IIETA International Information and Engineering Technology Association
Advancing the World of Information and Engineering

Journal Européen des Systèmes Automatisés
Vol. 58, No. 1, January, 2025, pp. 161-169
Journal homepage: http://ieta.org/journals/jesa

Smoothing and Denoising ECG Signals Based on Modified Smoothing Spline and Discrete Wavelet

Akram Jaddoa Khalaf[1*], Hayder Mazin Makki Alibraheemi[2], Shamam Alwash[3], Sarmad Ibrahim[3]

[1] Department of Biomedical Engineering, Faculty of Engineering, University of Babylon, Babil 51002, Iraq
[2] Department of Electronics and Communication Engineering, Faculty of Engineering, University of Al-Qadisiyah, Al Quadisiya 58002, Iraq
[3] Department of Electrical Engineering, Faculty of Engineering, University of Babylon, Babil 51002, Iraq

Corresponding Author Email: eng.akram@uobabylon.edu.iq

https://doi.org/10.18280/jesa.580118

Received: 29 November 2024
Revised: 31 December 2024
Accepted: 15 January 2025
Available online: 31 January 2025

Keywords:
denoising, discrete wavelet, ECG, filter, MIT-BIH-arrhythmia database, smoothing spline

ABSTRACT

Smoothing and denoising ECG signals are challenging in biomedical signal processing applications. This paper proposes a new smoothing and denoising ECG signal method based on a proposed modified smoothing spline (MSS) method and mean discrete wavelet (MDW). The traditional smoothing spline (TSS) method uses a certain smoothing parameter value for overall samples of the noisy signal with moderate performance. On the other hand, the proposed modified version of the smoothing spline method is based on utilizing a range of smoothing parameter values for each sample instead of a single value. The smoothing parameter values are selected according to the MDW value to improve the method's smoothing and denoising performance. The method is evaluated using the MIT-BIH-arrhythmia (MBA) database, and the obtained results illustrate a high smoothing and denoising performance without losing signal information. The signal-to-noise ratio (SNR), the Mean Square Error (MSE), and the Percent Root Mean Square Difference (PRD) are the parameters used to evaluate the method's performance. Based on white Gaussian noise at 10 dB input SNR, the results are 8.14 dB improved SNR, 0.0015 MSE, and 12.68 PRD. The evaluation results for the proposed method show a high performance compared with the TSS and the existing methods.

1. INTRODUCTION

The Electrocardiogram (ECG) is the process of graphical representation of the heart's electrical signal [1]. The ECG signal helps the cardiologist to diagnose heart diseases, which cause the most common deaths worldwide [2-4]. The ECG signal suffers from different types of noise, such as baseline, electromyography (EMG), (50 or 60) Hz AC power, and electrode motion, which affect its smoothness [5, 6]. So, the noisy signal results in inaccurate heart disease diagnostics by the cardiologists and the software applications. On the other hand, a smooth and denoised ECG signal improves the diagnosis of heart diseases and saves more lives. For the noisy ECG signals, the smoothing and denoising process, which is represented as a filter in the pre-processing signal operation, enhances the ECG signal quality for the next operations [7, 8].

Several smoothing and denoising filters are applied to the signal based on the type of noise. The baseline noise fr%uencies range between 0.1-0.5 Hz, the EMG noise frequencies range between 100-500 Hz, and the noise frequencies of the AC power is (50 or 60) Hz [9]. In digital signal processing, digital filters are the most essential operations to reduce different noises of the signal for software applications [10, 11]. An accurate signal estimation from the noisy signal is a significant subject in signal processing for many researchers. Therefore, various smoothing and denoising techniques have been proposed, such as bandpass, low pass, wavelet, and Savitzky Golay [2, 12].

In studies [13-16], methods based on wavelet transformation combined with different techniques are proposed to improve the smoothing performance. In the study [13], a new method that used empirical mode decomposition (EMD) and discrete wavelet transform (DWT) domains is presented. A window empirical mode decomposition domain decreases initial intrinsic mode function noise. In the study [14], a wavelet function (WF) as an optimal method for the DWT decomposition is proposed for ECG noisy signals. The method selects the best threshold to improve the wavelet performance. Three types of noise are applied in this method: electromyography, baseline drift, and power line. In the study [15], a combined DWT and Savitzky Golay filter (DWT-SG) is presented. The performance evaluation for each method alone is done before combining them. The combined method gets a better filtering performance than each of the DWT or Savitzky Golay methods. The parameters for the evaluation of the method are SNR and PRD. In the study [16], a method based on a Genetic Algorithm (GA) and Wavelet Transform (GA-WT) is proposed. The GA conducts systematic research to determine optimal WT levels of decomposition to reduce the noise. The MIT-BIH-arrhythmia (MBA) database is used for performance evaluation using the SNR and PRD parameters.

Another empirical mode decomposition was presented in the study [17] with an adaptive switching mean filter technique instead of DWT presented in the study [13]. A combined empirical mode decomposition and adaptive switching mean filter technique is presented. It removes the noise with low distortion, and the adaptive switching mean filter improves the ECH signal quality. White Gaussian noise, electromyogram, and power line interference are used to test the technique's performance based on signal-to-noise ratio (SNR).

A Majorization-Minorization denoising method based on the optimized overall ECG signal variance is proposed in the study [2]. After that, the output denoising signal is divided into segments using the bottom-up method. The method evaluation showed that the proposed method is a significant improvement. In the study [18], a new method is proposed using cooperative filtering denoising based on two stages by dividing the ECG signal into an equal segment. After that, an array from the similarly divided segments is filtered using Savitzky-Golay and polynomial fitting. The method evaluation was compared using cooperative filtering and without cooperative filtering, showing that the proposed method is effective. In the study [7], A new approach is presented based on two-stage filters. The first filter is a low-pass Butterworth filter, and the second stage is a zero-phase shift filter. The results show a high signal-to-noise ratio and low mean square error.

The abovementioned methods aim to smooth and denoise the ECG signals with high performance using different techniques. These methods operate with non-adjustable parameters for all ECG signal samples, which affect their performance. The traditional smoothing spline method is not efficient because the method performance is relative to the ECG signal and the single smoothing parameter value selected for all ECG signal samples. On the other hand, it has a high smoothing performance for a certain signal part according to the single smoothing parameter value. The smoothing parameter value affects the smoothing operation as a trade-off between eliminating the noise and losing some signal information or reducing some noise and maintaining the signal information. Conversely, the smoothing parameter for the proposed modified smoothing spline is selected based on the mean discrete wavelet (MDW) value of each sample for the ECG signal to improve the performance of the proposed smoothing and denoising method. The method performance is evaluated using MATLAB based on the MBA database. It is compared with the traditional methods to demonstrate the performance superiority of the proposed method. The SNR, MSE, and PRD assess the method's performance.

