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A Computational Model for Transverse Thermal Displacements in Symmetric Sandwich Beam by Using Higher Order Shear Deformation Theory

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ABSTRACT

The effect of change in the material orthotropy and length to thickness ratios on transverse thermal displacement of sandwich beam is studied and presented in this paper. Further, the thermal transverse displacements are also obtained for various coefficients of thermal expansion ratios. The parabolic shear deformation beam theory (PSDT), trigonometric shear deformation beam theory (TSDT) and classical beam theory (CBT) are used to obtain transverse thermal displacements. The equations of motion are developed by using virtual work principle. A three layer simply supported sandwich beam is considered for the analysis. A computer program in FORTRAN language is developed to evaluate central thermal displacements for various aspect ratios, various modular ratios and various thermal expansion coefficient ratios. The displacement field of the parabolic and trigonometric beam theories takes into account stretching, bending and effect of shear deformation. The transverse displacements obtained by parabolic, trigonometric and classical beam theories are compared with each other and the results available in the literature wherever possible. It has been noticed that the transverse thermal displacement is low for high degree of orthotropy and low coefficient of thermal expansion ratio results in low transverse thermal displacement.

1. INTRODUCTION

Sandwich beams consisting of face sheets and core material has wide application in civil, mechanical and aerospace engineering. The sandwich beams are light weight and have high stiffness. The durability and structural efficiency of sandwich beam is very high. Therefore, these beams have wide application in shipbuilding also.

In the literature, sandwich beams are analysed by using various structural theories. Sandwich beams with functionally graded material as core and homogeneous face sheets were analysed by Deng et al. [1]. The quadrature element method was used by author. The thermal post buckling behaviour of sandwich beam with functionally graded core was presented by Li et al. [2]. The core in this sandwich beam has negative poison's ratio with honeycomb. The sandwich beams with cellular core were analysed for deflections under dynamic thermal loads by Mamoon et al. [3]. The functionally graded beam with piezoelectric layers was studied for thermal buckling behaviour by Ellali et al. [4]. The sandwich beam having functionally graded core were studied for free vibration by using complementary functions method and presented by Yildirim [5]. The plane stress condition was taken into consideration by author. A review on analysis of functionally graded sandwich beam using analytical methods based on refined beam theories was addressed by Sayyad and Ghugal [6]. The thermal bending analysis of a sandwich beam with simple support in thermal environment was studied and hand over by Kulkarni and Ghugal [7]. The authors used order theory with cubic equation. A numerical study of sandwich beams on flexural buckling was studied by Chen et al. [8]. In the study, author examine thermally induced non-uniform cross sectional properties of sandwich beam. Sandwich beams with functionally graded material face sheets under flexural load were analysed by Theotokoglou and Mallios [9]. The vibration of sandwich beam with thermally induced non-uniform cross-sectional properties was studied by Chen et al. [10]. The sandwich beam with porous functionally graded material core in between two isotropic face sheets were analysed for free thermal vibration by Hung et al. [11]. The soft core sandwich plates were analysed for free vibration by Sayyad and Ghugal [12] by using four variable trigonometric theory. The authors considered the transverse shear in the theory rotary inertia. A refined trigonometric beam theory was developed by Sayyad et al. [13] for flexural examination of laminated and sandwich structural beams. Analysis of sandwich and composite laminated beams was examined and presented by Pawar et al. [14] with the use of novel shear and transverse deformation theory. A review of buckling, vibration and flexure of sandwich and composite laminated beam based on layer-wise theories, equivalent single layer theories, zig-zag theories and elasticity exact theory was hand over by Sayyad and Ghugal [15]. The curved sandwich and laminated beams were analysed and presented by Avhad and Sayyad [16] by using a new quasi-3D polynomial type beam theory. Homogeneous plates under flexure and vibration of thick plate by using higher order theory was presented by Murty [17, 18]. The exact benchmarking solution for laminated plates and plates under cylindrical bending was provided by Bhaskar et al. [19]. The thermal post buckling of sandwich beam resting on elastic foundation and acted upon by uniform thermal rise has been presented by Wang et al. [20]. A non-linear bending analysis of sandwich beam consisting of carbon nanotube reinforced composite sheet subjected to thermo-mechanical loading was presented by Lal and Markad [21]. The buckling of functionally graded sandwich beam in thermal environment with constant, linear and nonlinear thermal load was presented by Daikh et al. [22]. The passive constrained layer damping (PCLD) sandwich beams was analysed by Karmi et al. [23] in thermal environment. The frequency and temperature dependent of viscoelastic laws are considered by authors. A sandwich beam theory based on Bernoulli's hypothesis was developed by Krajcinovic [24] for sandwich beams under static loads. A sandwich beam with thick viscoelastic core was analyzed and presented by Cortes and Sarria [25] by using finite element approach. The non prismatic beams play a vital role in engineering. These beams are often subjected to dynamic and static loads. The free vibration of such non-prismatic beams was presented by Jebur and Alansari [26].

