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The Dynamic Motion of Hybrid Nanofluid Intertwined with Viscous Dissipation and Nonlinear Radiative Heat Flux Gliding Across a Disk

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ABSTRACT

Nanofluids and hybrid nanofluids are vital in heat transfer and fluid dynamics. This study explores the influence of flow parameters on these non-Newtonian fluids, emphasizing their symmetric behavior in thermal modeling. Analyzing steady two-dimensional flow, it integrates nonlinear radiative effects and viscous dissipation, enhancing understanding of their mechanics in spinning disk motion. This study investigates the influence of various flow parameters on the behaviour of nanofluids and hybrid nanofluids, specifically non-Newtonian symmetric fluids, within the context of heat transfer and fluid dynamics. The research models a steady two-dimensional flow influenced by nonlinear radiative effects and viscous dissipation over a spinning disk. The governing partial differential equations are transformed into ordinary differential equations using variable transformations and solved using the Runge-Kutta method combined with shooting techniques. The study reveals that increasing magnetic field strength reduces velocity profiles, while nonlinear thermal radiation significantly enhances the Nusselt number, temperature, and velocity but decreases local skin friction. These findings illustrate complex interactions between flow parameters and fluid behaviour. The results provide valuable insights into the thermal and dynamic characteristics of hybrid nanofluids, aiding advancements in heat transfer modelling and applications in engineering and industrial processes.

1. INTRODUCTION

The quest for hybrid nanofluids has ignited extraordinary intrigue due to their unparalleled thermal attributes and the vast array of potential applications in an abundance of engineering endeavours, particularly in mechanisms of heat transfer these extraordinary fluids birth from the union of multiple nano particles suspended in a foundational liquid, showcase remarkable thermal conductivity and heightened heat transfer capabilities when Juxtaposed which traditional single component liquids. This rise in thermal efficiency makes hybrid nanofluids particularly enticing for the use in cooling systems, energy storage solutions and various industrial domains where adept heat management is crucial. The incorporation of viscous dissipation alongside non linear radioactive heat flux introduces additional complexities to the flow dynamics, necessitating advanced mathematical modelling to accurately predict their behaviour across the range of operational context. Such modelling not only aids in fine tuning the design of heat transfer systems but also enriches our understanding of the core physical principles governing the efficacy of hybrid nanofluids paving the way of revolutionary paces in thermal management. These breakthroughs hold the promise of introducing next generation cooling systems that significantly boost energy efficiency and reduce operational costs across a wide spectrum of industries including automotive aerospace and electronics. The synthesis of these innovative cooling strategies could also reinforce sensible bolster sustainability initiatives by minimising energy consumption and lowering greenhouse gas emissions, thus nurturing more environmentally conscious industrial landscape.

The exploration of hybrid nano fluids has emerged as an essential frontier in the domain of thermal management fuelled by their remarkable thermal properties and wide ranging applications in engineering pursuits, particularly within heat transfer systems. Hybrid nanofluids characterised by the amalgamation of various types of nano fluids suspended in a fluid medium, demonstrate increased thermal conductivity and improved heat transfer capabilities when compared to traditional single component fluids. This remarkable surge in thermal performance positions hybrid nanofluids as enticing candidates for a multitude of applications such as cooling systems energy storage technologies and numerous industrial process where good heat control is crucial. The inclusion of factors like viscous dissipation and non linear radioactive heat flux introduces layers of complexity to flow dynamics of these fluids, demanding sophisticated mathematical modelling to accurately capture their behaviour in diverse operational context. Such modelling initiatives not only felicitate the better heat transfer system designs but also deepen our understanding of the core physical principles that dictate the effectiveness of hybrid nanofluids. This research could ultimately pave the way for the development of state of the art cooling technologies that enhance energy potency and reduce operational costs across various fields including automotive aerospace and electronics. Furthermore the implementation of these revolutionary cooling advancements could significantly support sustainability initiatives by lowering energy usage consumption and alleviating greenhouse gas emissions thereby fostering more environmentally conscious industrial landscape. The intricate dance of boundary layer dynamics within an electrically charged fluid profoundly influences the transfer magnetic field. Such explanation notably significantly enriched the fabric of collaborative research, thanks to their vast applications spanning engineering, commercial physics, astrophysics and aerodynamics.

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Mansur et al. [1] set forth and exhilarating journey into the captivating and intricate world of p permeable sheets that exhibit both contracting and expanding behaviours in the realm of nano fluid, while providing a sharp analysis of the heat transfer mechanisms that fuel these captivating phenomena. Esfe et al. [2] performed an experimental investigation on the thermal conductivity of Cu/TiO_2–water/EG hybrid nanofluid and formulated predictive models employing artificial neural networks and empirical correlations. Their findings revealed that the hybrid nanofluid exhibits superior thermal conductivity compared to mono nanofluids, with thermal performance significantly influenced by nanoparticle concentration and base fluid composition. Mahanta and Shaw [3] unveiled the wonders of a 3D linear stretchable sheet, intricately linked with convective boundary conditions. Shahzad et al. [4] Immersed themselves in a remarkably intricate 3 dimensional flow of magnetohydrodynamics, skillfully intervening the elements of heat generation within a permeable medium, thus illuminating the intricate dynamics of thermal and fluid interaction in this elaborate terrain. Alao et al. [5] look into the complex dance of heat and mass transfer within a chemically reactive fluid cascading down a semi-infinite vertical plate. Their study explored the underlying physical mechanisms governing the transport phenomena, considering the effects of chemical reactions, thermal diffusion, and fluid dynamics on the overall system behaviour.

Mushtaq and Mustafa [6] analyzed the flow and heat transfer characteristics of a nanofluid near a stretchable rotating disk under the influence of an axial magnetic field and convective boundary conditions, solving the transformed equations using the Runge-Kutta-Fehlberg method. Their findings highlight that increasing magnetic field strength suppresses velocity while enhancing thermal boundary layer thickness, significantly affecting heat transfer rates.

Hayat and Nadeem [7] investigated the heat transfer enhancement of Ag-CuO/water hybrid nanofluid, emphasizing its superior thermal conductivity compared to conventional nanofluids. Their study demonstrated that the presence of hybrid nanoparticles significantly improves heat transfer efficiency, with increasing solid volume fraction leading to higher Nusselt numbers and enhanced thermal performance.

Ghadikolaei et al. [8] analyse the nonlinear effects of thermal radiation in Magneto nano fluid dynamics, highlighted by Joule heating as it is glided past and inclined penetrable stretchable sheet. Raza [9] assesses the importance of radiative heat transfer and slip effects on the stagnation point dynamics flowing elegantly past a convective stretchable sheet. Alsagria et al. [10] ventured into the enchanting domain of MHD nanofluid dynamics simulation, emphasizing the significant role of dissipative heating. Shoaib et al. [11] journeyed into the captivating three-dimensional landscape of magnetohydrodynamic flow characteristics linked to second-grade fluids as they gracefully traverse an absorbent plate, unveiling the intricacies of fluid behaviour under such distinctive conditions. Sarkar et al. [12] probed into the radiative characteristics of nanofluids streaming over an inclined cylindrical surface while addressing the chemically reactive behaviours of nanofluids on a stretchable sheet in the backdrop of joule heating influences.

