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Pade Approximation of Line Parameters for the Analysis of Transient Processes in Uniform Lossy Lines

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ABSTRACT

The aim of the research is to develop and validate an efficient method for analyzing transient processes in uniform lossy lines based on the Pade approximation of line parameters. It is used the synthesis in the frequency – domain of the characteristic impedance or admittance and the exponential propagation function, which are realized by classical methods of Foster and Caurer. As a result of the combination of the Pade approximation with general transmission line equations, the problem of transient analysis of the uniform loss line is brought to the calculation of a lossless line and a lumped-parameters network. So, it allows the analysis in the time-domain of the transients in two-conductor or multi-conductor transmission lines. The presented simulation results demonstrate the validity and the efficiency of the proposed model. The practical value of this research lies in the development of a tool that will enable engineers and scientists to analyze transient processes in uniform lossy lines more accurately and efficiently. This can lead to improved design and optimization of power and data transmission systems, reduced losses, and increased reliability of electrical and telecommunication networks.

1. INTRODUCTION

In 21th century, the tasks associated with modeling real systems, whether they are electrical circuits, data transmission lines, or other physical systems, are becoming increasingly complex. Accurate mathematical models are essential for predicting the behavior of such systems, enhancing their efficiency and reliability. In this context, the Pade approximation offers significant advantages due to its ability to accurately describe system behavior based on a limited number of parameters. The main challenge faced by research on Pade approximation is the need to optimize the algorithms used for their application in real systems with limited computational resources. It is crucial to find the optimal balance between the accuracy of the approximation and the computational complexity of the algorithms. Extremely high accuracy may require substantial computational resources, making the model impractical for real-time use.

Allen et al. [1] explored the application of the Pade method for approximating transmission line parameters to streamline the modeling of transient processes. The research demonstrated that the Pade method provides a high level of accuracy in estimating transmission line parameters such as impedance, capacitance, and inductance. Furthermore, the study highlighted the method’s potential in improving the efficiency and precision of transient process modeling, making it a valuable tool for engineers and researchers in the field of electrical engineering.

Başar [2] conducted research applying the Pade method to develop rational functions that effectively modeled the behavior of transmission lines under various conditions, such as signal switching and load variations. The study revealed that the approximations generated by the Pade method surpassed the accuracy of traditional polynomial methods across diverse operational scenarios. Additionally, this approach proved to be robust and versatile, offering significant improvements in the predictive modeling of transmission line behavior under dynamic conditions, thereby enhancing the reliability and performance of electrical systems. Keleş and Tekalp [3] conducted a study on the application of the Pade method for generating rational functions that precisely approximate original functions. The findings indicated that the Pade method markedly enhanced accuracy in comparison to straightforward polynomial approximations. Furthermore, the research highlighted the method’s effectiveness in capturing the intricate behaviors of the original functions, making it a valuable technique for various analytical and computational applications where precision is crucial.

Ali et al. [4] conducted a comparative study on the precision of rational functions versus polynomial approximations of the same order. The study found that rational functions consistently demonstrated significantly higher accuracy than polynomial approximations of equivalent order. Additionally, the research underscored the superiority of rational functions in modeling complex systems, suggesting their greater efficacy in applications requiring high precision and reliability. In the study of Branin [5], the rate of convergence of the Pade method was assessed in comparison to alternative approximation techniques, particularly in scenarios featuring functions with poles. The results validated that the Pade method exhibited superior convergence speed, especially when dealing with functions containing poles, which posed challenges for other approximation methods.

In the investigation conducted by Wedepohl [6], transmission lines in electrical engineering and telecommunications were examined as concrete pathways for the propagation of signals. The study explored the fundamental operational principles and characteristics of these transmission lines, alongside their influence on signal transmission. The conclusions validated the essential role of transmission lines in electrical engineering and telecommunications, emphasizing their effectiveness in reliably transmitting signals over long distances.

In the study by Chang [7], the focus was on the modeling of transient processes within data transmission systems. This research is crucial for comprehending how these systems respond to various changes, such as signal activation or deactivation, load fluctuations, and other influencing factors. Various modeling methods and techniques were explored to capture the dynamic changes within the system, aiming to forecast their impact on overall system performance. The study's conclusions highlighted the critical importance of modeling transient processes in data transmission systems to understand their behavior under diverse conditions.

In the research conducted by Palusinski and Lee [8], the emphasis was placed on applying the Pade method to develop models capable of predicting system behavior under dynamic conditions. The study evaluated the effectiveness of the Pade method in comparison to other modeling and approximation techniques for predicting dynamic system behavior. The findings from the study showed that models constructed using the Pade method provided more accurate forecasts of system behavior under dynamic conditions than alternative modeling and approximation methods.

Detailed questions about the influence of Pade approximation on line parameters in the case of non-uniform losses and the impact of nonlinear effects on the accuracy of Pade approximation in analyzing transient processes remained unexplored. These include the method's influence on line parameters under conditions of non-uniform losses, where variations in resistance, inductance, or capacitance challenge accurate modeling of transient behavior. Additionally, the impact of nonlinear effects, such as high signal amplitudes or harmonics, on the precision of the Pade approximation remains insufficiently studied. While the method demonstrates superior accuracy compared to polynomial approaches, its limitations in handling complex systems with highly variable parameters are not fully understood. Addressing these gaps is critical for enhancing the applicability of the Pade approximation in fields like power transmission and telecommunications, where precise transient analysis is essential.

The aim of the research was to assess the effectiveness and limitations of Pade approximation of line parameters in analyzing transient processes in uniform lossy lines. The problematic questions include the impact of Pade approximation on the accuracy of analyzing transient processes in uniform lossy lines and the discussion of limitations and advantages of using Pade approximation compared to other methods.

