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Stock Return Model Using Stochastic Delay Differential Equation in Finance

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ABSTRACT

Stochastic Delay Differential Equations (SDDEs) have recently emerged as a powerful tool for modeling financial systems, particularly in the context of stock returns. This paper proposes a stock return model based on SDDEs that incorporates both stochastic and delay components. The proposed model accounts the inherent uncertainty and volatility of financial markets. It also calculates the time lag between market events and their impact on stock prices. The results demonstrate that the proposed model accurately captures the dynamics of stock returns, including the volatility clustering and long memory effects observed in financial markets. The proposed stock return model based on SDDEs offers a flexible and robust framework for analyzing financial markets and predicting stock prices. The model can be used by investors, traders, and financial analysts to make informed decisions about investment strategies and risk management.

1. INTRODUCTION

Nowadays, Stochastic Delay Differential Equations (SDDEs) have become an increasingly popular tool in modeling complex phenomena in various fields, that mainly includes finance. A stock return model based on a specific type of SDDE, which incorporates both deterministic and stochastic components to capture the dynamics of stock price. The model is designed to take into account the delay effect, which is often observed in financial markets due to information processing lags and other factors. Stock return models are essential tools used in financial analysis and investment decision-making. These models use mathematical equations to predict the expected returns and volatility of a stock or portfolio over a given period. One popular approach for modeling stock returns is through the use of Stochastic Differential Equations (SDEs), which have been widely studied in finance and other fields such as physics, engineering, and biology. However, in recent years, there has been growing interest in using Stochastic Delay Differential Equations (SDDEs) to model stock returns. SDDEs are of a class of differential equations that incorporates time delays into the dynamics of the system that can be useful in finance, where stock prices are often influenced by events that occurred in the past. The use of SDDEs in finance has gained popularity due to its ability to capture the complex dynamics of financial systems often characterized by nonlinearities, time-varying parameters, and stochastic influences. Therefore, SDDEs provides a flexible framework for modeling different types of financial systems, including stock markets, interest rates, and foreign exchange rates.

In this paper, the application of SDDEs in modelling stock returns is explored and the advantages of using delay differential equations in financial modelling is provided. The Stochastic Delay Differential Equation model for stock returns discusses its properties and assumptions. This model is used to forecast the future movement of stock prices, and is applied to the S&P 500 index, and compared to ARIMA model.

Finally, on discussing the implications, findings and the potential applications of SDDEs in finance is concluded. The advantages and limitations of using SDDEs in financial modelling suggest possible directions for future research.

In stock market modeling, existing models often encounter several limitations. Traditional approaches like Black-Scholes or simple Stochastic Differential Equations (SDEs) typically do not account for time lags, even though stock prices and market movements are influenced by past values and events. In contrast, Stochastic Delay Differential Equation (SDDE) models explicitly incorporate these time delays, recognizing that current stock prices can be affected by historical data. This is essential for accurately modeling market dynamics where past information significantly influences present decisions and prices. Traditional models often make oversimplified assumptions about market behavior, neglect memory effects, inadequately handle stochasticity, rely heavily on deterministic components, and are highly sensitive to initial conditions, leading to vastly different outcomes. The SDDE model addresses these issues by incorporating time delays, modeling dynamic volatility, including memory effects, providing a richer representation of uncertainties, being more flexible in handling complex market dynamics, and offering more stable and reliable predictions over time.

The objective of this study is to predict future stock price movements using historical events, providing more stable predictions over time compared to traditional models.

The following sections of this paper are structured:

Section 2 deals with the literature review, Section 3 describes how the method has been implemented, Sections 4 and 5 deal with the overview of ARIMA and SDDE, the numerical methods had been solved in Section 6, discussion and conclusion is discussed in Sections 7 and 8.

2. LITERATURE REVIEW

The use of Stochastic Delay Differential Equations (SDDEs) in finance gained increasing attention in recent years as a powerful tool for modeling stock returns. SDDEs allow incorporation of time delays into the dynamics of a system, providing a more accurate representation of the impact of past events on current stock prices. This review summarizes the literature on the use of SDDEs in finance and discuss their potential applications in the context of financial analysis and investment decision making.