2. TRADITIONAL SMOOTHING SPLINE METHOD

The traditional smoothing spline (TSS) method minimizes Eq. (1), which consists of two parts: the first part is the error measure, and the second is the roughness measure [19].

$$p\sum_{i}\left(y_i - s(x_i)\right)^2 + (1-p)\int\left(\frac{d^2s}{dx^2}\right)dx \qquad (1)$$

where,

p: the smoothing parameter

s: the smoothing spline

$s(x)$: the smoothing spline for the given x

x, y: the input data

The smoothing parameter (p) value changes the method smoothing operation between the highest value p=1 and the lowest value p=0. In the case of p=0, the smoothing operation is the best smoothed, and the spline will be as a line. However, the case of p=1 is the lowest smoothing operation, and the spline function will intersect all data points for the ECG signal. In other words, the range of p is from 0 to 1, where 0 produces the least square straight line, and 1 produces a cubic spline [19, 20].

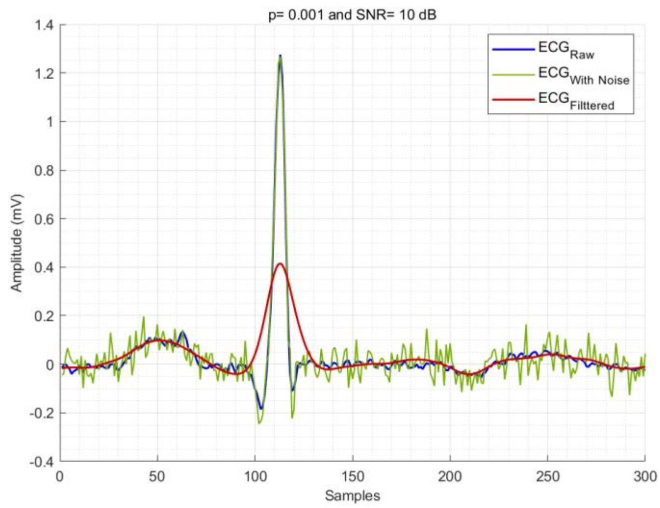


Figure 1. Smoothing a sampled ECG signal based on the TSS method using p = 0.001

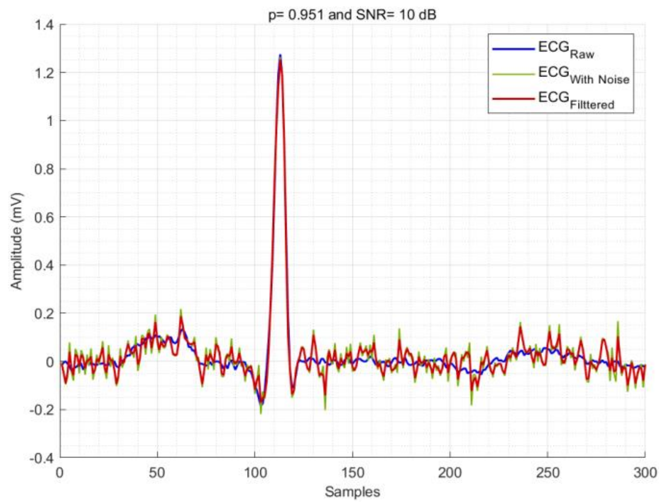


Figure 2. Smoothing a sampled ECG signal based on the TSS method using p = 0.951

Using the TSS method, two p-values are applied to a sample of ECG signal from the MBA database, as shown in Figure 1 and Figure 2. Selecting a low smoothing parameter value (p=0.001) produces a smooth signal and loses the signal information, as shown in Figure 1. This figure shows a high smoothness for the overall ECG signal. In contrast, the QRS level and shape are affected, which causes the signal information to be lost for this part of the signal. In contrast, selecting a high smoothing parameter value p=0.951 produces a low smoothing signal without losing signal information, as shown in Figure 2. This figure illustrates a noisy signal and low smoothing operation for the ECG signal without losing the

QRS level and shape that maintains the signal information. Selecting any p-value between the previous numbers is a trade-off between the performance of the smoothing operation and losing the signal information.

Therefore, a modified smoothing spline is proposed in this paper based on variable p-values to improve the smoothing and denoising performance without losing signal information.

3. PROPOSED SMOOTHING AND DENOISING METHOD

The proposed smoothing and denoising ECG signal method is illustrated in the block diagram shown in Figure 3 and is based on three stages. The first stage is the moving median filter, the second stage is the mean wavelet, and the third is a proposed modified smoothing spline method. The raw ECG signal $e[(n)]$ is the input for the proposed method, and after the three stages, the result is the smoothed and denoised ECG signal $s[(n)]$. These stages are described in the following paragraphs.

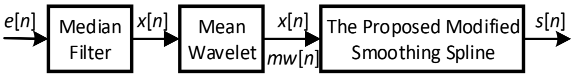


Figure 3. Proposed method block diagram

First, a moving median filter eliminates the baseline wander noise from the raw ECG signal. The database sampling rate is 360 samples per second, and the median filter window of 120 samples was chosen based on the database sampling rate. So, the window of the moving median is 1/3 of the database sampling rate. The raw ECG signal $e[(n)]$ is subtracted from the moving median output of each sample (n), using N number of samples for the filter window, as shown in Eq. (2) for the selected even N number of samples.

$$x[n] = e[n] - \left\lceil \frac{\left(\left(e\left[\frac{N}{2}\right]\right)^{th} sv + \left(e\left[\frac{N}{2}\right]+1\right)^{th} sv \right)}{2} \right\rceil \tag{2}$$

where, sv is the sorted value for the sampled in the filter window. Second, the general DWT (W) for the input signal $x[n]$ and the mother wavelet $\emptyset[n]$ are illustrated in Eq. (3) and Eq. (4), respectively [21, 22]:

$$W[tp, dp] = \sum_{n=-\infty}^{\infty} x[n]\, \emptyset_{tp,dp}[n] \tag{3}$$

$$\emptyset_{tp,dp}[n] = \left(\frac{1}{tp^{1/2}}\right) \times \emptyset\left[\frac{n-dp}{tp}\right] \tag{4}$$

where, tp is a translation parameter, and dp is a dilation parameter. The ECG signal $x[n]$ is decomposing based on Eq. (5) and Eq. (6) to determine the approximation coefficients $Ac[k]$ and the detail coefficients $Dc[k]$, respectively [23].

