

## New Meta-Heuristic Computer-Oriented Algorithms to Solve Unconstrained Optimization Problems



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### ABSTRACT

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Intelligent algorithms are among the most suitable methods for practical applications, as well as conjugate gradient algorithms (CGAs), which are very useful in solving multidimensional optimization problems. Therefore, combining optimization methods with heuristic algorithms is an exciting research idea to enhance the efficiency of solutions. This paper discusses the suitability of a new parameter in the conjugate gradient algorithm with the mayfly algorithm for various optimization problems. The proposed strategy is characterized by its high ability to reach the optimal point consistently, thanks to the careful choice of the step size resulting from the combination of the two methods. We also prove the global convergence of the algorithm based on basic assumptions, and finally the new method shows superior performance when compared with other algorithms in the same field.

## 1. INTRODUCTION

The difficulty is in tailoring optimization algorithms to the real-world applications that they find in several scientific and technological domains [1]. As a result, certain algorithms are only useful for solving complicated issues, which supports the idea that it's important to constantly adopt new techniques for improvement [1]. Meta-heuristic algorithms (MAs) are a class of optimization algorithms that are used to handle difficult issues that are beyond the scope of conventional techniques. These algorithms are used to search a wide search area in order to find the global optimum of a problem. They are inspired by natural processes such as evolution, swarm behavior, and genetics [2, 3]. Darwin's theory of evolution served as the basis for the development of the genetic algorithm (GA) recently [4]. GA and the differential evolution (DE) algorithm both employ the same operators, namely crossover and mutation. But DE uses a different approach [5]. The collective foraging activities of fish and bird species serve as an inspiration for particle swarm optimization (PSO) [6]. The ABC algorithm is a computer method that mimics honeybees' information-sharing capacities and foraging strategies [7]. Utilizing the laws of motion and gravity, the gravity search algorithm (GSA) is a computer optimization method [8]. Numerous sophisticated algorithms, as demonstrated in references [9-14], base their depiction on the actions of real-world occurrences and biological entities.

CGAs are classified as effective methods for reaching minimization points for optimization problems with the least possible iterations, but the solutions are considered local.

Therefore, the process of linking these algorithms with MAs gives them greater efficiency in finding minimizations of the same problems globally as in references [15-20].

The difference between conjugate gradient methods and mayfly optimization algorithm lies in how they approach and optimize solutions to mathematical optimization problems, especially in terms of the mathematical mechanism and theoretical basis behind each. Let's review the main differences:

Conjugate gradient methods are analytic algorithms that rely on gradients to optimize the objective function, and are commonly used in unconstrained optimization problems, especially when the objective function is a quadratic or linear function with derivative functions. They work by calculating gradient directions at each step, but instead of following the gradient directly, they use a technique that makes the direction at each step orthogonal (conjugate) to the previous directions, which makes them avoid excessive recompilation. As common applications of these algorithms, they are usually used in convex function optimization problems, such as least squares problems in machine learning, as well as in large vector optimization in machine learning. Their features are very effective in large problems where Hessian matrices are computationally expensive. They almost guarantee a local solution in quadratic function problems. The mayfly optimization algorithm is an evolutionary algorithm inspired by the natural behavior of mayflies in nature. It is a type of swarm intelligence algorithm, which uses the concept of attraction between individuals to update locations and minimize the value of the objective function. The algorithm's

working mechanism simulates the behavior of mayflies in terms of mating and movement, where groups of females and males form a "swarm", and each individual move towards other individuals according to rules based on their fitness, trying to minimize or optimize the value of the objective function. Applications of these algorithms are used in optimizing complex nonlinear functions, and in problems that require a broader exploration of the solution space, such as engineering system design, and multidimensional optimizations. Its features are suitable for highly complex and discontinuous optimization problems, and it can exit local solutions due to the mating and exploration mechanism.

The paper's structure is as follows: in Section 2, we shall provide a different formulation of the CGAs. In Section 3, under specific assumptions, we examine the theoretical components and determine the global convergence properties of the Fletcher and Reeves conjugate gradient algorithm (FR)-CGA. A detailed description of the original mayfly algorithm is provided in Section 4. To improve both algorithms' performance, we combined the FR-CGA and mayfly algorithms in Section 5. The sixth section of the research concentrates on the digital side. Seven unconstrained functions are treated with a unique combination algorithm mayfly optimization algorithm and conjugate gradient algorithm (MOA-CG), and the results are compared with the mayfly method.

## 2. A NEW FORMULA

Conjugate gradient is an analytic algorithm that relies on the properties of gradient and deliberateness to optimize a mathematical function (constrained by analyticity and differentiation), and fits linear or convex problems. The unconstrained minimization problem can be defined as follows:

$$\min_{x} f(x) \text{ where } f: R^n \rightarrow R \quad (1)$$

Since  $x$  is a vector and the function  $f$  is a differentiable and continuous function, and according to these data we can use the CGA to find the solution to the formula (1) and express the current iteration as follows:

$$x_{k+1} = x_k + \alpha_k d_k, k \geq 0 \quad (2)$$

Also denoted  $f$  at  $x_k$  ( $f_k = f(x_k)$ ). In addition, the gradient of the function is represented as  $g_k = g(x_k)$ , while the second derivative of the function is denoted as  $G_k = G(x_k)$  at iteration  $k$ , which corresponds to a Hessian matrix,  $\alpha_k$  is the step size of the iterative method in Eq. (2) and  $d_k$  is the search direction. To find the value of the step  $\alpha_k$ , which is used if the problems are general, through the powerful Wolfe-Powell (PWP) line search method to determine the appropriate dimension of the step as follows [21]:

$$\begin{cases} f(x_k + \alpha_k d_k) - f(x_k) \leq \delta \alpha_k g_k^T d_k. \\ |d_k^T g_{k+1}| \leq -\sigma d_k^T g_k, \quad 0 \leq \delta \leq \sigma \leq 1. \end{cases} \quad (3)$$

