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A Semi-Analytical Method for Solving Non-Linear Multi-Dimensional Acoustic Wave Equations

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ABSTRACT

In the current paper, a reliable iterative method has been presented and implemented to solve the non-linear acoustic wave equations in one, two, and three dimensions. The Temimi-Ansari method (TAM) is implemented to find exact solutions to linear problems. Moreover, for nonlinear problems, approximate solutions are generated, which converge to the exact solution. The static-point theory was used in the analysis of the convergence of the proposed method. This method has demonstrated its accuracy and efficiency in solving non-linear equations. The software applied to the computing in this research was $Mathematica^{®}10$. The findings indicate that the proposed iterative method is effective, dependable, time-efficient, and well-suited for addressing the given problems. Moreover, it shows potential for application to other nonlinear problems.

1. INTRODUCTION

The relationship between engineering problems and other sciences requires the integration of knowledge from multiple fields [1]. Engineers and scientists are able to analyze and understand engineering and scientific phenomena using equations and develop effective solutions to the problems facing the environment and society [2]. These equations help us understand the relationships between different variables, leading to new and effective solutions for engineering problems that promote innovation and technological advancement [3].

Many problems in engineering, science, and the environment can be expressed by nonlinear partial differential equations (PDEs) [4]. When the analytical solution is not feasible, solving a nonlinear PDE presents a challenge for engineers and scientists [5]. Consequently, numerous mathematicians and engineers have explored various methods and algorithms to tackle these problems, including, Finite Element Method [6] Complex variable boundary element method (CVBEM) [7], (TAM) [8], homotopy perturbation method (HPM) [9], and some other analytical and numerical methods [10, 11].

The acoustic wave equation describes the transmission of energy through a medium, including air, water, displacement, and acoustic intensity [12]. The speed of acoustic waves relies on the medium's qualities, such as density, elasticity and differing velocities in solids. Examples of acoustic waves include audible sound from speakers, seismic waves creating ground movements, and ultrasound used for medical imaging [13, 14].

Acoustic wave equations have numerous techniques to be used in solving them, for example. An Effective Discontinuous Galerkin Method [15], The Adomian decomposition approach [16], Variation iteration technique [17], Finite difference method [18], locally one method (LOD) [19], Normal mode analysis [20] Method of fundamental solutions [21], Explicit hybridizable discontinuous Galerkin method (HDGM) [22], the Concept of Perturbed Derivative Order [23], adaptive WKB approximation method [24] partial-low-rank method [25].

The TAM proposed by Temimi and Ansari in 2011 [26] to solve non-linear equations. This technique will be employed to address a range of differential equations, it includes second-order nonlinear ordinary differential equations in the field of physics [27], Falkner-Skan equations [28], non-linear multi-dimensional wave equations [29], differential algebraic equations [30], and nonlinear thin film flow issues [31].

This study primarily aims to employ the iterative TAM method for solving the nonlinear acoustic wave equation in one-dimensional (1D), two-dimensional (2D), and three-dimensional (3D) cases. The outcomes are compared with the exact resolution of this issue to show the efficacy and precision of the suggested approach. The following shows how the paper is divided. Section 2 consists of the standard formulation of the problem. Section 3 provides a comprehensive explanation of the core concepts underlying the suggested iterative technique. In Section 4 of the suggested technique, we will analyze the convergent. Section 5 presents a detailed analysis and discussion of the numerical simulation. Section 6 provides the conclusion.

2. THE FORMULATION OF THE ACOUSTIC WAVE EQUATIONS

The acoustic wave equation is one type of partial differential equation includes variable *t*, which may have one or more than one spatial variable (*x, y, ...*) and a scalar function *u*=*u*(*x, y, ...*). Acoustic wave equations have received great attention from researchers because of their great importance, including physical and engineering applications [32]. Our research will examine the linear and nonlinear acoustic wave equations, which can be described by one-dimensional, two-dimensional, and three-dimensional formulas:

$$\begin{gathered} u_{tt} = u_{xx} + F(x,t,u,u_x,u_t), \\ a < x < b, \qquad t > 0 \end{gathered} \tag{1}$$

with the conditions

$$u(x,0) = f_1(x), \qquad u_t(x,0) = f_2(x)$$

$$\begin{gathered} u_{tt} = u_{xx} + u_{yy} + F\big(x,y,t,u,u_x,u_y,u_t\big), \\ a < x,y < b, t > 0 \end{gathered} \tag{2}$$