The fascinating impact of fusion on MHD Casson fluid dynamics moving across a stretchable sheet within a permeable medium was thoroughly investigated by Mabood and Das [13]. Their findings revealed that increasing the magnetic field strength and nanoparticle concentration enhances thermal conductivity while influencing the boundary layer flow dynamics, making the study significant for advanced cooling and thermal management applications.

Idowu and Falodun [14] investigated the systematic flow of viscous and chemically active non-Newtonian fluids, deftly adjusting viscosity and thermal conductivity. Acharya et al. [15] investigated The Thermal-fluid process behavior of a magnetized TiO_2–$CoFe_2O_4$ water-based hybrid nanofluid in steady-state flow over a radiative rotating disk, emphasizing the influence of magnetohydrodynamics and thermal radiation on heat transfer efficiency. Waqas et al. [16] conducted a comprehensive study on the influence of magnetohydrodynamic (MHD) radiative Flow dynamics of a hybrid nanofluid over a rotating disk focusing on the interplay between electromagnetic forces, thermal radiation, and heat transfer mechanisms. Their research examined how variations in magnetic field intensity and radiative heat flux impact fluid motion, energy distribution, and overall thermal efficiency. By analysing key governing parameters, such as nanoparticle concentration, magnetic field strength, and radiation effects, the study provided valuable insights into optimizing heat transfer performance in advanced thermal systems. The findings contribute to a deeper understanding of MHD-based thermal management techniques, which are crucial in engineering applications such as energy systems, aerospace technology, and industrial cooling processes.

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Abdul Hakeem et al. [17] analyzed the three-dimensional viscous dissipative flow of nanofluids over a Riga plate, emphasizing the combined effects of magnetohydrodynamics and heat dissipation on fluid dynamics. Their study demonstrated that the incorporation of nanoparticles improves thermal conductivity while the Lorentz force generated by the Riga plate significantly influences velocity and temperature profiles, making the findings relevant for advanced thermal engineering applications. Al Mamun et al. [18] explored a numerical simulation to Inspect the periodic magnetohydrodynamic (MHD) flow of Casson nanofluid over a permeable extending surface, focusing on the Joint influences of magnetic fields and fluid rheology. Their findings revealed that increasing the magnetic field intensity and Casson parameter enhances flow resistance, while nanoparticle concentration significantly improves heat transfer efficiency, making the study valuable for biomedical and industrial fluid applications. Thamaraikannan et al. [19] examined the behaviours of nanofluids as they traversed a permeable channel shaped by MHD and periodic body acceleration.

Muntzir et al. [20] undertaken a numerical study on the magnetohydrodynamic (MHD) flow of Carreau nanofluid with gyrotactic microorganisms over various geometries including a plate, wedge, and stagnation point. Their analysis highlighted the influence of non-Newtonian fluid properties, magnetic fields, and bioconvection on heat and mass transfer, demonstrating the potential for enhanced microbial transport and thermal efficiency in engineering and biomedical applications. The subtle intricacies of heat transfer in magnetohydrodynamics, especially concerning the behaviour of non-Newtonian fluids interacting with a permeable shrinking and stretching sheet, were carefully dissected and explored by Vishalakshi et al. [21], illuminating the nuances of this compelling topic. Anwar Saeed et al. [22] explored the flow dynamics of ternary hybrid nanofluids over a spinning disk, incorporating the effects of nonlinear thermal radiation on heat transfer performance. Their study demonstrated that the inclusion of multiple nanoparticles enhances thermal conductivity and energy transport efficiency, while the nonlinear radiation significantly influences temperature distribution, making the findings crucial for high-performance thermal systems and industrial cooling.

Alqawasmi et al. [23] presented a numerical study on the flow of ternary hybrid nanofluids over a disk, considering the influence of a nonlinear heat source-sink and the Fourier heat flux model. Their findings demonstrated that the inclusion of three different nanoparticles enhances thermal conductivity and energy transport, while the nonlinear heat source-sink mechanism plays a crucial role in temperature regulation, making the study valuable for optimizing thermal management in industrial and engineering applications. Algehyne et al. [24] conducted an in-depth analysis of the Mechanical properties of magnetohydrodynamic (MHD) flow in a non-Newtonian, Maxwell fluid across a surface heated by bi-directional convection under conditions of mass flux. Their research highlighted the complex interactions between viscoelastic effects, magnetic field influences, and convective heating, demonstrating their significant impact on velocity distribution and thermal boundary layer formation. The findings provide valuable insights into optimizing heat transfer mechanisms in various advanced engineering applications, including industrial thermal management systems, polymer processing, and energy conversion technologies. Irshad et al. [25] Numerical simulations were conducted to analyze, the effects of magnetohydrodynamics (MHD) on the flow of a generalized- Newtonian fluid over a stretching sheet within a permeable medium. The analysis was performed using the finite difference method to capture flow behavior and thermal characteristics. Their findings highlighted that permeability and magnetic field strength significantly alter velocity and temperature profiles, demonstrating crucial implications for industrial coating processes and advanced heat transfer applications.

Ullah et al. [26] Examined the improvement in thermal performance of a ternary hybrid nanofluid (SiO_2 + Cu + MoS_2/H_2O) in symmetric flow over a nonlinear stretching surface using a hybrid Cuckoo Search-based artificial neural network approach. Their study revealed that the combination of multiple nanoparticles significantly improves heat transfer efficiency, while the AI-based optimization technique provides accurate predictions for fluid behavior, making the research valuable for advanced thermal management and engineering applications.

Ramzan et al. [27] carried out an analysis of ternary hybrid nanofluid flows over multiple geometries with non-isothermal and non-isosolutal boundary conditions. Their study demonstrated that the presence of three distinct nanoparticles enhances thermal and solutal transport properties, while variations in temperature and concentration gradients significantly impact flow dynamics, making the findings relevant for advanced heat transfer and fluid engineering applications.

Sudarmozhi et al. [28] investigated the boundary layer characteristics of a Maxwell fluid over a porous inclined vertical plate, emphasizing the effects of magnetohydrodynamics (MHD) and thermal radiation on heat transfer. Their findings indicated that the fluid's viscoelastic nature, along with magnetic and convective heat transfer influences, plays a crucial role in controlling temperature distribution and velocity profiles, making the study significant for industrial cooling and energy systems. Ayele et al. [29] examined the unsteady magnetohydrodynamic (MHD) flow of a hybrid nanofluid over a rotating disk, incorporating the effects of viscous dissipation and the Cattaneo–Christov heat flux model. Their study revealed that thermal relaxation time significantly influences heat transfer behavior, while the presence of hybrid nanoparticles enhances thermal conductivity, making the findings valuable for optimizing high-performance thermal systems and industrial cooling applications.