And the search direction  $d_{k+1}$  in which it is:

$$d_{k+1} = \begin{cases} g_{k+1} & \text{if } k = 0, \\ g_{k+1} + \beta_k d_k, & \text{if } k > 0, \end{cases} \quad (4)$$

One of the distinctive formulas for the parameter  $\beta_k$  is the formula presented by Fletcher and Reeves [22], which is distinguished by its theoretical performance, but requires tuning of its properties when applied numerically, which is known as:

$$\beta_k^{FR} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}$$

In order for the iterative points to come close to reaching the smallest point during the algorithm, the distance between these points is measured through the Euclidean distance, i.e.,  $\|\cdot\|$ . Several authors modified the FR parameter by using retrieval based on the shrinkage effect  $s_k = x_{k+1} - x_k$  and  $y_k = g_{k+1} - g_k$  with a discussion of the global convergence of these methods as in references [23-30].

Now, using the general CGA direction, the scale FR search direction is given as:

$$d_{k+1} = -g_{k+1} + \vartheta_k \beta_k^{FR} s_k \quad (5)$$

In order to take advantage of the Barzilai-Borwein strategy [31-33], we propose the following parameter  $\vartheta_k$  as a parameter to adjust the standardization of the FR parameter  $\beta_k^{FR}$ :

$$\vartheta_k = (1 - \mu_k) \frac{d_k^T y_k}{\|d_k\|^2 \|y_k\|^2} \quad (6)$$

We note that the division of a fraction is always positive because the numerator ( $d_k^T y_k$ ) is positive and the denominator ( $\|d_k\|^2 \|y_k\|^2$ ) is also positive, which leads to a positive value in relation to  $\vartheta_k$  because the arc  $(1 - \mu_k)$  has a diminishing value. The spectral scalars of the Barzilai-Borwein are the most popular ones, as they are:

$$\mu_k^1 = \frac{s_k^T s_k}{y_k^T s_k}; \mu_k^2 = \frac{s_k^T y_k}{s_k^T s_k} \quad (7)$$

And  $\mu_k = \max\{\mu_{min}^i, \min\{\mu_k^i, \mu_{max}\}\}$ ,  $i = 1, 2$ , where, the  $0 < \mu_{min} < \mu_{max} < \infty$ . Furthermore, in order to obtain  $|\mu_k| \leq 1$ .

## 3. GLOBAL CONVERGENCE

When comparing the theoretical convergence properties of a particular conjugate gradient method, it depends on the properties (convergence rate, ensuring convergence towards the optimal solution (Global vs Local Convergence), reliability and stability, computational efficiency, scalability). These properties will be available if the functions are less complex and also if certain conditions are available to achieve them, such as the global convergence of the algorithm. In this section, we will show the advantages of the proposed algorithm from the theoretical side, relying on some necessary and important conditions such as the basic assumption.

### Assumption 3.1

The existence of the value  $x_1$ , which is defined as the restricted level set  $S = \{x: f(x) \leq f(x_1)\}$  indicates the existence of a number ( $B > 0$ ) s. t.

$$\|x\| \leq B, \forall x \in S \quad (8)$$

If  $g$  is Lipschitz continuous and  $f$  is continuously differentiable in certain neighborhoods  $N$  of  $S$ , then there is a constant  $L \geq 0$  s. t.

$$\|g(x) - g(x_k)\| \leq L \|x - x_k\|, \forall x, x_k \in N \quad (9)$$

**Theorem 3.2**

If we assume that two new algorithms, Eqs. (5) and (6), produce  $x_{k+1}$  and  $d_{k+1}$ , and we use PWP (3) to yield  $\alpha_k$ , then the direction holds such that s. t.

$$d_{k+1}^T g_{k+1} \leq -c \|g_{k+1}\|^2, \forall k \geq 1 \quad (10)$$

**Proof:** Multiply both sides of Eq. (5) from the right side by  $g_{K+1}$ , we have:

$$d_{k+1}^T g_{k+1} = -\|g_{k+1}\|^2 + \vartheta_k \beta_k^{FR} s_k^T g_{k+1}$$

$$d_{k+1}^T g_{k+1} = -\|g_{k+1}\|^2 + (1 - \mu_k) \frac{d_k^T y_k}{\|d_k\|^2 \|y_k\|^2} \beta_k^{FR} s_k^T g_{k+1}$$

where,  $s_k^T g_{k+1} = s_k^T y_k + s_k^T g_k > s_k^T y_k$ , then

$$d_{k+1}^T g_{k+1} \leq -\|g_{k+1}\|^2 + (1 - \mu_k) \frac{d_k^T y_k}{\|d_k\|^2 \|y_k\|^2} \beta_k^{FR} s_k^T y_k$$

$$d_{k+1}^T g_{k+1} \leq -\|g_{k+1}\|^2 + (1 - \mu_k) \frac{\alpha_k (d_k^T y_k)^2}{\|d_k\|^2 \|y_k\|^2} \frac{\|g_{k+1}\|^2}{\|g_k\|^2}$$

$$d_{k+1}^T g_{k+1} \leq -\|g_{k+1}\|^2 + (1 - \mu_k) \alpha_k \frac{\|g_{k+1}\|^2}{\|g_k\|^2}$$

Upon simplification, a satisfactory descent is obtained for this algorithm in the following manner:

$$d_{k+1}^T g_{k+1} \leq -c \|g_{k+1}\|^2$$

where,  $c = \left[1 + (1 - \mu_k) \frac{\alpha_k}{\|g_k\|^2}\right] > 0$ .

