International Information and Engineering Technology Association
Advancing the World of Information and Engineering

Mathematical Modelling of Engineering Problems
Vol. 11, No. 12, December, 2024, pp. 3300-3306
Journal homepage: http://iieta.org/journals/mmep

Analyzing Oil Reservoir Dynamics: Leveraging Separated Variable Solution of Radial Diffusivity Equation with Constant Bottom Flux

Drilona Sauli[1*], Neime Gjikaj[2], Evgjeni Xhafaj[3], Robert Kosova[2], Esmeralda Zeqo[1]

[1] Department of Energy Resources, Polytechnic University of Tirana, Tirana 1000, Albania
[2] Department of Mathematics, Faculty of Information Technology, University “Aleksander Moisiu”, Durres 2000, Albania
[3] Department of Mathematical Engineering, Faculty of Mathematics and Physics Engineering, Polytechnic University of Tirana, Tirana 1000, Albania

Corresponding Author Email: drilona.sauli@fgjm.edu.al

https://doi.org/10.18280/mmep.111210

Received: 20 May 2024
Revised: 10 October 2024
Accepted: 20 October 2024
Available online: 31 December 2024

Keywords:
radial diffusivity equation, linear equation with separable variables, drainage volume, Amonica oilfield

ABSTRACT

The demand for oil and its subproducts is steadily increasing, making the study of oilfields and in particular oil wells a crucial aspect of exploitation engineering. Information obtained from fluid filtration and the study of pressure drop in reservoir conditions is of great importance in determining the productive capabilities of the oilfield reservoir. The behavior of fluid flow in a reservoir is usually modeled by a nonlinear partial differential equation (PDE), which is often simplified to a linear form in the petroleum industry for practical applications. This paper presents an analytical solution using the separable-variable technique for the constant-flow radial diffusivity equation, which describes the pressure drop in the near-wellbore region under conditions of constant oil production. The production data for the Amonica oilfield were provided by the Geological Institute of Oil and Gas in Fier, Albania. Our findings underscore the importance of depletion time, production rate, and reservoir radius in calculating pressure drops. A sensitivity analysis shows that the primary factors influencing the pressure profile are several parameters such as permeability, porosity, and viscosity. Additionally, we determined that the radial diffusivity equation solution for a finite constant flow rate during the initial transient flow period can be derived using the separable-variable method, which was approximated by the so-called linear solution. It is assumed that, in comparison to infinite reservoirs, the well radius is negligible, and the area near the wellbore can be treated as a point source.

1. INTRODUCTION

The radial diffusivity equation is a fundamental equation which is used in many different fields, including mathematics, physics, chemistry, and engineering [1]. In the context of oil wells, this equation is widely employed to simulate fluid movement through porous materials. One of the main solutions of the radial diffusivity equation is to maintain the flow of the fluid in the well at a constant rate during a finite period of time [2].

This solution is particularly useful in the petroleum industry for analyzing production wells or injection wells where a flow rate is held constant for a certain duration [3].

The well survey involves the production of a well with constant flow or with variable flow, from where we can simultaneously make a continuous record of the change of pressure values in near the bottom hole area as shown in Figure 1.

Considering near-bottom pressure as a function of time, it is possible to analyze values according to the known flow rate, and in this way reservoir parameters could be determined [4]. In the framework of this work, pressure levels were measured in the wells of the Amonica oilfield in Albania, and with these data, graphs are built for the progress of the layer pressure over the years [5].

The Amonica oilfield is characterized by two distinct hypsometric zones: an upper zone saturated with oil and a lower zone saturated with water [6].

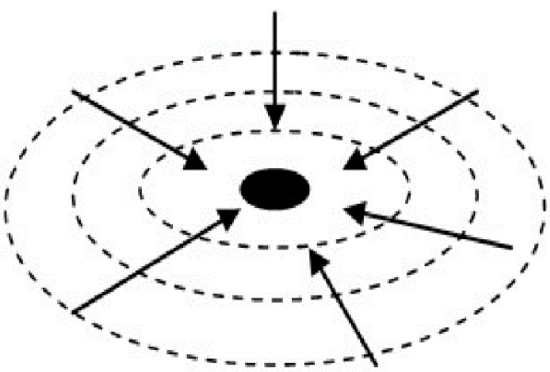

Figure 1. The cylindrical drainage region in a uniform reservoir

So, for better control of the progress of the layer pressure in time and to have values at points as uniformly distributed, it is necessary to carry out systematic measurements in observation wells. In this case, the exploitation and processing of this oilfield were based on the determination of a fluid extraction, which would then provide the opportunity to draw up conclusions for the processing of the oilfield as a whole [7].