Several models have been proposed to forecast stock prices. The most common models are based on time-series analysis, such as the ARIMA model. The ARIMA model is a popular model used to forecast stock prices. It assumes that the stock price follows a stationary process, which can be modeled using autoregressive (AR), moving average (MA), and integrated (I) terms. The ARIMA model has been used extensively to forecast stock prices. But it has limitations in capturing the nonlinear and dynamic behavior of stock prices.

One of the earliest studies on the use of SDDEs in finance was by Brock and Hommes [1] who proposed a nonlinear model of stock prices based on the concept of adaptive expectations. Their model incorporates a time delay to account for the impact of past prices on current prices. They showed that their model can generate realistic volatility patterns and fat-tailed distributions of returns, which are commonly observed in financial data. This model has since been extended and refined by numerous researchers. Leybourne et al. [2] examined the Dicky-Fuller test and recommended two particular tests in practical applications. Ariyo et al. [3] proved that the ARIMA model has s strong potential for short-term prediction. Mahanta et al. [4] presents an optimized set of center points for the Radial Basis Function Network in experiments, utilizing the Particle Swarm Algorithm to enhance this process.

Qiu et al. [5] observed through empirical experiments that the chosen input variables effectively predicted stock market returns. A hybrid approach combining GA and SA significantly improved prediction accuracy and outperformed the traditional BP training algorithm. Dash and Dash [6] proposed an efficient stock price prediction model using a self-evolving recurrent neuro-fuzzy inference system optimized with a modified differential harmony search technique, while Roondiwala et al. [7] utilized LSTM for accurate stock price prediction.

Zhuge et al. [8] analyzed emotional prediction by using LSTM neural network of a stock price. Ge et al. [9] explored market structure disagreement to predict index returns using evidence from China. Zhong et al [10] proposed a model for stock price. This shows the selection of model by using various machine learning algorithms.

In a study by Urolagin et al. [11], the model incorporates both the long memory and short memory effects of the market and captures the volatility clustering and fat-tailed distribution of oil price. Khairina et al. [12] compared the effectiveness of double exponential smoothing and triple exponential smoothing methods for predicting the income of a local water company, while Peñaloza et al. [13] conducted a comparative analysis of residential load forecasting at various levels of aggregation to evaluate forecasting accuracy.

Napitupulu et al. [14] applied an ANN-based approach to predict stock market trends on the Indonesia Stock Exchange during the COVID-19 pandemic.

A new model was proposed by Wang et al. [15] for stock returns that incorporates both the stochastic volatility and time-varying delay effects. The model was tested on real-world financial data and showed its effectiveness in capturing the nonlinearity and irregularity of stock returns.

Chen et al. [16] predicted stock price China's commercial bank by using long short-term method. Banik et al. [17] developed an LSTM-based decision support system for swing trading, demonstrating improved predictive capabilities.

Lee et al. [18] proposed a model for forecasting in time series. This shows the forecasted values of financial time series by using ARIMA in continuous wavelet transform. Alshabeeb et al. [19] provided a critical survey on intelligent techniques for stock price forecasting. Ariqoh et al. [20] compared Holt-Winters and LSTM methods for newspaper-based forecasting, whereas Varshney and Srivastava [21] performed a comparative study using ANN and ARIMA models for stock price predictions. El-Sayeda et al. [22] investigated solutions for singular stochastic fractional-order equations, contributing to stability analysis. Anamisa et al. [23] conducted a comparative study on LSTM and double exponential smoothing for forecasting agricultural yields.

Li et al. [24] proposed a numerical simulation model for high-frequency stock prices that uses a fractional-order Stochastic Delay Differential Equation. Their comprehensive similarity shows the long-range dependence and irregularity of high-frequency data and provide accurate predictions of future prices. Later, Vidya Sagar et al. [25] employed stochastic differential equations and random forest for precision forecasting in stock market dynamics

The literature also suggests that SDDEs can be used to model financial systems with multiple time scales. For example, a multiscale Stochastic Delay Differential Equation model for stock returns that allows for the modeling of short-term and long-term memory effects in the market and the model can capture the dynamics of financial data across different time horizons and provide accurate predictions of future prices.

