International Information and Engineering Technology Association
Advancing the World of Information and Engineering

Mathematical Modelling of Engineering Problems
Vol. 12, No. 3, March, 2025, pp. 1053-1063
Journal homepage: http://iieta.org/journals/mmep

Creation of a Mathematical Model for Bedding Manure Grinding and Homogenizing

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https://doi.org/10.18280/mmep.120332

Received: 30 October 2024
Revised: 13 January 2025
Accepted: 20 January 2025
Available online: 31 March 2025

Keywords:
anaerobic digestion, mathematical model, sludge digestion, waste treatment, modelling

ABSTRACT

Bacteria have to adapt to changing conditions in animal waste during anaerobic digestion. The presence of liquid and solid substances in the waste causes this. When a straw residue layer forms, it has negative effects. It reduces biogas release, methane concentration, and the quality of the sludge. It also increases the time the substrate spends in the bioreactor. The substrate is initially ground and cavitated, which changes the particle structure by creating micro-cracks. This reduces temperature differences and ensures equal load on the biocenosis. It also maximizes contact with bacteria and releases natural enzymes, which act as biological catalysts to enhance fermentation in a bioreactor. A macerator, geterorotor pump, and cavitation dispersant are used in a new technology, developed through analysis, to grind particles as large as 2.5mm and homogenize them to micron size using screw and knife devices. Mathematical models in this article describe substrate processing (screw/knife grinding, dispersion, homogenization) to optimize anaerobic fermentation in a bioreactor. Using known theoretical and experimental parameters, the authors derived the mathematical model through numerical methods. We used Mathcad simulations to get the operating parameter results for the processes, then tested these in SCADA Trace IS Mode 6.10.1.

1. INTRODUCTION

1.1 Relevance

Manure is the most important organic fertilizer containing nitrogen, phosphorus, potassium and other nutrients necessary for plants. In animal husbandry, when keeping cattle, bedding manure is mainly received. Processing and receiving from it a substrate for anaerobic fermentation in a bioreactor because of the presence of straw, grass and feed residues in it is a complex and labor-intensive process for most agricultural enterprises. Their presence in the substrate increases the biogas yield, and the methane and nutrient concentration in the sludge. Their disposal causes pollution in nearby settlements, affecting the environment, fields, water bodies, and atmosphere with methane and carbon dioxide emissions, contributing to global warming and the greenhouse effect.

1.2 Current problems and limitations

Current methods of grinding bedding manure often result in the formation of a solid layer of unprocessed straw and feed residues in the bioreactor. This prevents the release of biogas and significantly reduces its yield, as well as the quality of biofertilizer. Their processing on existing equipment requires high energy costs and has low efficiency [1, 2]. The main problems include:

- Low degree of particle grinding which reduces the surface area for interaction of the substrate with bacteria.
- High energy consumption of grinding equipment.
- Heterogeneity of the final product, which leads to uneven fermentation and reduced biogas yield.

1.3 Need for improvements

To improve anaerobic fermentation, it is necessary to develop technologies that will ensure:

- Efficient and uniform grinding of solid particles.
- Reduction of energy costs at the stage of substrate preparation.
- Obtaining a uniform homogeneous composition of the substrate to improve biochemical processes.

1.4 Novelty and contribution

Screw and knife devices, along with a cavitation disperser, are combined in this proposed technology for grinding and homogenization. The novelty of the approach lies in:

- Development of mathematical models for optimization of grinding processes, considering the structure and size of solid particles.
- Reduction of energy costs due to optimization of the operating parameters of the devices.

- Increased efficiency of biogas extraction due to improved homogeneity of the substrate.

1.5 Potential actual impact and application possibilities

The application of the proposed methods can lead to:

- Reduction of livestock waste emissions into fields and water bodies of nearby settlements, greenhouse gases because of more efficient anaerobic digestion.
- Increase in the energy efficiency of biogas plants.
- Production of high-quality biofertilizers, which contributes to the sustainable development of agriculture.

1.6 Broader implications

Improvement of bedding manure processing technologies has broad environmental and economic implications. The application of the proposed methods can significantly reduce environmental pollution, improve the economic sustainability of agricultural enterprises, and promote the development of biogas production technologies. Thus, the proposed study is aimed at overcoming the existing limitations in the processing of litter manure, which will make a significant contribution to the development of biogas production technologies, solving environmental problems and improving the economic sustainability of agricultural associations.

1.7 Research objectives

Dairy farms have manure for cattle comfort. Manure is mixed with straw and feed during animal stalling. Solid manure with straw, hay, and organic waste boosts installation productivity. Crop waste and grain processing waste like rapeseed straw and grain straw remain unused.

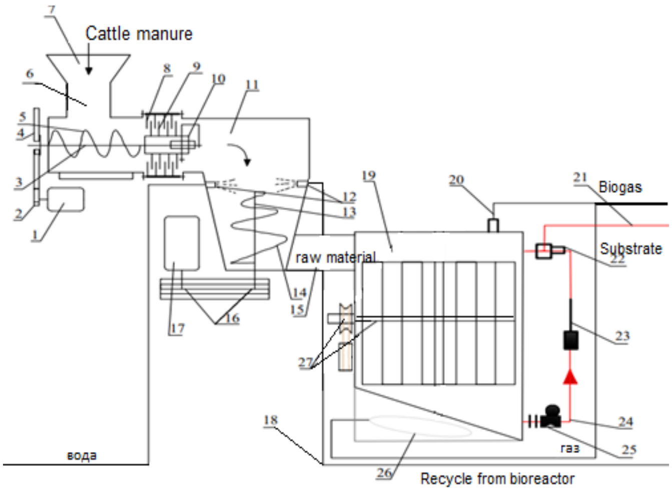
Accordingly, a higher biogas output ([1-3]) is achievable by incorporating dry mass relative to bedding manure in the raw materials.

Biogas plant efficiency improvements depend on component selection for homogeneity and the fineness of pre-grinding. Those solid particles, especially of plant origin, in the mixture should not exceed 12% and must first be grinded to a particle size >30 mm using screw, knife, tearing or flattening devices before feeding into the bioreactor [3, 4].

Utilizing screw and knife mechanisms enables the grinding of crop residues into tinier fragments, yet this step alone is insufficient. Cavitation dispersion comes into play for achieving a finer grind, transforming the raw materials into a uniform mixture. The structure is meticulously engineered to harness cavitation's destructive power, resulting in a consistent and homogeneous feedstock mass. With precisely controlled cavitation, intricate organic fiber bonds at the molecular scale (such as lignin and cellulose) are severed in the biological materials. Consequently, the raw material's dispersion is greatly enhanced, shrinking particle sizes to a range of 0.1-8 microns. This process facilitates the breakdown of biogenic materials by bacteria involved in biogas generation at various stages, as the uniform structure breaks down, increasing the surface area accessible to bacterial action on the biological feedstock [5-7].