with the conditions

$$u(x,y,0) = f_1(x,y), \qquad u_t(x,y,0) = f_2(x,y)$$

$$\begin{gathered} u_{tt} = u_{xx} + u_{yy} + u_{zz} \\ + F\big(x,y,z,t,u,u_x,u_y,u_z,u_t\big), \\ a < x,y,z < b, \qquad t > 0 \end{gathered} \tag{3}$$

with the conditions

$$u(x,y,z,0) = f_1(x,y,z), \qquad u_t(x,y,z,0) = f_2(x,y,z)$$

3. THE UNDERLYING CONCEPT OF THE TAM

In this section, the basic concepts of TAM will be presented.

Here is the nonlinear partial differential equation that will be presented [33]:

$$L\big(u(x,t)\big) = N\big(u(x,t)\big) + g(x,t) = 0 \tag{4}$$

with the boundary conditions

$$B(u, \tfrac{du}{dt}) = 0.$$

The independent variable is denoted as *x*, while the variable of time is denoted as *t*. The function *u*(*x, t*) represents the unknown function, whereas *g*(*x, t*) is a known given function. *L* denotes a linear operator, *N* represents a nonlinear operator, and *B*(.) stands for the boundary operator. Let us start by considering that this is a starting approximation for solving the Eq. (4).

The initial phase of the resolution approach is to solve the following initially value problem $u_0(x,t)$:

$$L(u_0(x,t)) = g(x,t), \text{with } B(u_0, \tfrac{du_0}{dt}) = 0 \tag{5}$$

The next iterative function $u_1(x,t)$ are obtained by solving the following equation:

$$L\big(u_1(x,t)\big) = N\big(u_0(x,t)\big) + g(x,t), B(u_1, \tfrac{du_1}{dt}) = 0 \tag{6}$$

In the same way, we find iterative function $u_2(x,t)$ by solving the subsequent issue

$$L(u_2(x,t)) = N(u_1(x,t)) + g(x,t), \qquad B(u_2, \tfrac{du_2}{dt}) = 0$$

In general, we can compute the subsequent iterations. $u_{n+1}(x,t)$ by solving the following problem

$$\begin{gathered} L(u_{n+1}(x,t)) = N\,(u_n(x,t)) + g(x,t), \\ B\left(u_{n+1}, \frac{du_{n+1}}{dt}\right) = 0 \end{gathered} \tag{7}$$

It should be noted that each u_n serves as a solution to Eq. (4). Additionally, increasing the number of iterations enhances the precision of the approximation results.

The solution for Eq. (4) can be obtained by $u = \lim_{n\to\infty} u_n$.

4. THE CONVERGENCE OF THE SUGGESTED ITERATIVE METHOD

To demonstrate the convergence analysis of the suggested method, it is necessary to examine instances of both linear and nonlinear acoustic wave equations. Here, we establish the next iteration utilizing the subsequent equation

$$\begin{aligned} v_0 &= u_0(x,t) \\ v_1 &= F[v_0] \\ v_2 &= F[v_0 + v_1] \\ v_{n+1} &= F[v_0 + v_1 + \cdots + v_n] \end{aligned} \tag{8}$$

where, v_n is the new iterations and F is the operator can be given as follows

$$F[v_k] = S_k - \sum_{i=0}^{k-1} v_i(x,t)\,, k = 1,2, ... \tag{9}$$

The term S_k means the solutions to the equation shown below

$$\begin{gathered} L\big(v_k(x,t)\big) + g(x,t) + N\left(\sum_{i=0}^{k-1} v_i(x,t)\right) = 0, \\ k = 1, 2, ..., \end{gathered} \tag{10}$$

Assuming the same specified circumstances of the problem, in this fashion, we possess $u(x,t) = \lim_{n\to\infty} u_n(x,t) = \sum_{n=0}^{\infty} v_n$. So, the representation of solution of the problem can be accessed it using Eq. (8) and Eq. (9) in the resulted series:

$$u(x,t) = \sum_{i=0}^{\infty} v_i(x,t) \tag{11}$$

The sufficient and required conditions for the convergence of the solution are outlined according to the proposed method. The accompanying theorems reveal the main findings for the used manner.