Various numerical and analytical methods have been developed to solve the diffusivity equation. It is well established that the equation can be expressed using dimensionless parameters, with the general solution representing dimensionless pressure as a function of two variables: dimensionless radius and time.

Several numerical and analytical methods are presented to solve the diffusivity equation. It is known that it can be expressed using dimensionless parameters, the general solution of which will be for dimensionless pressure as a function of two parameters, dimensionless radius and time [8].

The finite constant flow solution for all-time fluid flow was first introduced by Hurst and Van Everdingen in 1949. The researchers applied the Laplace transform method to the radial diffusivity equation for both finite constant flow and constant finite pressure scenarios [9].

The purpose of this research is to apply the solution of the radial diffusivity equation for wells with variable flow rates and for reservoirs limited in extent [10].

The mathematical approach applied to the diffusion problem takes into account the reduction of several parameters to study the nonlinear terms in engineering problems in the hydrocarbon industry. Consequently, we have a simplified expression for the diffusivity equation and support engineering decisions based on the various rock and fluid properties in oil reservoirs [11].

The importance of the dynamic study of the hydrocarbon reservoir by means of the radial diffusivity equation is to evaluate the behavior and predict its future performance. Our contribution, in this case, is that the use and processing of this resource should be based on the definition of fluid extraction, in which the change in the energy of the layer does not change and the water content is minimal [12, 13].

Subsequently, measurements were made of the well levels, which resulted in improved values with natural flow. This indicates a maximum effectiveness in the extraction of oil from this source and the impact on the increase of debit in the time interval of exploitation [14].

The results of the study show the importance of production rate and reservoir radius in calculating the pressure drop from the well to the contour. Analytical solutions are applicable to well test analysis and fluid flow prediction [15].

2. METHODOLOGY

The radial diffusivity equation with constant finite flow serves as the fundamental equation in well analysis. Despite the extension of the short transient flow period, the solution is strongly influenced by the reservoir boundary condition.

This research presents the finite constant flow solution for a well of Amonica oilfield, located within the no-flow boundary across all the geometric arrangements studied by Matthews, Brons, and Hazebroek, and for each value of flow time [16].

This solution leads to a general wellbore equation that is applicable to analyze any pressure developed near the bottom hole area.

In order to determine the pressure measurement in accordance with the collector properties of the well drainage area, the water-oil contact, as well as the physico-chemical properties of the fluids contained in the layer, hydrodynamic studies of the wells are carried out [17].

Thus, in the phase of using fountains of wells in the source of Amonica, Albania, the hydrodynamic studies for the wells were carried out Am-7, 9, 16, 22, 30, 31, 32, 8, 15, 16, 18, 21 as well as the operating pressures were evaluated using direct measurement with a manometer [18, 19].

Based on this study, the optimal flows for the initial phase in the wells Am-7, 9, and 16 are from 45-52 m^3/day, and for these flow values, the pressures were from 10-30 atm. In this paper, such studies are presented for reservoirs with fluids of small and constant viscosity, mainly for unsaturated oil [20].

Beginning with the static pressure equilibrium and constant flow conditions, the solution to the radial diffusivity equation is often described in terms of the bottom-hole flow pressure, which evolves over time following a change in the well's flow rate from 0 to Q as shown in Figure 2.

The resulting pressure drop (Figure 3) can typically be divided into three distinct stages, based on the duration of flow, the reservoir geometry, and the reservoir's characteristics [21].