Yet, employing screw and knife mechanisms to shred plant residues into minute fragments proves insufficient. During anaerobic digestion, the varying presence of liquid and solid materials within the substrate compels bacteria to adjust to fluctuating environments. This adaptation notably diminishes biogas production, lowers methane levels, and prolongs the substrate's duration in the bioreactor [8]. By employing cavitation destruction, this issue can be addressed effectively, reducing temperature inconsistencies while also achieving an equal distribution of load on the biocenosis and maximizing the contact area of bacteria over time.

Following an analysis, a method for the coarse and fine milling of bedded manure has been suggested. This involves designing screw and knife devices for coarse grinding to a size of up to 2.5 mm, followed by fine milling and homogenization into a uniform substance as small as 5 microns through the use of a macerator, a geterorotor pump, and a cavitation dispersant [9-12] (Figure 1).



Note: 1, 17 electric motors; 2, 4, 16 pulleys; 3, 13 shafts; 5 screws; 6 nozzle; 7 receiving bins; 8 fixed knives; 9 movable knives; 10 automatic shutdown; 11 mixing chamber; 12 nozzles; 14 conical screw mixer; 15 unloading window; 18 pipes with circulating water; 19 psychrophilic mode bioreactor; 20 torch for sending the released gas to the burner; 21 pipe for sending the substrate for fermentation to the thermophilic mode bioreactor; 22 dispersant; 23 geterorotor pump; 24 macerator; 25 coupling; 26 burner; 27 automatic mixer.

Figure 1. Functional diagram of the unit for coarse and fine grinding of cattle bedding manure

1.8 Description

The initial processing unit involves both coarse and fine grinding. Manure from bedding is first channeled into the receiving hopper for screw and blade grinding, after which it is directed to the mixing chamber. This chamber also receives recycled sludge and water. In a cone mixer, the sediment is combined with water, and then the blend is gravity-fed through a discharge outlet into the psychrophilic mode bioreactor. Inside, it undergoes heating and mixing using a stirrer and is broken down with a macerator, a geterorotor pump, and a dispersant. During the fine grinding phase, the daily input is held back from progressing to the next stage until the substrate achieves a homogeneous consistency. Once this condition is met, pump 21 helps transfer it through a heat exchanger to the bioreactor, accommodating psychrophilic, mesophilic, and thermophilic temperature ranges at 25℃, 40℃, and 56℃, respectively.

This article further examines the proposed functional layout for the initial preparation and grinding of bedding manure within a biogas facility.

2. MATERIAL AND METHODS

Mathematical description and methods for calculating coarse grinding of cattle waste in screw and knife grinders.

Screw grinding and calculation method.

Studies of the granulometric composition of grinded particles of manure straw obtained in a screw chopper were performed by scientists [4], who established that the percentage of straw particles in the dose depends on humidity and rotor rotation speed, i.e., at n=200, it is 55% and straw splitting is 18.9%, and at n = 600 it is 86.7% and straw splitting is 45%.

In a one-factor experiment program, various aspects of the chopper-spreader were studied, such as productivity, degree of grinding, and power consumption. The factors examined were rotor speed, number of blades, angle of shear plate installation, and moisture content of the manure. From the group of control factors for the experiment, the following were selected: x_1 rotor rotation speed rpm, x_2 number of blades, pcs, x_3 installation angle of the shear plate, x_4 and manure moisture (Table 1).

The response function is the estimated indicator *Eoc*. It was chosen as an optimization criterion.

$$Eoc = \frac{N_{spec}}{Q}\left[1 + a_{cf}\frac{lcf}{L}\right] \tag{1}$$

where, N_{spec} is the power; Q is the performance; L is the average length of the stems of unchopped straw; a_{cf} is the share of the coarse fraction in the grinded product; l_{cf} is average length of coarse particles.

Determination of specific energy consumption for grinding bedding manure with a screw grinder.

Table 1. Levels of factor variation

Code	Name of Factors	Factor Levels		
		Lower -1	Main 0	Upper +1
x_1	n – rotor speed rpm	200	600	1000
x_2	z – number of blades, pcs.	2	4	6
x_3	y – installation angle of the shear plate	17	61	105
x_4	W – moisture content of manure	18	47	76

The resulting regression equation looks like:

$$\begin{aligned} Y_N = 18.439 - 0.011x_1 + 0.00001x_1^2 - 1.269x_2 \\ - 0.096x_3 + 0.0003x_3^2 - 0.272x_4 \\ + 0.001x_4^2 + 0.0005x_1x_2 \\ + 0.000003x_1x_4 + 0.003x_2x_3 \\ - -0.005x_2x_4 - 0.0005x_3x_4 \end{aligned} \tag{2}$$

According to the research results, it was found that with an optimal speed value of 549.85 rpm, an optimal installation angle of the shear plate of 79.3, and an optimal number of blades of 6, sufficient productivity and crushing of bedding manure up to 50-100 mm are ensured. The moisture content of the manure can be anything within the range of 18-76%, since the chopper will provide the required dimensions within the range of 50-100 mm.

Analysis of the received dependencies shows that with an increase in the rotor speed from 200 to 1000 revolutions, the average length of grinded particles decreased from 20.1 to 4.1 cm, and the grinding degree increased from 1.21 to 5.92. The throughput of the unit increases from 0.109 to 1,240 t/h. It is also noted that along the length (50-100 mm) grinding of straw particles in manure is achieved at n=550 rpm, the number of blades is 6 and the installation angle of the shear plate is 79°.

Also, the reduction ratio depends on the angle of installing a shear plate, for example, when grinding bedding manure with a moisture content of 76% at 600 drum revolutions, at φ =17° and z=6, the grinding coefficient is λ=5.92, and when the angle of the shear plate is φ=105°, λ=3.11.

As a result, it was established that λ the grinding coefficient increases with a larger number of blades. For example, with two blades λ=3.47; with six blades λ=4.1. When grinding manure with a moisture content of W=76%, the maximum grinding ratio is achieved at 500-700 rpm of the shredder rotor. However, the grinding degree received in this case is not sufficient for advanced processing of manure in the bioreactor, so the resulting dose is subsequently sent to knife grinding.

Knife grinding and calculation method

The mathematical calculation of knife grinding of bedding manure can be divided into the following stages [4]:

1). Calculation of the grinding time. The grinding time is determined by the formula:

$$t = V/Q \tag{3}$$

where, *t* is grinding time (h), *V* is volume of manure, m^3, Q is crusher productivity (m^3/h);

The particle size is determined by the formula:

$$d = 2\pi v/(Pq) \tag{4}$$

where, *d* is particle size (mm), *π* is mathematical constant equal to 3.14, *v* is the blade rotation speed (rpm), *P* is the crusher power (kW), *q* is the manure density (kg/m^3).