Pau et al. [30] examined the flow of an engine oil-based Casson tri-hybrid nanofluid over a rotating disk. Their study focused on analyzing the fluid dynamics and heat transfer characteristics. It focuses on effects like viscous dissipation and radiative flux, enhancing heat and mass transfer rates compared to Casson hybrid and nanofluids. The studies of Anjum et al. [31] and Anjum et al. [32] presents a non-similar Keller Box analysis of magnetochemically radiative Buongiorno's nanofluid flows over a stretched surface, addressing the unexplored effects of thermal radiation, chemical reaction, and magnetic parameters on heat and mass transfer. By employing the Keller Box method, the research offers a comprehensive multi-physics analysis, demonstrating that increasing magnetic fields enhance nanoparticle concentration, while rising thermal radiation and chemical reactions significantly influence velocity, temperature, and concentration profiles, with results showing a 99.9% compatibility rate with previous studies. Parige et al. [33] explored the interplay between Fourier heat flux and Joule heating in ternary hybrid nanofluids. Their study focused on analyzing the combined effects on heat transfer and fluid dynamics. Particularly in the context of a rotating circular porous disk, reveals significant insights into heat transfer dynamics.

All these existing studies have extensively explored the thermal and fluid dynamics of nanofluids and hybrid nanofluids, focusing on their heat transfer characteristics, convective behavior, and radiative effects. Research has also addressed the impact of viscous dissipation, magnetic fields, and nonlinear radiation in Newtonian and some non-Newtonian fluids. However, a significant gap remains in analyzing the symmetric behavior of non-Newtonian hybrid nanofluids under spinning disk motion with integrated nonlinear radiative effects and viscous dissipation. While previous studies have investigated flow over stretching/shrinking sheets and rotating disks, the coupled effects of these parameters on heat transfer efficiency, Nusselt number variation, and local skin friction reduction in symmetric thermal modeling are not well understood. Additionally, the influence of hybrid nanofluids in such a setting, particularly their interaction with external magnetic fields and nonlinear heat radiation, requires further investigation to optimize their industrial and engineering applications.

The current investigation stands out for its innovative approach, analysing the flow characteristics of both hybrid nanofluids and standard nanofluids concurrently, while factoring in nonlinear thermal radiation, viscous dissipation, and the Cattaneo-Christov model. This study meticulously assessed the thermophysical properties of nanomaterials, including Ag, $MnZnFe_2O_4$, and Cu, utilizing pure water as the foundational fluid. The thermal enhancement of the Fourier heat flux was evaluated within the context of a nonlinear radiative transfer scenario. A MATLAB program was deployed to analyze the parametric elements, shedding light on the physics underlying the issue. To the best of our knowledge, no existing literature has ventured into such an analysis.

Therefore, the primary aim of this study is to investigate the profound effects of magnetic fields, viscous dissipation, nonlinear thermal radiation, and Cattaneo-Christov heat flux on the motion of hybrid nanofluids traversing past a rotating disk. The current analysis was numerically resolved using the Runge-Kutta method combined with a shooting technique. The substantial influence of flow parameters is illustrated through graphical representations, while the effects of engineering relevance are systematically tabulated.

2. MATHEMATICAL MODEL

Envision a two-dimensional, steady flow of a nanofluid where the hybrid concoction of water serves as the foundational fluid. The methodical movement takes place within a disk that twirls with an angular velocity denoted as Ω. A Cattaneo-Christov heat flux was integrated into the flow dynamics. The temperatures across the rotating disk are evenly spread, symbolized as T_w, while the ambient stream is represented as T_∞. The effects of viscous dissipation and thermal conduction were deemed to hold considerable importance. A magnetic field with consistent distributions was imposed on the flow, aligned parallel to the z-axis. An external magnetic field B_0 of uniform strength was presumed to play a significant role within the flow, while the induced magnetic field was dismissed due to the minimal Reynolds number.

Figure 1 illustrates the study of nanofluids, focusing on a mathematical model for laminar, incompressible, two-dimensional flow of a ternary hybrid nanofluid, which consists of nanomaterials suspended in pure water as the base fluid. The setup features a circular porous disk, emphasizing the physical significance of the geometry in influencing fluid dynamics. The ambient temperature is denoted as T_∞, while T_w represents the wall temperature of the disk. The velocity components (u,v,w) in the (r,ϕ,z) framework. The boundary layer approximation remains applicable, transforming the governing equations of the problem [23].

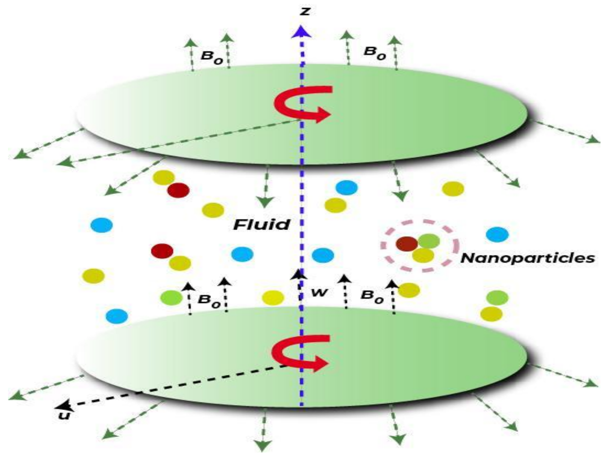


Figure 1. Visualization of the physical model

The formulated model is established on key underlying assumptions, encompassing the effects of thermal radiation, internal heat generation, Joule heating, an externally imposed magnetic field, viscous dissipation, and Fourier heat conduction, all of which collectively govern the flow dynamics The system comprises two rotating disks, each with an angular velocity Ω, while the boundary layer approximation is employed to simplify the governing equations. Furthermore, the porous medium, depicted in the upper and lower sections of the schematic, is defined by its permeability, allowing fluid to infiltrate through its interconnected voids, thereby augmenting convective heat transfer and optimizing fluid transport characteristics. This configuration enables a comprehensive investigation of the interactions between the nanofluid and the rotating disks under varying thermal and hydrodynamic conditions.