$$Ac[k] = \sum_{n} x[n] \cdot h[2k-n] \tag{5}$$

$$Dc[k] = \sum_{n} x[n] \cdot g[2k-n] \tag{6}$$

where, $2k$ is the downsampling of the signal by 2. The h and g are the low and high pass convolutional filters. The mean wavelet ($mw[n]$) for the DWT can be calculated by Eq. (7), representing the output to the next stage for N_w=30.

$$mw[n] = \frac{1}{N_w}\sum_{i=1}^{N_w} \left| Ac_l\left[k - \frac{N_w}{2} + i\right] \right| \tag{7}$$

where, N_w is the number of samples for the moving mean window.

Third, the traditional smoothing spline equation is modified by the range of p_j values based on the mean wavelet of the signal to apply the suitable p-value for each sample of the noisy ECG signal, as shown in Eq. (8). The proposed equation smooths and denoises the signal with high performance without losing any signal information. The modified method is called modified smoothing spline (MSS).

$$p_j \sum_{i} \left(y_i - s(x_i)\right)^2 + (1-p_j)\int \left(\frac{d^2 s}{dx^2}\right) dx \tag{8}$$

The range of p_j can be chosen depending on the performance improvement needed for smoothing and denoising operation. Increasing the p_j range values improves the smoothing and denoising operation without losing signal information. On the other hand, it will add complexity to the smoothing and denoising process. The low range of p_j remains several noises with the original signal; moreover, the signal loses information.

The mean discrete wavelet is divided into a range of levels (lv) based on the minimum and maximum values. These levels select the p_j value for the proposed MSS method. So, the values of p_j=p_1, p_2, …, p_{lv}. The level number is computed by incrementing the level number to reach the best performance. In this paper, using the MBA database, the best performance is achieved at twenty levels (lv = 20) without adding more complexity. The overall proposed method flowchart is described in Figure 4, which can be illustrated as follows:

(1) Reading the raw ECG signal from the MBA database.
(2) Adding noise with SNR=0-20 dB, noise power=0-20 dB, or power line 50Hz noise for each time.
(3) Applying the median filter with a window removes the baseline wandering, as shown in Figure 5.
(4) Applying MDW with Sym3, level 3, and mean window 30 samples for the ECG signal, as shown in Figure 5, after testing different types of wavelets and different levels for the best wavelet transformation performance based on the ECG signals.
(5) Dividing the $mw[n]$ by levels based on a selected range (lv levels).
(6) According to the lv levels number, the p_j values are calculated between $p_j > 0$ and $p_j < 1$ based on Eq. (9) with α as a very small number close to zero (less than 0.05) as the initial p_j value.

$$p_j = \alpha + \frac{(j-1)}{lv} \tag{9}$$

(7) Applying the smoothing spline with a suitable p_j value

for the sample based on the $mw[n]$ level.

(8) Measuring SNR, MSE, and percent root mean square difference to evaluate the method's performance.

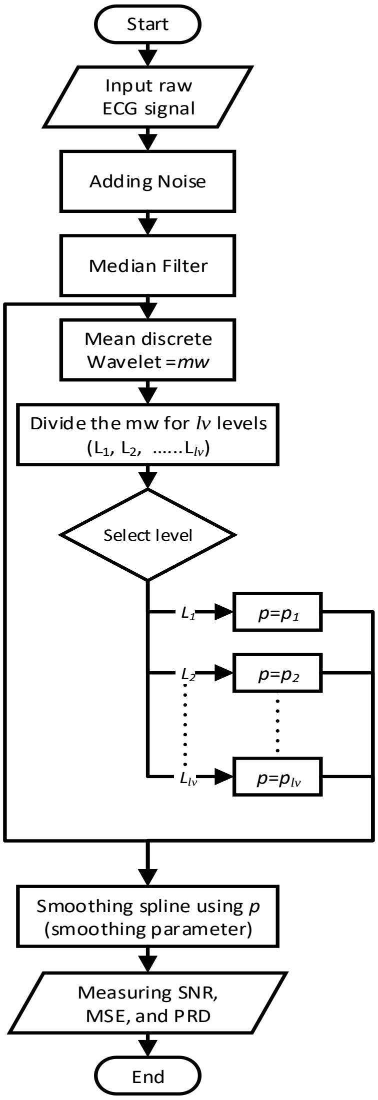


Figure 4. The proposed method flowchart

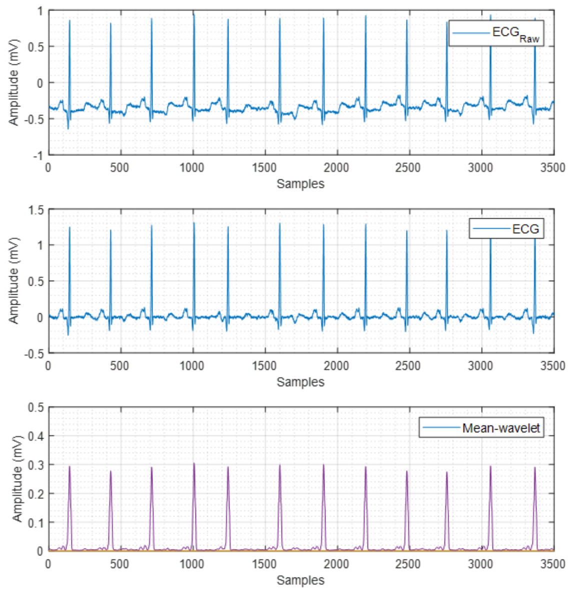


Figure 5. The median filter and MDW for a sample ECG signal

4. RESULTS AND DISCUSSIONS

The MBA database consists of 48 records with two signals that are 30 minutes long [24]. In this paper, the MBA database is used for the method evaluation after selecting a 10-second from the first ECG signal. These ECG signals are processed using the traditional smoothing spline and the proposed method.