2). Calculation of the knife grinding of bedding manure in the Python program with the following input data:

Python capacity=10 # m^3/h, power =100 # kW, speed =1000 # rpm.

feed_rate =5 # m^3/h.

The program will produce the following results: grinding time 0.5 h, productivity 20 m^3/h, specific energy intensity of grinding 5 kW/m^3, particle size 2.5 mm. Thus, when particle size is 2.5 mm the dose is supplied to cavitation destruction to obtain a homogeneous environment.

Cavitation destruction and calculation method

To describe the necessary biotechnological processes for producing biogas, we use a tree structure graph [10, 13] (Figure 2).

When constructing a dynamic model of grinding and dispersing bedding manure, we accept the above assumptions. Taking them into account, a dynamic model of the biotechnological processing of bedding manure will look like (Figure 3).

Using fuzzy logical equations that connect the membership functions of input and output variables at the technological and raw material level with the biogas release *Q*:

$$Q = fQ(A, B) \tag{5}$$

where,

A=fa(a1,a2,a3,a4,a5).

B=fb(b1,b2,b3,b4,b5). b2=f(w1,w2).

A is the quality of the feedstock; *B* is the quality of the technological process; a1-origin of raw materials; a2-moisture content of raw materials; a3-presence of impurities; a4-raw material viscosity; a5-density of raw materials; b1-reactor loading volume; b2-temperature quality; b3-mixing quality; b4-raw material processing time; b5-optimal grinding; t1-temperature regime; t2-temperature stability.

Thus, knowing the values of the theoretical parameters of the desired function (Table 2), and using approximation tools, we will perform the analysis. First, let us make a table of auxiliary quantities:

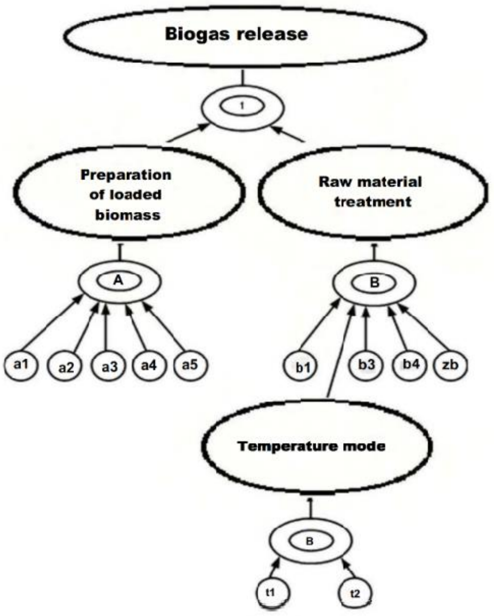


Figure 2. Graphics of biotechnological processes for obtaining biogas

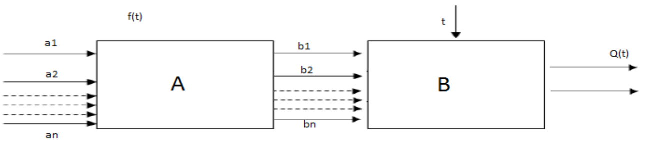


Figure 3. Dynamic model of the bedding manure processing

Table 2. Auxiliary quantities

i	x_i	Y_i	$1/x_i$	$1/x_{2i}$	y_i/x_i
1	1	200	1	1	200
2	2	42.19	0.5	0.25	21.095
3	3	19.44	0.33333333	0.11111111	6.48
4	4	13.22	0.25	0.0625	3.305
5	5	10.93	0.2	0.04	2.186
6	6	9.93	0.16666667	0.02777778	1.655
∑∑	21	295.71	2.45	1.49138889	234.721

Let us calculate the coefficients *a* and *b* for the hyperbolic regression equation $\hat{y} = \alpha + \frac{b}{x}$ using the known formulas:

$$b = \frac{n\sum\frac{y_i}{x_i} - \sum\frac{1}{x_i}\sum y_i}{n\sum\frac{1}{x_i^2} - \left(\sum\frac{1}{x_i}\right)^2} = \frac{6 \times 234.721 - 2.45 \times 295.71}{6 \times 1.49138889 - 2.45^2} \approx 232.1369 \qquad (6)$$

$$a = \frac{1}{n}\sum y_i - \frac{b}{n}\sum\frac{1}{x_i} = \frac{1}{6} \times 295.71 - \frac{232.1369}{6} \times 2.45 \approx -45.5042 \qquad (7)$$

Thus, the required regression equation has the form:

$$f(x) := 161.11 \cdot x^{-1,705} \; x := (1\ 2\ 3\ 4\ 5\ 6)\hat{y} = -45.5042 + \frac{232.1369}{x} \qquad (8)$$

x is the number of passes, and *a* is the particle size of the grinded mixture after x-th pass.

Let us analyze the graph of the regression equation using the Mathcad program (Figure 4).

$$x := (1\ 2\ 3\ 4\ 5\ 6).$$
$$f(x) := -45.5042 + \frac{232{,}1369}{x}.$$
$$[(x > 0) \text{ and } (y > 0)].$$

Correlation index:

$$\mathrm{R} = \sqrt{1 - \frac{\sum(\mathrm{y_i} - \hat{\mathrm{y}}_\mathrm{i})^2}{\sum(\mathrm{y_i} - \bar{\mathrm{y}}_\mathrm{i})^2}} = \sqrt{1 - \frac{15.194042}{27976.6806}} \approx 0.9725 \qquad (9)$$

Determination index: $R^2 = 0.9725^2 \approx 0.9457$.
Average approximation error:

$$\bar{\mathrm{A}} = \frac{1}{\mathrm{n}}\sum\left|\frac{\mathrm{y_i} - \hat{\mathrm{y}}_\mathrm{i}}{\mathrm{y_i}}\right| \times 100\% = \frac{4.033}{6} \times 100\% \approx 67.2172\% \qquad (10)$$

Analyzing the graph, we can conclude that the function $\hat{y} = -45.5042 + \frac{232{,}1369}{x}$ describes the grinding process only up to $x <= 4$ or up to a grinding of 14 microns. With large refinement, the approximation error increases, and the data becomes incorrect.

To find a function describing the grinding process after approximately 14 μn, we will use the “Trend Line” function in Excel (Figure 5).

The average approximation error is 17.89%, which shows a good “fit” of the resulting function to the parameters under study. Let us model this function in Mathcad (Figure 6).