**Theorem 3.3**

Assuming that Assumption (3.1) A is satisfied, let us consider any CGA (2)-(4), where  $d_{k+1}$  satisfies condition (10) and  $\alpha_k$  is determined using Eq. (3).

$$\sum_{k \geq 1} \frac{1}{\|d_k\|^2} < +\infty \quad (11)$$

Then, we have

$$\lim_{k \rightarrow \infty} \inf \|g_k\| = 0 \quad (12)$$

In this discourse, we shall proceed to expound upon the theory of global convergence by drawing upon the conditions posited in preceding proof theories.

**Theorem 3.4**

Assuming that the conditions in Assumption 3.1, B are met. Assuming that  $|\mu_k| \leq 1$ , there exists a positive constant ( $i=1,2$ ) for each  $k$  that is less than or equal to zero. These constants are designated as  $\gamma_1$  and  $\gamma_2$ , respectively, with  $\gamma_1 \leq \|g_k\| \leq \gamma_2$ . Then, the PWP search determines the updated CGA and  $\alpha_k$ ,

where  $g_k$  is equal to zero for a given  $k$  or  $\lim_{k \rightarrow \infty} \inf \|g_k\| = 0$ .

**Proof:** Due to the fulfilment of the descent state, we have  $\|d_k\| \neq 0$ . This observation is made in conjunction with the Lipschitz condition.

$$\|y_k\| = \|g_{k+1} - g_k\| \leq L \|s_k\|$$

$$\|d_{k+1}\| = -g_{k+1} + \vartheta_k \beta_k^{FR} s_k$$

$$\|d_{k+1}\| \leq \|g_{k+1}\| + |\vartheta_k| |\beta_k^{FR}| \|s_k\|$$

where,

$$\begin{aligned} |\vartheta_k| &= \left| \frac{d_k^T y_k}{\|d_k\|^2 \|y_k\|^2} - \mu_k \frac{d_k^T y_k}{\|d_k\|^2 \|y_k\|^2} \right| \\ &= \frac{2\alpha_k}{\|s_k\| \|y_k\|} \leq 2\alpha_k L \end{aligned}$$

Given the assumption that  $D$  known and  $|\beta_k^{FR}| \leq \frac{\gamma_2^2}{\gamma_1^2} \equiv E$ , the updated search direction can be expressed:

$$\|d_{k+1}\| \leq \gamma_2 + 2\alpha_k L E$$

This implies  $0 < \sum_{k=1}^{\infty} \frac{(g_k^T d_k)^2}{\|d_k\|^2} < \infty$  when  $\sum_{k=1}^{\infty} \frac{\|g_k\|^4}{\|d_k\|^2} < \frac{1}{c^2} \frac{(g_k^T d_k)^2}{\|d_k\|^2} < \infty$ .

**4. THE MAYFLY OPTIMIZATION ALGORITHM**

The mayflies within the MOA swarms exhibit sexual dimorphism, with distinct separation of male and female individuals. The male mayflies consistently exhibit robustness, leading to enhanced performance in optimization tasks. In a manner akin to the constituents of swarms within the PSO algorithm, the constituents within the multi-objective optimization algorithm would proceed to modify their positions based on their respective current positions  $p(t)$  and velocities  $v(t)$  during the ongoing iteration.

$$p_k(t+1) = p_k(t) + v_k(t+1) \quad (13)$$

Both male and female mayflies were to update their positions using the identifier (13). Nevertheless, the manner in which their velocity is updated would vary [34].

**4.1 The locomotion patterns exhibited by male mayflies**

During iterations, male mayflies in swarms participated in either exploitation or exploration strategies. The velocity will be adjusted based on the current fitness values  $f(x_k)$  and the prior best fitness values in the trajectories  $f(x_{h_k})$ . When  $f(x_k) > f(x_{h_k})$ , the male mayflies will adjust their velocities based on their present velocities, the distance between their current location and the global best position, and their previous best trajectories:

$$\begin{aligned} v_k(t+1) &= g \cdot v_k(t) + a_1 e^{-\beta r_p^2} [x_{h_k} - x_k(t)] \\ &\quad + a_2 e^{-\beta r_g^2} [x_g - x_k(t)] \end{aligned} \quad (14)$$

The variable  $g$  exhibits a linear fall from its maximum value

to a lesser value. The variables  $a_1$ ,  $a_2$  and  $\beta$  are utilized to equilibrate the values. The variables  $r_p^2$  and  $r_g^2$  are utilized to quantify the Euclidean distance between individuals and the associated historical best status, as well as the global best position among swarms. The Cartesian distance is a representation of the second norm of the distance array:

$$\|x_k - x_j\| = \sqrt{\sum_{i=1}^n (x_{ki} - x_{ji})^2} \quad (15)$$

Conversely, in the event that  $f(x_k)$  is less than  $f(x_{h_k})$ , the male mayflies would proceed to adjust their velocities from the present value using a random dance coefficient  $d$ .

$$v_k(t + 1) = g \cdot v_k(t) + d \cdot r_1 \quad (16)$$

where,  $r$  is a uniformly distributed random number in the range  $[-1, 1]$  [34].

#### 4.2 The locomotion patterns exhibited by female mayflies

The female mayflies would modify their velocities via an alternative approach. From a biological perspective, female mayflies possessing wings typically have a lifespan ranging from one to seven days. Consequently, these female mayflies exhibit a sense of urgency in locating male counterparts in order to engage in mating and facilitate their own reproductive processes. Consequently, the individuals would adjust their velocities in accordance with their desired male mayfly partners for mating purposes. According to the MOA, the highest-ranking female and male mayflies would assume the role of the primary mates, while the second-ranking female and male mayflies would assume the role of secondary mates, and so forth. For the female mayfly indexed by  $i$ , if the  $f(y_k) < f(x_k)$ :

$$v_k(t + 1) = g \cdot v_k(t) + a_3 e^{-\beta r_{mf}^2} [x_k(t) - y_k(t)] \quad (17)$$

where,  $a_3$  represents an additional constant that serves the purpose of equilibrating the velocities. The variable  $r_m$  denotes the Cartesian distance between the two entities. On the contrary, if  $f(y_k) > f(x_k)$ , female mayflies have been shown to adjust their velocities by a random dance movement, so updating their current speed  $fl$ :

$$v_k(t) = g \cdot v_k(t) + fl \cdot r_2 \quad (18)$$

where,  $r_2$  is uniform distribution in the range  $[-1, 1]$ .