A computer with Windows 10, core i7 processor, 32 GB RAM, and MATLAB R2023b performs the methods results. The performance evaluation is based on the performance parameters: the signal-to-noise ratio, mean square error, and percent root mean square difference, as shown in Eqs. (10)-(14), respectively.

$$SNRin = 10\log\left(\frac{\sum_{i=1}^{nt}\left(S_1(i)\right)^2}{\sum_{i=1}^{nt}\left(S_2(i) - S_1(i)\right)^2}\right) \quad (10)$$

$$SNRimp = 10\log\left(\frac{\left(\sum_{i=1}^{nt}\left(S_2(i) - S_1(i)\right)^2\right)}{\sum_{i=1}^{nt}\left(S_3(i) - S_1(i)\right)^2}\right) \quad (11)$$

$$SNRout = 10\log\left(\frac{\sum_{i=1}^{nt}\left(S_3(i)\right)^2}{\sum_{i=1}^{nt}\left(S_3(i) - S_1(i)\right)^2}\right) \quad (12)$$

$$MSE = \left(\frac{\left(\sum_{i=1}^{nt}\left(S_1(i) - S_3(i)\right)^2\right)}{nt}\right) \quad (13)$$

$$PRD = \left(\sqrt{\frac{\sum_{i=1}^{nt}\left(S_1(i) - S_3(i)\right)^2}{\sum_{i=1}^{nt}\left(S_1(i)\right)^2}}\right) \times 100 \quad (14)$$

where,

S1: The input ECG signal.
S2: The ECG signal after adding noise.
S3: The filtered ECG signal.
SNRin: The input SNR after adding noise.
SNRimp: The improved SNR.
SNRout: The output SNR after smoothing.
MSE: Mean square error.
PRD: Percent root mean square difference.
nt: Total no. of samples.

The mean value of the performance parameters for all 48 sample records is calculated. This value is a better evaluation of the methods' performance. Therefore, all parameters for performance evaluation of the methods presented in this section are the mean values.

The evaluation for TSS and proposed methods depends on three types of adding noise: white Gaussian noise with a signal-to-noise ratio (WGN-SNR) value from 0 to 20 dB for the input signal, white Gaussian noise with a power value noise (WGN-P) from 0 to 20 dB, and power-line noise with 50 Hz. The smoothing methods TSS and the proposed MSS method are applied to the signal after adding one of these noises each time to evaluate the performance parameters.

Based on the original ECG signals from the MBA database (record No. 100), for testing the TSS method, two smoothing parameter values are applied to the raw ECG signal after adding white Gaussian noise with SNR=10 dB. The TSS method results for p=0.001 and p=0.951 were presented in

Figure 1 and Figure 2, respectively. As discussed previously, a low p-value smooths the signal from any ripple. Still, it negatively affects the QRS in amplitude and shape, so the signal loses some of the information. A high p-value maintains the signal information but has a low smoothing of the noisy signal.

For testing the proposed MSS method, applying a range of p_j values based on the MDW of the ECG signal smooths and denoises the noisy signal without losing signal information. As shown in Figure 6, the output signal is smoother than the raw ECG signal before adding the noise.

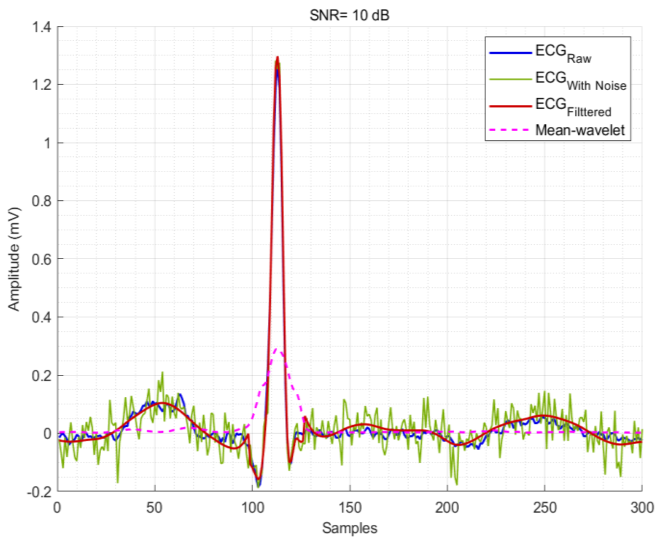
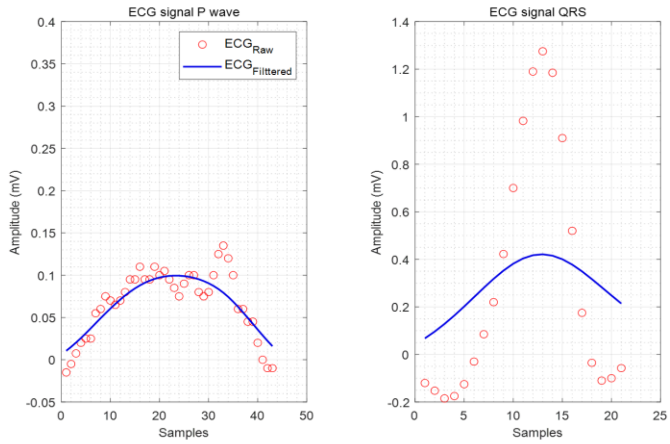


Figure 6. Smoothing and denoising a sampled ECG signal based on the proposed MSS method

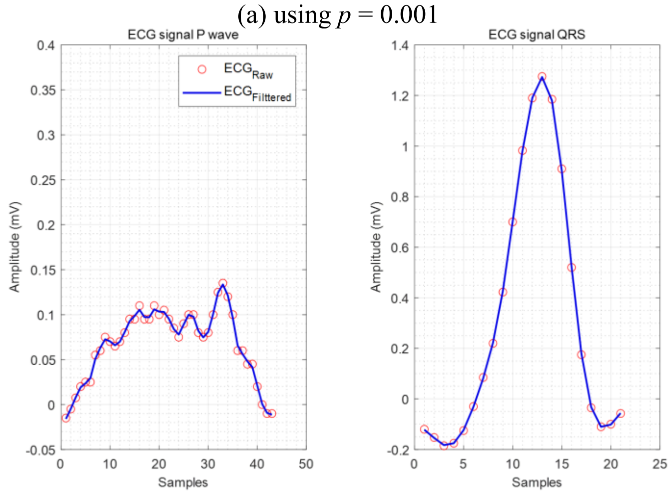
The P and QRS waves are extracted for the ECG signal to evaluate the smoothing and denoising operation for the TSS and the proposed MSS methods. The P and QRS waves extracted from Figure 1 and Figure 2 for the TSS method with two smoothing parameters (p=0.001 and p=0.951) are shown in Figure 7. With a low p-value for the smoothing parameter, the TSS method smooths the P wave with the best performance while losing the QRS. For the TSS method based on a high p-value, the TSS method smooths the QRS with the best performance, but the P wave has a noise and a low smoothing performance. The proposed MSS method using a range of p=0.001 to p=0.951 with step 0.0475 (based on lv=20 levels and α=0.001) is shown in Figure 8, which is extracted from Figure 6. By evaluating the TSS method for the record No. 100 samples, the SNRimp values are between 0.664 to 2.385 dB based on the lowest to highest p-values. Moreover, the MSE values are between 0.0024 to 0.0016. On the other hand, the proposed method evaluation values for this record are SNRimp = 8.20 dB and MSE = 0.000428. Therefore, it can be concluded that the proposed method has a higher smoothing and denoising performance than the TSS method according to the SNRimp and MSE values.