2. MATERIALS AND METHODS

The use of the distributed parameter modeling method had enabled the analysis and consideration of key characteristics of transmission lines in mathematical models, such as their length, inductance, capacitance, and resistance. These parameters were essential for describing the behavior of electromagnetic signals and transient processes in lossy transmission lines. This approach allowed for more accurate modeling and analysis of dynamic processes occurring in energy or data transmission networks. When using the statistical approach, the effectiveness of combining the Pade method with generalized transmission line equations was investigated. This approach contributed to improving the accuracy and efficiency of analyzing transient processes in various electromagnetic systems, including data and power transmission networks.

Research was conducted using the algorithm development method to explore the properties of materials required for creating algorithms applicable to implementing the Pade method in approximating line parameters considering losses. This approach involved analyzing and determining suitable mathematical models and computational methods to ensure effective and accurate approximation of line parameters considering losses. The application of approximation order optimization allowed for considering the determination of the optimal order, achieving a balance between accuracy and computational complexity. This process involved analyzing various approximation orders to select one that provided sufficient modeling accuracy with minimal computational burden.

Advanced simulation facilities were used to evaluate the application of the Padé approximation in the modeling of transmission line transients. The simulations were performed using ATP (Alternative Transients Program) software, a robust tool for electromagnetic transient analysis. Key parameters included line lengths, resistances, capacitances, inductances, and conductances that reflected real transmission systems such as a 5 cm two-wire line and a 150 km long 110 kV multi-wire overhead line. Assumptions included homogeneity of line materials and an idealized ground conductor in some scenarios, and some losses were approximated to simplify the models. The simulations also included boundary and initial conditions such as periodic voltage sources and specific load impedances to replicate practical operating conditions. By synthesizing the characteristic impedance and exponential propagation functions using the Padé method, the transient behavior under different conditions was evaluated to understand the accuracy and efficiency of the method.

The integration method into existing software tools was used to develop and implement the developed approximation algorithms into specialized software tools, such as electromagnetic field modeling packages. This process involved not only creating algorithms but also integrating them into existing software products to enrich their functionality and enhance electromagnetic field modeling capabilities. Verification and validation methods were applied to assess the reliability of the developed techniques by comparing the simulation results with analytical solutions or experimental data. This process ensured confidence in the accuracy and precision of applying the methods in real-world conditions and enabled the detection and correction of potential errors or inaccuracies in the modeling.

The method of comparative analysis was used to evaluate the effectiveness and accuracy of the Pade approximation method compared to other methods for analyzing transient processes in homogeneous lossy lines. The analysis conducted allowed for identifying the advantages and disadvantages of each method, as well as determining the areas where they are most effectively applied in various scenarios of modeling and analyzing electromagnetic systems. This approach helped in selecting the most suitable methods for specific tasks in the field of modeling and analyzing electromagnetic systems, contributing to the development of efficient and accurate methods for assessing electromagnetic phenomena and improving the quality of engineering solutions.

3. RESULTS

3.1 The lossy uniform transmission line modelling according to the Thevenine-Norton structure

The analysis of two-conductor uniform lossy lines involves solving generalized equations in the frequency domain, taking into account the boundary conditions of voltages and currents [2]. Figure 1 shows a two-conductor uniform lossy line of length l (where ground is considered the first conductor), the modelling of which is based on these analysis conditions:

1. Boundary conditions in Eqs. (1)-(2):

$$v(0,t) = v_1(t); (l,t) = v_2(t), \tag{1}$$

$$i(0,t) = i_1(t); -i(l,t) = i_2(t). \tag{2}$$

2. The initial conditions in Eq. (3):

$$v(x,0) = 0; i(x,0) = 0; 0 \leq x \leq l. \tag{3}$$

The parameters R, L, C, and G denote the values per unit length of the line, which typically vary with frequency [1, 2]. So, in the frequency domain in Eq. (4):

$$R \equiv R(s); L \equiv L(s); C \equiv C(s); G \equiv G(s). \tag{4}$$

In the complex frequency domain, the solution of the telegraph equations is expressed in terms of complex frequencies and complex amplitudes. This allows for incorporating both active and reactive signal components, alongside phase shifts. In this base the solution of the telegraphic equations can be expressed in the complex frequency domain as follows Eqs. (5)-(6):

$$v(x,s) = exp[-\gamma(s)x]\, A(s) + exp[+\gamma(s)x]\, B(s), \tag{5}$$

$$Z(s)i(x,s) = exp[-\gamma(s)x]\, A(s) - exp[+\gamma(s)x]\, B(s). \tag{6}$$

V(*x*, *s*) and *I*(*x*, *s*) are the voltage and current at a point x along the transmission line in the frequency domain s.

From Figure 1:

$$Z(s) = \sqrt{\frac{R+Ls}{G+Cs}}, Y(s) = Z^{-1}(s), \tag{7}$$

$$Z\gamma(s) = \sqrt{(R+Ls)(G+Cs)}. \tag{8}$$

Z(s) and *Y*(s) denote the per-unit-length impedance and admittance matrices in the frequency domain.

Setting:

$$v(x,s) \equiv v_x(s), i(x,s) \equiv i_x(s), \tag{9}$$

and successively adding and subtracting Eqs. (5) and (6), the authors obtain the solutions in Eqs. (10)-(11):

$$v_1(s) = Z(s)i_1(s) + E_1(s), \tag{10}$$

$$v_2(s) = Z(s)i_2(s) + E_2(s), \tag{11}$$

where,

$$v_2(s) = Z(s)i_2(s) + E_2(s), \tag{12}$$

$$E_2(s) = exp[-\gamma(s)l] + [v_1(s) + Z(s)i_1(s)]. \tag{13}$$

By comparing Eqs. (10)-(11) with Eqs. (12)-(13), the authors obtain Eqs. (14)-(15):

$$E_1(s) = exp[-\gamma(s)l] + [2v_2(s) - E_2(s)], \tag{14}$$

$$E_2(s) = exp[-\gamma(s)l] + [2v_1(s) - E_1(s)]. \tag{15}$$

Therefore, for Eqs. (10)-(11) is made a correspondence through equivalent network presented in Figure 2, which contains the Thevenine-structure, where $E_1(s)$ and $E_2(s)$ are given in the Eqs. (14)-(15).