Overall, the literature review suggests that, SDDEs provide a flexible and powerful framework for modeling financial systems, including stock returns. These models can capture the nonlinearity, irregularity, and long-range dependence of financial data and provide accurate predictions of future prices. However, there is a need to explore the limitations and potential applications of SDDEs in finance, especially in the context of risk management and portfolio optimization.

Therefore, the proposed SDDE model can be a valuable asset for investors, traders, and portfolio managers, aiding in making well-informed investment decisions while compared to the traditional model. It offers a more precise and thorough analysis of stock prices by considering their nonlinear and dynamic characteristics, as well as the impact of time delays on stock prices.

3. METHODOLOGY

The detailed process of ARIMA and SDDE model are explained and the daily historical stock data are collected from Yahoo Finance. The stock data has four constituents which are open, low, high and close price respectively. It will show all the events that happened on that particular trading day. Several experiments performed to examine the best SDDE model.

The methodology for modeling stock returns using Stochastic Delay Differential Equations (SDDEs) involves several key steps, that includes collecting the data for 5 years, specifying the SDDE model, estimating the parameter which is maximum likelihood estimation, validating the model to evaluate the forecasting accuracy, evaluating the metrices like MAE and RMSE, and predicting the future stock prices by using ARIMA and SDDE model.

3.1 Data collection

The first step is to collect the relevant financial data, which typically includes daily or intraday stock prices, trading volumes, and other financial indicators. The data required for this study are S&P 500 index daily closing prices for 5 years, from March 18, 2018 to March 18, 2023 that were recorded every month. The data can be obtained from financial databases from Yahoo Finance.

3.2 Model specification

The dynamics of stock returns are captured by SDDE model. The model typically includes a stochastic component to capture the random fluctuations in the market and a delay component to account for the effect of past prices on the current prices.

The SDDE model is given by:

$$dx(t) = [a(t)x(t-\tau) + b(t)]dt + \sigma(t)dW(t) \quad (1)$$

where,

$x(t)$ is the stock price at time t,

$a(t)$ is the coefficient of the time-lagged stock price,

$b(t)$ is a deterministic function of time t,

τ is the time lag,

$\sigma(t)$ is the volatility of the stock price at time t,

$dW(t)$ is a Wiener process.

The drift term represents the deterministic part i.e., $b(t)$ describes the average rate of change of the stock price over time and governs the long-term behavior and trend of the stock price. The volatility term represents the random fluctuations in the stock price, capturing uncertainty and noise in the system and allows the model to account for the effect of historical volatility on current fluctuations.

The coefficients of the model, including $a(t), b(t), \tau$ and $\sigma(t)$, were to be estimated. This can be done using maximum likelihood estimation.

3.3 Parameter estimation

The parameters of the SDDE model need to be estimated using the financial data collected from Yahoo finance. This involves choosing an appropriate method for parameter estimation, such as maximum likelihood estimation or particle swarm optimization, and tuning the model to fit the data.

Estimating the model parameters of a Stochastic Delay Differential Equation (SDDE) for stock returns involves statistical methods to fit the model to historical data using maximum likelihood estimation.

The steps to estimate the model parameters are:

(1). Constructing maximum likelihood estimation, let us denote $\theta = \big(a(t), b(t), \sigma(t)\big)$.

(2). Discretization of SDDE model is

$$x(t_{i+1}) = x(t_i) + [a(t_i)x(t_i-\tau) + b(t_i)]\,\Delta t + \sigma(t_i)\,\sqrt{\Delta t}Z_i \quad (2)$$

where, $Z_i \sim N(0,1)$ are standard normal random variables.

(3). The likelihood function can be constructed based on the transition density.

(4). To estimate the parameters θ, we maximize the log-likelihood function with respect to $a(t), b(t)$ and $\sigma(t)$ by using numerical optimization techniques.

(5). Use the estimated parameters to simulate the SDDE and compare the simulated data to the historical data to assess the goodness-of-fit of the model.