3. GOVERNING EQUATIONS

The vector forms of the governing equation are [6].

$$\nabla . u = 0$$
$$\rho_f (u.\Delta)u = -\nabla p + \mu_f \nabla^2 u + JXB,$$
$$(u.\Delta)T = \alpha_f \nabla^2 T + \tau[D_B(\nabla T) + \frac{D_T}{T_\infty}(\nabla T^2)]$$

The approximation of the boundary layer is deemed to be valid, and consequently, the governing equation pertinent to the issue is expressed as follows [23]:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\rho_{hnf}\left(u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z}\right) + \frac{\partial p}{\partial r} = \mu_{hnf}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right) - \sigma_{hnf}B_0^2 u \quad (2)$$

$$\rho_{hnf}\left(u\frac{\partial v}{\partial r}-\frac{uv}{r}+w\frac{\partial v}{\partial z}\right) = \mu_{hnf}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}+\frac{\partial^2 v}{\partial z^2}\right) - \sigma_{hnf}B_0^2 u \tag{3}$$

$$\rho_{hnf}\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)+\frac{\partial p}{\partial r} = \mu_{hnf}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right) \tag{4}$$

$$\left(\rho c_p\right)_{hnf}\left(u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)+\frac{\partial p}{\partial r} = K_{hnf}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right) - \frac{\partial q_r}{\partial z}+Q_0(T-T_\infty) + \frac{\mu_{hnf}}{\left(\rho c_p\right)_{hnf}}\left(\frac{\partial u}{\partial z}\right)^2 - \Gamma_1\left(u^2\frac{\partial^2 T}{\partial r^2}+w^2\frac{\partial^2 T}{\partial r^2} + 2uw\frac{\partial^2 T}{\partial r\partial z}+\left(u\frac{\partial u}{\partial r}+w\frac{\partial w}{\partial r}\right)\frac{\partial T}{\partial r} + \left(u\frac{\partial u}{\partial r}+w\frac{\partial w}{\partial r}\right)\frac{\partial T}{\partial z}\right) \tag{5}$$

4. BOUNDARY CONDITIONS

The boundary conditions governing the three-dimensional flow are specified as follows [2, 6, 7, 15, 16]:

$$u=0, v=\Omega r, w=w_0, T=T_w, C=C_w \; at \; z=0 \tag{6}$$

$$u\longrightarrow 0, v\longrightarrow 0, T\longrightarrow T_\infty, P\longrightarrow P_\infty, C\longrightarrow C_\infty \; at \; z \longrightarrow \infty \tag{7}$$

To evaluate the radiative heat flux, the Rosseland diffusion approximation, as formulated by Alao et al. [5], is employed. This approximation is based on the assumption that the radiative heat transfer within the medium occurs primarily through diffusion, which is valid for optically thick media where radiation is scattered and absorbed multiple times before escaping. The Rosseland model simplifies the radiative heat flux expression by relating it to the temperature gradient, thereby transforming the radiative heat transfer term into a conduction-like form. This method is extensively utilized in thermal analysis as it effectively approximates radiative effects while eliminating the need for complex integral formulations of the radiative transfer equation.

$$q_r = -\frac{4\sigma^*}{3K^*}\frac{\partial T^4}{\partial z} \tag{8}$$

where, $\sigma*$ denotes the Stefan-Boltzmann constant, $K*$ represents the coefficient of mean absorption.

Under the premise that the temperature variations present within the flow are minimal T^4 to an extent that is articulated in a nonlinear representation as:

$$\frac{\partial q_r}{\partial z}=\frac{\partial}{\partial z}.\left(\frac{4\sigma^*}{3K^*}.4T^3.\frac{\partial T}{\partial z}\right) \tag{9}$$

In order to facilitate the simplification of the governing partial differential equations (PDEs), appropriate similarity transformation variables are employed. The specific variables utilized are:

$$u=r\Omega f', \mathrm{v}=\mathrm{r}\Omega \mathrm{g}, \mathrm{w}=\sqrt{2\Omega\nu_f}f, P = P_\infty + 2\Omega\mu_f P(\eta), \quad \eta=\sqrt{\frac{2\Omega}{\nu_f}}z, \theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{10}$$

Utilizing the aforementioned equation facilitates the derivation of the subsequent dimensionless representation of the equations:

$$\frac{M_1}{M_4}(2fg'-2f'g)+2g''-\frac{M_5}{M_4}Mf'=0 \tag{11}$$

$$\frac{M_1}{M_4}(2fg'-2f'g)+2g''-\frac{M_5}{M_4}Mf'=0 \tag{12}$$

$$\frac{M_2}{M_3}f\theta'+\frac{1}{Pr}\left(\theta''+\frac{4Nr}{3M_3}\frac{d}{d\eta}\left(1+\theta(\eta)(\theta_w-1)\right)^3\frac{d\theta(\eta)}{d\eta}\right) + \frac{M_2}{M_3}f\theta'+PrQr(f^2\theta''+ff'\theta') + \Delta_r\theta+Ec(f'')^2=0 \tag{13}$$

Subject to:

$$f=-S, f'=0, g=1, \theta=1, at \; \eta=0 \tag{14}$$

$$f'\rightarrow 0, g\rightarrow 0, \theta\rightarrow 0, at \; \eta\rightarrow\infty \tag{15}$$

where, defined parameters are:

$$M_1=\frac{\rho_{hnf}}{\rho_f}, M_2=\frac{(\rho c_p)_{hnf}}{(\rho c_p)_f}, \; M_3=\frac{K_{hnf}}{K_f},$$

$$M_4=\frac{\mu_{hnf}}{\mu_f}, \; M_5=\frac{\sigma_{hnf}}{\sigma_f}$$

Table 1 showcases the fascinating thermophysical characteristics of a variety of nanomaterials alongside the base fluid, kerosene oil, as detailed by Alqawasmi et al. [23]. This table highlights essential attributes such as thermal conductivity, density, heat capacity, and Prandtl number for each substance. These attributes are vital for comprehending the thermal dynamics and flow properties of nanofluids.

Table 1. The captivating thermophysical characteristics of nanomaterials combined with the base fluid [23]

Properties	Thermal Conductivity	Heat Capacity	Density	Prandtl Number
Ag-Silver	428.9	235.1	1050*10	-
Cu-Copper	401.1	385.2	89.33*100	-
$MuZnFe_2O_4$ −Manganese zin ferrite	3.91	1049.9	47*100	-
Kerosene oil	0.151	2090.1	78.3*10	21

Silver (Ag): Silver shines with an impressive thermal conductivity of 429 W/m·K, placing it among the elite thermal conductors of common metals. Nevertheless, its heat capacity (235 J/kg·K) and density (10,500 kg/m^3) remain relatively modest when juxtaposed with other materials in the table. The absence of a specified Prandtl number for silver likely stems from its exceptional conductivity, indicating that its thermal diffusivity might overshadow momentum diffusivity in standard applications.

Copper (Cu): Copper, too, showcases remarkable thermal conductivity (401 W/m·K), albeit a notch below silver. It boasts a higher heat capacity (385 J/kg·K) yet a lower density (8,933 kg/m^3) than silver. Similar to silver, copper's Prandtl number remains unspecified but is expected to mirror analogous behavior, where thermal diffusivity significantly influences the heat transfer mechanism.

Manganese Zinc Ferrite (MnZnFe): This intriguing nanomaterial possesses a notably lower thermal conductivity (3.9 W/m·K) in comparison to silver and copper. Nonetheless, it flaunts an exceptionally high heat capacity (1,050 J/kg·K) and a comparatively reduced density (4,700 kg/m^3). Although the Prandtl number is not indicated, the low thermal conductivity implies that this material may not excel in heat transfer applications as its metallic counterparts.