After going through literature, it has been noticed that a study on transverse thermal displacement of sandwich beam in thermal environment when aspect ratio, modular ratio i.e. degree of orthotropy and thermal expansion coefficient ratio changes is unavailable or lacking in the literature.

Hence, the purpose of this work is to examine the transverse thermal displacements when degree of orthotropy and coefficient of thermal expansion ratio changes. This novel approach will be useful in satellite structures and automotive engineering where low thermal coefficient and high stiffness is required.

2. MATHEMATICAL FRAMEWORK

In the mathematical framework, the concept of virtual work principle is used to derive the equations of motion and boundary conditions. The integration by parts is used for further solution. The simply supported symmetric sandwich beam is considered for thermal analysis. The effect of thermal load is included in strain. The Hook's law and corresponding constitutive relationship is taken in to consideration during analysis. The thermal load or temperature variation is assumed to be linear across the thickness of the beam. The z axis is along the thickness of sandwich beam and assumed as positive in downward direction. Further, close-form Navier's technique is adopted to develop analytical solution. A computer program in FORTRAN-90 has been developed to obtain thermal central displacements in sandwich beam when aspect ratio, modular ratio and coefficient of thermal expansion ratio changes.

3. THE SANDWICH BEAM

The coordinate system and configuration of the sandwich beam is shown in Figure 1(a). The sandwich beam (0/core/0) has soft core in between two orthotropic layers as shown in Figure 1(b). The two orthotropic layers have fibres parallel to x axis. The angle between fibres and x axis is zero. The two orthotropic layers are known as face sheets.

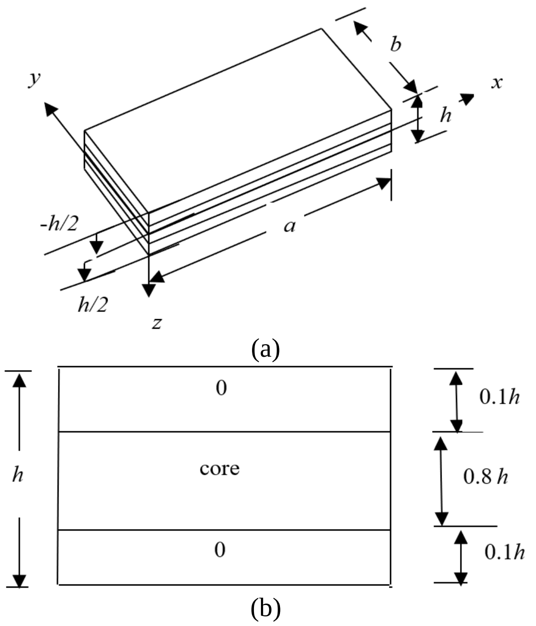

Figure 1. (a) The coordinate system and geometry of sandwich beam; (b) Thickness of each layer of sandwich beam

The upper and lower face sheets are 0^0 laminated orthotropic layers having a thickness of 0.1 h each. The soft core has a thickness of 0.8 h. Where h is the overall thickness of sandwich beam. The z axis is along the thickness of the sandwich beam and considered as positive in the downward direction. The face sheets are thin and soft core is lightweight and thick. The upper surface of sandwich beam represents (z=-h/2). The upper surface is subjected to thermal load $T(x, z)$. The coordinates of sandwich beam can be expressed as given below

$$0 \leq x \leq a, 0 \leq y \leq b, -\frac{h}{2} \leq z \leq \frac{h}{2} \tag{1}$$

4. DISPLACEMENT FIELD OF PSDT

The displacement field or kinematic of the PSDT is based on the following assumption:

1. The components of displacement along x axis and z axis are represented by u and w respectively.
2. Since the sandwich beam is a dimensional problem the displacement along y is considered as zero.
3. The axial displacement (u) along x axis consists of stretching (u_0), bending $\left(-z\frac{\partial w(x)}{\partial x}\right)$, and shear component $z\left(1-\frac{4z^2}{3h^2}\right)$.
4. The shear component in the TSDT is $\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$.
5. The shear component in the classical beam theory is zero.
6. The transverse thermal displacement (w) is considered as a function of x only.
7. The beam is acted upon by thermal load only.
8. No body forces are considered in the thermal analysis.
9. The perfect bond is assumed between layers of beam.