Theorem 1: Let F represent an operator, as defined in Eq. (9) that maps a Hilbert space H to itself. The solution can be expressed using a series formula $u_n(x,t) = \sum_{i=0}^{n} v_i(x,t)$ converges if $\exists 0 < r < 1$ such that $\|F[v_0 + v_1 + \cdots +$

$v_{i+1}]\| \leq r\|F[v_0 + v_1 + \cdots + v_i]\|$ (that is $\|v_{i+1}\| \leq r\|v_i\|$) $\forall i = 0, 1, 2, \ldots$.

This theory is not simply a case of the fixed-point theory in Banach spaces; instead, this specific condition is beneficial for studying convergence. Proof: See reference [34].

Theorem 2: If the series solution $u(x,t) = \sum_{i=0}^{\infty} v_i(x)$ converges, it will provide an accurate representation of the solution to the current nonlinear problem. Proof: See reference [34].

Theorem 3: Let the series solution $\sum_{i=0}^{\infty} v_i(x)$ which may be represented by Eq. (11), converge to the solution $u(x,t)$. If the current problem can be approximated by the truncated series $\sum_{i=0}^{n} v_i(x,t)$, the maximum error $E_n(x,t)$ is computed using the following expression:

$$E_n(x,t) \leq \frac{1}{1-r} r^{n+1} \|v_0\| \tag{12}$$

Proof: See reference [34].

Theorems 1 and 2 assert that the solutions found using either the Eq. (7) (for the TAM) or Eq. (8) will converge to the exact solution, given the constraint that there exists a value of r between 0 and 1 such that $\|F[v_0 + v_1 + \cdots + v_{i+1}]\| \leq r\|F[v_0 + v_1 + \cdots + v_i]\|$ (that is $\|v_{i+1}\| \leq r\|v_i\|$) $\forall i = 0, 1, 2, 3, \ldots$. In another meaning, the parameters can be defined for each i.

$$\beta_i = \begin{cases} \frac{\|v_{i+1}\|}{\|v_i\|}, & \|v_i\| \neq 0 \\ 0, & \|v_i\| = 0 \end{cases} \tag{13}$$

The series solution $\sum_{i=0}^{\infty} v_i(x,t)$ converges to the precise solution $u(x,t)$, if and only if $0 \leq \beta_i < 1$, for each $i = 0,1,2,3, \ldots$. The estimated error of maximum truncation, as stated in Theorem 3, may be expressed as $\|u(x,t) - \sum_{i=0}^{n} v_i\| \leq \frac{1}{1-\beta} \beta^{n+1} \|v_0\|$, where $\beta = \max\{\beta_i, i = 0,1, \ldots, n\}$.

5. NUMERICAL EXAMPLES

Now, we examine the proposed solution to certain linear and nonlinear acoustic wave equations in one, two, and three dimensions.

Example 1. Let's examine the given 1D linear acoustic wave equation by [35]

$$u_{tt}(x,t) = \frac{1}{V^2} u_{xx}(x,t), \quad \text{with } u(x,0) = 3 \sin \pi\theta, \quad u_t(x,0) = 0 \tag{14}$$

will be solved by using the suggested method, where V=1 [30].

Let's begin by examining the given problem as it is stated

$$L(u(x,t)) = u_{tt}(x,t), \quad N(u(x,t)) = v^2 u_{xx}(x,t), \quad g(x,t) = 0 \tag{15}$$

The main problem might be expressed as

$$L(u_0(x,t)) = 0, \quad \text{with } u_0(x,0) = 3\sin \pi\theta, \quad u_{0t}(x,0) = 0 \tag{16}$$

The subsequent issues can be derived from the overarching general relationship.

$$L(u_{n+1}(x,t)) = g(x,t) + N(u_n(x,t)) = 0, \quad \text{with } u_{n+1}(x,0) = 3 \sin \pi\theta, \; u_{(n+1)t}(x,0) = 0 \tag{17}$$

$$u_{0tt}(x,t) = 0, \quad \text{with } u_0(x,0) = 3 \sin \pi\theta, \; u_{0t}(x,0) = 0 \tag{18}$$

To solve the problem set out in Eq. (18) we obtain $u_0 = 3\sin \pi\theta$.