Figure 2. Finite constant flow solution, constant produced

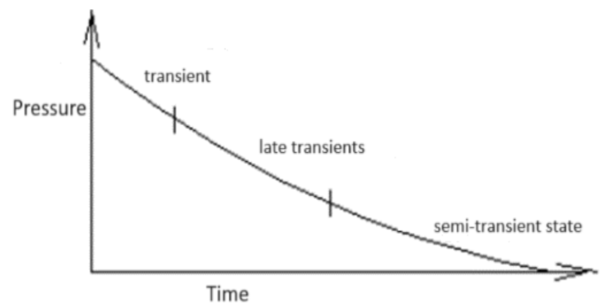

Figure 3. The resulting flow pressure drop at the bottom of the well

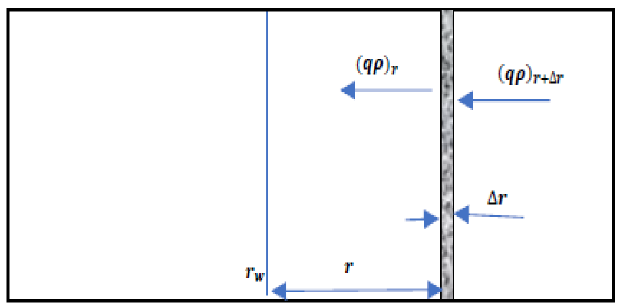

Figure 4. Radial fluid flow near a production well

Initially, the pressure can be described by the time-dependent solution of the diffusion equation, assuming that the pressure near the well's bottomhole is unaffected by the boundaries of the drainage area, and vice versa. This scenario corresponds to an infinite reservoir condition.

During transient flow, the reservoir's boundaries seem limitless. Subsequent to the transient stage, the so-called later transient phase is passed (Figure 4).

At this stage, as the drainage boundaries begin to influence the flow, it is essential to determine an approximate constant flow solution for the later transient phase in wells with non-flowing boundaries. The shape of the drainage area and the well's location relative to these boundaries are key factors that significantly impact the flow behavior [22].

2.1 Analytical solution with constant finite flow

In stabilized flow conditions with no-flow contours, the flow rate stays constant even as the end-well pressure changes over time, making the calculations much easier [23]. The solution is based on the following boundary and starting conditions as shown in Table 1.

Table 1. Starting and edge values for the diffusion equations

Initial Conditions		Boundary Conditions
$p = p_l$	$t = 0$	R
$p = p_l$	$t = \infty$	for each r
$\lim_{r\to 0} r\dfrac{\partial p}{\partial r} = \dfrac{Q\mu}{2\pi kh}$	$t > 0$	

Condition (a) states that prior to production, the pressure throughout the drainage volume is equal to the initial reservoir pressure, $p - l$.

Condition (b) specifies that the pressure at the outer boundary remains unaffected by pressure changes near the bottomhole.

Condition (c) addresses the source within the boundary conditions, indicating that the rock formation is uniform and isotropic, with full well penetration facilitating radial flow. The fluid is assumed to have steady viscosity and a slight, constant compressibility.

The derived solution is valuable for modeling the flow of unsaturated oil. By developing a simplified pressure analysis theory based on these assumptions, several limitations can be overcome. This allows for the inclusion of factors such as partial well completions and high-compressibility fluid flow. Under these conditions, the radial diffusivity equation for a compressible fluid takes the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \rho}{\partial r}\right) = \frac{1}{\chi}\frac{\partial \rho}{\partial t} \tag{1}$$

$$\frac{\partial^2 \rho}{\partial r^2} + \frac{1}{r}\frac{\partial \rho}{\partial r} = \frac{1}{\chi}\frac{\partial \rho}{\partial t} \tag{2}$$

where, $\chi = \frac{k}{\varphi\mu\beta}$ is the diffusivity coefficient of the layer.

The equations above are currently expressed in terms of liquid density, a parameter that is not typically measured under reservoir conditions. Therefore, we will re-express them in terms of pressure. To achieve this, we begin with the equation of state for a compressible fluid in porous media:

$$\rho = \rho_0\left[1 + \beta\left(P - P_0\right)\right] \tag{3}$$

$$\frac{\partial \rho}{\partial r} = \rho_0\beta\frac{\partial P}{\partial r} \text{ and } \frac{\partial^2 \rho}{\partial r^2} = \rho_0\beta\frac{\partial^2 P}{\partial r^2} \text{ and } \frac{\partial \rho}{\partial t} = \rho_0\beta\frac{\partial P}{\partial t}$$