#### 4.3 Mating of mayflies

Each female and male mayfly in the upper half of the population would be paired for mating and produce a pair of offspring for each individual. The progeny would exhibit random variations resulting from the evolutionary process inherited from their parental generation.

$$offspring1 = L * male + (1 - L) * female \quad (19)$$

$$offspring2 = L * female + (1 - L) * male \quad (20)$$

Here,  $L$  will also take random numbers in Gauss distribution [34].

## 5. NEW MOA-CG ALGORITHM

Within this segment of the paper, we present a novel algorithm (MOA-CG) that relies on the conjugate gradient algorithm put forth by Eqs. (5) and (6). We have demonstrated that this algorithm has adequate descent and convergence to the minimum point with the algorithm (MOA) in order to identify optimal solutions for the optimization functions. Combining the two algorithms gives us their advantages together, i.e., finding analytical solutions using algorithmic methods inspired by nature that are close to ideal within complex solution spaces.

The suggested algorithm is available for viewing in the paragraph that follows:

### 5.1 MOA-CG algorithm

**Step 1:** Initialized mayfly algorithm parameter (nPop= 20, Max-Iter=2000,  $g=0.8$ ,  $a1=1.0$ ,  $a2=1.5$ ,  $a3=1.5$ ,  $\beta=2$ ,  $d= 5$ ,  $fl=1$ ).

Give initial CG parameter: variable  $x_0 \in R^n$ ,  $\delta \in [0,0.5]$  and  $\sigma \in [\delta, 1]$ . Let  $k = 0$ ,  $d_0 = -g_0$ .

**Step 2:** The best value of step to algorithm results: set  $\alpha_k$  from (PWP) in formula (3) and compute the  $\vartheta_k$  from Eq. (6).

**Step 3:** Evaluate the parameters used by the new search direction (4)-(5).

**Step 4:** Find the new point happened  $x_{k+1}$  as formulas (2)-(4).

**Step 5:** Compute population based on  $f(x)$  and find the global best (gbest).

**Step 6:** If the maximum iteration has not been achieved, compute the following:

- The Eqs. (17)-(18) can be utilized to determine the speed and position of individual female mayflies.
- The Eqs. (13)-(15) can be utilized to determine the speed and position of individual male mayflies.
- Arrange the mayflies into categories and assign them a ranking based on the function  $f(x)$ .
- Produce male and female progeny, denoted as Eqs. (19)-(20) respectively.
- The process of offspring development involves the random division of offspring into male and female individuals.
- Replace the least desirable elderly persons with the most exemplary fresh ones.
- The pbest and gbest values are updated.

**Step 7:** If the sum of duplicate values has reached its maximum, terminate the process and display the optimal value without any adjustments in the variable  $k$ . Proceed to Step (2).

## 6. ARITHMETIC RESULTS

This stage included testing the new algorithms on a set of seven test functions that were sourced from different sources [35, 36]. After being modified to conform to the new method, the application was run on a PC running an Intel Core i5 CPU using MATLAB R2018b. Three main categories apply to the findings. The new MOA-CG algorithm's results were contrasted with the mayfly algorithm's first results. MOA under the condition of  $n=25$ , as outlined below:

- 1- Best cost.
- 2- Standard deviation.
- 3- Mean.

**Table 1.** The best cost of the new MOA-CG algorithm compared to the MOA algorithm

Function Name	MOA-CG	MOA
<i>Sphere</i>	1.3984e-19	1.9159e-18
<i>Alpine 1</i>	3.6166e-12	3.3201e-09
<i>Chung Reynolds</i>	9.9005e-38	2.0952e-35
<i>Cosine Mixture</i>	-4995	-4995
<i>Deflected Corrugated Spring</i>	-1	-1
<i>Styblinski-Tang</i>	-1703.8473	-1788.6677
<i>Rastrigin</i>	15.9193	18.9042

**Table 2.** The standard deviation of the new MOA-CG algorithm compared to the MOA algorithm

Function Name	MOA-CG	MOA
<i>Sphere</i>	24.5918	21.699
<i>Alpine 1</i>	13.7991	12.9897
<i>Chung Reynolds</i>	4354.4405	3346.6964
<i>Cosine Mixture</i>	309.2886	399.5781
<i>Deflected Corrugated Spring</i>	178.2457	175.3428
<i>Styblinski-Tang</i>	137.2658	242.4148
<i>Rastrigin</i>	53.321	58.3351

**Table 3.** The mean of the new MOA-CG algorithm compared to the MOA algorithm

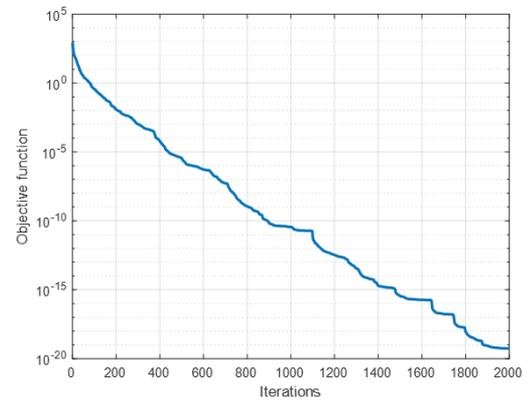
Function Name	MOA-CG	MOA
<i>Sphere</i>	1.7161	1.7112
<i>Alpine 1</i>	1.1843	1.0693
<i>Chung Reynolds</i>	150.144	125.9651
<i>Cosine Mixture</i>	-4940.0053	-4916.3963
<i>Deflected Corrugated Spring</i>	15.9547	17.2023
<i>Styblinski-Tang</i>	-1688.6974	-1728.4473
<i>Rastrigin</i>	40.4155	50.4466

The objective is to determine the minimum value  $f_{min}$  of the function that is associated with the tables illustrating the efficiency of the newly developed algorithms.