The first iteration can be found by assessing the following problem

$$u_{1tt}(x,t) = v^2 u_{0xx}(x,t) \quad \text{with } u_1(x,0) = 3\sin \pi\theta, \; u_{1t}(x,0) = 0. \tag{19}$$

Then, the solution of Eq. (19) is

$$u_1 = 3\sin \pi\theta - \frac{3}{2} t^2 v^2 3\sin \pi\theta$$

The second iteration is

$$u_{2tt}(x,t) = v^2 u_{1xx}(x,t) \quad \text{with } u_2(x,0) = 3\sin \pi\theta, \; u_{2t}(x,0) = 0. \tag{20}$$

Then the solution of Eq. (20) is

$$u_2 = 3\sin \pi\theta - \frac{3}{2} t^2 v^2 \sin \pi\theta + \frac{1}{8} t^4 v^4 \sin \pi\theta$$

$$u_3 = 3\sin \pi\theta - \frac{3}{2} t^2 v^2 \sin \pi\theta + \frac{1}{8} t^4 v^4 \sin \pi\theta - \frac{1}{240} t^6 v^6 \sin \pi\theta$$

$$u_4 = 3\sin \pi\theta - \frac{3}{2} t^2 v^2 \sin \pi\theta + \frac{1}{8} t^4 v^4 \sin \pi\theta - \frac{1}{240} t^6 v^6 \sin \pi\theta + \frac{t^8 v^8 \sin \pi\theta}{13440}$$

$$u_5 = 3\sin \pi\theta - \frac{3}{2} t^2 v^2 \sin \pi\theta + \frac{1}{8} t^4 v^4 \sin \pi\theta - \frac{1}{240} t^6 v^6 \sin \pi\theta + \frac{t^8 v^8 \sin \pi\theta}{13440} - \frac{t^{10} v^{10} \sin \pi\theta}{1209600}$$

Finally, by taking the limit

$$u(x,t) = \lim_{n \to \infty} u_n$$

$$u(x,t) = 3\sin \pi\theta - \frac{3}{2} t^2 v^2 \sin \pi\theta + \frac{1}{8} t^4 v^4 \sin \pi\theta - \frac{1}{240} t^6 v^6 \sin \pi\theta + \frac{t^8 v^8 \sin \pi\theta}{13440} - \frac{t^{10} v^{10} \sin \pi\theta}{1209600} + \cdots$$

Thus, we continue with more iterations until we obtain the exact solution $= 3\sin \pi\theta \text{Cos}\pi vt$.

Example 2. Examine the one-dimensional, nonlinear acoustic wave equation

$$u_{tt} - u u_{xx} = 2 - 2(x^2 + t^2) \quad \text{with } u(x,0) = x^2, \; u_t(x,0) = 0. \tag{21}$$

Eq. (21) will be solved by the TAM.

The following form are provided as,

$$L\big(u(x,t)\big) = u_{tt}(x,t),$$
$$N\big(u(x,t)\big) = uu_{tt}, \tag{22}$$
$$g(x,t) = 2 - 2(x^2 + t^2)$$

The initial problem is

$$L(u_0) = 2 - 2(x^2 + t^2), \text{ with } u_0(x,0) = x^2,\ u_{0t}(x,0) = 0 \tag{23}$$

The subsequent problems will rely on the generalized iterative formula

$$L\big(u_{n+1}(x,t)\big) = g(x,t) + N\left(u_n(x,t)\right) = 0 \text{ with } u_{n+1}(x,0) = x^2,\ u_{(n+1)t}(x,0) = 0 \tag{24}$$

will be solving the following initial problem

$$u_{0tt}(x,t) = 2 - 2\,(x^2 + t^2), \text{ with } u_0(x,0) = x^2,\ u_{0t}(x,0) = 0 \tag{25}$$

we get

$$u_0 = t^2 - \tfrac{t^4}{6} + x^2 - t^2x^2.$$

The initial iteration $u_1(x,t)$ can be found by solving

$$u_{1tt} = uu_{0xx} + 2 - 2(x^2 + t^2) \text{ with: } u_1(x,0) = x^2,\ u_{1t}(x,0) = 0 \tag{26}$$

The solution of Eq. (26) with initial conditions, will be

$$u_1 = t^2 - \frac{7t^6}{90} + \frac{t^8}{168} + x^2 - \frac{t^4x^2}{3} + \frac{t^6x^2}{15}.$$