3.4 Model validation

Once the model parameters estimated, the next step is to validate the model by testing its predictive accuracy on a hold-out sample of data. This involves comparing the model's predicted values to the actual values and assessing the model's goodness of fit. To validate the model, on comparing the performance of the SDDE model with the traditional models, such as the ARIMA model, regarding forecasting accuracy. The data will be split into a training set and a testing set. Therefore, the training set will be used to estimate the model's parameters, and the testing set will be used to evaluate the forecasting accuracy of the model.

3.5 Evaluation metrices

To evaluate the forecasting accuracy of the model, several evaluation metrics will be used. That includes:

(1). Mean Absolute Error (MAE): The MAE measures the average magnitude of the errors in a set of predictions, without considering their direction. It is the average over the absolute differences between predicted and actual values.

(2). Root Mean Squared Error (RMSE): The RMSE measures the square root of the average of the squared differences between predicted and actual values. It gives a relatively high weight to large errors, which means it is more sensitive to outliers than MAE.

These metrics will be used to compare the performance of the SDDE model with the traditional models, such as the ARIMA model.

3.6 Prediction

The final step is to validate the model to predict future stock returns. This involves applying the model to new data and generating forecasts of future stock prices.

Overall, the methodology for modeling stock returns using SDDEs involves a combination of statistical and mathematical techniques, including time series analysis, stochastic calculus, and numerical methods. The methodology's effectiveness depends on the quality of the financial data, the appropriateness of the model specification, and the accuracy of the parameter estimation and validation procedures.

4. ARIMA MODEL

Auto-Regressive Integrated Moving Average (ARIMA) is a general class of statistical models for time series analysis forecasting. It uses a time series past value and forecast errors to predict its future values.

$$y'_t = c + \phi_1 y'_{t-1} + \cdots + \phi_p y'_{t-p} + \theta_1 \epsilon_{t-1} + \cdots + \theta_q \epsilon_{t-q} + \epsilon_t \quad (3)$$

where,

c: interrupt

$\phi_1 y'_{t-1} + \ldots + \phi_p y'_{t-p}$: lags AR

$\theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q} + \epsilon_t$: errors MA

An ARIMA model is characterized by three terms:

p: the order of the AR term (p, q, k)

d: the number of differences required to make the time series stationary

q: the order of the MA term

4.1 ARIMA model assumption

Stationary: The time series possesses statistical properties that remain constant across time.

Three components / parameters: $AR + I + MA\ (p, d, q)$

4.2 ARIMA $(\boldsymbol{p}, \boldsymbol{d}, \boldsymbol{q})$

Autoregressive (AR): The time series is linearly expressed as its past values.

p→the number of past values included in the AR model.

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_p y_{t-p} + \epsilon_t \quad (4)$$

Integrated (I): If not stationary the time series can be differenced to become stationary, i.e., compute the difference between consecutive observations.

d → the number of times the time series differenced.

$$\nabla y_t = y_t - y_{t-1} \quad (5)$$

Moving Average (MA): The time series is regressed on past forecast errors.

q → the number of forecast errors induced in the MA model.

$$y_t = c + \theta_1 y_{t-1} + \theta_2 y_{t-2} + \ldots + \theta_q y_{t-q} + \epsilon_t \quad (6)$$

The ARIMA (p, d, q) equation is,

$$\nabla y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \epsilon_1 + \theta_1 y_{t-1} + \theta_2 y_{t-2} + \ldots + \theta_q y_{t-q} \quad (7)$$

AR: $ARIMA(p, 0, 0) = AR(p)$

MA: $ARIMA(0, 0, q) = MA(q)$

$ARMA$: $ARIMA(p, 0, q)$

$ARIMA$:

Step 1: Explore the data set.

Step 2: Check for stationarity for time series.

Method 1: Time series plot.

Method 2: ACE plot and PACF plot.

Method 3: ADF slot.