Kerosene Oil: Serving as the foundational fluid, kerosene oil exhibits a minimal thermal conductivity of 0.15 W/m·K, which hampers its heat transfer capabilities. On the flip side, its heat capacity (2,090 J/kg·K) and density (783 kg/m^3) are quite commendable. The Prandtl number for kerosene oil is documented as 21, suggesting that momentum diffusivity prevails over thermal diffusivity in the fluid, shaping its convective heat transfer characteristics.

The parameters are:

Magnetic parameter is $M = \frac{\sigma_f B_0^2}{\Omega \rho_f}$.

Thermal radiation is $Nr = \frac{4\alpha^* T_\infty^3}{3K^* K_f}$.

Temperature ratio parameter is $\theta_w = \frac{T_w}{T_\infty}$.

Prandtl number is $Pr = \frac{\mu_f (\rho c_p)_f}{\rho_f K_f}$.

Thermal relaxation parameter is $Q_r = 2\Gamma_1 \Omega$.

Heat generation parameter is $\Delta_r = \frac{Q_0}{\Omega (\rho c_p)_f}$.

Eckert number is $Ec = \frac{(\Omega)^2}{c_p (T_w - T_\infty)}$.

5. METHODOLOGY

A comprehensive numerical methodology was formulated based on the transformed Eqs. (1)-(13) subject to the conditions outlined in (6), (7) and (14)-(15) The boundary value problem was addressed through the application of the Runge-Kutta method in conjunction with the shooting technique to resolve the transformed ordinary differential equations (ODEs). This methodology was implemented in MATLAB to solve the system of equations, demonstrating notable simplicity and a rapid convergence rate. An iterative framework was employed to handle the nonlinear system, ensuring computational efficiency. The shooting method was utilized by first transforming the third-order differential equation into an equivalent system of first-order equations, as outlined below.

Table 2 presents a comprehensive set of parameters essential for analyzing fluid dynamics and thermal transfer. It includes viscosity (M) values ranging from 0.2 to 3, along with corresponding entries for additional parameters such as *Po*, *Pr*, *Nr*, Δ*r*, and *Ec*, which play crucial roles in characterizing the thermophysical behavior of the system. The table also incorporates dimensionless quantities, including *Qr*, which likely represents the heat transfer rate, and $-f'(0)$, which is presumed to be a velocity gradient or shear stress evaluated at a specific boundary. Moreover, the Nusselt number (*Nu*) is presented as a critical parameter that characterizes the ratio of convective to conductive heat transfer within the system. The dataset facilitates the identification of trends in *Qr* and *Nu* as functions of *M* and other governing parameters, thereby offering insights into their interdependencies within the broader framework of fluid dynamics and heat transfer analysis. By examining these variations, the table helps elucidate the underlying physical mechanisms that influence heat and momentum transport in the system.

Table 2. Comparison outcomes for and Prandtl numbers pertinent to the Nusselt number (*Nu*)

$\phi_1 = \phi_2 = \phi_3$	*S*	*Pr*	Alqawasmi et al. [23]	Present Results
0	1	0.64	0.720242	0.72024
	-1		0.090675	0.090673
	2	0.71	1.437812	1.43781
	0		0.325969	0.325967
	-1		0.011404	0.011402
	1	6.2	6.272213	6.272215
	0		0.933725	0.933727
	-1		0.000211	0.000209
	1	6.2	4.657438	4.657439
	0		0.821517	0.821518
	-1		0.001638	0.001639

6. RESULTS WITH DISCUSSION

A comprehensive review of the existing literature indicates that studies investigating hybrid nanofluids alongside conventional nanofluids are relatively limited. This highlights a significant research gap in understanding the combined thermal and fluid dynamic behavior of such advanced working fluids. To address this, a comprehensive mathematical model was developed, incorporating key physical effects such as viscous dissipation, externally applied magnetic fields, thermal radiation, and other governing parameters that significantly influence heat and momentum transport. The resulting system of transformed nonlinear differential equations exhibits both temperature dependency and coupling characteristics, necessitating a robust numerical approach for accurate solutions to solve these ordinary differential equations (ODEs), a hybrid numerical approach was implemented, integrating the Runge-Kutta method with the shooting technique to ensure accurate and efficient computations. This computational approach ensures efficient convergence while maintaining high accuracy in capturing the intricate interactions between the flow, thermal fields, and imposed external forces. The adopted methodology enables a systematic exploration of hybrid nanofluid behavior under varying physical conditions, contributing valuable insights to the field of heat transfer and fluid mechanics. The numerical values assigned to the physical parameters are as follows: *M*=1.0, *Nr*=0.5, *Qr*=2.0, *Pr*=2.2, *S*=0.3, Δ_r=0.6, *Ec*=0.4.

In this study, the numerical values used in tables and graphical representations remain constant unless explicitly stated otherwise. This ensures consistency in analyzing the effects of various governing parameters on fluid flow behavior. The intricate dynamics of fluid motion and heat transfer can be comprehensively understood by systematically examining how controlled parameters influence velocity and temperature distributions. By maintaining key variables at fixed values, the study isolates the impact of specific factors, thereby enabling a more precise evaluation of their role in shaping the flow characteristics. Furthermore, to validate the robustness and accuracy of the adopted methodology, the obtained results are systematically compared with existing literature. This comparative analysis serves to confirm the reliability of the computational approach, ensuring that the study aligns with established theoretical and empirical findings. Through such an assessment, the study not only reinforces the credibility of its results but also contributes to the broader understanding of fluid flow phenomena by demonstrating the consistency of observed trends with prior research.