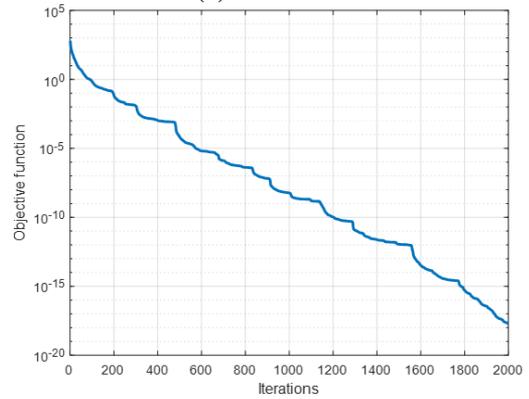
Table 1 shows that the new technique performs significantly better in calculations to determine the smallest point of the seven functions. Table 2 compares the proposed algorithm to the original algorithm using the standard deviation as one of the measures of point dispersion. The proposed algorithm performs better than the original algorithm in this case. Table 3 compares the proposed algorithm to the average and finds that the new algorithm has a higher point concentration than the original algorithm. This suggests to us that the original algorithm was not as accurate as we thought it was. Figures 1-7 show the comparison between the suggested and original algorithms when used with the seven test functions used in the article.

The preceding tables show that the new method performs best in achieving the least limit of the seven functions described, regardless of how diverse the functions and their dimensions are.

Although the combination of conjugate gradient algorithm and maybug algorithm is effective, there are some drawbacks and challenges that may face this algorithm, including (sensitivity to parameter selection, high computational cost, convergence to local solutions, difficulty adapting to dynamic environments). These drawbacks make it necessary to conduct additional studies to improve the flexibility and efficiency of the algorithm in various applications.

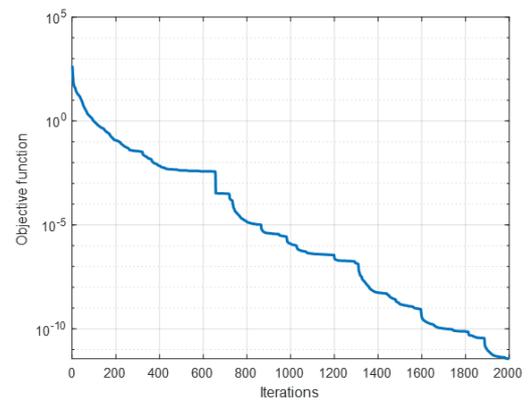


(a) MOA-CG

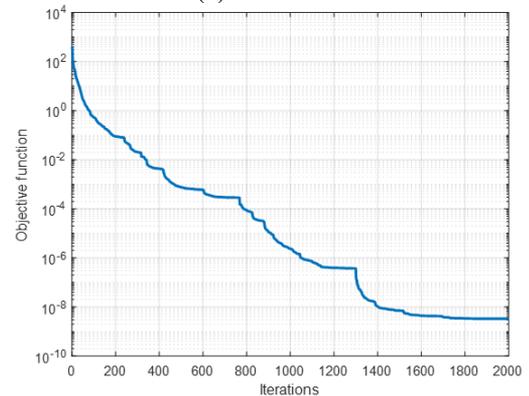


(b) MOA

**Figure 1.** Comparison of the performance of the two algorithms (MOA-CG vs. MOA) with respect to the function (Sphere)

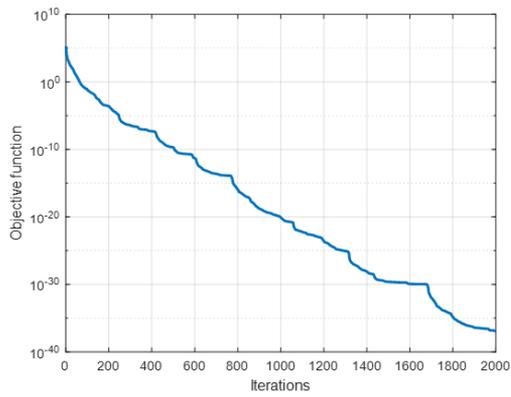


(a) MOA-CG

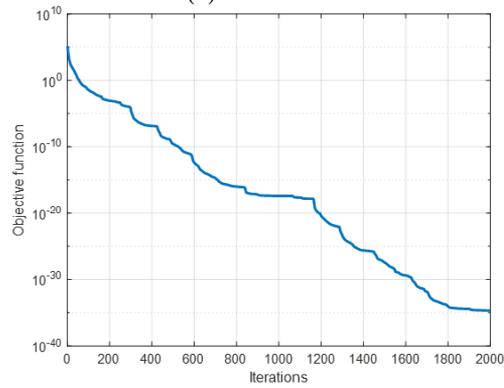


(b) MOA

**Figure 2.** Comparison of the performance of the two algorithms (MOA-CG vs. MOA) with respect to the function (Alpine 1)