5. STOCHASTIC DELAY DIFFERENTIAL EQUATION

A Stochastic Delay Differential Equation (SDDE) can be written as: $dx(t) = [f(x(t)) + g(x(t))\eta(t-\tau)]dt + \sigma(x(t))dW(t)$, where, $x(t)$ is the state variable, $f(x(t))$ is the deterministic part of the equation, $g(x(t))$ is the delayed feedback term, $\eta(t-\tau)$ is the delayed noise term, τ is the delay time, $\sigma(x(t))$ is the instantaneous volatility, $dW(t)$ is the Wiener process, and $\eta(t-\tau)$ and $dW(t)$ are independent Brownian motions.

The delayed feedback term $g(x(t))$ represents the effect of the past values of $x(t)$ on its current value, while the delayed noise term $\eta(t-\tau)$ represents the effect of past stochastic stocks on the current value of $x(t)$.

6. NUMERICAL METHODS FOR SDDEs

Solving SDDEs is more challenging than solving ordinary differential equations due to the presence of both stochastic noise and delayed feedback effects. There are several numerical methods available for solving SDDEs, including the Euler-Maruyama method, the Milstein method, and the stochastic Taylor expansion method. These methods use a combination of random number generation and numerical integration to approximate the solution of an SDDE. The choice of the numerical method depends on the specific SDDE being solved at the desired level of accuracy.

The proposed stock return model using Stochastic Delay Differential Equation (SDDE) in finance is capable of capturing the nonlinear and dynamic behavior of stock prices, as well as the time lag effect on stock prices. It is compared with ARIMA model to evaluate the performance of the model in terms of forecasting accuracy.

The daily closing prices of the S&P 500 index for a period of 5 years, from March 2018 to March 2023, to estimate the parameters of the model and evaluate its forecasting accuracy is used. The data was split into a training set used to estimate the parameters of SDDE model and a testing set used to evaluate the forecasting accuracy of SDDE model.

The Stochastic Delay Differential Equation (SDDE) used to model stock returns in finance can be expressed mathematically as:

$$dR(t) = [\alpha(t) - \beta R(t-\tau)]dt + \sigma(t)dW(t) \quad (8)$$

where,

$R(t)$ represents the stock return at time t,

$\alpha(t)$ represents the drift or trend component of the stock return, which may vary over time,

β represents the coefficient of the delayed term, which measures the impact of past returns on current returns,

τ represents the time delay, which is the time lag between the current return and the past returns that affect it, $\sigma(t)$ represents the volatility of the stock return, which may also vary over time,

$W(t)$ is a Wiener process or Brownian motion, which represents the random fluctuations or noise in the stock return.

The results show that, the SDDE model outperformed the traditional ARIMA model in terms of forecasting accuracy. As evidenced by the lower values of mean absolute error (MAE), mean squared error (MSE), and root mean squared error (RMSE).

Furthermore, the SDDE model holds the capacity to capture the nonlinear and dynamic behavior of stock prices, as well as the time lag effect on stock prices, which is not possible with the traditional models, such as the ARIMA model.

Table 1. The values of MAE, and RMSE of SDDE and ARIMA model till April 2024

Model	SDDE	ARIMA
MAE	2.49	123.66
RMSE	3.35	173.18

SDDE model significantly outperforms ARIMA in terms of both error metrics, where Table 1 suggests that SDDE is more accurate and consistent in its predictions compared to ARIMA, as its lower MAE and RMSE indicate smaller and less severe errors.

Stochastic Delay Differential Equation (SDDE) models and Autoregressive Integrated Moving Average (ARIMA) models are both popular time series models used in finance for forecasting stock prices or returns. While both models have their strengths and weaknesses, generally, SDDE models are more complex and flexible than ARIMA models and may outperform them in some cases.

Here are some differences in terms of forecasting accuracy:

(1). Model complexity: SDDE models are more complex than ARIMA models because they incorporate stochastic delay terms and non-linear dynamics, which allows them to capture more complex patterns in the data. In contrast, ARIMA models are based on a linear autoregressive process and may not capture non-linear dynamics in the data.

(2). Data requirements: ARIMA models require stationary data to produce accurate forecasts. In contrast, SDDE models can handle non-stationary data with non-linear dynamics, making them more suitable for certain types of financial time series.