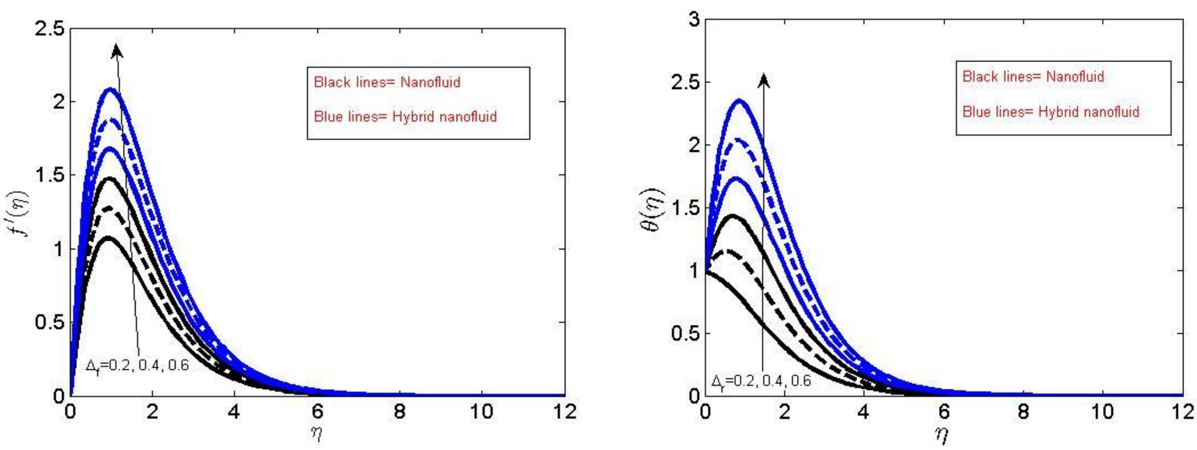
The insights that are meticulously laid out in the comprehensive analysis found in Table 3 serve to eliminate the complex and intricate interplay that exists among variety of flow parameters along with their consequential effects on pivotal engineering benchmarks ultimately revealing notable trends that have possesses the potential to guide and influence both future scholarly research endeavours and practical applications in the captivating rime of fluid dynamics which is rich with possibilities. These fascinating elevations strongly imply that developing a more intense and nuanced understanding of the intricate relationships that exist among these various parameters could significantly pave the way for creative designs and markedly improved efficiency in array of systems that are fundamentally reliant on the principles of fluid dynamics and their applications in real world scenarios.

The compelling information that is meticulously present in Table 2 unfold and intriguing and detailed comparison of various Nusselt number *Nu* values, which in turn reveals strikingly harmonious relationship between the enlightening discoveries made by Alqawasmi et al. [23]. And the current findings that span a diverse range of Prandtl numbers, showcasing a rich tapestry of thermal dynamics. This firm consistency in the result not only accentuates the reliability and trustworthiness of both investigations but also strongly implies that the sophisticated techniques employed are robust and resilient for thoroughly exploring the intricate phenomena of heat transfer within the fascinating realm of fluid dynamics.

Such a synchrony of data not only serves to bolster the credibility and integrity of experimental methodologies that have been meticulously implemented but also establishes a solid and composed foundation for future enquiries that will focus on significantly enhancing thermal efficiency in systems that are analogous in nature and function. This remarkable coming together of data points vividly highlights the profound significance of collaborative explanation and investigation in advancing our comprehensive understanding of thermal dynamics thereby clearing a clear and optimistic path for innovative and groundbreaking solutions that can revolutionise various engineering realms.

Table 3. The significance of flow variables on the scale of engineering impact

M	P_o	Pr	N_r	Δ_r	E_c	Q_r	$-f'(0)$	Nu
0.0							1.919901383	0.612419717
0.5							1.477169702	0.612459717
1.0							1.281962577	0.506919717
	2.0						1.987291042	0.494982647
	4.0						2.082510184	0.496182647
	6.0						2.142918985	0.560022647
		2.0					2.104045097	0.560827227
		3.2					2.208356978	0.503944358
		4.2					2.376494986	0.514006231
			0.2				2.054554162	1.798019237
			0.4				2.002016306	2.465066431
			0.6				4.372640918	3.132131295
				0.1			2.250527179	0.014189128



(a) The velocity profiles

(b) The velocity and temperature landscapes

Figure 2. The influence of the heat generation parameter on the velocity and temperature distributions

Figure 2 serves as a vivid illustration of the remarkably significant impact that the generation of heat has on both the velocity and temperature profiles, showcasing the intricate interplay between these variables. An extraordinarily marked increase in the parameter associated with heat generation leads to a substantial enhancement in the temperature profiles, which in turn correlates with the acceleration of the velocity profiles throughout the system. The emergence of thermal generation sparks a remarkable surge in the heat transfer coefficient particularly within the realm of a single-phase system, thereby unleashing a chain reaction of effects. This captivating occurrence ultimately nurtures an escalation in thermal conductivity all the while driving a significant shrinkage in the physical dimensions required for heat exchangers. Thus amplifying their overall efficiency and functionality.

The nonlinear thermal radiation parameter is vividly observed in Figure 3, presenting a striking visual representation of these influences. And elevation in the parameter referred to as *Nr* evidently contributes to the enhancement of temperature profiles signifying a direct link between the two. Thermal radiation acts as a crucial parameter that enables the transformation of thermal energy into electromagnetic energy which is essential for a plethora of applications. In this framework the kinetic energy chiefly characterised by the erratic motions of both molecules and atoms can indeed be acknowledged as a unique manifestation of thermal energy.

As the value of *Nr* increases, it significantly enhances the temperature profile, indicating a direct relationship between the two. Thermal radiation serves as a crucial parameter that facilitates the transformation of thermal energy into electromagnetic energy, which is essential for various applications. In this context, kinetic energy primarily manifests through the erratic motions of molecules and atoms, representing a distinct form of thermal energy. Consequently, as *Nr* increases, there is a corresponding growth in the thermal boundary layer and a notable rise in the temperature profile. This illustrates the deep interconnection between these factors in the thermodynamic behaviour of the system.

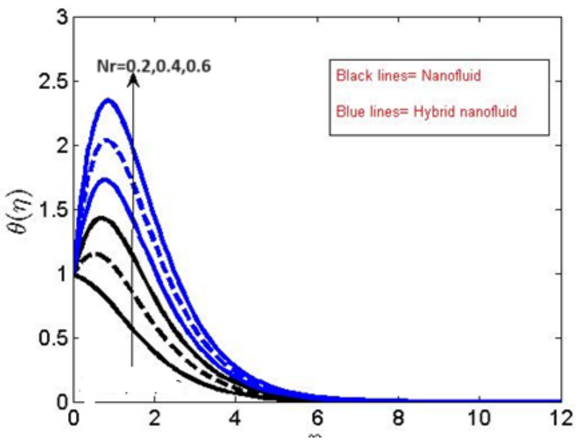
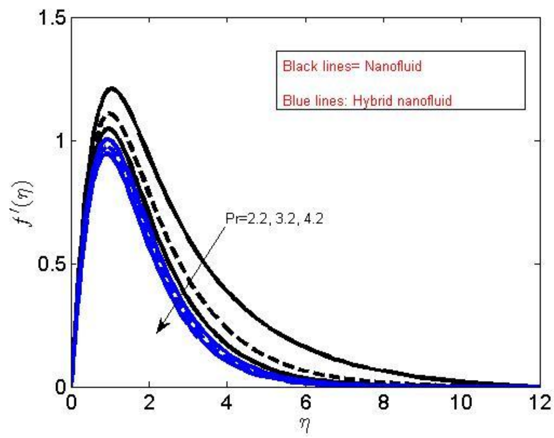
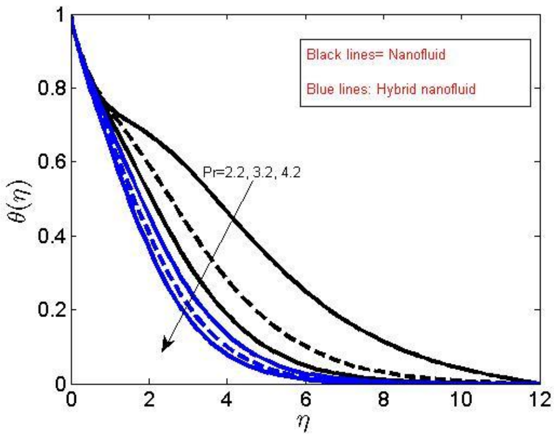