After taking the first and second derivatives of Eq. (3) and substituting into Eq. (2) we obtain a nonlinear differential equation of the second order:

$$\begin{gathered}\rho_0\beta\left(\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r}\right) = \rho_0\beta\frac{1}{\chi}\frac{\partial P}{\partial t} \\ \frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{1}{\chi}\frac{\partial P}{\partial t}\end{gathered} \tag{4}$$

Let denote $x = \frac{r^2}{\chi t}$, so if we take derivatives in terms of x, applying chain rule of derivatives, we obtain:

$$\frac{\partial P}{\partial t} = \frac{\partial P}{\partial x}\frac{\partial x}{\partial t}, \quad \frac{\partial x}{\partial t} = \frac{-r^2}{\chi t^2} \text{ and } \frac{\partial P}{\partial t} = \left(\frac{-r^2}{\chi t^2}\right)\frac{\partial P}{\partial x}$$

So the right term of the nonlinear Eq. (4) takes form as:

$$\frac{1}{\chi}\frac{\partial P}{\partial t} = \frac{-r^2}{\chi^2 t^2}\frac{\partial P}{\partial x} \tag{5}$$

In the same way, we find the two terms of the left side of Eq. (4):

$$\frac{\partial P}{\partial r} = \frac{\partial P}{\partial x}\frac{\partial x}{\partial r}, \quad \frac{\partial x}{\partial r} = \frac{2r}{\chi t} \text{ and } \frac{\partial P}{\partial r} = \frac{2r}{\chi t}\frac{\partial P}{\partial x}$$

So

$$\frac{1}{r}\frac{\partial P}{\partial r} = \frac{2}{\chi t}\frac{\partial P}{\partial x} \tag{6}$$

$$\begin{gathered}\frac{\partial^2 P}{\partial r^2} = \frac{\partial}{\partial r}\left(\frac{\partial P}{\partial r}\right), \quad \frac{\partial^2 P}{\partial r^2} = \frac{\partial}{\partial r}\left(\frac{\partial P}{\partial x}\frac{2r}{\chi t}\right) \\ \frac{\partial^2 P}{\partial r^2} = \frac{\partial}{\partial r}\left(\frac{2r}{\chi t}\right)\frac{\partial P}{\partial x} + \frac{\partial}{\partial r}\left(\frac{\partial P}{\partial x}\right)\frac{2r}{\chi t}\end{gathered}$$

According to the chain rule of derivation we get:

$$\begin{gathered}\frac{\partial^2 P}{\partial r^2} = \frac{2}{\chi t}\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left(\frac{\partial P}{\partial x}\right)\frac{\partial x}{\partial r}\frac{2r}{\chi t} \\ \frac{\partial^2 P}{\partial r^2} = \frac{2}{\chi t}\frac{\partial P}{\partial x} + \frac{4r^2}{\chi^2 t^2}\frac{\partial^2 P}{\partial x^2}\end{gathered} \tag{7}$$

Through first derivative and the second derivative we determined each of the nonlinear terms of Eq. (2) and by substituting Eqs. (5)-(7) we can write Eq. (4) as:

$$\begin{gathered}\frac{4}{\chi t}\frac{\partial P}{\partial x} + \frac{4r^2}{\chi^2 t^2}\frac{\partial^2 P}{\partial x^2} = \frac{-r^2}{\chi^2 t^2}\frac{\partial P}{\partial x} \\ \frac{4}{\chi t}\frac{\partial P}{\partial x} + \frac{4r^2}{\chi^2 t^2}\frac{\partial^2 P}{\partial x^2} + \frac{r^2}{\chi^2 t^2}\frac{\partial P}{\partial x} = 0\end{gathered} \tag{8}$$

Multiply by χt on both sides of Eq. (8), and substitute $x = \frac{r^2}{\chi t}$ we transform Eq. (8) as linear equation of the second order:

$$(4+x)\frac{\partial P}{\partial x}+4x\frac{\partial^2 P}{\partial x^2}=0 \quad (9)$$