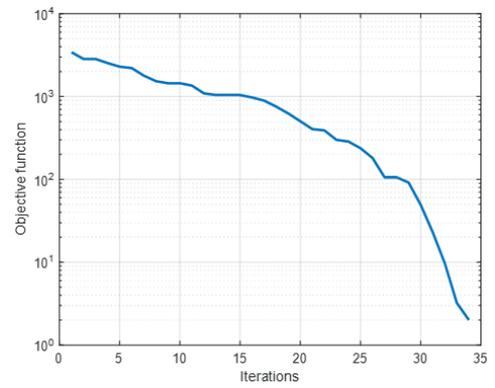


(a) MOA-CG

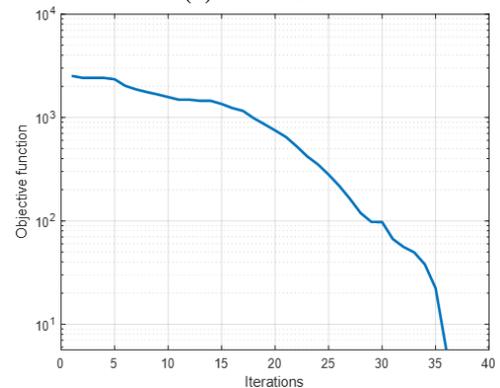


(b) MOA

**Figure 3.** Comparison of the performance of the two algorithms (MOA-CG vs. MOA) with respect to the function (Chung Reynolds)

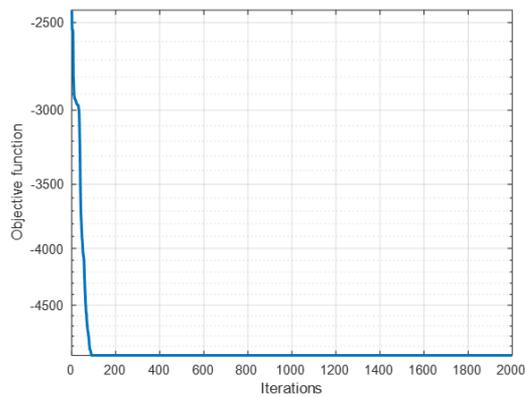


(a) MOA-CG

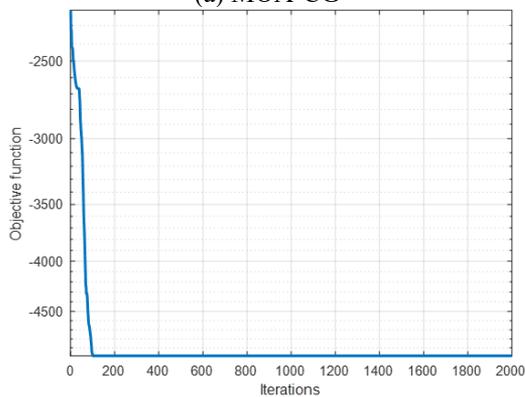


(b) MOA

**Figure 5.** Comparison of the performance of the two algorithms (MOA-CG vs. MOA) with respect to the function (Deflected Corrugated Spring)

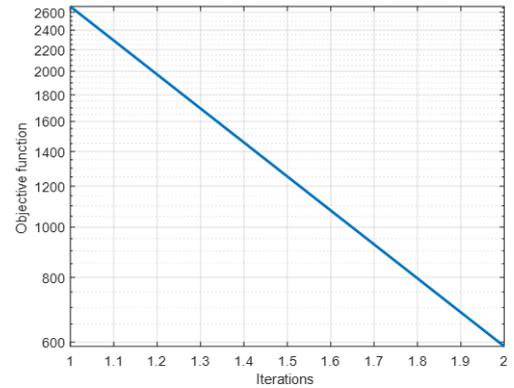


(a) MOA-CG

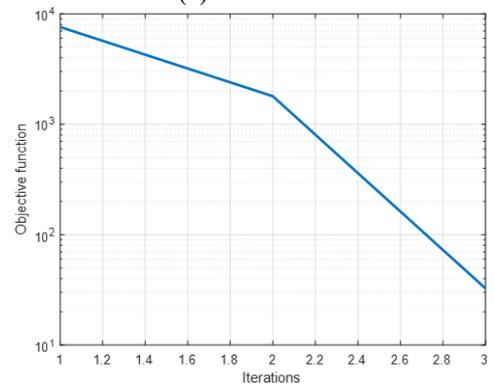


(b) MOA

**Figure 4.** Comparison of the performance of the two algorithms (MOA-CG vs. MOA) with respect to the function (Cosine Mixture)

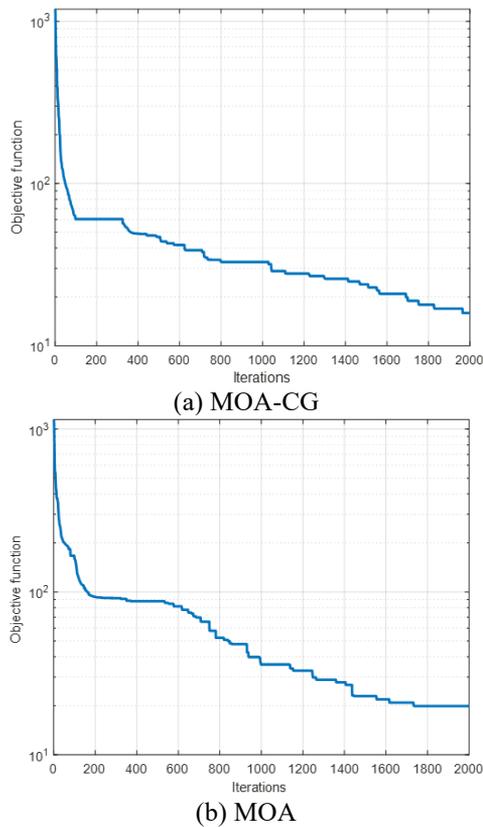


(a) MOA-CG



(b) MOA

**Figure 6.** Comparison of the performance of the two algorithms (MOA-CG vs. MOA) with respect to the function (Styblinski-Tang)



**Figure 7.** Comparison of the performance of the two algorithms (MOA-CG vs. MOA) with respect to the function (Rastrigin)

In the field of optimization algorithms, there are several recent research trends that focus on improving the performance of traditional and nature-inspired algorithms and adapting them to the increasing challenges in complex applications. These trends include:

- 1) Integrating machine learning with optimization algorithms (Machine learning-enhanced optimization)
- 2) Automatic parameter tuning (automatic parameter tuning)
- 3) Hybrid approaches
- 4) Increasing efficiency in large and complex environments (Scalability and complexity handling)
- 5) Multi-objective optimization (multi-objective optimization)
- 6) Context-aware optimization (context-aware optimization)
- 7) Practical applications in artificial intelligence and machine learning (AI and machine learning applications)
- 8) Reliable and robust optimization (reliable and robust optimization)

These trends reflect the increasing efforts to make optimization algorithms more intelligent and adaptable to modern complex applications, and to direct them to solve real and effective problems in various scientific and industrial fields.

