Length  $C_0$  and Arc Length  $L$  for different values of  $n_1$  results in Table 1. The higher the number of sections  $n_1$  used, the closer is the value of  $C_0$  to  $L$ .

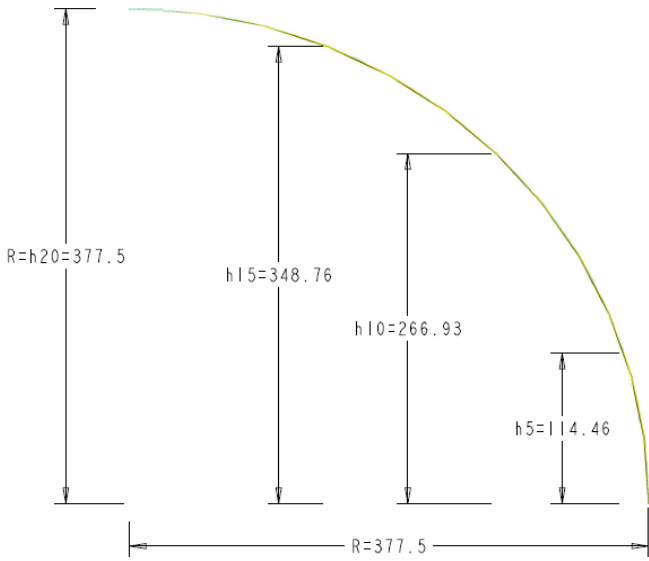
A more comprehensive analysis and comparisons of the generated data from the developed Mathematical models are as shown in Table 2. Figure 10 shows some of the points of reference on the developed section of the sphere as previously depicted in Figure 3 while Figure 11 shows the variation of the linearized sectional height to the actual equivalent sectorial arc length. Figures 12 and 13 illustrate the level of sphericalness achieved by using different number of sections, that is,  $n_1=10$  and  $n_1=30$  for spherical shapes formation which is equivalent to  $n_1=20$  and  $60$  respectively for the corresponding hemispherical shapes. The developed spherical tanks at different levels of welding/fabrication are as shown in Figure 14 to 20.

**Table 1.** Table of values of  $C_0$  approaching  $L$  as  $n_1$  increases

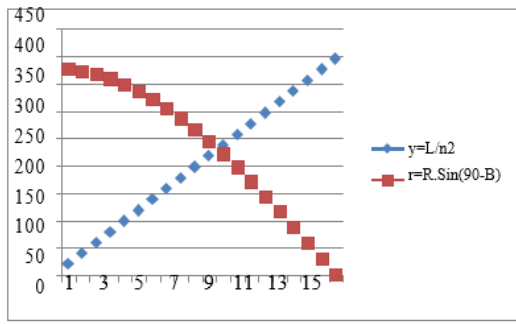
R	n1	A=360/n1	$C_0 = \sqrt{((2 \cdot R^2) - 2 \cdot R \cdot R \cdot \cos(A))}$	$L = (A \cdot \pi \cdot R) / 180$	L-C <sub>0</sub>
377.5	5	72.000	443.778	474.380	30.603
	10	36.000	233.308	237.190	3.882
	15	24.000	156.973	158.127	1.154
	20	18.000	118.108	118.595	0.487
	25	14.400	94.627	94.876	0.250
	30	12.000	78.919	79.063	0.144
	35	10.286	67.678	67.769	0.091
	40	9.000	59.237	59.298	0.061
	45	8.000	52.666	52.709	0.043
	50	7.200	47.407	47.438	0.031
	55	6.545	43.102	43.125	0.023
	60	6.000	39.514	39.532	0.018