We apply the same process for the iteration $u_2(x,t)$ as follows

$$u_{2tt} = uu_{1xx} + 2 - 2(x^2 + t^2), \text{ with: } u_2(x,0) = x^2, u_{2t}(x,0) = 0. \tag{27}$$

By solving Eq. (27), we get

$$u_2 = t^2 - \frac{37t^8}{2520} + \frac{61t^{10}}{37800} + \frac{7t^{12}}{17820} - \frac{271t^{14}}{3439800} + \frac{t^{16}}{302400} + x^2 - \frac{2t^6x^2}{45} + \frac{t^8x^2}{210} + \frac{t^{10}x^2}{405} - \frac{t^{12}x^2}{1485} + \frac{t^{14}x^2}{20475},$$

$$u_5 = t^2 - \frac{149t^{10}}{113400} + \frac{241t^{12}}{2494800} + \frac{17t^{14}}{540540} - \frac{11309t^{16}}{13621608000} - \frac{110821t^{18}}{189464184000} - \frac{4019t^{20}}{16590420000} + \cdots + x^2 - \frac{2t^{10}x^2}{14175} + \frac{t^{12}x^2}{155925} + \frac{2t^{14}x^2}{1216215} + \frac{t^{16}x^2}{8687250} + \frac{2t^{18}x^2}{65786175} - \frac{5351t^{20}x^2}{259602367500}\cdots$$

The exact solution is converged by this series when

$$u(x,t) = \lim_{n\to\infty} u_n(x,t) = x^2 + t^2.$$

The convergence of the suggested technique is proved using the provided procedure in Eqs. (9)-(12). The iterative scheme for Eq. (21) can be written

$$v_0(x,t) = u_0(x,t) = t^2 - \frac{t^4}{6} + x^2 - t^2x^2$$

Using the technique (TAM), the operator $F[v_k]$ as represented by Eq. (10) with the term S_k is defined. Thus, the aim of this study is to exercise that proposition by finding a solution to the next challenge

$$v_{ktt}(x,t) = \left(\sum\nolimits_{i=0}^{k-1} v_{ixx}(x,t)\right)\left(\sum\nolimits_{i=0}^{k-1} v_i(x,t)\right) + 2 - 2(x^2 + t^2),$$