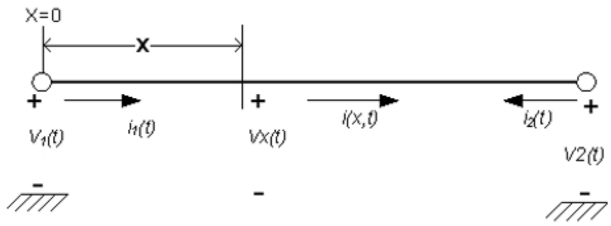


Figure 1. Uniform lossy line with one conductor (Ground is the other conductor)

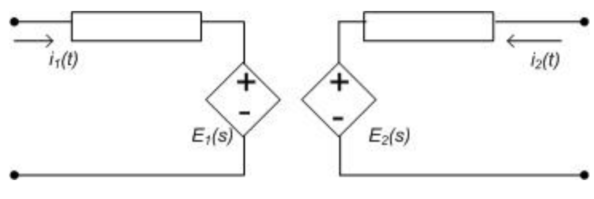


Figure 2. The equivalent network of the loss uniform line with one conductor according to Thevenin – structure

Complex frequencies are represented as the sum of a real part (real frequency) and an imaginary part (complex part). This representation allows for the consideration of both the magnitude and phase of the signal. Analyzing signals in uniform lossy lines is critical due to variations in parameters such as time and distance. By expressing the respective currents, the authors obtain Eqs. (16)-(17):

$$i_1(s) = Y(s)v_1(s) - exp[-\gamma(s)l]\,[Y(s)v_2(s) + i_2(s)], \tag{16}$$

$$i_2(s) = Y(s)v_2(s) - exp[-\gamma(s)l]\,[Y(s)v_1(s) + i_1(s)] \quad (17)$$

Denoting:

$$J_1(s) = exp[-\gamma(s)l]\,[Y(s)v_2(s) + i_2(s)], \quad (18)$$

$$J_2(s) = exp[-\gamma(s)l]\,[Y(s)v_2(s) + i_2(s)]. \quad (19)$$

Gives:

$$i_1(s) = Y(s)v_1(s) - J_1(s), \quad (20)$$

$$i_2(s) = Y(s)v_1(s) - J_2(s). \quad (21)$$

Comparing the Eqs. (18)-(19) and Eqs. (20)-(21), the authors obtain Eqs. (22)-(23):

$$J_1(s) = exp[-\gamma(s)l]\,[2i_2(s) + J_2(s)], \quad (22)$$

$$J_2(s) = exp[-\gamma(s)l]\,[2i_1(s) + J_2(s)]. \quad (23)$$

The Eqs. (22)-(23) have an equivalent two-port network shown in Figure 3.

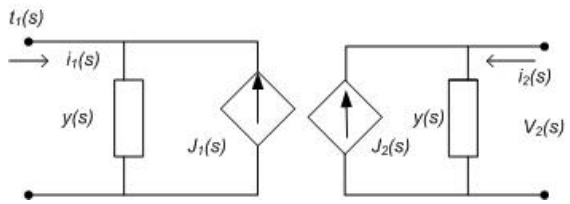


Figure 3. The equivalent network of the lossy uniform line with one conductor according to Norton-structure

The use of the complex frequency domain allows for a detailed examination of these systems, providing a comprehensive understanding of signal behavior within the transmission line. Expanding on the theory of telegraphic equations applied to multiconductor transmission lines, the research investigates a uniform lossy line with n-coupled conductors, depicted in Figure 4 [2].

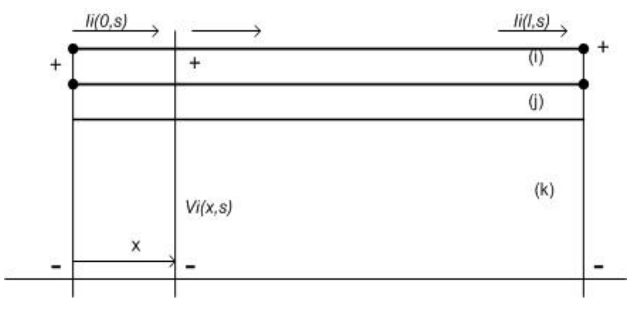


Figure 4. The lossy uniform line with n-conductors and the coordinative system

Assuming the conductor of line parallel with the ground in the medium homogeneous and the initial conditions zero, authors denoted: Ri(s), internal resistance of the i-th line conductor; Li(s), internal inductance of the i-th line conductor; zg(s), earth complete impedance. "In and Dn represent unit matrices of order n×n, with elements defined as follows:"

$$[I_n]_{ii} = 1;\ [I_n]_{ij} = 0;\ i \neq j;\ i,j = 1,2,\ldots,n$$

$$[D_n]_{ii} = 1;\ [D_n]_{ij} = 0;\ i \neq j;\ i,j = 1,2,\ldots,n$$

$Z_c(s) = [R(s) + sL(s)]1_n$is a diagonal $n\times n$ order matrix of the line conductor's complete impedance; *Rg(s)* is $n\times n$ order matrix, which represents the internal earth resistance; $Z_c(s) = [R_g(s) + sL_g(s)]$ the Earth impedance matrix of size $n\times n$ is computed using Carson's formulas [2, 3, 6].