**Table 2.** More comprehensive results of mathematical analysis of sections of the developed sphere

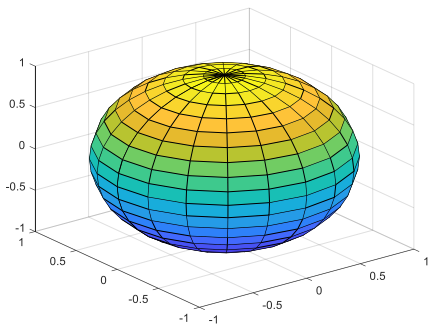
n1 =		n2 =	20.00	R =	377.50	B =	90.00		x <sub>0</sub> = 2.πR/n1
n1 (th)	n2	n2 (th)	R	L	y = L/n2	B = 90/n2	h = R.Cos(90- B)	r = R.Sin(90- B)	x = 2.πr/n1
1	20	1	377.50	592.98	19.77	4.50	29.62	376.34	78.82
2	20	2	377.50	592.98	39.53	9.00	59.05	372.85	78.09
3	20	3	377.50	592.98	59.30	13.50	88.13	367.07	76.88
4	20	4	377.50	592.98	79.06	18.00	116.65	359.02	75.19
5	20	5	377.50	592.98	98.83	22.50	144.46	348.76	73.05
6	20	6	377.50	592.98	118.60	27.00	171.38	336.35	70.45
7	20	7	377.50	592.98	138.36	31.50	197.24	321.87	67.41
8	20	8	377.50	592.98	158.13	36.00	221.89	305.40	63.96
9	20	9	377.50	592.98	177.89	40.50	245.17	287.05	60.12
10	20	10	377.50	592.98	197.66	45.00	266.93	266.93	55.91
11	20	11	377.50	592.98	217.42	49.50	287.05	245.17	51.35
12	20	12	377.50	592.98	237.19	54.00	305.40	221.89	46.47
13	20	13	377.50	592.98	256.96	58.50	321.87	197.24	41.31
14	20	14	377.50	592.98	276.72	63.00	336.35	171.38	35.89
15	20	15	377.50	592.98	296.49	67.50	348.76	144.46	30.26
16	20	16	377.50	592.98	316.25	72.00	359.02	116.65	24.43
17	20	17	377.50	592.98	336.02	76.50	367.07	88.13	18.46
18	20	18	377.50	592.98	355.79	81.00	372.85	59.05	12.37
19	20	19	377.50	592.98	375.55	85.50	376.34	29.62	6.20
20	20	20	377.50	592.98	395.32	90.00	377.50	1.00	0.21



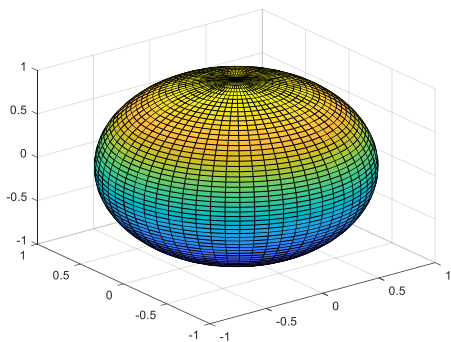
**Figure 10.** Points of reference on the developed section of the sphere



**Figure 11.** Variation of the linearized sectional height to the actual equivalent sectorial arc length



**Figure 12.** Sphericalness achieved with  $n_1=10$



**Figure 13.** Sphericalness achieved with  $n_1=30$



**Figure 14.** First section being welded to the upper hemisphere



**Figure 15.** 10 section-sphere under development with lower hemisphere



**Figure 16.** Developed sphere of 10 sections



**Figure 17.** 30-Section 225 liter capacity spherical tank under development



**Figure 18.** Complete 30-Section 225liter capacity spherical tank developed



**Figure 19.** Body-filed 30-Section 225liter capacity spherical tank developed



**Figure 20.** Grey-Coated 30-Section 225liter capacity spherical tank developed

## 5. CONCLUSIONS

This paper has developed mathematical and procedural frameworks that will facilitate the development of spherical storage tanks. The effect of the number constituent sections on the sphericalness obtainable was also illustrated by developing a 225-litre capacity spherical storage tank using ten and twenty sections respectively. It is hoped that the framework developed will encourage the development and usage of spherical tanks when they are needed and not approximate them to cylindrical tanks with hemispherical caps.

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## NOMENCLATURE

$\tau_{r\theta}$ , $\tau_{\theta r}$ , $\tau_{r\theta}$	Shear Stresses in different planes in N/m <sup>2</sup>
$\tau_{\theta r}$ , $\tau_{\theta\theta}$ , $\tau_{\theta\theta}$	Normal Stress in N/m <sup>2</sup>
$\sigma_{rr}$	Normal Stress in N/m <sup>2</sup>
$n_1$	Number of Sections
$n_2$	Number of Minor Circles
$X_0$	Equivalent base (arc) length of each section of the sphere in mm
$R$	Great Circle Radius in mm
$\alpha$	Angle subtended by an arc of the section in degrees
$L$	Length of an Arc of a constituent great circle
$C_0, C_n$	Equivalent Chord Length in mm
$\beta$	Angle subtended by a minor arc