Denoting with $y = \frac{\partial P}{\partial x}$ and hence $y' = \frac{\partial y}{\partial x}$, and Eq. (9) takes the form of a linear equation with separable variables:

$$(4+x)y+4x\frac{\partial y}{\partial x}=0 \quad (10)$$

Now we proceed with solving Eq. (10), so:

$$4x\frac{\partial y}{\partial x}=(-4-x)y\,,\ \frac{\partial y}{\partial x}=\left(\frac{-4}{4x}-\frac{4}{4x}\right)y$$

$$\frac{dy}{y}=\left(\frac{-1}{x}-\frac{1}{4}\right)dx\,,\ \int\frac{dy}{y}=-\int\frac{1}{x}dx-\frac{1}{4}\int dx$$

$$\ln y=-\ln x+\ln e^{\frac{-x}{4}}+\ln C_1$$

$$y=C_1\frac{e^{\frac{-x}{4}}}{x} \quad (11)$$

Since we denoted $y = \frac{\partial P}{\partial x}$ then Eq. (11) takes form as:

$$\frac{\partial P}{\partial x}=C_1\frac{e^{\frac{-x}{4}}}{x} \quad (12)$$

Solving Eq. (12) we obtain the pressure distribution in the layer

$$P=C_1\int\frac{e^{\frac{-x}{4}}}{x}dx+C_2 \quad (13)$$

P - pressure distribution field in the layer
C_1 and C_2 are integration constants that we determine from the boundary conditions

(a) Initial conditions: $t = 0$, $P = P_l$, $x = \frac{r^2}{\chi t}$

$$P=P_l+C_1\int\frac{e^{\frac{-x}{4}}}{x}dx$$
$$P=C_1\int\frac{\chi t}{r^2}e^{\frac{-x}{4}}dx+C_2 \quad (14)$$

The constant C_1 is related to the commissioning of the well.
We know that the flow of oil expressed as a function of density is [1]:

$$Q=\frac{2\pi kh(\rho_c-\rho_w)}{\mu\beta\ln\frac{r_c}{r_w}} \quad (15)$$

where, $Q = F * v = 2\pi rh,\ v$ the filtration velocity according to Darcy's law is given by: $v = \frac{k}{\mu}\frac{dP}{dr}$ and from the displacement in Eq. (15) of the debit we have:

$$Q=\frac{2\pi kh}{\mu}\cdot\frac{2r}{\chi t}C_1\cdot\frac{e^{\frac{-x}{4}}}{\frac{r^2}{\chi t}} \quad (16)$$

By substituting Eq. (12) and the filtration rate according to Darcy's law in the above equation, we get:

$$Q=\frac{4\pi kh}{\mu}e^{\frac{-r^2}{4\chi t}}*C_1 \quad (17)$$

(b) Initial conditions: $t \to \infty$ and $p = p_l$

$$C_1=\frac{Q\mu}{4\pi kh}$$
$$P=P_l+\frac{Q\mu}{4\pi kh}\int\frac{e^{\frac{-x}{4}}}{x/4}d\left(\frac{x}{4}\right) \quad (18)$$

$$P=P_l+\frac{Q\mu}{4\pi kh}\int\frac{e^{-u}}{u}d(u) \quad (19)$$

$\int\frac{e^{-u}}{u}d(u) = -E_i(u)$ exponential integral and $E_i(u) = ln\frac{1}{u} - 0.5772$ where the number 0.5772 is Euler's constant

$$P(r,t)=P_l-\frac{Q\mu}{4\pi kh}\left[\ln\frac{4\chi t}{r^2}-0.5772\right] \quad (20)$$

Solving the constant flow rate diffusivity equation.

3. RESULTS AND DISCUSSION

The study presents and discusses the findings of the analytical solution using separable variables for the radial diffusivity equation with finite constant discharge. The critical reservoir radius, defined as the maximum radius of a reservoir with specific properties of the formation rock and reservoir fluid, was used to analyze the results. This radius is determined from both linear and nonlinear solutions [24, 25].

This is a convenient technique because it can incorporate the influences of oilfield reservoir variations and complex natural or hydraulic fracture patterns [26]. We initiated our study with the finite constant flow solution in a reservoir with infinite boundaries. The approximation is verified with various applications for which Laplace transform reference solutions are available [27].