## 7. CONCLUSIONS

Based on the data presented in the preceding section, we draw the conclusion that the new algorithm, MOA-CG,

performs better than the traditional algorithms, MOA, when it comes to finding minimization points for known testing functions (i.e., in terms of the number of iterations and the time it takes to reach the optimal point). Furthermore, the traditional proposed CGA converged more strongly than when the FR parameter was used; however, when algorithms with comprehensive behavior were introduced to reach the minimization of functions, its efficiency increased in convergence and decreased the number of iterations. Traditional methods (such as conjugate gradient) provide local convergence, high computational speed and efficiency on specific problems such as quadratic or linear functions. Modern algorithms inspired by nature provide high flexibility in exploring global solutions and scalability, but they require more computations and may be less stable in terms of theoretical convergence. Therefore, the choice of algorithm often depends on the nature of the specific problem, whether it is convex, linear, or non-convex and complex. It is feasible to combine new conjugate gradient techniques with the mayfly method in future study. The conjugate gradient algorithm suggested in the paper can also be used with other clever algorithms.

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## REFERENCES

- [1] Kaveh, A., Mahdavi, V.R. (2014). Colliding bodies optimization: A novel meta-heuristic method. *Computers & Structures*, 139: 18-27. <https://doi.org/10.1016/j.compstruc.2014.04.005>
- [2] Mirjalili, S. (2016). SCA: A sine cosine algorithm for solving optimization problems. *Knowledge-Based Systems*, 96: 120-133. <https://doi.org/10.1016/j.knosys.2015.12.022>
- [3] Rashedi, E., Nezamabadi-Pour, H., Saryazdi, S. (2009). GSA: A gravitational search algorithm. *Information Sciences*, 179(13): 2232-2248. <https://doi.org/10.1016/j.ins.2009.03.004>
- [4] Mirjalili, S., Mirjalili, S.M., Lewis, A. (2014). Grey wolf optimizer. *Advances in Engineering Software*, 69: 46-61. <https://doi.org/10.1016/j.advengsoft.2013.12.007>
- [5] Kaveh, A., Dadras, A. (2017). A novel meta-heuristic optimization algorithm: Thermal exchange optimization. *Advances in Engineering Software*, 110: 69-84. <https://doi.org/10.1016/j.advengsoft.2017.03.014>
- [6] Zhao, W., Wang, L., Zhang, Z. (2019). A novel atom search optimization for dispersion coefficient estimation in groundwater. *Future Generation Computer Systems*, 91: 601-610. <https://doi.org/10.1016/j.future.2018.05.037>
- [7] Castillo, O., Melin, P., Ontiveros, E., Peraza, C., Ochoa, P., Valdez, F., Soria, J. (2019). A high-speed interval type 2 fuzzy system approach for dynamic parameter adaptation in metaheuristics. *Engineering Applications of Artificial Intelligence*, 85: 666-680.