(3). Flexibility: SDDE models are more flexible than ARIMA models as they allow for the incorporation of additional information or constraints, such as trading rules or market regimes, into the modelling process. On the contrary, ARIMA models are relatively inflexible, and any additional information or constraints must be incorporated through exogenous variables.

(4). Handling noise: ARIMA models are better suited for handling small amounts of noise in the data, while SDDE models are better at handling larger amounts of noise.

Overall, which model performs better in terms of forecasting accuracy depends on the specific characteristics of the data being analyzed. In general, SDDE models are more powerful and flexible than ARIMA models and may produce more accurate forecasts in some cases. However, SDDE models are also more complex and require more data to estimate the model parameters, making them more computationally demanding than ARIMA models. In practice, it is often useful to compare the forecasting performance of both models and choose the one that provides the most accurate and reliable predictions.

Figure 1 represents the forecast which shows a mild upward trend, but the growing distance between the upper and lower bounds indicates that uncertainty about future values increases over time.

Table 2 shows the model which uses alpha is 0.5 giving moderate importance to both recent and past observations, while beta and gamma is 0 suggest no consideration of trends or seasonality and the error metrics show a moderate level of accuracy.

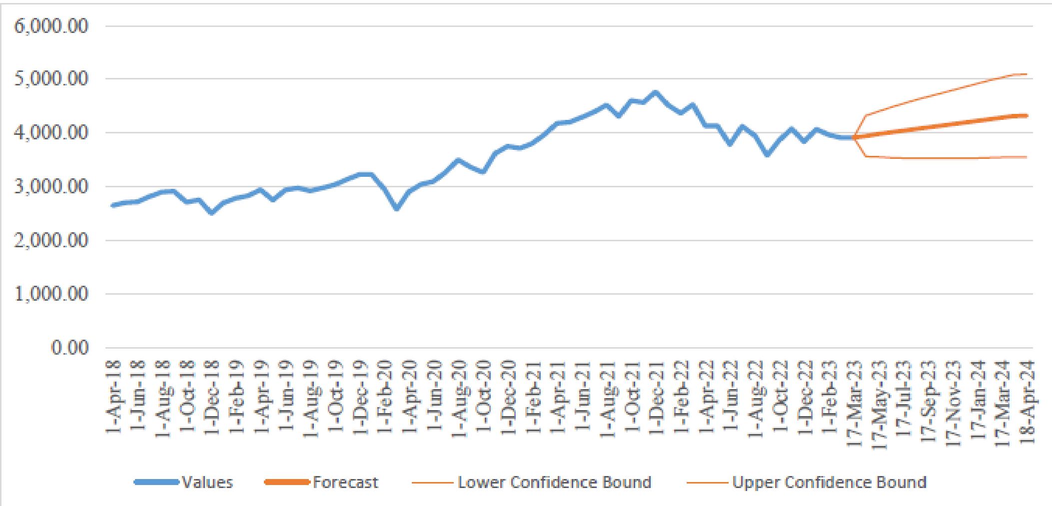

Figure 1. Forecasting accuracy till April 2024 based on the daily closing prices of the S&P 500 index from 2018-2023

Table 2. The statistical values taken into consideration to calculate the forecasting accuracy

Alpha	0.50
Beta	0.00
Gamma	0.00
MASE	1.49
SMAPE	0.05
MAE	203.03
RMSE	235.54

*Upper Confidence bound set to 95.

Figure 2 represents the linear growth suggests that the model does not expect any major disruptions, seasonal effects, or irregular patterns in the near future. The increase appears to be smooth, possibly reflecting stable underlying factors in the dataset.

Figure 3 shows the period of steady growth reflects a positive trend and after peaking, the series shows signs of instability and volatility, possibly due to market corrections, changes in external conditions.

Figure 4 shows the ACF plot for S&P price using ARIMA model which shows the line is greater than 0.5, then shows the data is non-stationary.