Based on the above assumptions the kinematic or displacement field of the PSDT can be written as below

$$u(x,z) = u_0(x) - z\frac{\partial w(x)}{\partial x} + z\left(1-\frac{4z^2}{3h^2}\right)\varphi_x(x) \tag{2}$$

$$w(x,z) = w(x) \tag{3}$$

where, $u(x,z)$ represents the displacement along x axis and $w(x,z)$ represents transverse displacement along z direction. The axial displacement along x is the function x and z, whereas the transverse displacement along z is the function of x. The mid-plane displacement $u_0(x)$ and shear slope φ_x are the functions of x only. The displacement field has cubic term representing the effect of shear deformation. The present theory (PSDT) excludes the transverse normal effect. The transverse displacement along z direction is the function of x only. This is the limitation of the present theory.

4.1 Strain displacement relation

The strains (normal and shear) are evaluated from fundamentals of theory of elasticity. These strains are shown by following equations

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + z\left(1 - \frac{4z^2}{3h^2}\right)\frac{\partial \varphi_x}{\partial x} \tag{4}$$

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \left(1 - \frac{4z^2}{h^2}\right)\varphi_x \tag{5}$$

4.2 Constitutive relations (CR)

The constitutive relation for sandwich beam is expressed by the following equation

$$\begin{Bmatrix}\sigma_x \\ \tau_{zx}\end{Bmatrix}_k = \begin{bmatrix}\bar{Q}_{11} & 0 \\ 0 & \bar{Q}_{55}\end{bmatrix}_k \begin{Bmatrix}\varepsilon_x - \alpha_x T \\ \gamma_{zx}\end{Bmatrix}_k \tag{6}$$

The reduced stiffness coefficients in the above equation are represented by $\bar{Q}_{ij}^{(k)}$. These coefficients are computed by using the following equation

$$\bar{Q}_{11}^{(k)} = \frac{E_1^{(k)}}{\left(1 - \mu_{12}^{(k)}\mu_{21}^{(k)}\right)}, \bar{Q}_{55}^{(k)} = \bar{G}_{13}^{(k)} \tag{7}$$

The Young's modulus, shear modulus and Poisson's ratio are represented by E, G and μ_{ij}, respectively.

4.3 Thermal variation (Through thickness)

The temperature distribution across the thickness (h) of the soft core sandwich beam is considered as given below

$$T(x,z) = \frac{2z}{h}T_1(x) \tag{8}$$

The change in temperature is represented by T in the above Eq. (8). The temperature change is considered as a function of x and z. The thermal load T_1 is linearly varying through the depth of sandwich beam. The thermal load T_1 is considered as a function of x. The sinusoidal distribution of temperature changeis as given below

$$T(x,z) = \left(\frac{2z}{h}T_1\right) sin\left(\frac{m\pi x}{a}\right) \tag{9}$$

The positive integer of sine series is represented by m in the above Eq. (9). This is further used in Navier's solution of simply supported sandwich beam.

5. EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

The concept of virtual work displacements in analytical form is used to obtain equations of motion. The principle when applied to beam leads to following equation

$$b\int_{z=-\frac{h}{2}}^{z=\frac{h}{2}}\int_{x=0}^{x=a}(\sigma_x\delta\varepsilon_x + \tau_{zx}\delta\gamma_{zx})\,dxdz - \int_{x=0}^{x=a} q(x)\delta w dx = 0 \tag{10}$$

The unknown displacement variables and corresponding variation is denoted by δ in the above equation. The governing equations of static equilibrium or equations of motion are deduced from the above Eq. (10) by using integrating by parts and setting the coefficients of $\partial u_0, \partial w$ and $\partial \varphi_x$ to zero separately. The obtained equations of motion are as given below

$$-A_{11}\frac{\partial^2 u_0}{\partial^2 x^2} + B_{11}\frac{\partial^3 w}{\partial x^3} - \left(B_{11} - \frac{4}{3h^2}E_{11}\right)\frac{\partial^2 \phi_x}{\partial x^2} + TB_{11}\frac{2}{h}\frac{\partial T_1}{\partial x} = 0 \tag{11}$$

$$-B_{11}\frac{\partial^3 u_0}{\partial x^3} + D_{11}\frac{\partial^4 w}{\partial x^4} - \left(D_{11} - \frac{4}{3h^2}F_{11}\right)\frac{\partial^3 \phi_x}{\partial x^3} + \frac{2}{h}TD_{11}\frac{\partial^2 T_1}{\partial x^2} = q \tag{12}$$

$$\begin{aligned}&-\left(B_{11} - \frac{4}{3h^2}E_{11}\right)\frac{\partial^2 u_0}{\partial x^2} + \left(D_{11} - \frac{4}{3h^2}F_{11}\right)\frac{\partial^3 w}{\partial x^3} - \left(D_{11} + \frac{16}{9h^4}H_{11} - \frac{8}{3h^2}F_{11}\right)\frac{\partial^2 \phi_x}{\partial x^2} \\ &+ \left(TD_{11} - \frac{4}{3h^2}TF_{11}\right)\left(\frac{2}{h}\right)\frac{\partial T_1}{\partial x} + \left(A_{55} - \frac{4}{h^2}D_{55}\right)\phi_x - \left(D_{55} - \frac{4}{h^2}F_{55}\right)\frac{4}{h^2}\phi_x = 0\end{aligned} \tag{13}$$