The term external inductance (or inductive resistance) refers to the presence of inductance that exists outside the primary components of a circuit, such as wires or PCBs. This phenomenon can occur due to factors such as coil windings or the elongation of wire lengths [9, 10]. L, C are the real, symmetric (independent of frequency) matrices per unit length of external inductance and external capacitance, computed for static conditions and assuming lossless conductor, perfect earth and lossless medium:

$$C = CL = \mu_0\varepsilon_0 I_n, \quad (24)$$

Therefore

$$C = \mu_0\varepsilon_0 L^{-1}, \quad (25)$$

where, *Y(s)=G+sC* is a complete admittance symmetric matrix of line $V_x(s)$, $I_x(s)$ are respectively the vectors of phase voltages and currents of the conductors in the point x of the line:

$$V_x(s) = [V_{x1}(s), V_{x2}(s), \ldots\ V_{xn}(s)], \quad (26)$$

$$I_x(s) = [I_{x1}(s), I_{x2}(s), \ldots\ I_{xn}(s)]. \quad (27)$$

Considering the line equation for the TEM mode propagation in line:

$$\frac{\partial V_x(s)}{\partial x} = -Z(s)I_x(s), \quad (28)$$

$$\frac{\partial I_x(s)}{\partial x} = -Y(s)V_x(s). \quad (29)$$

The authors obtain Eqs. (30)-(31):

$$\frac{d^2V_x(s)}{dx^2} = Z(s)Y(s)V_x(s), \quad (30)$$

$$\frac{d^2I_x(s)}{dx^2} = Z(s)Y(s)I_x(s). \quad (31)$$

By employing variable transformation techniques to generate the general solution for multiconductor transmission line equations [7], the authors converted them into mode quantities as Eqs. (32)-(33):

$$\hat{V}(x) = \hat{T}_V\hat{V}_m(x), \quad (32)$$

$$\hat{I}(x) = \hat{T}_I\hat{I}_m(x). \quad (33)$$

The $n\times n$ complex matrices $\hat{T}_V$ and $\hat{T}_I$ define a change of variables between the actual phasor line voltages and currents $\hat{V}$ and $\hat{I}$, and the mode voltages and currents $\hat{V}_m$ and $\hat{I}_m$. For the case of phase complete transposition line, the matrix of eigenvalues results:

$$T = T_V = T_I. \quad (34)$$

During the process of linear rearrangement from phase to modal coordinates [6], the authors derived as shown in Eqs. (35)-(36):

$$V_x(s) = T v_{mx}(s), \quad (35)$$

$$I_x(s) = T i_{mx}(s), \quad (36)$$

where, $v_{mx}(s)$, $i_{mx}(s)$ are respectively voltage and modal currents vectors at point x of line: m=1, 2, …, n.

The T matrix in case of n=3 and for the preliminary condition has the form of Clark rearrangement:

$$T = \frac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & \sqrt{3} & 1 \\ \sqrt{2} & 0 & -2 \\ \sqrt{2} & -\sqrt{3} & 1 \end{bmatrix}; \mathrm{T}^{-1} = \frac{1}{\sqrt{6}}\begin{bmatrix} \sqrt{2} & \sqrt{2} & \sqrt{2} \\ \sqrt{3} & 0 & \sqrt{3} \\ 1 & -2 & 1 \end{bmatrix}. \quad (37)$$

After diagonalized the symmetric matrices *Z(s)* and *Y(s)*; expressing the characteristic resistances in modal coordinates and diagonalized too, admitting for m=1, 2, 3:

$$z(s) \equiv z_m(s); y(s) \equiv y_m(s), \quad (38)$$

$$V_1(s) \equiv V_m^{(1)}(s); V_2(s) \equiv V_m^{(2)}(s); \mathrm{E}_1(s) \equiv \mathrm{E}_m^{(1)}(s); \mathrm{E}_2(s) \equiv \mathrm{E}_m^{(2)}(s), \quad (39)$$

$$i_1(s) \equiv i_m^{(1)}(s); i_2(s) \equiv i_m^{(2)}(s); J_1(s) \equiv J_m^{(1)}(s); J_2(s) \equiv J_m^{(2)}(s). \quad (40)$$

The conditions of frequency-independent imply that the external inductance and external capacitance remain unchanged over a wide frequency range, which can be crucial for various applications, especially in high-frequency circuits. The authors obtain the two-port equivalent networks in modal coordinates according to Thevenin and Norton structure [11]. In the same way, using T and T-1 eigenvalue matrix, can be the transform in modal coordinates of the source network and the load at the end of the line.

3.2 Pade approximation of the characteristic impedance (admittance) and exponential propagation function

Each function F(y), which is expanded in Mac Lauren series in relation with y, around point y=0, formed Eq. (41):

$$F_k(y) = 1 + m_1 y + m_2 y^2 + \ldots + m_k y^k. \quad (41)$$

Putting in correspondence with rational function of (n, n) order by Pade approximation formed Eqs. (42)-(43):

$$F_p(y) = \frac{a_n y^n + a_{n-1} y^{n-1} + \ldots + a_1 y + 1}{b_n y^n + b_{n-1} y^{n-1} + \ldots + b_1 y + 1}, \quad (42)$$

$$F_p(y = \infty) = \frac{a_n}{b_n}; F_p(y = 0) = 1, \quad (43)$$

where, the moments are computed as below:

$$m_k = \frac{\frac{d^k F(y)}{dy^k}}{k!}\Big|_{y=0}. \quad (44)$$

On this basis, the authors can transform the characteristic impedance (admittance) into the form Eq. (42), after expanding it as shown in Eq. (41) [12, 13]. Therefore, if the authors perform expansion around point s=∞ in the relation with 1/s:

$$Y(s) = \sqrt{\frac{C}{L}}\left(\frac{1+\frac{G}{Cs}}{1+\frac{R}{Ls}}\right)^{1/2}, \quad (45)$$

$$F(y) = \left(\frac{1+ay}{1+by}\right)^{1/2} \text{ for } a = \frac{G}{C}, b = \frac{R}{L}. \quad (46)$$