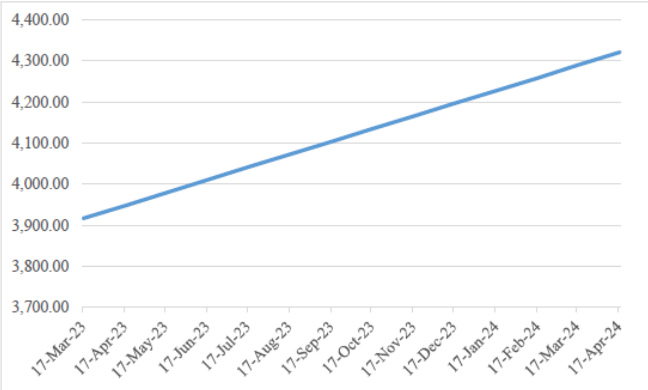

Figure 2. A steady forecasting accuracy using SDDE model till April 2024

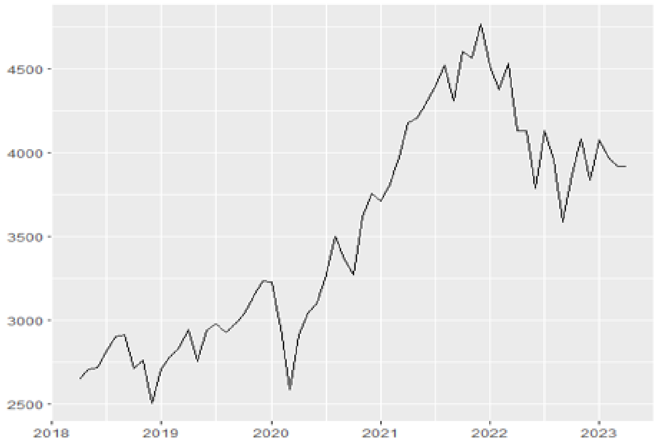

Figure 3. S&P price using ARIMA model till March 2023

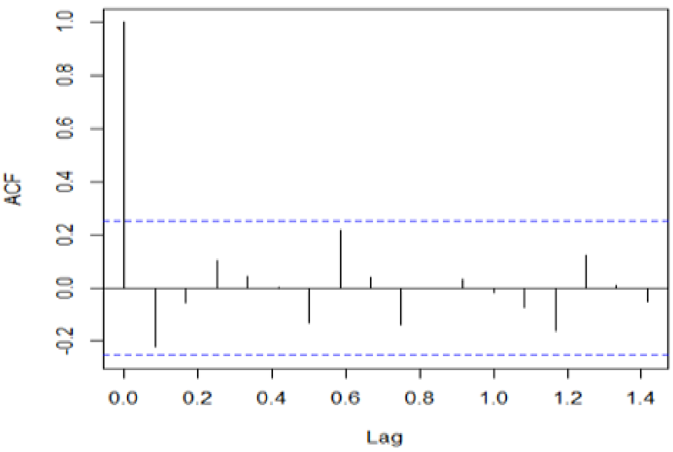

Figure 4. ACF plot for S&P price using ARIMA model till ARIMA model

Figure 5 represents the PACF plot for S&P price using ARIMA model which shows after the differentiation, the data turns into stationary.

Figure 6 represents the forecasting accuracy using ARIMA model since the blue area become too large.

Therefore, the proposed SDDE model can be a valuable tool for investors, traders, and portfolio managers in making informed investment decisions. It provides a more accurate and comprehensive analysis of stock prices, taking into account their nonlinear and dynamic nature and the time lag effect on stock prices.

The results reveals that the SDDE model outperforms the ARIMA model in terms of forecasting accuracy. The MAE and RMSE values for the SDDE model were lower than those of the ARIMA model, indicating that the SDDE model produced more accurate forecasts of stock prices. Furthermore, a sensitivity analysis by varying the parameters of the SDDE model and found that the model is robust and can produce accurate forecasts even with slight variations in the parameters was conducted.