The associated boundary conditions along edges $x = 0$ and $x = a$ are as follows

$$A_{11}\frac{\partial u_0}{\partial x} - B_{11}\frac{\partial^2 w}{\partial x^2} + \left(B_{11} - \frac{4}{3h^2}E_{11}\right)\frac{\partial \phi_x}{\partial x} - TB_{11}\frac{2}{h}T_1 = 0, \text{ or } u_0 \text{ is prescribed.} \tag{14}$$

$$-B_{11}\frac{\partial u_0}{\partial x} + D_{11}\frac{\partial^2 w}{\partial x^2} - \left(D_{11} - \frac{4}{3h^2}F_{11}\right)\frac{\partial \phi_x}{\partial x} + TD_{11}\frac{2}{h}T_1 = 0, \text{ or } \frac{dw}{dx} \text{ is prescribed.} \tag{15}$$

$$B_{11}\frac{\partial^2 u_0}{\partial x^2} - D_{11}\frac{\partial^3 w}{\partial x^3} + \left(D_{11} - \frac{4}{3h^2}F_{11}\right)\frac{\partial^2 \phi_x}{\partial x^2} - \frac{2}{h}TD_{11}\frac{\partial T_1}{\partial x} = 0, \text{ or } w \text{ is prescribed.} \tag{16}$$

$$\begin{aligned}&\left(B_{11} - \frac{4}{3h^2}E_{11}\right)\frac{\partial u_0}{\partial x} - \left(D_{11} - \frac{4}{3h^2}F_{11}\right)\frac{\partial^2 w}{\partial x^2} + \left(D_{11} + \frac{16}{9h^4}H_{11} - \frac{8}{3h^2}F_{11}\right)\frac{\partial \varphi_x}{\partial x} \\ &- \left(TD_{11} - \frac{4}{3h^2}TF_{11}\right)\frac{2}{h}T_1 = 0, \text{ or } \varphi_x \text{ is prescribed.}\end{aligned} \tag{17}$$

In Eqs. (14) through (17), the natural boundary conditions are represented by left hand side equations and the kinematic boundary conditions are shown by right-hand side ones. The stiffness coefficients $(A_{ij}, B_{ij}, \ldots\ldots\ldots)$ in the above equations are defined as follows

$$(A_{11}, B_{11}, D_{11}, E_{11}, F_{11}, H_{11}) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{Q}_{11}^{(k)} (1, z, z^2, z^3, z^4, z^6)\, dz \tag{18}$$

$$(A_{55}, D_{55}, F_{55}) = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \bar{Q}_{55}^{(k)} (1, z^2, z^4)\, dz \tag{19}$$

$$(TB_{11}, TD_{11}, TF_{11}) = \sum_{k=1}^{N} (\alpha_x) \int_{z_k}^{z_{k+1}} \bar{Q}_{11}^{(k)} (z, z^2, z^4) dz \tag{20}$$

For three layers symmetric sandwich beam u_0, B_{11}, E_{11} and $TB_{11} = 0$.

6. APPLICATION OF THEORY

The performance or efficiency of parabolic (PSDT) shear deformation beam theory is examined by applying it to sandwich beam subjected temperature field. A three-layer soft core sandwich beam (simply supported) is considered for thermal analysis. The central transverse displacements are evaluated for different aspect ratios and modular ratios. The effect of change in coefficient of thermal expansion ratio on transverse displacement is also examined for different aspect ratios. The material properties are as shown below:

Material properties for orthotropic layer (0^0)

The Graphite-Epoxy material is considered for orthotropic layer [19].

$$\frac{E_L}{E_T} = 25, \frac{G_{LT}}{E_T} = 0.5, \frac{G_{TT}}{E_T} = 0.2,$$
$$\mu_{LT} = \mu_{TT} = 0.25, \frac{\alpha_T}{\alpha_L} = 1125$$

The direction parallel to the fibre is represented by *L* (Longitudinal) and the direction perpendicular to the fibre is represented by *T* (Transverse). The thermal expansion coefficient in the fibre direction is shown by α_L. The thermal expansion coefficient in the transverse direction is represented by α_T.