For y=1/s, calculating the (2n-1) moments of function as shown in Eq. (44), the Pade approximation of impedance and admittance has the form Eqs. (47)-(48):

$$Z_P(s) = \sqrt{\frac{L}{C}} \frac{s^n + a_1 s^{n-1} + \ldots + a_n}{s^n + b_1 s^{n-1} + \ldots + b_n}, \quad (47)$$

$$Y_P(s) = \sqrt{\frac{C}{L}} \frac{s^n + a_1 s^{n-1} + \ldots + a_n}{s^n + b_1 s^{n-1} + \ldots + b_n}, \quad (48)$$

from where the authors can determine the functions if impedance (admittance) as below Eqs. (49)-(50):

$$Z_P(s) = Z(s = 0) = \left(\frac{R}{G}\right)^{1/2}, \quad (49)$$

$$Y_P(s) = Y(s = 0) = \left(\frac{G}{R}\right)^{1/2}. \quad (50)$$

The coefficients a_j, b_j in Eqs. (47)-(48), depending by the sort of F(y) and the regarding moments are computed Eqs. (51)-(52):

$$\begin{matrix} \frac{1-a_n}{b_n} & m_1 & m_2 & \cdots & m_{n-1} \\ m_1 & m_2 & m_3 & \cdots & m_n \\ m_2 & m_3 & m_4 & \cdots & m_{n+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{n-1} & m_n & m_{n+1} & \cdots & m_{2n-2} \end{matrix} \begin{matrix} b_n \\ b_{n-1} \\ b_{n-2} \\ \cdots \\ b_1 \end{matrix} = - \begin{matrix} m_n \\ m_{n+1} \\ m_{n+2} \\ \cdots \\ m_{2n-1} \end{matrix}, \quad (51)$$

$$\begin{matrix} a_1 \\ a_2 \\ a_3 \\ \cdots \\ a_{n-1} \end{matrix} = \begin{matrix} 1 & 0 & 0 & \cdots & 0 \\ m_1 & 1 & 0 & \cdots & 0 \\ m_2 & m_1 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ m_{n-2} & m_{n-3} & m_{n-4} & \cdots & 1 \end{matrix} \begin{matrix} b_1 \\ b_2 \\ b_3 \\ \cdots \\ b_{n-1} \end{matrix} + \begin{matrix} m_1 \\ m_2 \\ m_3 \\ \cdots \\ m_{n-1} \end{matrix}. \quad (52)$$

In analogue manner is performed the Pade expanding Eq. (42) of the characteristic impedance (admittance) in series y=s for s=0, where the coefficients a_j, b_j are computed according the above matrices in Eqs. (53)-(54):

$$Z_P(s) = \sqrt{\frac{R}{G}} \frac{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + 1}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + 1}, \quad (53)$$

$$Y_P(s) = \sqrt{\frac{G}{R}} \frac{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + 1}{b_n s^n + b_{n-1} s^{n-1} + \ldots + b_1 s + 1}. \quad (54)$$

Let’s denote:

$a = \frac{R}{L}$; $b = \frac{G}{C}$; $\tau = l(LC)^{\frac{1}{2}}$; $\tau_1 = \frac{\tau}{l}$; $A = \gamma(s = 0) \cdot l = l \cdot (RG)^{\frac{1}{2}}$; $\gamma(s = \infty) \cdot l = s \cdot \tau$; $\alpha_1 = -m_{i+1}(LC)^{\frac{1}{2}}$; $i = 1,2, \ldots$

Let’s transform the function:

$$\gamma(s) = [(R + Ls)(G + Cs)]^{\frac{1}{2}} = s\tau_1 \left[\left(1 + \frac{a}{s}\right)\left(1 + \frac{b}{s}\right)\right]^{\frac{1}{2}} = s\tau_1 + s\tau_1 \left[\left(1 + \frac{a}{s}\right)^{\frac{1}{2}}\left(1 + \frac{b}{s}\right)^{\frac{1}{2}} - 1\right], \quad (55)$$

By setting $y = \frac{1}{s}$ for $s \to \infty$ and y=0, and expanding in a Maclaurin series with respect to y [5], the authors computed the moments in Eq. (44) and obtained Eq. (56):

$$\sqrt{\left(1 + \frac{a}{s}\right)\left(1 + \frac{b}{s}\right)} = 1 + \sum_{i=1}^{\infty} \frac{m_i}{s^i}, \quad (56)$$

In the same way as in paragraph 2, the authors calculate respectively α_j, β_j coefficients for the exponential propagation function by Pade approximation:

$$exp[-\gamma(s) \cdot l] = exp(-s\tau)\, exp(\alpha_0) \frac{s^n + a_1 s^{n-1} + \ldots + a_n}{s^n + b_1 s^{n-1} + \ldots + b_n}, \quad (57)$$

Solving by the boundary conditions and transforming further as function of impedance [14], the authors obtain the expression:

$$exp[-\gamma(s) \cdot l] = exp(-s\tau) \frac{\left(\frac{1}{2}\right) Z_0\, exp(\alpha_0) F(s)}{\left(\frac{1}{2}\right) Z_0} = exp(-s\tau) \left[\left(\frac{1}{2}\right) Z_0\right] y_e(s), \quad (58)$$

where,

$$Z_0 = Z(s = \infty) = \left(\frac{L}{C}\right)^{\frac{1}{2}},\ Z_e(s) = 2Z_0\, exp(\alpha_0)\, F(s), \quad Y_e(s) = \frac{1}{\frac{1}{2} Z_0} \cdot exp(\alpha_0)\, F(s). \quad (59)$$

The line model according to Thevenine structure, together with data of characteristic impedance synthesis are shown in Figure 5.

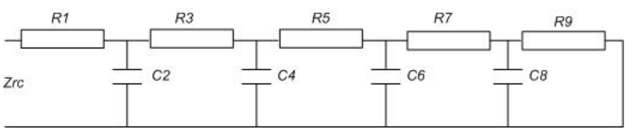


Figure 5. Thevenine-structure line model with characteristic impedance data, synthesis, and exponential propagation function

The synthesis of function in Eq. (58) can be achieved using lossless lines with characteristic impedance Z_0 by incorporating the time delay τ and synthesizing the function $Z_e(s)$ or $Y_e(s)$ through Foster’s or Cauer’s methods [8, 9].