After testing the TSS and MSS methods, for evaluation of the methods' performance, three types of adding noise are presented as follows:

First, a WGN-SNR from 0 to 20 dB is added to the input signals (raw ECG) to calculate the performance parameters (SNRimp, SNRout, MSE, and PRD). In a case study, the raw ECG signal, the noisy ECG signal by adding a white Gaussian noise with signal-to-noise ratio WGN-SNR=10 dB to the raw ECG signal, and the filtered noisy ECG signal based on the TSS method with p=0.951 are presented in Figure 9. On the other hand, the ECG raw signal, the noisy ECG signal by adding the same noise to the raw ECG signal, and the filtered noisy ECG signal based on the proposed MSS method are presented in Figure 10.



(a) using p = 0.001



(b) using p = 0.951

Figure 7. Smoothing P and QRS waves based on the TSS method

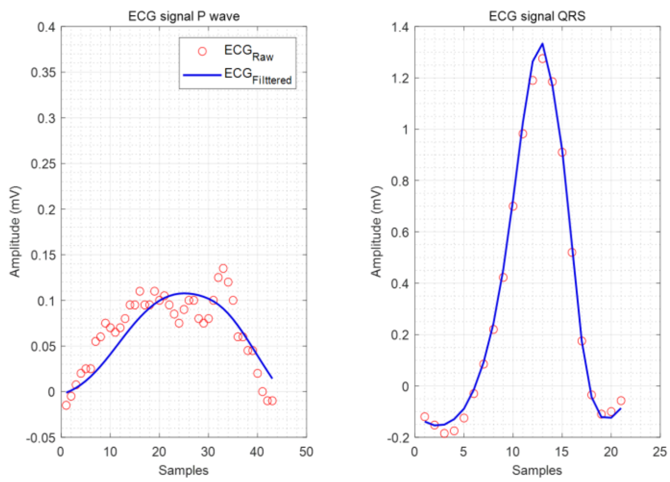


Figure 8. Smoothing and denoising P and QRS waves using a range of p-values based on the MSS method

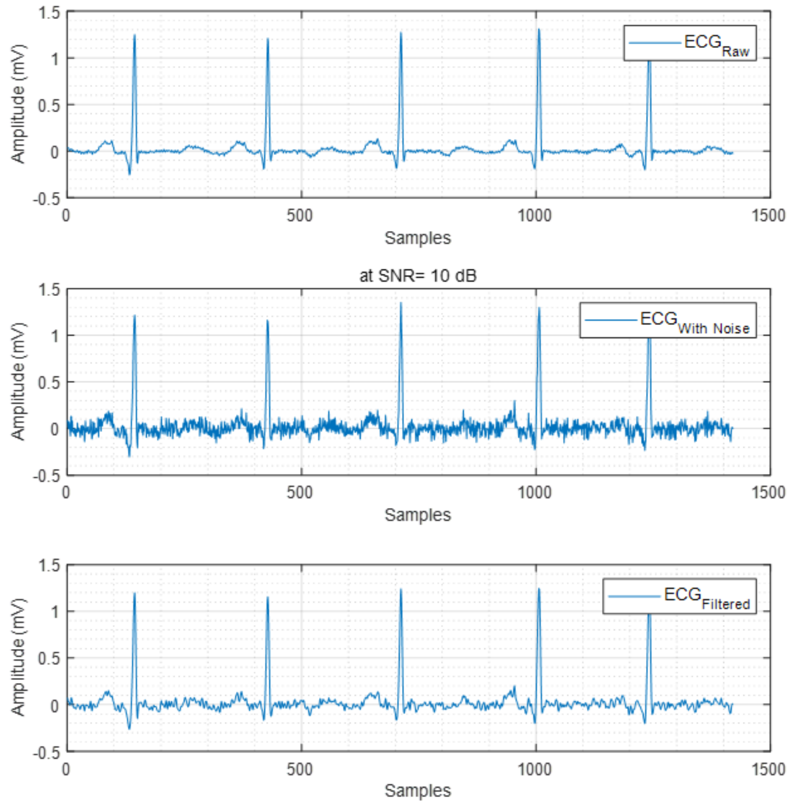


Figure 9. The raw, noisy, and filtered ECG signals using the TSS method with $p = 0.951$