Pressure studies are often conducted to determine the extent of communication between wells, such as in pulsation studies. In these cases, transient pressures induced in one well are recorded at a remote well [28].

The solution to the equation for transient fluid flow is not valid for the entire drainage area with respect to the well's position relative to the contour. For a short period, Eq. (20) is applicable only when the reservoir is considered infinite [29].

From a theoretical standpoint, for the finite constant flow solution, the flow rate (Q) in the transient flow equations and the semi-steady state flow equation remains the same. However, in practice, it is often difficult to maintain a constant flow rate from a well over an extended period.

In this case, the flow rate in Eq. (20) is equal to the final flow rate, and the flow time is expressed as the effective flow time. The use of effective flow time, defined as the ratio of cumulative production to the final flow rate, is commonly applied in well pressure analysis in petroleum engineering [30].

To observe how the fluid flow in the layer changes depending on the distance "r" and the time "t," we consider the deposit of petroleum as a circular area, used by a central well with radius "rw" and "r".

It is known that the pressure will drop at the same rate throughout the ring from "r" to "rw" [31]. Therefore, we can express the flow rate (Q) in the wellbore as the sum of the flow rate (Q(r)) entering the ring and the expansion of the oil within that ring [32].

To analyze this problem, we will examine the fluid flow progression over time at a specific distance from the well, as illustrated in (Figure 5). It appears that Q(r) increases slowly from zero and then tends towards "Q", when time increases the flow of the well is actually fed from more and more distant areas of the layer. So, the bottom hole is only a transitional area that practically no longer participates in production after a considerable time.

For the analytical solution, we utilized the finite element method to visualize the pressure profiles over time (Figure 6). Typically, pressure analysis depends on various mathematical solutions to the linear diffusion equation, incorporating a priori assumptions regarding heterogeneity, fluid properties, and boundary conditions. However, in practical situations, these idealized constraints may not be present, which can render analytical solutions nearly impossible to obtain [33].

The numerical solution at the wellbore and the analytical solution are virtually indistinguishable, indicating a good agreement during the middle and late times. However, there is a significant discrepancy in the early time region due to a faulty assumption made in developing the analytical approach [34].

The analytical solution assumes that the well can be represented as a point, which may not hold true mathematically (Figure 7). Nevertheless, the model studied assumes that the radius is vanishingly small. This assumption is physically reasonable, given that the size of the wellbore is considerably smaller than the radius of the volume being drained [35].

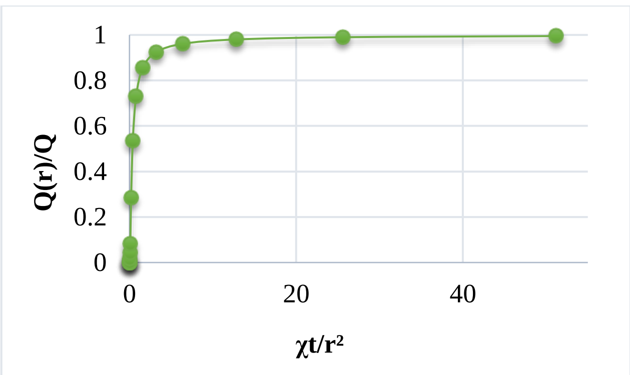

Figure 5. The change of debit over time (by author)