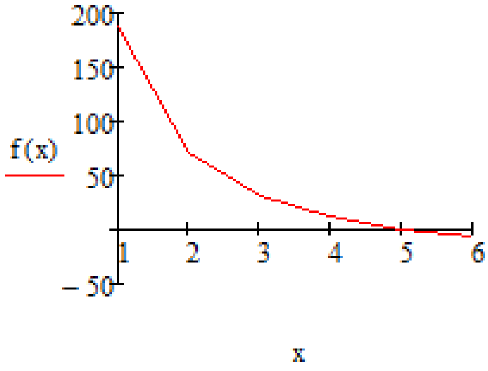


Figure 4. Regression equation graph

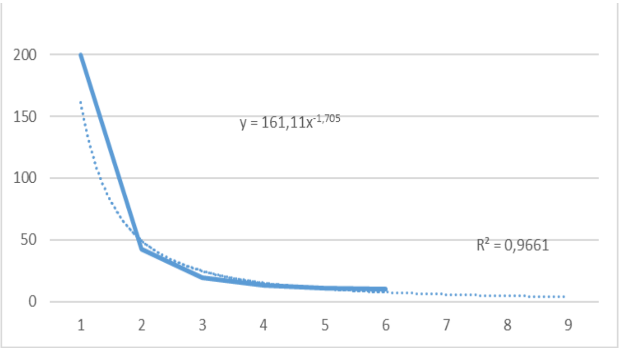


Figure 5. Trend line function

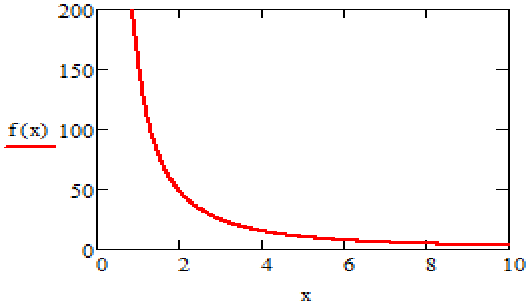


Figure 6. Implementation of a power function in Mathcad

Thus, processing cattle excrement (grinding) is described by a system of equations.

$$\begin{cases} f(x) = 45.5042 + \dfrac{232.1369}{x} \\ f(x) = 161.11x^{-1.705} \end{cases} \tag{11}$$

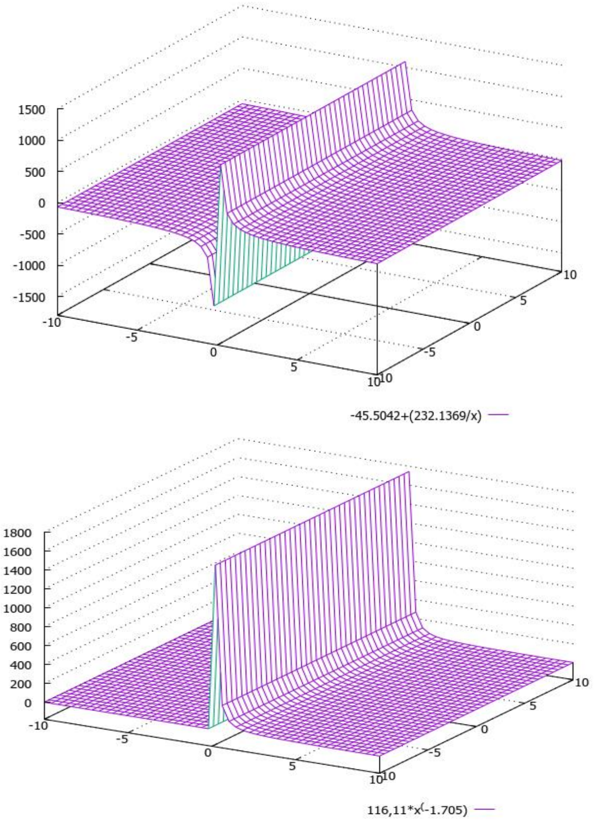


Figure 7. Processing cattle excrement

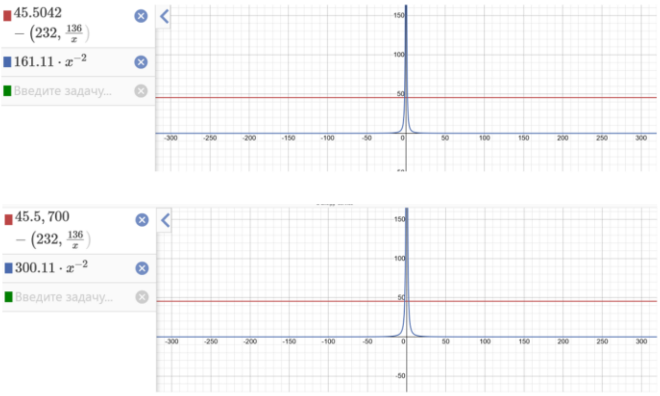
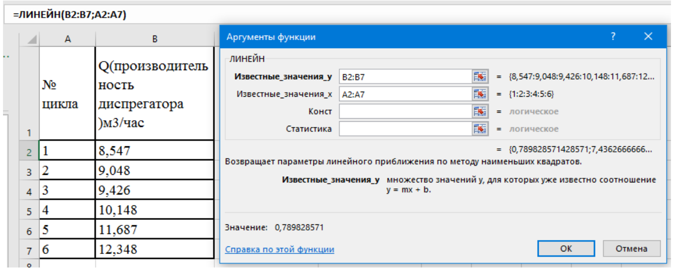


Figure 8. System limit definitions


=ЛИНЕЙН(B2:B7;A2:A7)

	A	B
1	№ цикла	Q(производитель ность диспрегатора)м3/час
2	1	8,547
3	2	9,048
4	3	9,426
5	4	10,148
6	5	11,687
7	6	12,348

Аргументы функции

ЛИНЕЙН

Известные_значения_y B2:B7 = {8,547:9,048:9,426:10,148:11,687:12...

Известные_значения_x A2:A7 = {1:2:3:4:5:6}

Конст = логическое

Статистика = логическое

= {0,789828571428571;7,4362666666...

Возвращает параметры линейного приближения по методу наименьших квадратов.

Известные_значения_y множество значений y, для которых уже известно соотношение y = mx + b.

Значение: 0,789828571

Figure 9. Linear regression

Let us model this function in Mathcad (Figure 7).

To ascertain the system's conditions, the point where the graphs intersect is identified (see Figure 8).

The system of equations based on the obtained data accepts the following restrictions

$$\begin{cases} f(x) = 45.5042 + \dfrac{232.1369}{x}, x < 4(f(x) >= 18) \\ f(x) = 161.11x^{-1.705}, x > 3(f(x) < 18) \end{cases} \tag{12}$$

We need to find the relationship between the productivity of the disperser and the number of passes performed. The relationship of the parameters in the table is described by a linear equation, so we will use the linear regression analysis tool in Excel, the "ЛИНЕЙН" (LINEIN) function (Figure 9).

The required inequality takes the form where *x* is the number of passes $Q=0.79x+7.436$.

To illustrate the general formula for processing raw materials using selected parameters such as processing duration and loaded volume, we establish the processing time equation, determining the required number of cycles per pass as N=V/4.69. Here, V represents the volume of raw materials loaded (in liters), and 4.69 refers to the system pipes' total volume.

The processing time for a single pass is calculated as Tob=N*9.61, where 9.61 corresponds to the total duration of one cycle.