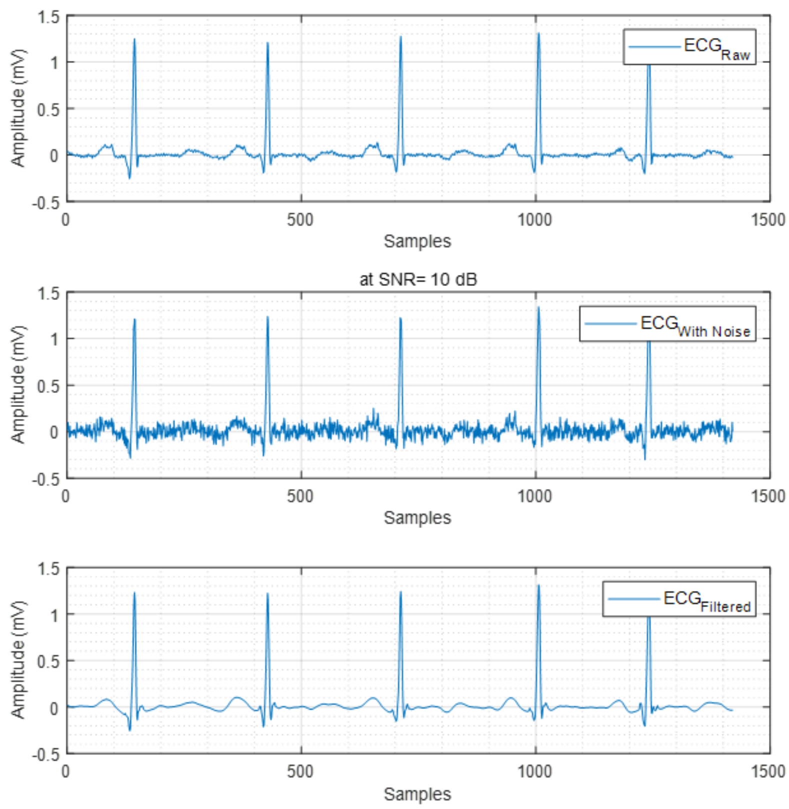


Figure 10. The raw, noisy, and filtered ECG signals using the proposed MSS method

From these figures, the TSS method smooths the noisy ECG with low smoothing based on the high *p*-value. The ECG signal output for the proposed method is highly smooth compared to the TSS method and smoother than the raw ECG signal.

The SNRimp is calculated for the WGN-SNR from 0 to 20 dB using Eq. (11) for all records samples. The average SNRimp for the proposed MSS method compared with the TSS method, WF [14] (soft and hard), DWT-SG [15], and GA-WT [16] are shown in Figure 11. The existing methods [14-16] applied a range of WGN-SNR between 0 to 15 dB and 0 to 20 dB, so the proposed and traditional methods can be compared with these methods. It can be seen from Figure 11 that the SNRimp for the proposed MSS method started at 1.44 dB in the highest WGN-SNR at 20 dB, and the improvement reached 10.45 dB for the zero dB WGN-SNR. On the other hand, the TSS method has the lowest SNR improvement, around 3 dB, in the WGN-SNR range. Meanwhile, the existing methods [14-16] have reached the highest improvement of 8.6 and 4.6 dB for SNRin of 0 and 20 dB, respectively. It can be concluded that the proposed method surpasses the existing methods by varying the SNR inputs from 0 to 20 dB. The superiority of the proposed method has been achieved due to the use of the variable soothing parameter (p) for each ECG signal sample based on the mean wavelet value. So, the smoothing parameter is low for the P and the T waves and high for the QRS wave. The proposed method reduces the noise and produces a smoother ECG signal than the original ECG signal before adding noise.

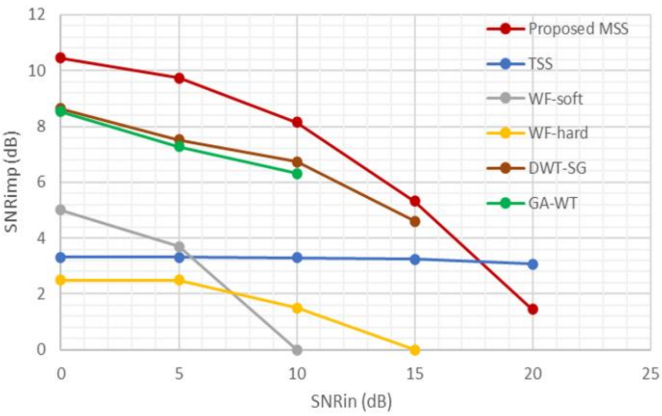


Figure 11. Comparison of methods performance for SNRimp based on SNRin

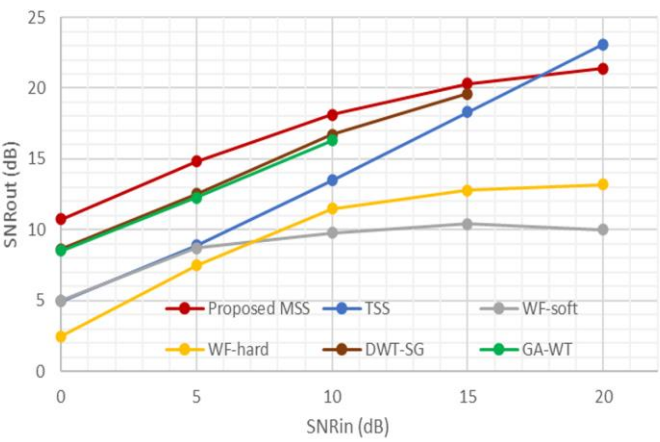


Figure 12. Comparison of methods performance for SNRout based on SNRin

However, compared to the TSS method, the proposed method has the highest performance for all ranges of the SNRin except the high SNRin value (at 20 dB). On the other hand, the TSS performance for all SNRin ranges is low except for 20 dB. In addition, the average SNRout for the MSS, TSS, and existing methods presented in Figure 12 also demonstrates the superiority of the proposed method over other methods.

The MSE for all sampling records is calculated using Eq. (13) for the proposed MSS method compared with the TSS method and WF [14] method, and the averages of MSE values are shown in Figure 13. From the MSE values, it can be concluded that the proposed MSS method has the lowest MSE values compared with the TSS and WF methods, particularly at the low SNRin based on the range of WGN-SNR starting from 0dB to 20dB for the noisy ECG signal.