Figure 3. Influence of thermal radiation parameters on temperature distributions

Figure 4 showcases the vividly pronounced and elaborate impacts of the Prandtl number, commonly referred to as *Pr*, which serves as a crucial dimensionless metric that intricately defines the complex relationship between momentum diffusion and thermal diffusion in the realm of fluid dynamics. When one considers the condition where *Pr* is significantly less than one, represented as *Pr*<<1, this situation indicates a dominant pre-eminence of thermal diffusivity over momentum diffusivity; conversely, when the Prandtl number is markedly greater than one, denoted as *Pr*>>1, the behaviour of the fluid is predominantly governed by the principles of momentum diffusivity rather than thermal aspects. Therefore, when we examine engine oil, which characteristically exhibits high viscosity in conjunction with remarkably low thermal conductivity, it becomes evident that such oil possesses a momentum diffusivity that remarkably surpasses its thermal diffusivity, leading to unique fluid dynamics. Hence, as it has been meticulously observed in the intricately detailed Figure 4, an elevated Prandtl number correlates with a notable reduction in both the velocity and temperature profiles, indicating a profound relationship between these physical properties.



(a) The velocity profile



(b) The thermal landscape

Figure 4. The influence of the Prandtl number on the profiles of velocity and temperature

Figure 5 illustrates, in a comprehensive manner, the considerable and multifaceted effects exerted by the magnetic parameter on the velocity profile denoted as (g) along with the associated dynamics of the velocity profile itself. An increase in the value of the magnetic parameter results in a substantial diminishment of both the velocity profile g and its corresponding dynamics, showcasing a clear interaction between magnetic forces and fluid motion. The diminishing effect that is observed can be directly attributed to the adverse influence exerted by the Lorentz force on the intricate flow dynamics, which plays a crucial role in shaping the behaviour of the fluid. The Lorentz force, at its core, cultivates a discreet and muted flow behaviour within the boundary layer, thus weaving a more intricate tapestry of interaction between magnetic fields and fluid dynamics. Consequently, as one witnesses a bolstering of the magnetic parameter, it incites a marked decline in both the momentum boundary layer and the velocity profile, ultimately showcasing the elaborate dance of these physical phenomena.

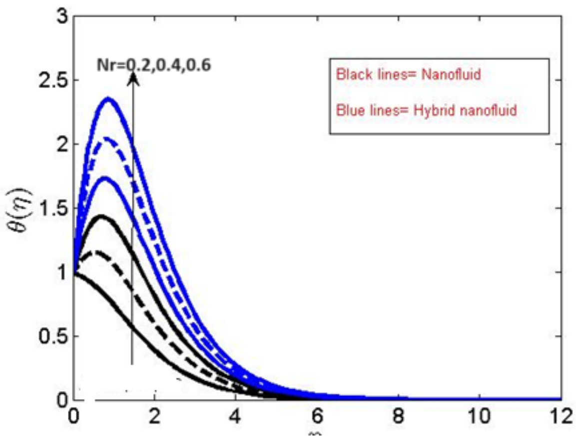
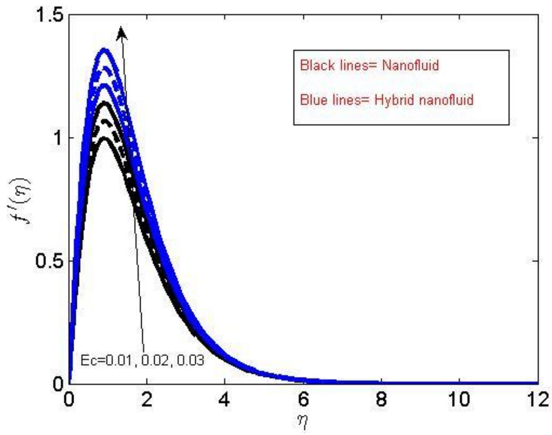
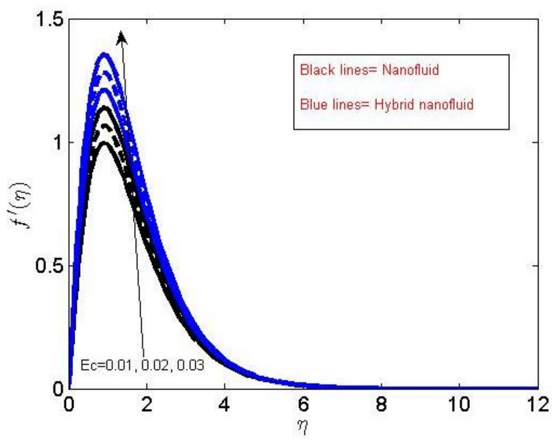


Figure 5. The influence of the magnetic parameter on the velocity profiles



(a) The velocity profiles



(b) The temperature landscape

Figure 6. Influence of the Eckert number on the profiles of velocity and temperature

Figure 6 artfully depicts and elucidates the profound influence that the Eckert number (*Ec*) wields on the intricate velocity and temperature profiles, unveiling the nuanced relationship between these elements. The occurrence of viscous dissipation stands as a pivotal player in flows defined by remarkably high velocities, underscoring its essential role in the realm of fluid dynamics. A remarkable boost in the speed and heat profiles can directly associated with the rise in viscous dissipative term denoted by *Ec*, showcasing the intricate web of these elements. As the Eckert number claims, the ensuring rapid increase in viscous flow at moderate speed leads to noticeable thickening of both the momentum and thermal boundary layers, underscoring the significant influence of viscous forces and fluid dynamics.

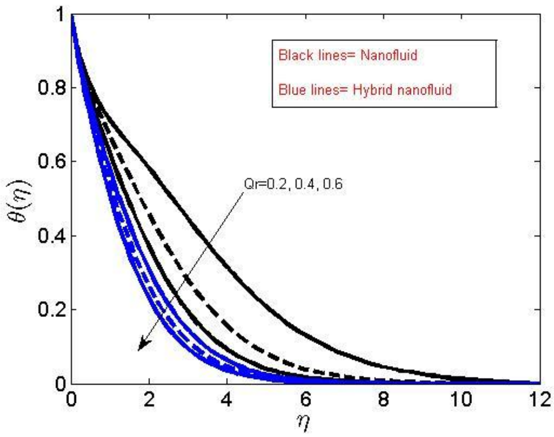


Figure 7. Influence of suction/injection on thermal profiles

Figure 7 illustrates the effects of suction and induction parameter (*S*) on the velocity profile presenting a striking visualization of these interactions. It has been observed that an increased value of S correlates with the notable reduction in the velocity profile suggesting a complex dance of forces at play.