Next, we express Tob as Tob=V*9.61/4.69*x=2.049*V*x, representing the full duration required to complete the raw material processing task. The equation for X is X=Tob/(2.049*V)=0.49 Tob/V. This derived value of X is then substituted back into the system of equations.

$$\begin{gathered} f(x) = 45.5042 + \frac{232.1369*V}{0.49*T_{06}}, x \le 4(x \ge 14\mu n) \\ f(x) = 161.11\left(\frac{0.49*T_{06}}{V}\right)^{-1.705}, x > 4(x < 14\mu n) \end{gathered} \tag{13}$$

Simplifying we get

$$\{f(x) = 45.5042 + 437.7\frac{V}{T_{об}}, x \leq 4 (x \geq 14\mu n)$$
$$f(x) = 161.11\left(\frac{0{,}49 * T_{Об}}{V}\right)^{-1.705}, x > 4 (x < 14\mu n) \tag{14}$$

The time needed to grind a given value (µn) can be calculated using this function:

$$T_{об} = \frac{437.7V}{L + 45.5042} \tag{15}$$

In this context, V represents the volume of raw materials being processed (liters), L denotes the target output dimension (microns), and Tob signifies the necessary duration to finish the process (seconds).

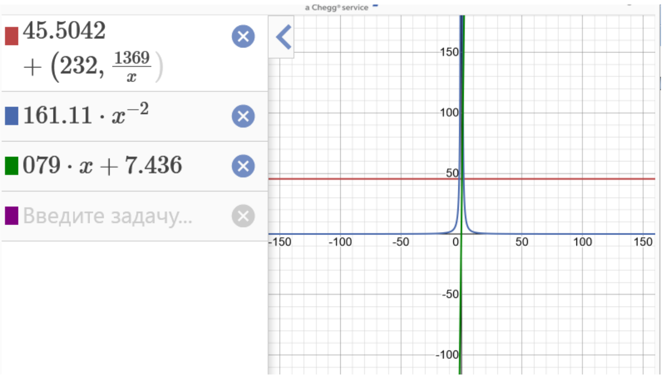
To display the general mathematical model of the biotechnological process for processing cattle excrement, it is necessary to add performance parameters to the function. Considering Q=0.79x+7.436, then x=(Q-7.436)/0.79, we get a system of equations:

$$\begin{gathered} f(x) = -45.5042 + \frac{183.39}{Q - 7.436} \\ = -45.5042 + \frac{183.39}{Vp\omega^2 t - 7.436} \\ \frac{437.7V}{T_{об}} = \frac{183.39}{(Q * 1000/3600) - 7.436} \\ 437.7V * \left(\left(Q * \frac{1000}{3600}\right) - 7.436\right) - 183.39T_{об} = 0 \end{gathered} \tag{16}$$

We receive from the equations an inversely proportional relationship between grinding sizes and productivity.

Figure 10 shows the results of solving the resulting system of equations in the program.

$$\begin{gathered} \mathrm{f}^1 = -45.5042 + \frac{232.1369}{\mathrm{x}} \\ \mathrm{f}^2 = 161.11 \times \mathrm{x}^{-1.705} \\ \mathrm{Q} = 0.79 * \mathrm{x} + 7.436 \end{gathered}$$



🟥 $45.5042 + \left(232, \frac{1369}{x}\right)$

🟦 $161.11 \cdot x^{-2}$

🟩 $079 \cdot x + 7.436$

🟪 Введите задачу...

Figure 10. System of equations for processing raw materials

Modeling the grinding degree of the substrate and the biogas release from the bioreactor

For the mathematical description and modeling of the technological process of substrate methane fermentation in bioreactors, the author chose the Conto model [14], which considers the hydrodynamics and heat exchange of the fermentable substrate and uses the K_6 coefficient, which describes the effect of bubbling mixing on the efficiency of the process.

$$\begin{gathered} \frac{dB}{d\tau} = V \cdot v; \\ V = \frac{B_0 \cdot S \cdot K_6}{\tau_{бр}} \cdot \left(1 - \frac{K}{\mu_T \cdot \tau_{бр} - 1 + K}\right); \\ \frac{dQ}{q\tau} = -u\frac{dQ}{d\tau} - k \cdot \Delta F_{зм} \cdot (T_T - T); \\ Qd\tau = k \cdot \Delta Fзм \cdot T1 - T; \end{gathered} \tag{17}$$

where, V-volume of biogas release (m^3); V-speed; K_b-bubbling mixing coefficient; T_{br}-fermentation time; K-kinetic parameter; Q is the amount of heat required to heat the bioreactor (J); u is the speed of movement of the hot flow through the pipe (m/s); k-heat transfer coefficient (W/(m^2*K)); $\Delta F_{зм}$-heat transfer area of the bioreactor (m^2); T_m-hot flow temperature (K); T is the temperature of the fermentable substrate in the bioreactor (K);

μ_m depends on the temperature of the fermentation process:

$$\mu_m = 0.013 \cdot T - 0.129 \tag{18}$$

where, T-temperature (°C).

Kinetic parameter for bedding manure:

$$K = 0.8 + 0.016 \cdot e^{0.06 \cdot S_0} \tag{19}$$

where, S_0-concentration of organic matter.

The concentration of organic matter is determined by the formula:

$$S_0 = \frac{G_{dry} \cdot S_1}{W_6} \tag{20}$$

$$G_{dry} = G_6 * C_{dry} \tag{21}$$

determined by the formula:

$$W_6 = G_6/\rho_6 \tag{22}$$

where, ρ_b-substrate density (kg/m^3).

Biomass density:

$$p_6 = 1000 + 2.4 \cdot (100 - b) \tag{23}$$

where, b-humidity of the fermented substrate (%).

The daily biogas release from the reactor depends on the daily methane release:

$$L_{БГ} = \frac{L_{CH4}}{C_{CH4}^{\%}} * 100 \tag{24}$$

where, L_{CH4}-daily methane release into biogas (m^3/day).

$$L_{CH4} = \frac{L_{0CH4 * V_{реакт}^{min}}}{k_3} * 100 \tag{25}$$

where, L_{0CH4}-specific daily methane release (m^3/day), V_{min}-90%.

The minimum reactor volume is determined by the formula:

$$V_{min} = W_6 \cdot T_{6p} \tag{26}$$

The specific daily methane yield is determined by the formula:

$$L_{0CH4} = \frac{B_0 \cdot S_0}{T_{\text{бр}}} \cdot \frac{1 - K}{T_{\text{бр}} \cdot \mu_{\text{М}} - 1 + K} \tag{27}$$

The use of this mathematical model allows to determine operating parameters: raw material loading temperature; humidity; process time; volume of biogas produced; stirring frequencies and effective structural and geometric dimensions of the bioreactor; mixing device; heating systems [11, 15].