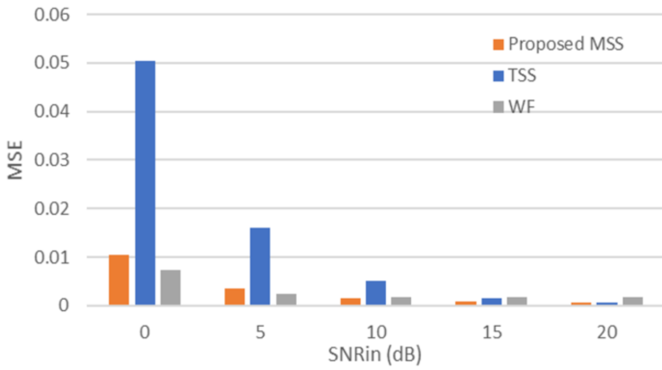


Figure 13. Comparison of methods performance for MSE based on SNRin

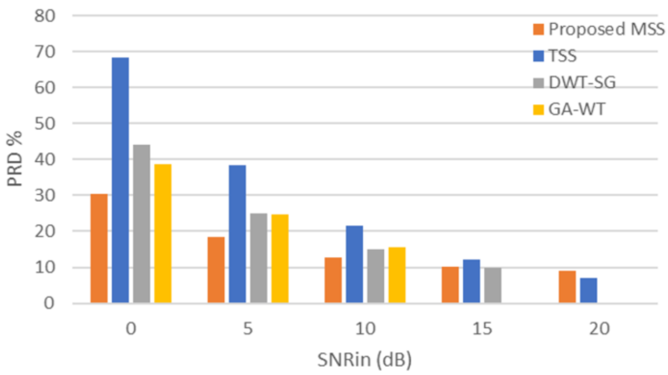


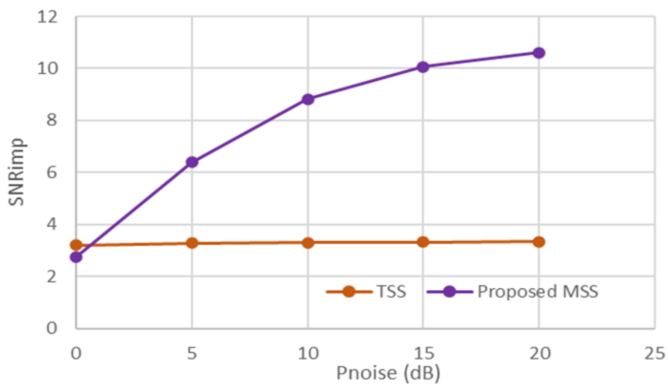
Figure 14. Comparison of methods performance for PRD using SNRin

Figure 14 shows the PRD values calculated using Eq. (14) for the proposed MSS method compared with the TSS, DWT-SG [15], and GA-WT [16] methods. It can be seen from Figure 14 that the PRD values for the proposed method are lower than the other methods' PRD values. The PRD difference between MSS and TSS methods reached more than 30% for the lowest WGN-SNR, while the difference between MSS and existing methods reached more than 25%.

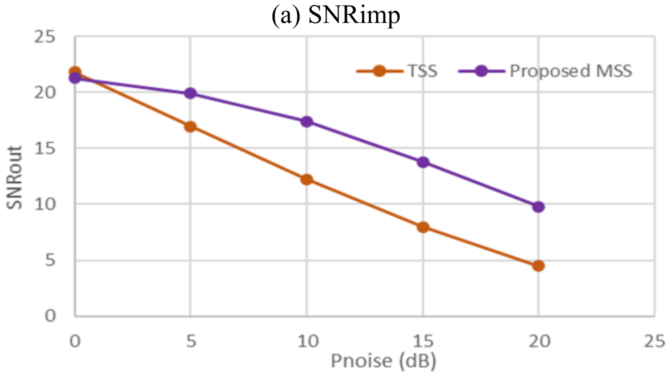
In the second type, noise is added to the raw ECG signals using the WGN with a power (WGN-P) from 0 to 20 dB. The average SNRimp, SNRout, MSE, and PRD for all sample records are calculated using Eqs. (11)-(14) after smoothing the noisy ECG signals based on the proposed MSS and TSS methods. The results are presented in Figure 15.

From these results, the values of SNRimp, SNRout, MSE, and PRD at SNRin=10 dB obtained from the proposed method are 8.14 dB, 18.13 dB, 0.0015, and 12.68%, while these values are 3.29 dB, 13.5 dB, 0.005, and 21.65% for the case of TSS method.

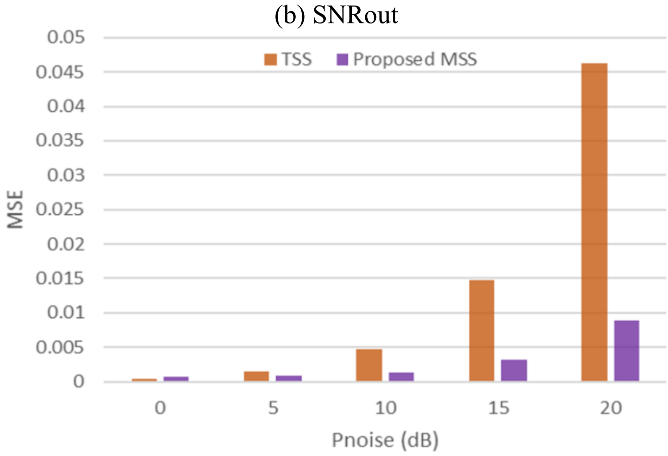
Third, the power-line noise with 50 Hz is added to the samples of the raw ECG signals at 3dB input SNR. The MSS and TSS methods are applied to the noisy signal, and the average SNRimp, SNRout, MSE, and PRD are determined using Eqs. (11)-(14). The results are presented in Table 1. These results show that using the proposed MSS instead of other methods improves the performance parameters, so the smoothing and denoising performance will be improved.



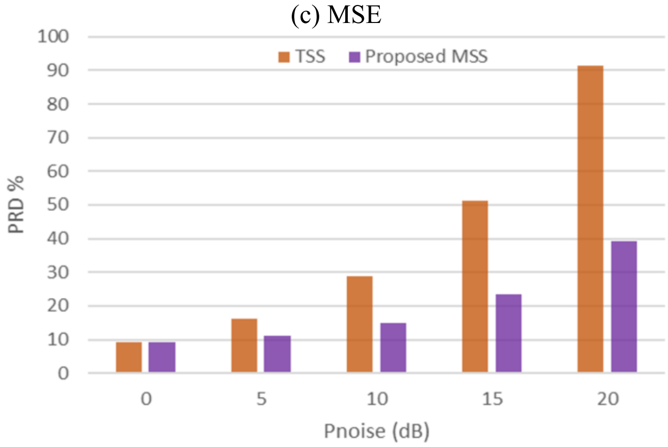
(a) SNRimp



(b) SNRout



(c) MSE



(d) PRD

Figure 15. Comparison of MSS and TSS methods performance using WGN-P

From the overall results, the proposed MSS method demonstrates a significant improvement compared with the TSS and the existing methods.