The properties of material are as given below [13]:

For 0^0 layers: $(Q_{11} = 25), (Q_{55} = 0.5)$
For soft core: $(Q_{11} = 4), (Q_{55} = 0.06)$

The thermal expansion coefficients:

$$\left(\frac{\alpha_L}{\alpha_0} = 0.333\right), \left(\frac{\alpha_T}{\alpha_0} = 1\right)$$

For soft core: $\alpha^{core} = \alpha_L = \alpha_T = 1.36\alpha_0$.

The normalization factor is α_0 for the thermal expansion coefficients.

6.1 Closed-form Navier technique

Below mentioned are the edge conditions used for sandwich beam (0/core/0) along the edges $x = 0$ and $x = a$.

$$w = 0, M_x = 0, N_x = 0, M_x^s = 0 \tag{21}$$

Navier's technique is used to get displacement variables. The solution form for (u_0), (w) and (φ_x) that satisfies the boundary conditions exactly as given below

$$u_0(x) = \sum_{m=1,3,5}^{\infty} u_{0m} \cos\left(\frac{m\pi x}{a}\right) \tag{22}$$

$$w(x) = \sum_{m=1,3,5}^{\infty} w_m \sin\left(\frac{m\pi x}{a}\right) \tag{23}$$

$$\varphi_x(x) = \sum_{m=1,3,5}^{\infty} \varphi_{xm} \cos\left(\frac{m\pi x}{a}\right) \tag{24}$$

where, u_{0m}, w_m and φ_{xm} are to be determined and known as series coefficients. The thermal load is expanded given below.

$$T_1(x) = \sum_{m=1}^{\infty} T_{1m} \sin\left(\frac{m\pi x}{a}\right) \tag{25}$$

where, *m* is positive integer and T_{1m} is the coefficient of single Fourier sine series expansion for thermal load as follows

$$(TB_{11}, TD_{11}, TF_{11}) = \sum_{k=1}^{N} (\alpha_x) \int_{z_k}^{z_{k+1}} \bar{Q}_{11}^{(k)} (z, z^2, z^4) dz \tag{26}$$

The intensity of thermal load is denoted by T_1 in the above Eq. (25). Substitution of Eqs. (22) through (25) into equations of motion (11), (12) and (13) gives simultaneous equations as given below

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{Bmatrix} u_{0m} \\ w_m \\ \varphi_{xm} \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} \tag{27}$$

The stiffness coefficients $(k_{11}, k_{12,} \ldots.)$ of the stiffness matrix $[k]$ in Eq. (27) are defined as follows

$$k_{11} = A_{11}\frac{m^2\pi^2}{a^2}, k_{12} = k_{21} = -B_{11}\frac{m^3\pi^3}{a^3},$$
$$k_{13} = k_{31} = \left(B_{11} - \frac{4}{3h^2}E_{11}\right)\frac{m^2\pi^2}{a^2}$$
$$k_{22} = D_{11}\frac{m^4\pi^4}{a^4}, k_{23} = k_{32} = -\left(D_{11} - \frac{4}{3h^2}F_{11}\right)\frac{m^3\pi^3}{a^3}$$
$$k_{33} = \left(D_{11} + \frac{16}{9h^4}H_{11} - \frac{8}{3h^2}F_{11}\right)\frac{m^2\pi^2}{a^2} + \left(A_{55} - \frac{4}{h^2}D_{55}\right) - \left(D_{55} - \frac{4}{h^2}F_{55}\right)\frac{4}{h^2}$$

The force vector elements $\{f_1, f_2, f_3\}$ in Eq. (27) are as follows

$$f_1 = -TB_{11}\frac{2}{h}\frac{m\pi}{a}T_{1m},\ f_2 = \frac{2}{h}TD_{11}\frac{m^2\pi^2}{a^2}T_{1m}$$
$$f_3 = -\left(TD_{11} - \frac{4}{3h^2}TF_{11}\right)\frac{m\pi}{a}\left(\frac{2}{h}\right)T_{1m}$$

After solving the above set of algebraic equations, the values of u_{0m}, w_m and ϕ_{xm} can be determined. Once obtained the values of u_{0m}, w_m and φ_{xm} one can then calculate all central thermal displacements within the beam by using Eqs. (5)-(9) and (11).

6.2 Numerical results and discussion

Thermal central displacements are computed in the soft core sandwich beam. The boundary condition is simply supported and the sandwich beam is subjected to pure thermal load varying linearly through the thickness of sandwich beam.