- <https://doi.org/10.1016/j.engappai.2019.07.020>
- [8] Zhao, W., Wang, L., Zhang, Z. (2019). Atom search optimization and its application to solve a hydrogeologic parameter estimation problem. *Knowledge-Based Systems*, 163: 283-304. <https://doi.org/10.1016/j.knosys.2018.08.030>
- [9] Rodríguez, L., Castillo, O., Soria, J., Melin, P., Valdez, F., Gonzalez, C.I., Martinez, G.E., Soto, J. (2017). A fuzzy hierarchical operator in the grey wolf optimizer algorithm. *Applied Soft Computing*, 57: 315-328. <https://doi.org/10.1016/j.asoc.2017.03.048>
- [10] Patel, V.K., Savsani, V.J. (2015). Heat transfer search (HTS): A novel optimization algorithm. *Information Sciences*, 324: 217-246. <https://doi.org/10.1016/j.ins.2015.06.044>
- [11] Draa, A., Bouzoubia, S., Boukhalfa, I. (2015). A sinusoidal differential evolution algorithm for numerical optimisation. *Applied Soft Computing*, 27: 99-126. <https://doi.org/10.1016/j.asoc.2014.11.003>
- [12] Garg, H. (2019). A hybrid GSA-GA algorithm for constrained optimization problems. *Information Sciences*, 478: 499-523. <https://doi.org/10.1016/j.ins.2018.11.041>
- [13] Salajegheh, F., Salajegheh, E. (2019). PSOG: Enhanced particle swarm optimization by a unit vector of first and second order gradient directions. *Swarm and Evolutionary Computation*, 46: 28-51. <https://doi.org/10.1016/j.swevo.2019.01.010>
- [14] Kaveh, A., Bakhshpoori, T. (2016). Water evaporation optimization: A novel physically inspired optimization algorithm. *Computers & Structures*, 167: 69-85. <https://doi.org/10.1016/j.compstruc.2016.01.008>
- [15] Al-Kawaz, R.Z., Al-Bayati, A.Y., Jameel, M.S. (2020). Interaction between un updated FR-CG algorithms with an optimal Cuckoo algorithm. *Indonesian Journal of Electrical Engineering and Computer Science*, 19(3): 1497-1504. <https://doi.org/10.11591/ijeecs.v19.i3.pp1497-1504>
- [16] Jameel, M., Yayla, O. (2021). PSO based blockchain committee member selection. In 2021 6th International Conference on Computer Science and Engineering (UBMK), Ankara, Turkey, pp. 725-730. <https://doi.org/10.1109/UBMK52708.2021.9559004>
- [17] Ahmed, H.I., Hamed, E.T., Saeed Chilmeran, H.T. (2020). A modified bat algorithm with conjugate gradient method for global optimization. *International Journal of Mathematics and Mathematical Sciences*, 2020(1): 4795793. <https://doi.org/10.1155/2020/4795793>
- [18] Khaleel, E.S., Hamed, E.T., Ahmed, H.I. (2021). Performance evaluation of firefly algorithm with unconstrained optimization issues. *International Journal of Nonlinear Analysis and Applications*, 12: 893-901. <https://doi.org/10.22075/ijnaa.2021.5518>
- [19] Najm, H.Y., Ahmed, H.I. (2022). Improving jellyfish swarm algorithm for solving unconstrained optimization problems. In 2022 3rd Information Technology to Enhance e-learning and Other Application (IT-ELA), Baghdad, Iraq, pp. 226-231. <https://doi.org/10.1109/IT-ELA57378.2022.10107944>
- [20] Zhang, L., Zhou, W., Li, D. (2006). Global convergence of a modified Fletcher-Reeves conjugate gradient method with Armijo-type line search. *Numerische Mathematik*, 104(4): 561-572. <https://doi.org/10.1007/s00211-006-0028-z>
- [21] Zoutendijk, G. (1970). Nonlinear programming, computational methods. *Integer and Nonlinear Programming*, 4(1): 37-86. <https://doi.org/10.1137/0304019>
- [22] Fletcher, R., Reeves, C.M. (1964). Function minimization by conjugate gradients. *The Computer Journal*, 7(2): 149-154. <https://doi.org/10.1093/comjnl/7.2.149>
- [23] Al-Baali, M. (1985). Descent property and global convergence of the Fletcher—Reeves method with inexact line search. *IMA Journal of Numerical Analysis*, 5(1): 121-124. <https://doi.org/10.1093/imanum/5.1.121>
- [24] Ahmed, H.I., Al-Kawaz, R.Z., Al-Bayati, A.Y. (2019). Spectral three—Term constrained conjugate gradient algorithm for function minimizations. *Journal of Applied Mathematics*, 2019(1): 6378368. <https://doi.org/10.1155/2019/6378368>
- [25] Dauda, M.K., Mamat, M., Mohamed, M.A., Hamzah, N.S.A. (2019). Hybrid conjugate gradient parameter for solving symmetric systems of nonlinear equations. *Indonesian Journal of Electrical Engineering and Computer Science*, 16(1): 539-543. <https://doi.org/10.11591/ijeecs.v16.i1.pp539-543>
- [26] Hamed, E.T., Al-Kawaz, R.Z., Al-Bayati, A.Y. (2020). New investigation for the Liu—Story scaled conjugate gradient method for nonlinear optimization. *Journal of Mathematics*, 2020(1): 3615208. <https://doi.org/10.1155/2020/3615208>
- [27] Al-Kawaz, R.Z., Abubakar, A.B., Ibrahim, A.H., Khammahawong, K. (2023). Algorithm for solving nonlinear monotone operator equations with applications. *Journal of Nonlinear Functional Analysis*, 2023(37): 1-15. <https://doi.org/10.23952/jnfa.2022.37>
- [28] Al-Arbo, A.A., Al-Kawaz, R.Z. (2021). A fast spectral conjugate gradient method for solving nonlinear optimization problems. *Indonesian Journal of Electrical Engineering and Computer Science*, 21(1): 429-439. <https://doi.org/10.11591/ijeecs.v21.i1.pp429-439>
- [29] Jameel, M., Al-Bayat, A.Y., Algamal, Z.Y. (2022). New conjugacy parameter for Cg iterative descent method. *NeuroQuantology*, 20(6): 3139. <https://doi.org/10.14704/nq.2022.20.6.NQ22316>
- [30] Chilmeran, H.T.S., Al-Kawaz, R.Z. (2023). A new scalar of conjugate gradient methods for solving unconstrained minimization. *European Journal of Pure and Applied Mathematics*, 16(1): 233-242. <https://doi.org/10.29020/nybg.ejpam.v16i1.4619>
- [31] Raydan, M. (1997). The Barzilai and Borwein gradient method for the large scale unconstrained minimization problem. *SIAM Journal on Optimization*, 7(1): 26-33. <https://doi.org/10.1137/S1052623494266365>
- [32] Althobaiti, A., Sabi'u, J., Emadifar, H., Junsawang, P., Sahoo, S.K. (2022). A scaled Dai—Yuan projection-based conjugate gradient method for solving monotone equations with applications. *Symmetry*, 14(7): 1401. <https://doi.org/10.3390/sym14071401>
- [33] Sabi'u, J., Aremu, K.O., Althobaiti, A., Shah, A. (2022). Scaled three-term conjugate gradient methods for solving monotone equations with application. *Symmetry*, 14(5): 936. <https://doi.org/10.3390/sym14050936>
- [34] Zervoudakis, K., Tsafarakis, S. (2020). A mayfly optimization algorithm. *Computers & Industrial Engineering*, 145: 106559. <https://doi.org/10.1016/j.cie.2020.106559>

- [35] Jamil, M., Yang, X.S. (2013). A literature survey of benchmark functions for global optimisation problems. *International Journal of Mathematical Modelling and Numerical Optimisation*, 4(2): 150-194. <https://doi.org/10.1504/IJMMNO.2013.055204>
- [36] Zhao, J., Gao, Z.M. (2020). The negative mayfly optimization algorithm. *Journal of Physics: Conference Series*, 1693(1): 012098. <https://doi.org/10.1088/1742-6596/1693/1/012098>