3.3 Case study

Example 1. A theoretical model of two-conductor transmission line. Let`s have a line with parameters (PUL R=2.5 Ω/cm; C=4×10^{-12} F/cm, L=108 H/cm, G=5×10^{-4} Ω^{-1}/cm and its length l=5 cm. An active resistance of 100Ω is connected at the line’s output, while at the input, there is a periodic voltage source, shown in Figure 6, in series with an internal resistance of 100Ω [15, 16]. It`s performed the Caurer synthesis using the Pade approximation of the characteristic impedance, then is performed the $Z_{RC}(s)$ function synthesis, in form of lumped RC-parameters given in Figure 5. The synthesis of the exponential propagation function is done for n=2 in an admittance $y_e(s)$, which is performed according to Foster [17, 18]. After performing the transient simulation through ATP software (Figures 6 and 7), in Figure 8 are shown the input and output voltages in line, and the current in line also.

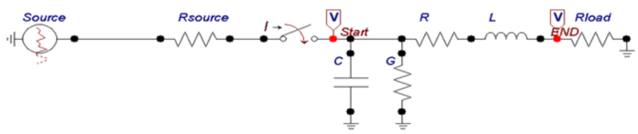


Figure 6. The model of line in ATP software

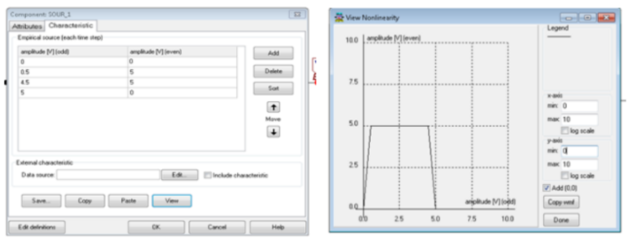
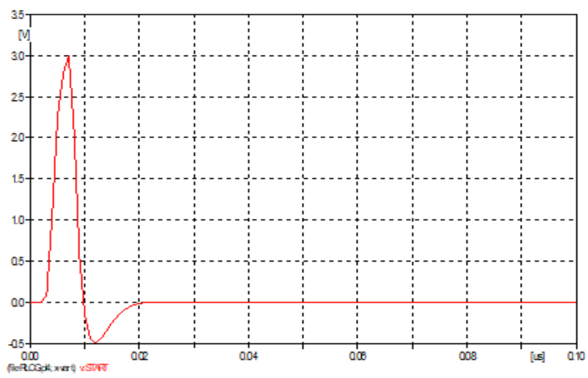
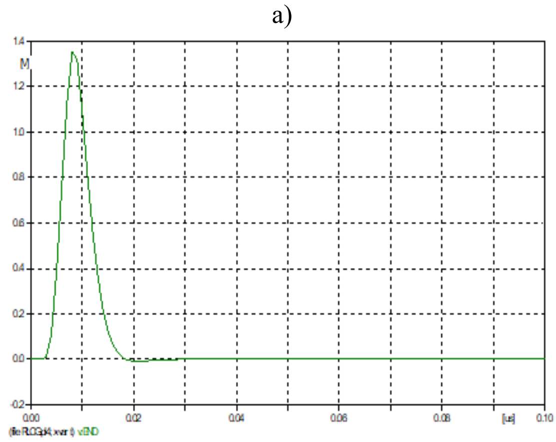


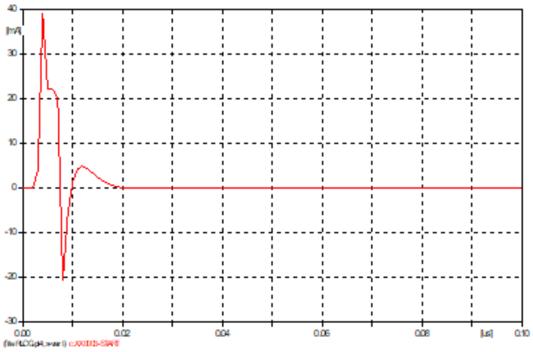
Figure 7. The impuls of voltage source



a)



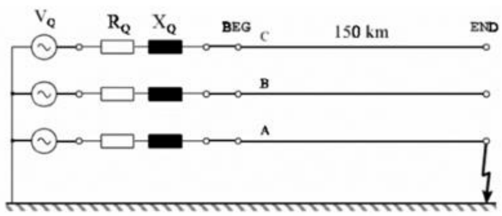
b)



c)

Figure 8. a) Voltage at the beginning of line; b) Voltage at the end of line; c) Current in line

Example 2. A model of an overhead 110 kV multiconductor transmission line. It`s analyzed a 110 kV transmission line with l=150 km, in the case of short circuit with ground of phase A, as shown in Figure 9.



Note: The parameters of line are: Z_1' =0.12+0.3871 Ω/km, C_1' =9 nF/km (positive sequence) and Z_0' =0.27+1.3927 Ω/km, C_0' =4.912 nF/km (zero sequence). The source network: V_Q=110/√3 kV (r.m.s values); R_Q=0.331 Ω; X_Q=3.331 Ω.

Figure 9. Three phase overhead 110 kV transmission line

Using the Pade approximation, there are calculated the modal parameters and the exponential propagation function of line. The model of line and the results of simulations in ATP software are shown in Figures 10 and 11.