Table 1. Comparison of methods performance parameters using the power line noise

	Proposed MSS	TSS	WF [14]
SNRimp	11.99	0.533	-
SNRout	15.40	5.35	13
MSE	0.0008	0.011	0.001
PRD	17.13	63.96	-

5. CONCLUSIONS

The traditional smoothing spline method filters the ECG signal based on the smoothing parameter value. The TSS smoothing parameter values affect the filtered signal between a high smoothing signal with low information or low smoothing without losing signal information. This paper proposes a new ECG signal smoothing and denoising ECG signal method based on the proposed modified smoothing spline method and mean discrete wavelet. The proposed modified smoothing spline method uses variable smoothing parameter values for smoothing and denoising the noisy ECG signal. Selecting the smoothing parameter value for each sample of the ECG signal is based on the MDW value to improve the smoothing and denoising process. Based on the performance evaluation, the proposed MSS method smooths and denoises the ECG signal with high performance without losing the signal information. The proposed and traditional methods are evaluated using average values of the performance parameters (SNRimp, SNRout, MSE, and PRD) for all records samples. The results showed the superiority of the proposed method compared to traditional and existing methods.

REFERENCES

[1] Berkaya, S.K., Uysal, A.K., Gunal, E.S., Ergin, S., Gunal, S., Gulmezoglu, M.B. (2018). A survey on ECG analysis. Biomedical Signal Processing and Control, 43: 216-235. https://doi.org/10.1016/j.bspc.2018.03.003

[2] Yadav, O.P., Ray, S. (2016). Smoothening and segmentation of ECG signals using total variation denoising-minimization-majorization and bottom-up approach. Procedia Computer Science, 85: 483-489. https://doi.org/10.1016/j.procs.2016.05.195

[3] Luo, S., Johnston, P. (2010). A review of electrocardiogram filtering. Journal of Electrocardiology, 43(6): 486-496. https://doi.org/10.1016/j.jelectrocard.2010.07.007

[4] Tulyakova, N., Trofymchuk, O. (2022). Real-time filtering adaptive algorithms for non-stationary noise in electrocardiograms. Biomedical Signal Processing and Control, 72: 103308. https://doi.org/10.1016/j.bspc.2021.103308

[5] Jain, S., Paul, S. (2023). Design of filters using current amplifiers for removal of noises from ECG signal. Procedia Computer Science, 218: 1888-1904. https://doi.org/10.1016/j.procs.2023.01.166

[6] Alla, M.R., Nayak, C. (2024). A robust ECG signal enhancement technique through optimally designed adaptive filters. Biomedical Signal Processing and Control, 95: 106434. https://doi.org/10.1016/j.bspc.2024.106434

[7] Waheed, M. (2023). Pre-processing of ECG signal based

on multistage filters. Journal of Integrated Circuits and Systems, 18(2): 1-6. https://doi.org/10.29292/jics.v18i2.698
[8] Gadawe, N.T., Hamad, R.W., Qaddoori, S.L. (2024). Realization of IIR digital filter structures for ECG denoising. Journal Européen des Systèmes Automatisés, 57(2): 599-608. https://doi.org/10.18280/jesa.570228
[9] Chandra, M., Goel, P., Anand, A., Kar, A. (2021). Design and analysis of improved high-speed adaptive filter architectures for ECG signal denoising. Biomedical Signal Processing and Control, 63: 102221. https://doi.org/10.1016/j.bspc.2020.102221
[10] Khaing, A.S., Naing, Z.M. (2011). Quantitative investigation of digital filters in electrocardiogram with simulated noises. International Journal of Information and Electronics Engineering, 1(3): 210-216. https://doi.org/10.7763/IJIEE.2011.V1.33
[11] Saha, S., Barman Mandal, S. (2024). FPGA implementation of IIR elliptic filters for de-noising ECG signal. Biomedical Signal Processing and Control, 96: 106544. https://doi.org/10.1016/j.bspc.2024.106544
[12] Duranta, D.U.S., Ali, M.S., Das, A.A., Rahman, M.M., Ahsan, M.M., Miah, M.S., Islam, M.K. (2023). Enhancing Atrial Fibrillation detection accuracy: A wavelet transform filtered single lead ECG signal analysis with artificial neural networks and novel feature extraction. Machine Learning with Applications, 12: 100472. https://doi.org/10.1016/j.mlwa.2023.100472
[13] Kabir, M.A., Shahnaz, C. (2012). Denoising of ECG signals based on noise reduction algorithms in EMD and wavelet domains. Biomedical Signal Processing and Control, 7(5): 481-489. https://doi.org/10.1016/j.bspc.2011.11.003
[14] Aqil, M., Jbari, A., Bourouhou, A. (2017). ECG signal denoising by discrete wavelet transform. International Journal of Online and Biomedical Engineering, 13(9): 51-68. https://doi.org/10.3991/ijoe.v13i09.7159
[15] Samann, F., Schanze, T. (2019). An efficient ECG denoising method using discrete wavelet with Savitzky-Golay filter. Current Directions in Biomedical Engineering, 5(1): 385-387. https://doi.org/10.1515/cdbme-2019-0097
[16] Azzouz, A., Bengherbia, B., Wira, P., Alaoui, N., Souahlia, A. (2023). The effectiveness of optimal discrete wavelet transform parameters obtained using the genetic algorithm for ECG signal denoising. Revue d'Intelligence Artificielle, 37(6): 1387-1396. https://doi.org/10.18280/ria.370602
[17] Rakshit, M., Das, S. (2018). An efficient ECG denoising methodology using empirical mode decomposition and adaptive switching mean filter. Biomedical Signal Processing and Control, 40: 140-148. https://doi.org/10.1016/j.bspc.2017.09.020
[18] Liu, B., Li, Y. (2021). ECG signal denoising based on similar segments cooperative filtering. Biomedical Signal Processing and Control, 68: 102751. https://doi.org/10.1016/j.bspc.2021.102751
[19] Pollock, D.S.G. (1993). Smoothing with cubic splines. Queen Mary and Westfield College, Department of Economics.
[20] Wang, Y. (2011). Smoothing Splines: Methods and Applications. CRC Press.
[21] Mallat, S.G. (1989). A theory for multiresolution signal decomposition: The wavelet representation. IEEE Transactions on Pattern Analysis and Machine Intelligence, 11(7): 674-693. https://doi.org/10.1109/34.192463
[22] Heil, C.E., Walnut, D.F. (1989). Continuous and discrete wavelet transforms. SIAM Review, 31(4): 628-666. https://doi.org/10.1137/1031129
[23] Edwards, T. (1991). Discrete wavelet transforms: Theory and implementation. Stanford University.
[24] Moody, G.B., Mark, R.G. (2001). The impact of the MIT-BIH arrhythmia database. IEEE Engineering in Medicine and Biology Magazine, 20(3): 45-50. https://doi.org/10.1109/51.932724