The results of calculating the biogas release on grinded and non- grinded substrate from manure-free manure of anaerobic digestion using the Conto method are presented in Figure 11.

Analyzing the curves obtained in Figure 11, a significant effect from grinding bedding manure is visible, increasing the biogas release by 2.7 times. In the calculations, the following values were taken: the density of the grinded substrate, Pmeas=1001 kg/m^3, and the density of the non-grinded Pconstant=1020 kg/m^3 of the substrate.

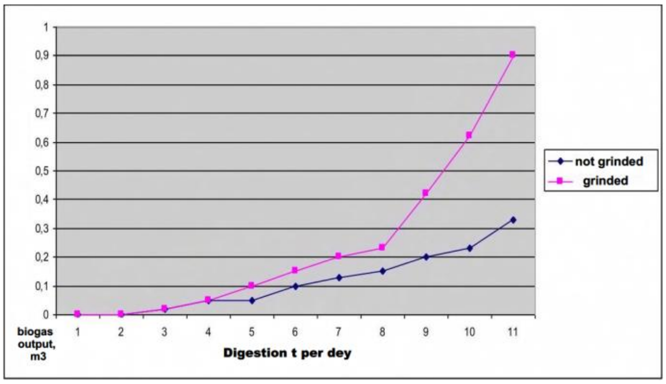


Figure 11. Biogas release from the substrate with grinded and with non-grinded and solids

3. RESULTS AND DISCUSSION

A computer simulation was conducted using an automated system designed to regulate methane fermentation within the Scada Trace Mode 6.1 framework, utilizing the hardware and software package titled "Automated control system for the methane fermentation process," which was created at DULATY University (see Figure 12). The input variables considered include the volume of raw materials loaded, the target particle size, the volume of piping in the raw material processing unit, the initial temperature of the raw materials, and the target temperature for the raw material.

Among the output variables are the number of operational cycles needed; the count of total mass runs; the duration of raw material grinding; heating time; methane concentration; and carbon dioxide concentration. As a guiding strategy for the experiment, we will generate a matrix that encompasses both initial and final values of these parameters. To establish a connection between the size of output particles and biogas concentration, we assume that the volume of the loaded raw materials, the piping volume in the raw material processing unit, the initial temperature of the raw materials, and the desired temperature remain constant, focusing solely on variations in the target particle dimensions.

Consequently, the input values to be used for the experiment will be illustrated in Figure 12.

Testing of the data received from computer modeling of the grinding of livestock waste on the collected waste treatment plant showed a slight deviation, however, this may be caused by many factors not considered, the instability of the parameters of the raw materials supplied for processing. Based on the results, we filled out Table 3 with output parameters.

МИНИСТЕРСТВО ЮСТИЦИИ РЕСПУБЛИКИ КАЗАХСТАН

УДОСТОВЕРЕНИЕ АВТОРА

№ 103153

Настоящим удостоверяется, что Айтбаева Замира Кишкембаевна (KZ)

и Тлебаев Манат Бейшенович (KZ); Тажиева Рысты Нарбаевна (KZ); Доумчариева Жанагул Есеновна (KZ); Байжарикова Марина Аитмухановна (KZ)

является(ются) автором(ами) полезной модели

(11) 2943

(54) Способ переработки органических отходов

(73) *Патентообладатель:* Республиканское государственное предприятие на праве хозяйственного ведения "Таразский государственный университет имени М.Х.Дулати" Министерства образования и науки Республики Казахстан (KZ)

(21) 2016/0566.2
(22) 21.10.2016

Вице-министр юстиции
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Figure 12. Hardware and software complex "Automated control system for the methane fermentation process" introduction of parameter values for a computer experiment

Table 3. Results of the obtained values of the computer experiment

Parameter	Number of Experiments (N)							
	1	2	3	4	5	6	7	8
Quantity of materials loaded	600	600	600	600	600	600	600	600
Target particle dimensions	2000	1000	200	100	50	20	10	5
Capacity of the piping within the raw material handling system	5	5	5	5	5	5	5	5
Heating element power	50	50	50	50	50	50	50	50
Initial temperature of raw materials	10	10	10	10	10	10	10	10
Desired raw material temperature	25	25	25	25	25	25	25	25

After the experiments, a computer analysis of the data and testing was performed; a subprogram for controlling the grinding degree of bedding manure in the raw material processing device is responsible for the grinding and homogeneity of the fermentable substrate fed into the bioreactors [12, 16-20]. The subroutine controls the grinding degree at each pass; if further grinding of the size of solid particles of the substrate stops, it stops working to save energy (Figures 13 and 14).

According to the calculation results of the algorithm for the fermentation process of cattle manure at a raw material moisture content of 92%, a temperature of 25°C, and a fermentation time of 10 days, at a temperature of 37°C, and a fermentation time of 9 days, and at a temperature of 52°C, and a fermentation time of 8 days, the following results were obtained: optimal value of the microorganism growth rate 0.1960 days, biomass density 1019, organic matter concentration 28.5320 kg/m^3, kinetic parameter for cattle manure 0.8089, biogas output rate 6.7589; biogas output volume from the bioreactor 37.8498 m^3, methane output volume from the bioreactor 26.4948 m^3.

Findings

1). Evaluation of the technological procedure for the unit designed for initial processing of bedding manure within a hardware and software system demonstrated the process's manageability alongside the grinding outcomes. Figure 15 illustrates the control system schematic for grinding bedding manure using the screw, knife, and cavitation disperser.