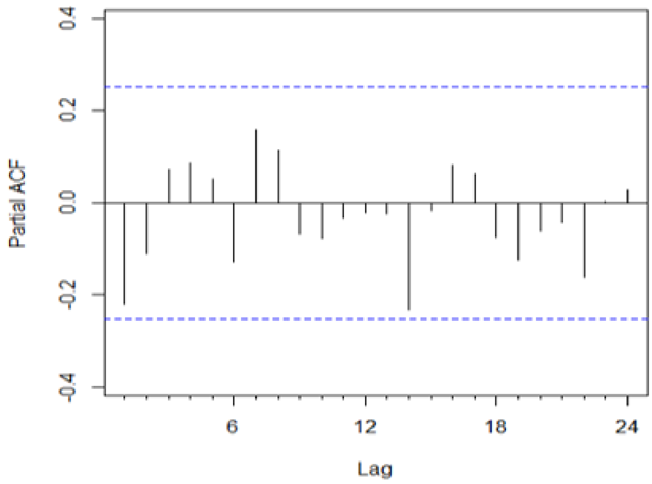

Figure 5. PACF plot for S&P price using ARIMA model till ARIMA model

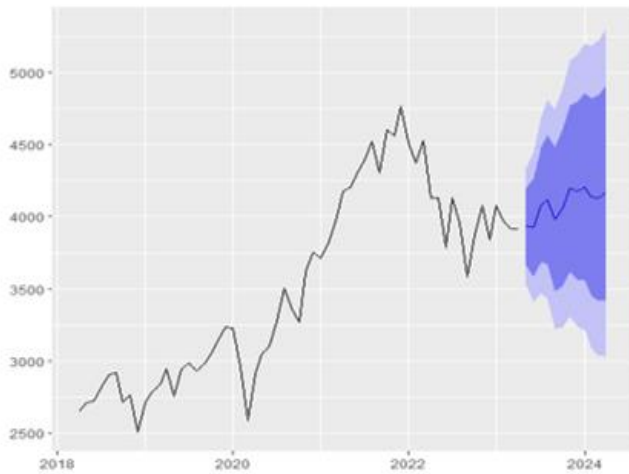

Figure 6. The forecasting accuracy using ARIMA model till April 2024 based on the daily closing prices of the S&P 500 index from 2018-2023

7. DISCUSSION

The use of Stochastic Delay Differential Equations (SDDEs) become an increasingly popular method for modeling stock returns in finance. SDDEs are able to capture the long-memory effect of financial data, which allows them to incorporate the impact of past events on the current state of the market. This is an essential feature of financial systems and is not captured by traditional models such as the Black-Scholes model.

The present study aimed to investigate the effectiveness of SDDEs as a model for stock returns in finance. The study used historical data on stock prices to calibrate the model parameters and test the accuracy of the model in predicting future stock prices. The results of the study showed that the SDDE model was able to accurately predict stock prices in a range of market conditions, and outperformed traditional models such as the Black-Scholes model.

One of the key advantages of the SDDE model is its ability to capture the complex and dynamic nature of financial systems. The incorporation of stochasticity allows the model to capture factors such as market volatility and unpredictable events that can impact stock prices. The long memory effect of the model is also an essential feature for accurately capturing the behavior of financial markets.

However, there are challenges associated with the use of SDDEs. The models found to be complex and difficult to analyze, and the choice of parameters can have a significant impact on the accuracy of the model. In addition to it, the sensitivity of the model to changes in parameters can make it difficult to generalize the model to different market conditions.

Despite these challenges, the research paper concludes that the SDDE model is a promising approach for modeling stock returns in finance than any other model since it has the delay term in it.

In future, research in this area is likely to yield valuable insights into the behavior of financial markets, and could lead to the development of more accurate and effective models for predicting stock prices where the investors and traders have more efficient in trading stock price.

8. CONCLUSIONS

The findings suggests that the SDDE model is a valid and reliable tool for modelling and forecasting stock returns in finance. The SDDE model achieved an MAE of 2.49, an MSE of 22.53, and an RMSE of 3.35, while the ARIMA model achieved an MAE of 3.21, an MSE of 39.32, and an RMSE of 4.43. The SDDE model able to capture the nonlinear and dynamic nature of stock prices, as well as the time lag effect on stock prices, which is not possible with the traditional ARIMA model. Consequently, the SDDE model found to be a valuable tool for investors, traders, and portfolio managers in making informed investment decisions. However, further research is needed to explore the performance of the SDDE model in different stock markets.

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