The central thermal transverse displacements are obtained for different aspect ratios, i.e., length to thickness ratios $\left(S = \frac{a}{h}\right)$, various modular ratios $\left(\frac{E_1}{E_2}\right)$ and various coefficient of thermal expansion ratios $\left(\frac{\alpha_2}{\alpha_1}\right)$. These central thermal displacements are given in following dimensionless form for the discussion purpose

$$\bar{w}\left(\frac{a}{2}, 0\right) = \frac{hw}{\alpha_L T_1 a^2}$$

Table 1. Normalized transverse central thermal displacements for different aspect ratios under sinusoidal and uniform thermal load in symmetric sandwich beam

Beam Configuration	Aspect Ratio	PSDT $\bar{w}$	TSDT $\bar{w}$	HBT [7]	CBT $\bar{w}$
		Sinusoidal Thermal Load			
0/core/0	100	0.2924	0.2922	0.2924	0.2924
	50	0.2924	0.2923	-	0.2924
	25	0.2924	0.2927	-	0.2924
	20	0.2924	0.2931	-	0.2924
	12.5	0.2924	0.2951	-	0.2924
	10	0.2924	0.2986	0.2923	0.2924
	6.25	0.2923	0.2699	-	0.2924
	5	0.2922	0.2812	-	0.2924
	4	0.2917	0.2839	0.2882	0.2924
	2	0.2499	0.2859	-	0.2924
		Uniformly Distributed Load			
0/core/0	100	0.3571	0.3576	0.3607	0.3572
	50	0.3571	0.3571	-	0.3572
	25	0.3572	0.3569	-	0.3572
	20	0.3572	0.3593	-	0.3572
	12.5	0.3572	0.3609	-	0.3572
	10	0.3572	0.3654	0.3606	0.3572
	6.25	0.3571	0.3292	-	0.3572
	5	0.3570	0.3433	-	0.3572
	4	0.3565	0.3467	0.3566	0.3572
	2	0.3170	0.3492	-	0.3572

The effect of change of aspect ratios on transverse displacement is shown in the Table 1. The results of transverse thermal displacements computed by PSDT and TSDT shows a significant difference within aspect ratios 2 to 20. This is due to shape function of parabolic and TSDT. The results obtained by TSDT are higher as compared to results obtained by PSDT for thick sandwich beam. These displacements are more or less similar after aspect ratio 20. The results computed by CBT are irrespective of aspect ratio. The results obtained by these three theories are same for aspect ratio 100. This indicates that higher order theories (PSDT and TSDT) are applicable to thin and moderately thick beams, whereas CBT is applicable to thin beam. The assumptions of classical beam theory ignore the shear deformation effect, whereas actually the shear deformation effect is very prominent in laminated sandwich beams. The higher order theories (PSDT and TSDT) include the shear deformation effect in the theory; therefore, these theories are applicable to thick and thin plate. The use of higher order theory is practical take away and becomes necessary for the analysis of sandwich beams. Figure 2 shows the effect of the change of aspect ratio on transverse thermal displacement under sinusoidal thermal load. The transverse thermal displacements obtained by PSDT, TSDT, and CBT are compared with the results available in the literature [7] wherever possible, as shown in Table 1.

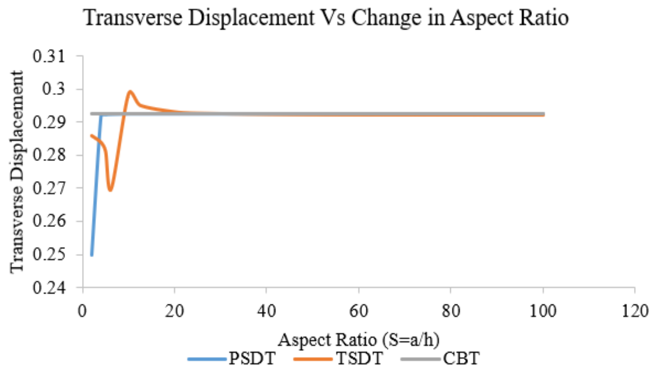

Figure 2. Effect of change of aspect ratio on central transverse thermal displacement

Table 2. Normalized central transverse displacement under sinusoidal thermal load for various modular ratios for aspect ratio 4

Beam Configuration	$\frac{E_1}{E_2}$	S	PSDT $\bar{w}$	TSDT $\bar{w}$	CBT $\bar{w}$
0/core/0	5	4	0.4870	0.5026	0.4875
	10	4	0.3866	0.7013	0.3872
	15	4	0.3385	0.3146	0.3391
	20	4	0.3103	0.2984	0.3109
	25	4	0.2917	0.2839	0.2924
	30	4	0.2786	0.2728	0.2792
	35	4	0.2688	0.2642	0.2695
	40	4	0.2612	0.2575	0.2619

Table 3. Normalized central transverse displacement under sinusoidal thermal load for various modular ratios for aspect ratio 10