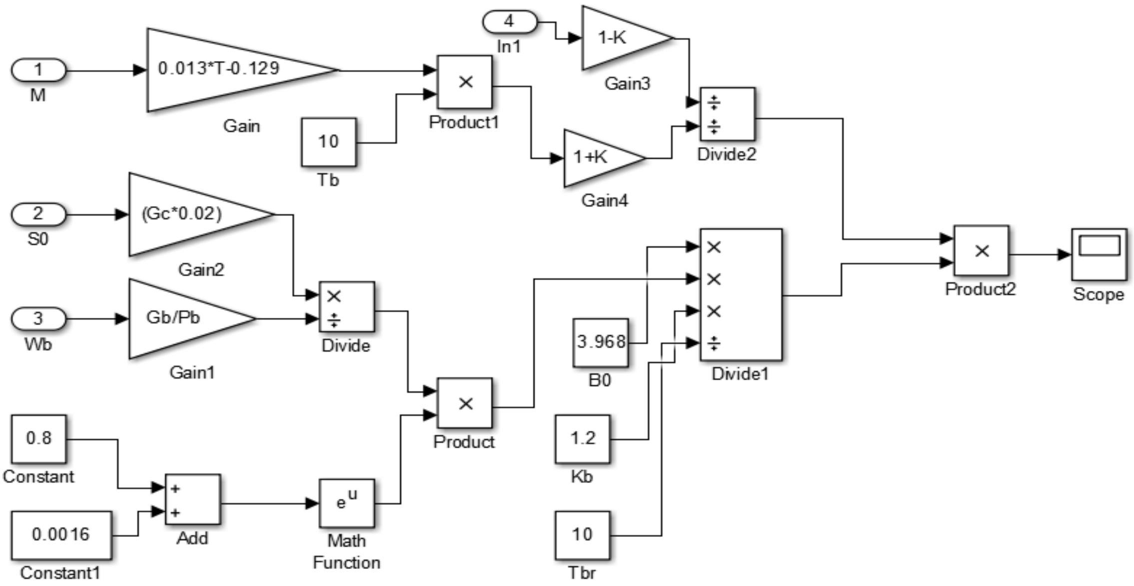
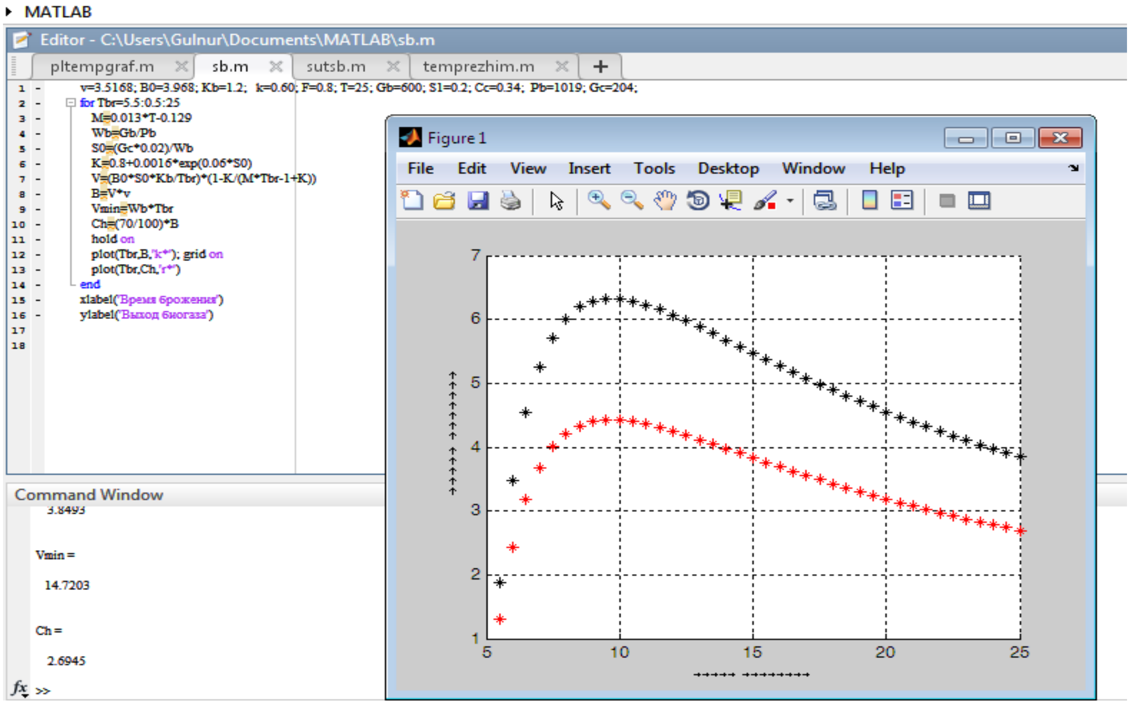


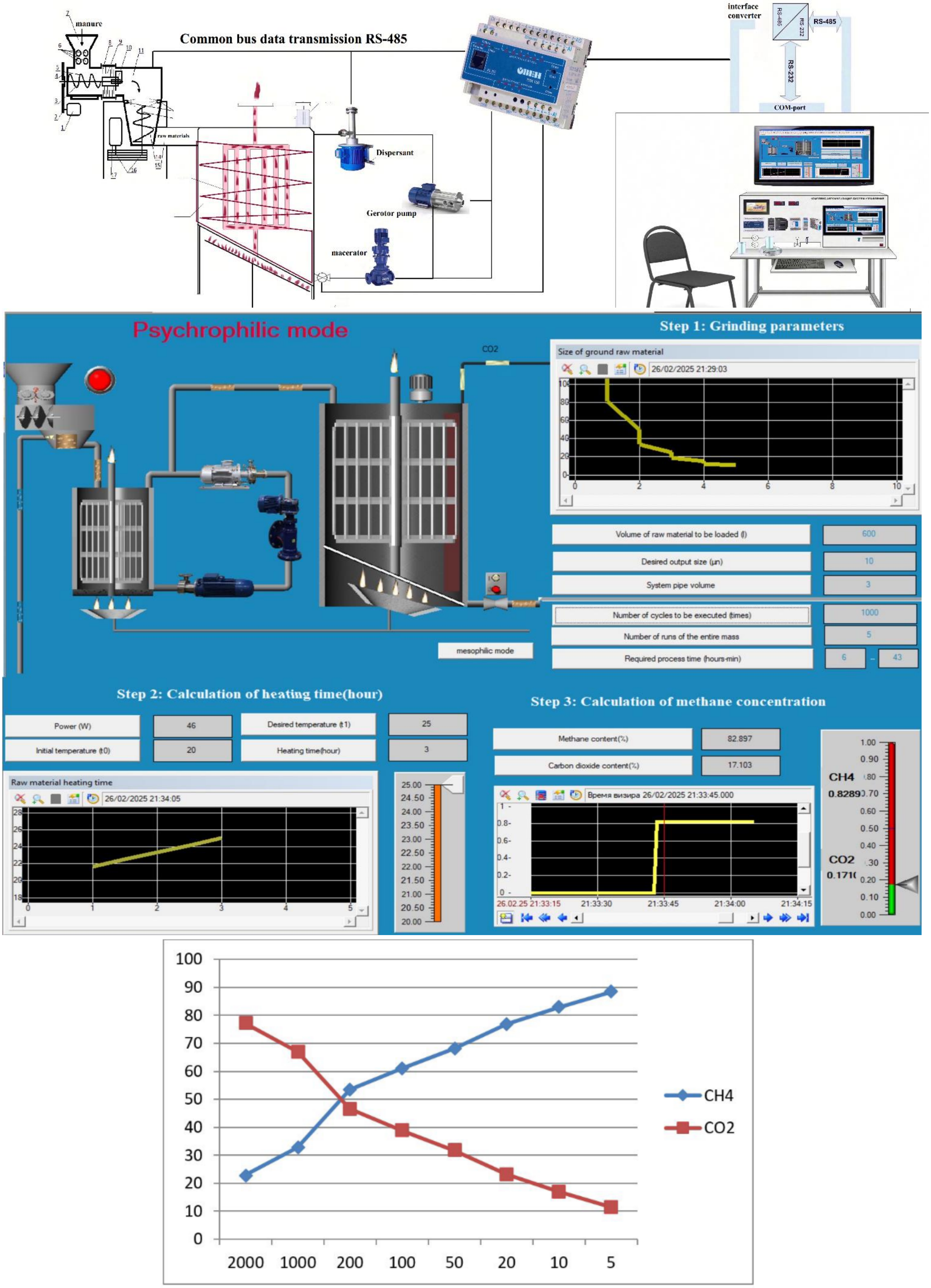
Figure 13. Schematic diagram of the biomass fermentation process model in the MATLAB environment



```
v=3.5168; B0=3.968; Kb=1.2;  k=0.60; F=0.8; T=25; Gb=600; S1=0.2; Cc=0.34;  Pb=1019; Gc=204;
for Tbr=5.5:0.5:25
   M=0.013*T-0.129
   Wb=Gb/Pb
   S0=(Gc*0.02)/Wb
   K=0.8+0.0016*exp(0.06*S0)
   V=(B0*S0*Kb/Tbr)*(1-K/(M*Tbr-1+K))
   B=V*v
   Vmin=Wb*Tbr
   Ch=(70/100)*B
   hold on
   plot(Tbr,B,'k*'); grid on
   plot(Tbr,Ch,'r*')
end
xlabel('Время брожения')
ylabel('Выход биогаза')
```

```
   3.8493


Vmin =

  14.7203


Ch =

   2.6945

>>
```