with

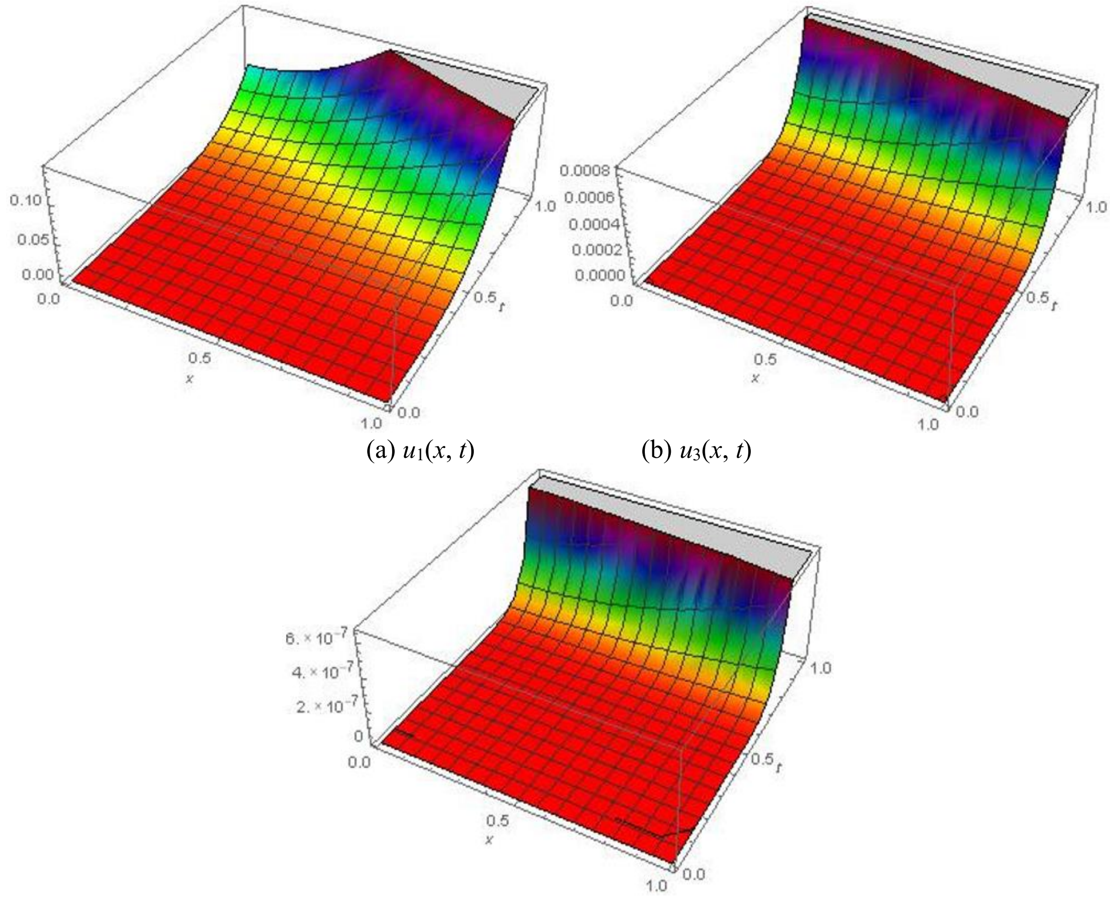
$$v_k(x,0) = x^2, \quad v_{kt}(x,0) = 0, \quad k \geq 1$$
$$v_1 = \frac{t^4}{6} - \frac{7t^6}{90} + \frac{t^8}{168} + t^2x^2 - \frac{t^4x^2}{3} + \frac{t^6x^2}{15},$$
$$v_2 = \frac{7t^6}{90} - \frac{13t^8}{630} + \frac{61t^{10}}{37800} + \frac{7t^{12}}{17820} - \frac{271t^{14}}{3439800} + \frac{t^{16}}{302400} + \frac{t^4x^2}{3} - \frac{t^6x^2}{9} + \frac{t^8x^2}{210} + \frac{t^{10}x^2}{405} - \frac{t^{12}x^2}{1485} + \frac{t^{14}x^2}{20475},$$
$$v_3 = \frac{37t^8}{2520} - \frac{83t^{10}}{28350} - \frac{739t^{12}}{2494800} + \frac{4171t^{14}}{37837800} - \frac{28177t^{16}}{6810804000} - \frac{110821t^{18}}{189464184000}\ldots\ldots + \frac{2t^6x^2}{45} - \frac{t^8x^2}{126} - \frac{32t^{10}x^2}{14175} + \frac{2t^{12}x^2}{2673} - \frac{17t^{14}x^2}{405405} - \frac{t^{16}x^2}{368550} - \frac{703t^{18}x^2}{546531300} + \cdots$$

The above type of duplicates is used for the computation of the values of β_i for the Eq. (13), in which we can get

$$\beta_0 = \frac{\|v_1\|}{\|v_0\|} = 0.0294663 < 1$$
$$\beta_1 = \frac{\|v_2\|}{\|v_1\|} = 0.0106408 < 1$$
$$\beta_2 = \frac{\|v_3\|}{\|v_2\|} = 0.00427883 < 1$$
$$\beta_3 = \frac{\|v_4\|}{\|v_3\|} = 0.00228477 < 1$$
$$\beta_4 = \frac{\|v_5\|}{\|v_4\|} = 0.00142926 < 1,$$

The β_i values for $i \geq 0$ and $\forall(x,t): x \in \mathbb{R},\ 0 < x, t \leq 1$ are all smaller than 1, indicating that the suggested iterative approach meets the condition for convergence.

We can conduct an additional investigation to evaluate the accuracy of the approximate solution (u_n), by employing an absolute error relationship $Absr_n = N\big[Abs[w - u_n]\big]$,where $w = x^2 + t^2$ is the exact solution. The *3D* graph of the $Absr_n$ for the approximate solution found by the recommended iterative procedure is shown in Figure 1. Increasing the number of iterations also cuts down on mistakes and improves the accuracy of the answers. Furthermore, Table 1 presents the absolute error values ($Absr_n$) derived from TAM for the iterations with n={1, 3, 5}. The reduction of error is readily apparent with an increase in the number of repetitions, particularly when t=1.