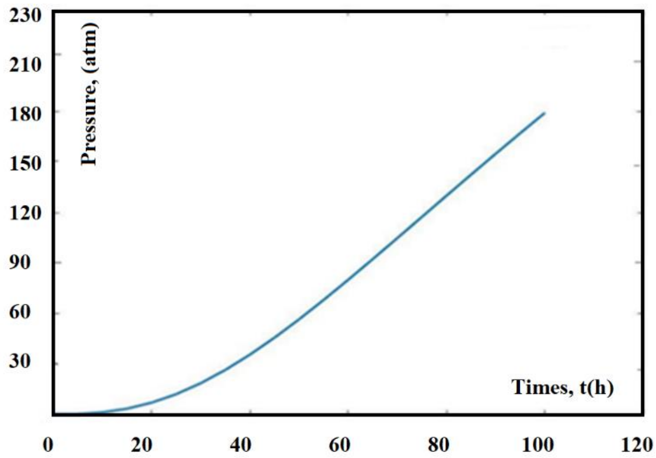

Figure 6. Pressure versus time for the analytical solution

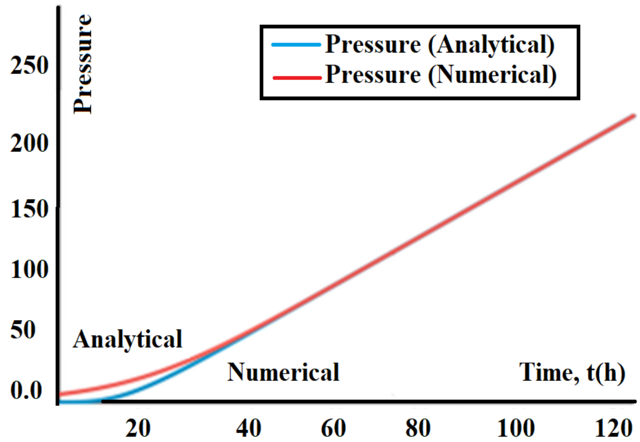

Figure 7. Comparison of the numerical model with the analytical solutions

4. CONCLUSIONS

This study highlights the effectiveness of using the radial diffusivity equation with constant finite flow to analyze oil wells. By utilizing pressure and flow rate data, reservoir properties and production decline rates can be accurately estimated.

The analytical solution facilitates the optimization of production strategies, enabling the maximization of oil recovery while minimizing operational costs. This approach is particularly advantageous for existing wells, offering valuable insights for designing interventions aimed at enhancing productivity. Additionally, enhanced oil recovery (EOR) techniques, such as water or gas injection, can be effectively evaluated using this method, thereby improving their efficiency in extracting additional oil.

The finite element technique provides a reliable way to visualize reservoir pressure profiles over time. The accuracy of the analytical solution is validated through comparison with numerical solutions, demonstrating its robustness. The applicability of the proposed procedure is further illustrated through its successful implementation on test wells in the Amonica oilfield.

However, the analytical solutions have limitations when applied to fluid flow problems in non-uniform reservoirs, as they are sensitive to variations in parameters such as porosity, permeability, and fluid viscosity. Changes in porosity and permeability within the reservoir can result in discontinuities in the properties of the porous media. Studies have observed that porosity tends to decrease as the distance from the wellbore center increases, while permeability is often considered uniform.

The problem is further compounded when accounting for non-ideal effects in the wellbore. Moreover, heterogeneous reservoirs are inherently complex and cannot be easily analyzed using conventional methods developed for homogeneous reservoirs.

In conclusion, while the proposed analytical solutions and methodologies are effective for homogeneous and isotropic reservoirs, further research and development are required to address the challenges posed by heterogeneous and anisotropic reservoirs, ensuring broader applicability in diverse field conditions.

5. FUTURE RESEARCH DIRECTIONS

Future research should focus on investigating the impact of varying fluid properties and multi-phase flow scenarios on the accuracy of the radial diffusivity equation, providing a comprehensive understanding of reservoir dynamics. Integrating advanced data analytics and machine learning techniques with the equation can enhance predictive capabilities and real-time decision-making. Exploring the integration of the radial diffusivity equation with other simulation methods will create holistic approaches to reservoir management.

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NOMENCLATURE

k	permeability, mD
P	pressure, psi
P_l	initial reservoir pressure, psi
P_{wf}	bottomhole flowing pressure, psi
Q	cumulative production, m^3/day
r	reservoir radius, ft
r_c	contour radius, ft
r_w	wellbore radius, ft
t	time, hr
P_0	atmospheric pressure, psi
x	cartesian spatial coordinate vector
h	thickness, ft
C_1	integration constant
C_2	integration constant

Greek symbols

β	elastic compression coefficient, psi^{-1}
χ	diffusivity coefficient of the layer
ρ	fluid density, $lb/ft^3,\ 1\,lb = 16.0815\,kg$
ρ_c	contour fluid density, lb/ft^3
	wellbore fluid density, lb/ft^3
ρ_0	fluid density in atmospheric pressure, lb/ft^3
ϕ	porosity, fraction
μ	fluid viscosity, cp