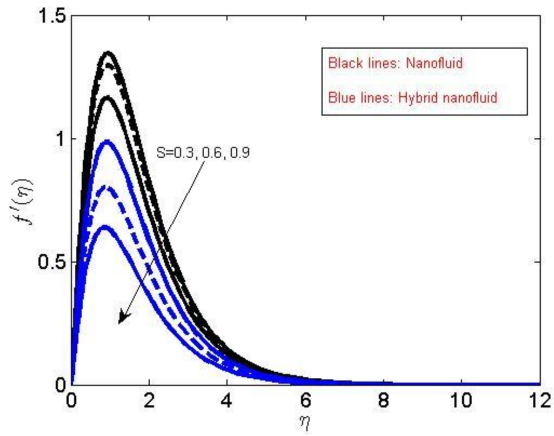


Figure 8. Influence of the thermal relaxation parameter on velocity distributions

Figure 8 is crafted to depict and elucidate the effect of relaxation parameter on the temperature profile showcasing how variations in this parameter can sway thermal dynamics. Heightened value of *Qr* has been seen to increase amplify the temperature profile indicating that the thermal distribution among fluid particles within the boundary layer is significantly enriched due to the thermal emissions arise from the fluids, thus eliminating the yellow operate relationships at play in thermal fluid mechanics.

7. CONCLUSION

We meticulously illuminated the intricate dynamics that govern the multifaceted motion of hybrid nanofluids, delving deeply into the profound and far-reaching implications that are brought about by the Cattaneo-Christov heat flux, which introduces an exceptionally fascinating layer of complexity, alongside a nonlinear thermal radiation phenomenon that, without a doubt, plays a pivotal and essential role in shaping our comprehensive understanding of these intricate fluid behaviour's within this specific and nuanced context. The influence of heat transmission on the intricate behaviour and motion of the hybrid nanofluid was thoroughly examined, while we subjected this complex system to the intricate influence of a uniformly distributed and carefully considered magnetic field, which not only complicates the analysis significantly but also provides a rich and vibrant tapestry of interactions that we sought to unravel and understand during our detailed investigation. In our comprehensive and methodical approach, we adopted a meticulously crafted two-dimensional steady flow model, which greatly facilitated our exploration into the systematic and nuanced characterization of the flow dynamics under consideration, thereby allowing us to capture the subtle nuances and complexities of this multifaceted system. This flow model culminated in the emergence of a complex and challenging system of partial differential equations (PDEs), which we later adeptly transformed into ordinary differential equations (ODEs) by employing similarity transformation variables that significantly streamline the mathematical treatment process and enhance the clarity of our insightful findings. To derive meaningful and deeply insightful solutions that could effectively inform our understanding of these intricate dynamics, we resorted to robust and sophisticated numerical methodologies, specifically implementing the highly regarded Runge-Kutta method in conjunction with innovative shooting techniques that greatly enhance the accuracy and efficiency of our computations, ensuring that our results are both reliable and richly informative. The pivotal and groundbreaking discoveries emerging from our extensive investigation are succinctly summarized as follows:

a) An increase in the magnetic parameter intensifies the Lorentz force, which acts as a resistive force opposing the motion of the hybrid nanofluid. This results in a noticeable reduction in velocity profiles, demonstrating the complex interaction between electromagnetic forces and fluid dynamics. The suppression of velocity due to the magnetic field influence highlights the significance of magnetohydrodynamic (MHD) effects in controlling flow behavior, particularly in applications involving electromagnetic flow regulation and thermal management systems.
b) A rise in thermal radiation enhances the energy exchange within the fluid, leading to an increased thermal distribution and a subsequent expansion of the thermal boundary layer. The thickening of the thermal boundary layer significantly influences heat transfer characteristics. It directly affects the temperature gradient near the surface. The interaction between radiative heat transfer and convection is crucial for thermal performance. This interplay is essential in applications such as high-temperature fluid transport, energy conversion systems, and industrial cooling processes.
c) Furthermore, a rise in the Eckert number (*Ec*) significantly expands both the velocity and temperature profiles, showcasing the enhanced kinetic and thermal behaviour of the hybrid nanofluid under varying conditions.
d) Lastly, an elevated Prandtl number effectively reduces the thickness of both the momentum and thermal boundary layers, indicating a refined balance between momentum transport and thermal diffusion in the fluid system.

8. APPLICATIONS AND FUTURE SCOPE

The expansive and intricate landscape of opportunities that lies before us, coupled with the undeniable and compelling promise that is deeply embedded within these cutting-edge and innovative technologies, possesses the incredible potential to act as a powerful catalyst for revolutionary advancements in the specialized domains of cooling systems that are meticulously engineered for electronic devices, the substantial enhancement of performance metrics associated with solar energy collectors, as well as the progressive development of more efficient and remarkably effective heat exchangers that find their application across an astonishingly diverse array of industries. By artfully and seamlessly incorporating hybrid nanofluids into these complex and multifaceted systems, we could potentially observe a truly remarkable and unprecedented reduction in energy consumption levels, an extraordinary enhancement of operational efficiencies that were previously deemed unattainable or even beyond reach, and ultimately, a significant and meaningful contribution to the relentless pursuit of more sustainable and environmentally responsible industrial practices that our planet is in dire need of and desperately craves. Moreover, the ongoing and unwavering research endeavors that are diligently directed towards the thorough exploration of nanofluids not only unlocks a gateway to exhilarating advancements within the expansive field of biomedical applications, such as the meticulous and precise targeted delivery of pharmaceutical drugs to specific and vital sites within the human body, along with the substantial and noteworthy improvement of imaging techniques that are employed in medical diagnostics, but it also brilliantly highlights and showcases the remarkable versatility and extraordinary adaptability of these captivating substances across an impressively wide-ranging spectrum of fields and disciplines.

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NOMENCLATURE

(u, v, w)	Components of velocity in a specified (r, ϕ, z) direction
P	Pressure of the fluid
ρ_{hnf}	Density characteristics of the hybrid nanofluid
μ_{hnf}	Dynamic viscosity attributes of the hybrid nanofluid
σ_{hnf}	Electrical conductivity properties of the hybrid nanofluid
B_0	Magnitude of the magnetic field
c_p	Specific heat at constant pressure conditions
K_{hnf}	Thermal conductivity parameters of the hybrid nanofluid
T	Temperature of the fluid
q_r	Radiative heat transfer flux
Q_0	Coefficient of heat generation
T_∞	Temperature of the free stream
Γ_1	Coefficient of Fourier heat transfer flux
Ω	Angular velocity measurement
T_w	Temperature at the wall interface
P_∞	Pressure of the free stream - Components of velocity in a specified direction