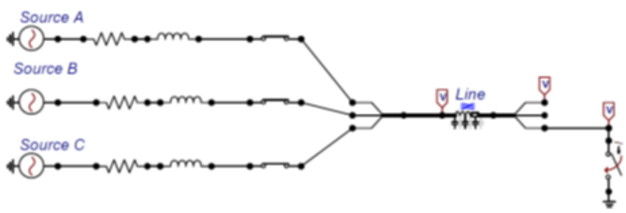
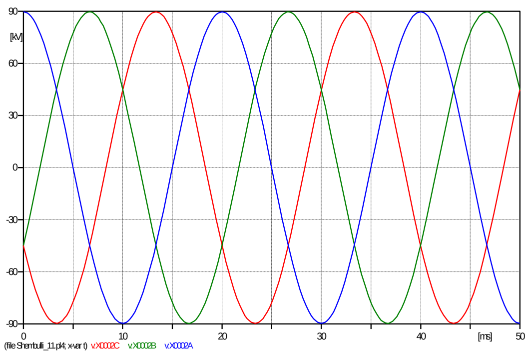
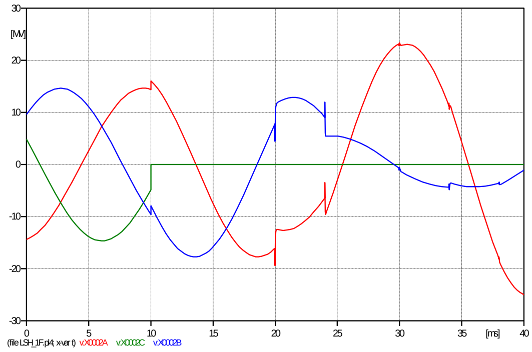


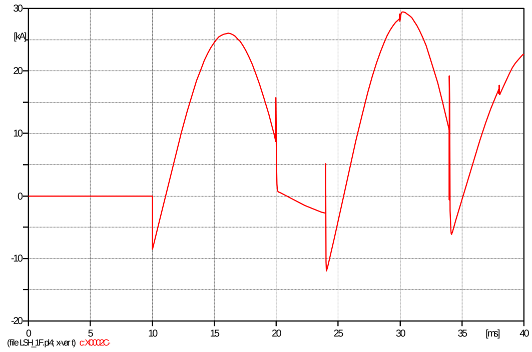
Figure 10. The model of line in ATP software



a)



b)



c)

Figure 11. The results of simulations for voltages and currents: a) Voltages at the beginning of line; b) Voltages at the end of line for the damage phase C and the other phases A and B; c) Current at the point of short-circuit in phase C of line

In the context of analyzing transient processes in homogeneous lossy lines, the Pade method is employed to approximate the parameters of these lines [19, 20]. This method accounts for losses due to the resistance of the line material, as well as other factors affecting the dynamic processes within the line.

The Pade approximation method, while offering superior accuracy compared to polynomial methods, has certain limitations and challenges. One significant drawback is its sensitivity to the order of approximation, as higher-order expansions can increase computational complexity and potentially lead to numerical instability. The method may also struggle with accurately modeling systems with non-uniform losses or nonlinear effects, as these conditions introduce complexities not inherently addressed by the standard Pade formulation. Furthermore, its reliance on rational functions can result in inaccuracies when applied to highly dynamic or chaotic systems with abrupt parameter variations. Additionally, the integration of the Pade approximation into existing modeling frameworks can pose implementation challenges, particularly in real-time applications where computational efficiency is critical. Understanding and addressing these limitations is essential to maximize the method's utility in practical scenarios.

4. DISCUSSION

The combination of the Pade method with general transmission line equations simplifies the analysis of transient processes in homogeneous lossy lines by approximating line parameters as rational fractions. Comparative analysis confirmed the method's accuracy and efficiency in modeling transient electromagnetic processes compared to traditional approaches. By enabling time-domain analysis, the method facilitates the study of dynamic behaviors in two-wire and multi-wire transmission lines under real-world conditions, helping identify transient effects caused by external disturbances. The simulation results validate the reliability of the proposed model for real-world applications, supporting its use in system design and optimization. Proper analysis of transient processes, especially for overhead lines in power systems, is essential to ensure reliability, safety, and prevention of issues such as accidents or energy losses caused by events like lightning or short circuits.

The results of this study highlighted the significance of utilizing the analysis of transmission line parameters to comprehend their impact on electromagnetic processes in power transmission systems. The careful determination of parameters such as resistance, inductance, capacitance, and conductivity contributed to enhancing design and improving energy transmission efficiency. Moreover, the investigation of transient process durations and resistance to short circuits enabled the prediction and prevention of potential operational issues in transmission lines, which were critical for ensuring their reliability and safety. These findings were applicable in engineering practice, where they enhanced the processes of designing and maintaining power transmission systems, as well as contributed to further scientific research in the field of electrical engineering and power systems. This aspect was also investigated by Alonso Fernández et al. [20] and Esho et al. [21]. Alonso Fernández et al. concluded that accurately determining transmission line parameters such as resistance, inductance, capacitance, and conductance was essential. These parameters were crucial for optimizing design and enhancing the efficiency of energy transmission, thereby reducing energy losses. Fernández's research focused on analyzing transient response times, providing a deeper understanding of the dynamic behavior of transmission lines. This analysis helped predict and address potential issues that could arise during line operation. Esho et al. concentrated on studying resistance to short circuits, which was vital for improving the reliability and safety of transmission lines. Accurate analysis of these parameters allowed for better planning of preventive measures to avoid accidents and mitigate risks. The findings from his research could be applied to improve the design and maintenance processes of power transmission systems. The proposed model aided engineers in making more informed decisions when developing and operating transmission lines.