Beam Configuration	$\frac{E_1}{E_2}$	S	PSDT $\bar{w}$	TSDT $\bar{w}$	CBT $\bar{w}$
0/core/0	5	10	0.4875	0.4892	0.4875
	10	10	0.3872	0.3899	0.3872
	15	10	0.3391	0.3428	0.3391
	20	10	0.3109	0.3157	0.3109
	25	10	0.2924	0.2986	0.2924
	30	10	0.2792	0.2881	0.2792
	35	10	0.2695	0.2836	0.2695
	40	10	0.2619	0.2951	0.2619

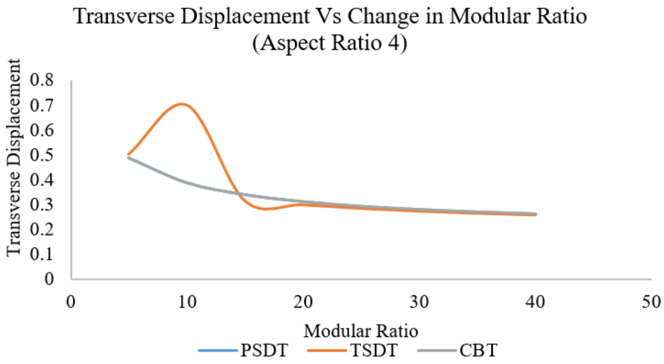

Figure 3. The effect of change of modular ratio on transverse displacement beam subjected to single sine thermal load for aspect ratio 4

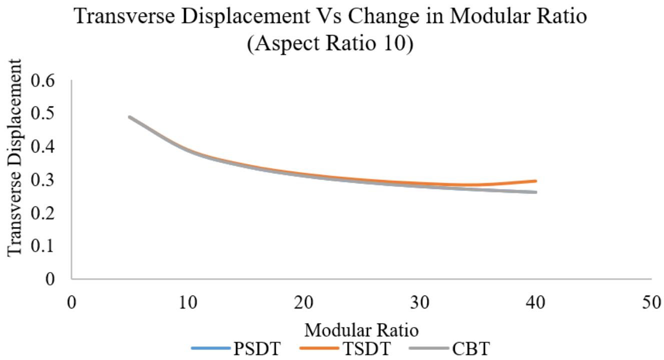

Figure 4. The effect of change of modular ratio on transverse displacement of beam subjected to single sine thermal load for aspect ratio 10

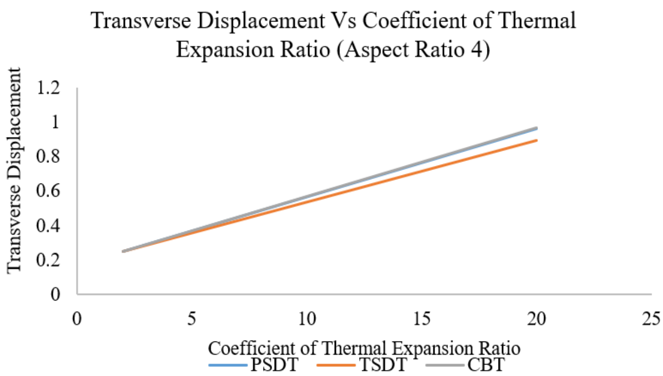

Figure 5. The effect of change of coefficient of thermal expansion ratio on transverse displacement of beam subjected to single sine thermal load for aspect ratio 4

The change in modular ratio, i.e., material orthotropy affects the transverse displacement as shown in Tables 2 and 3. The results of transverse displacements obtained by TSDT for aspect ratio 4 shows a considerable variation as compared to PSDT results when modular ratio changes from 5 to 20. However, these results are more or less similar for aspect ratio 10. This is clearly noted from Figures 3 and 4. It is noted that for low modular ratio the transverse displacement is high and for high modular ratio the transverse displacement is low. Thus, these results are useful in selection of material for sandwich beam. The sandwich beams used in automotive engineering need low thermal transverse displacement and high stiffness. The modular ratio between 25 to 40 shows lower thermal deformations. This high modulus is preferable in automotive engineering for better thermal performance.

The effect of change of thermal expansion coefficient ratio on transverse displacement is shown in Tables 4 and 5. The ratio of coefficient of thermal expansion is considered from 2 to 20. It is noted that the transverse thermal displacements are directly proportional to coefficient of thermal expansion ratios. This linear variation is shown in Figure 5. The transverse displacement is low for low ratio of coefficient of thermal expansion. The sandwich beams used in satellite structures, automotive engineering, suspension components in vehicle need low or minimum thermal expansion and high stiffness. This practical take away is useful for future study.