Figure 14. Dependence of the daily output of biogas and methane from the reactor on the temperature of the fermentation process (at 92% humidity)



Psychrophilic mode

Step 1: Grinding parameters

Size of ground raw material 26/02/2025 21:29:03

Volume of raw material to be loaded (l): 600
Desired output size (µn): 10
System pipe volume: 3
Number of cycles to be executed (times): 1000
Number of runs of the entire mass: 5
Required process time (hours-min): 6 – 43

mesophilic mode

Step 2: Calculation of heating time(hour)

Power (W): 46
Initial temperature (t0): 20
Desired temperature (t1): 25
Heating time(hour): 3

Raw material heating time 26/02/2025 21:34:05

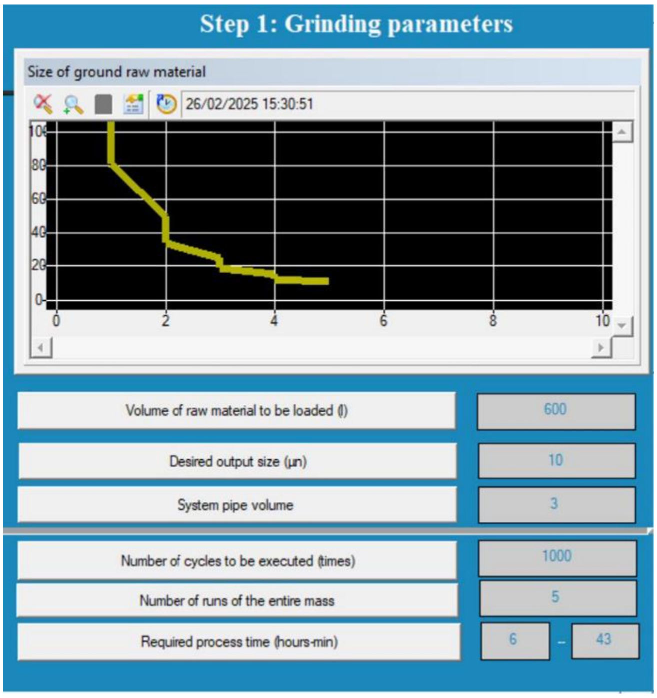
Step 3: Calculation of methane concentration

Methane content(%): 82.897
Carbon dioxide content(%): 17.103

Время визира 26/02/2025 21:33:45.000

CH4 0.8289
CO2 0.171

Figure 15. Control circuit for grinding bedding manure in screw and knife grinders and then in a cavitation disperser



Volume of raw material to be loaded (l)	600
Desired output size (µn)	10
System pipe volume	3
Number of cycles to be executed (times)	1000
Number of runs of the entire mass	5
Required process time (hours-min)	6 – 43

Figure 16. General view of the grinding trend with a description of objects

Figure 17. Installation, startup and control of a device for processing bedding manure and methane digestion in psychrophilic temperature conditions in production

2). Grinding of organic raw materials. Upon initiating the profiler, the system's starting parameters are configured. These include the amount of materials introduced, the desired output dimensions, and the capacity of the pipes within the raw material processing unit. The process also involves visualizing the container's fill level as it combines organic raw materials with water. Here, the pump for pumping biomass into the screw is in the "Off" position. When you press it, all the biomass goes into a container for temporary storage, grinding and fermentation of biomass. So, to start the grinding process, you need to switch the clutch switch to the "On" position. The elements of the organic raw material processing unit will start working, and the biomass itself in the temporary storage tank, using a sine wave generator, will begin cyclic movements in the direction from top to bottom. The "Trend" element displaying the graph will show step by step the relationship between processing time and raw material size. And in the fields for displaying output data, such as the number of required cycles and the process execution time, the results obtained are displayed in Figure 16.

3). Under actual manufacturing scenarios, various uncontrollable or hard-to-manage elements alongside the primary factors examined in a computer simulation can affect process metrics. Consequently, the ideal conditions determined through computer experimentation need further validation and refinement during implementation in a real production environment, as illustrated in Figure 17.

4. CONCLUSIONS

The authors revealed that in cavitation destructors, raw materials, before being fed into a biogas plant, are subjected to preliminary crushing, complex processing, and exposure to powerful high-frequency (frequency about 20kHz) hydrodynamic radiation. The impact destroys intracellular membranes, which allows extracting the contents of fibers and plant grains and thus significantly accelerates the depth and speed of raw material processing. The destruction of intracellular membranes allows the contents of plant grains and fibers and thereby significantly accelerates and deepens the processing of raw materials. When using a cavitation destructor to increase the biogas release, besides the degree of raw material grinding, the structure of particles as microcracks also changes, which allows bacteria to perform anaerobic digestion even faster.

Computer mathematical modeling has shown that the grinding of manure organic raw materials by screw, knife and cavitation dispersion is determined by the obtained approximating function $y = -45.5042 + \frac{2321369}{x}$, which describes the grinding process only up to x<=4 or up to grinding at 14 microns. Then we found that the data was incorrect. For the finer grinding of solids in the substrate, an approximating function is used in the following system of equations:

$$\begin{cases} f(x) = 45.5042 + \dfrac{232.1369}{x} \\ f(x) = 161.11x^{-1.705} \end{cases}$$

The researchers have developed software in the Trace Mode IS, which visually determines the optimal values and controls the following parameters: raw material grinding time; heating time; regulation of heating element power; starting and stopping the mixer; supply and selection of substrate; temperature stabilization; calculation of methane concentration. (The program has a security certificate of the Republic of Kazakhstan No. 11169 dated on June 29, 2020).

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