The paper emphasized the importance of accurately determining transmission line parameters. The study demonstrated that precise modeling of parameters such as resistance, inductance, capacitance, and conductance played a critical role in optimizing the design and improving the efficiency of power transmission systems. The mathematical models utilized for these purposes provided a deeper understanding of the dynamic behavior of transmission lines, enabling the forecasting and mitigation of potential issues, as well as enhancing the design and maintenance processes of these systems. These findings underscored the necessity of employing modern mathematical models and methods for calculating transmission line parameters in engineering practice. This approach not only increased the reliability and safety of the lines but also reduced energy losses, which was particularly crucial given the growing energy consumption and the escalating demands for efficiency in power systems. The research conducted by Sorgucu [22], Alajrash et al. [23] confirmed the importance of considering multiple factors when analyzing the influence on inductance. The findings of Sorgucu [22] indicated that the shape and dimensions of conductors had a significant impact on the inductance of transmission lines. Different diameters or shapes of conductors were observed to result in varying levels of inductance. Additionally, positioning conductors in close proximity was found to enhance the magnetic field, thereby increasing inductance. These discoveries underscored the importance of precise conductor placement in the design of electrical systems. In the study by Alajrash et al. [23], the significance of assessing the stress state in the application area of power transmission lines was highlighted. Mathematical models were employed to accurately estimate energy losses in the form of heat generated when current flowed through conductors. This understanding was crucial for optimizing system performance and minimizing energy losses. Understanding the effects of magnetic fields on electrical systems was essential for the development and efficient operation of electrical equipment. Overall, the research underscored the importance of considering conductor geometry and arrangement in the calculation of inductance, as well as the critical role of mathematical models in evaluating energy losses and optimizing the performance of electrical systems.

The results of this study confirmed the necessity of employing complex models for high-voltage transmission lines, considering the influence of various factors such as the surrounding environment. This underscored the importance of adopting a more detailed and comprehensive approach to modeling such systems. The study also validated that the capacitance of the transmission line was influenced by the geometry of the conductors and the spacing between them, emphasizing the significance of conductor shape and arrangement. The application of capacitance calculation models aided in assessing the process of electrical charge accumulation and transmission between conductors, which was critical for understanding the transmission line's characteristics and optimizing its performance. This issue was also investigated by Kalli [24], Heinzel et al. [25]. Kalli [24] confirmed that the classical capacitance model for parallel conductors considered not only the distance between the conductors but also their sizes and the dielectric permittivity of the medium between them. This approach enabled a more precise estimation of the transmission line's capacitance and its electrical characteristics. The conductivity of insulation materials between the conductors was found to affect the leakage current and the overall resistance of the transmission line; high conductivity could lead to increased energy losses and reduced efficiency. Heinzel et al. [25] concluded that mathematical models for conductivity calculation allowed for assessing the effectiveness of insulation materials and their impact on transmission line operation. This was essential for selecting optimal materials and ensuring reliable system performance. Therefore, the studies emphasized the importance of considering various factors, such as conductor sizes, dielectric permittivity, and the conductivity of insulation materials, in the analysis and design of electric power transmission lines.

The findings of this study emphasized the significance of applying the concept of the complex frequency domain for analyzing dynamic systems, particularly transmission lines. It was confirmed that representing signals as complex numbers, where the real part represented their amplitude and the imaginary part denoted their phase, facilitated effective analysis of their behavior. One of the key conclusions drawn was that utilizing the complex frequency domain enabled the examination of systems with varying frequencies. This capability allowed researchers to analyze signals with different frequencies and phases, which was essential for understanding and modeling dynamic processes in transmission lines. The benefits of this approach included providing a more comprehensive description of signal behavior and enhancing understanding of their interaction with transmission lines. These insights opened up new avenues for developing efficient methods of analysis and control for electromagnetic systems, potentially leading to enhancements in the performance and reliability of technical systems.

This aspect was also investigated by Rojhani and Shaker [26], Adeleke et al. [27]. The research of Rojhani and Shaker [26] demonstrated that the utilization of the complex frequency domain was particularly advantageous for analyzing transient processes in transmission lines, where signals could vary in frequencies and phase characteristics. Transitioning from the time domain to the complex frequency domain provided additional insights into signal behavior, facilitating the assessment of spectral, frequency, and phase characteristics. Similarly, Adeleke et al. [27] concluded that the complex frequency domain significantly simplified the analysis and comprehension of signal behavior in transmission lines. This approach aided engineers in gaining a better understanding of system responses to external influences and in anticipating potential issues in signal transmission through transmission lines. Thus, the complex frequency domain played a pivotal role in the design and operation of electronic systems, offering a deeper understanding and more effective management of their behavior.

5. CONCLUSIONS

The object of this paper is the analysis of electromagnetic transients in transmission lines, which are characterized as circuits with distributed parameters. Based on the Pade mathematical approximation, the generalized equations of the line in the frequency domain, have been synthesized through the synthesis methods in equivalent circuits of Thevenin or Norton. The experiment confirmed that the use of Pade approximation ensures high accuracy and reliability of the obtained line parameters. This allows for a more detailed modeling and analysis of transient processes in the lossy line. This procedure is applied for the characteristic impedance (admittance) and exponential propagation function.

The facts presented in this paper are applicable to uniform lossy lines, which through the presented procedure, can be transformed into lossless lines and lumped-parameters circuits. The performed simulations prove the accuracy of the presented model for the transient processes of voltages and currents in the line. The study confirms that employing Pade approximation for determining line parameters is an effective method for analyzing transient processes. This approach enables the consideration of losses in the line and ensures more accurate results compared to alternative approximation methods. This study developed efficient methodologies and strategies for the development and validation of an effective approach to analyzing transient processes in uniform lossy lines based on the Pade approximation of line parameters.

The study is limited to the analysis of transient processes in homogeneous lossy lines, it may not account for some other factors or conditions that could affect the accuracy of the results. Future research directions based on the results of this work may focus on extending the Padé approximation method to analyze transients in more complex scenarios such as inhomogeneous transmission lines, multiwire systems, and networks with significant nonlinear effects. In addition, exploring the integration of this method with new computational techniques such as machine learning can further improve the accuracy and efficiency of transient analysis. Exploring the applicability of this approach to broader engineering fields such as modern telecommunication networks and renewable energy systems can provide valuable insights. Finally, studying environmental factors and their influence on the performance of transmission lines using the proposed model can pave the way for more sustainable and reliable designs in the power sector.

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