Table 4. Dimensionless transverse thermal displacements for various ratios of coefficients of thermal expansions under sinusoidal thermal load for aspect ratio (S) 4

Beam Configuration	$\frac{\alpha_2}{\alpha_1}$	S	PSDT $\bar{w}$	TSDT $\bar{w}$	CBT $\bar{w}$
0/core/0	2	4	0.2524	0.2480	0.2527
	3	4	0.2917	0.2839	0.2924
	4	4	0.3310	0.3197	0.3320
	5	4	0.3703	0.3556	0.3716
	6	4	0.4096	0.3914	0.4112
	7	4	0.4490	0.4273	0.4508
	8	4	0.4883	0.4632	0.4904
	9	4	0.5276	0.4990	0.5301
	10	4	0.5669	0.5349	0.5697
	11	4	0.6062	0.5708	0.6093
	12	4	0.6456	0.6066	0.6489
	13	4	0.6849	0.6425	0.6885
	14	4	0.7242	0.6784	0.7281
	15	4	0.7635	0.7142	0.7678
	16	4	0.8028	0.7501	0.8074
	17	4	0.8422	0.7859	0.8470
	18	4	0.8815	0.8218	0.8866
	19	4	0.9208	0.8577	0.9262
	20	4	0.9601	0.8935	0.9658

Note: Modular ratio is considered as 25

Table 5. Dimensionless transverse thermal displacements in beam subjected to single sine thermal load for various coefficients of thermal expansions ratios for aspect ratio (S) 10

Beam Configuration	$\frac{\alpha_2}{\alpha_1}$	S	PSDT $\bar{w}$	TSDT $\bar{w}$	CBT $\bar{w}$
0/core/0	2	10	0.2527	0.2563	0.2527
	3	10	0.2924	0.2986	0.2924
	4	10	0.3320	0.3410	0.3320
	5	10	0.3716	0.3834	0.3716
	6	10	0.4112	0.4258	0.4112
	7	10	0.4508	0.4682	0.4508
	8	10	0.4904	0.5106	0.4904
	9	10	0.5300	0.5530	0.5301
	10	10	0.5697	0.5954	0.5697
	11	10	0.6093	0.6378	0.6093
	12	10	0.6489	0.6802	0.6489
	13	10	0.6885	0.7226	0.6885
	14	10	0.7281	0.7650	0.7281
	15	10	0.7677	0.8073	0.7678
	16	10	0.8074	0.8497	0.8074
	17	10	0.8470	0.8921	0.8470
	18	10	0.8866	0.9345	0.8866
	19	10	0.9262	0.9769	0.9262
	20	10	0.9658	1.0193	0.9658

Note: Modular ratio is considered as 25

7. CONCLUSIONS

The effect of change of aspect ratio, change of degree of orthotropy and change of coefficient of thermal expansion ratio on central transverse displacements is studied by using parabolic, trigonometric and classical beam theories. Transverse displacements obtained for thick beam by using TSDT shows a noticeable change as compared to other theories. It has been noticed that the trigonometric and parabolic theories are suitable for thick and thin sandwich beam. The transverse thermal displacements computed by classical beam theory for various aspect ratios are irrespective of aspect ratio. This is because of eliminating the shear deformation effect in the theory. This theory (CBT) is applicable to thin beams only. Further, it is observed that transverse displacement is low for high modular ratio. The low coefficient of thermal expansion material for face sheets and soft core may be used. The satellite structures need minimum thermal expansion high stiffness.

Thus, the usefulness of the effect of degree of orthotropy on transverse thermal displacement is seen in satellite structures. It is seen that the transverse displacement is directly proportional to coefficient of thermal expansion ratio. The low ratio of coefficient of thermal expansion results in low transverse thermal displacement. This will be useful in automotive engineering where sandwich beam is used to improve the thermal performance required. The performance of these theories (PSDT and TSDT) may be improved in the future by adding transverse normal effect in the theory and its application to sandwich beams subjected to combined thermal and mechanical load.

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NOMENCLATURE

ε_x	Normal strain
γ_{zx}	Shear strain
σ_x	Normal stress
τ_{zx}	Transverse shear stress
E	Young's modulus
G	Shear modulus
μ_{ij}	Poisson's ratio
$A_{ij}, B_{ij}, ..$	Stiffness coefficients
α_0	Normalized factor for thermal expansion coefficient
$[k]$	Stiffness coefficient
$\{f\}$	Force vector
$\left(S = \frac{a}{h}\right)$	Aspect ratio
$\left(\frac{E_1}{E_2}\right)$	Modular ratio
$\left(\frac{\alpha_2}{\alpha_1}\right) = \frac{\alpha_T}{\alpha_L}$	Coefficient of thermal expansion ratio