(a) $u_1(x, t)$ (b) $u_3(x, t)$

(c) $u_5(x, t)$

Figure 1. Three-dimensional plotted graph for the $Absr_n$ at n=(1, 3, 5) of example 2, where t=1

Table 1. The $Absr_n$ for example 2 computed using the TAM method over three iterations with t=1

x	Abs. Errs. for u_1	Abs. Errs. for u_3	Abs. Errs. for u_5
0	0.0718254	0.00118746	0.00000219871
0.1	0.0744921	0.00121631	0.00000223977
0.2	0.0824921	0.00130285	0.00000236293
0.3	0.0958254	0.0014471	0.0000025682
0.4	0.0114492	0.00164903	0.00000285559
0.5	0.138492	0.001908667	0.00000322508
0.6	0.167825	0.002226	0.00000367668
0.7	0.202492	0.00260103	0.00000421039
0.8	0.242492	0.00303375	0.00000482621
0.9	0.287825	0.00352417	0.00000552414
1	0.338492	0.00407229	0.00000630417

Example 3. Consider the 2D linear acoustic wave equation shown below [36]

$$u_{tt} = \frac{1}{V^2}(u_{xx} + u_{yy}), \quad \text{with: } u(x, y, 0) = \text{Cos}(-x - y), \quad u_t(x, y, 0) = -\sqrt{2}\,\text{Sin}(-x - y). \tag{28}$$

The three proposed iterative method will be used to solve Eq. (28), $V = 1$ [31]

$$u_{0tt} = 0, \text{with } u_0(x, y, 0) = \text{Cos}(-x - y),$$

$$u_{0t}(x, y, 0) = -\sqrt{2}\text{Sin}(-x - y)$$

Then

$$u_0 = \text{Cos}(x + y) + \sqrt{2}\,t\,\text{Sin}(x + y).$$

The initial iteration $u_1(x, y, t)$ can be found by solving

$$u_{1tt} = u_{0xx}, \text{ with } u_1(x, y, 0)\text{Cos}(-x - y),\ u_{1t}(x, y, 0) = -\sqrt{2}\text{Sin}(-x - y)$$

We get

$$u_1 = \mathrm{Cos}(x+y) - t^2\mathrm{Cos}(x+y) + \sqrt{2}t\mathrm{Sin}(x+y) - \frac{1}{3}\sqrt{2}\, t^3\mathrm{Sin}(x+y),$$

The iteration $u_2(x,y,t)$ can be obtained by solving this problem

$$u_{2tt} = u_{1xx} + u_{1yy}, \quad \text{with } u_2(x,y,0)\mathrm{Cos}(-x-y), u_{2t}(x,y,0) = -\sqrt{2}\mathrm{Sin}(-x-y),$$

then

$$u_2 = \mathrm{Cos}(x+y) - t^2\mathrm{Cos}(x+y) + \frac{1}{6}t^4\mathrm{Cos}(x+y) + \sqrt{2}t\mathrm{Sin}(x+y) - \frac{1}{3}\sqrt{2}t^3\mathrm{Sin}(x+y) + \frac{t^5\mathrm{Sin}(x+y)}{15\sqrt{2}},$$

$$u_5 = \mathrm{Cos}(x+y) - t^2\mathrm{Cos}(x+y) + \frac{1}{6}t^4\mathrm{Cos}(x+y) - \frac{1}{90}t^6\mathrm{Cos}(x+y) + \frac{t^8\mathrm{Cos}(x+y)}{2520} - \frac{t^{10}\mathrm{Cos}(x+y)}{113400} + \sqrt{2}t\mathrm{Sin}(x+y) - \frac{1}{3}\sqrt{2}t^3\mathrm{Sin}(x+y) \ldots$$

When the solution converges to the exact solution, then

$$u(x,y,t) = \mathrm{Cos}(\sqrt{2}t - x - y) = \mathrm{Cos}(x+y) - t^2\mathrm{Cos}(x+y) + \frac{1}{6}t^4\mathrm{Cos}(x+y) - \frac{1}{90}t^6\mathrm{Cos}(x+y) + \frac{\mathrm{tCos}(x+y)}{2520} - \frac{t^{10}\mathrm{Cos}(x+y)}{113400} + \sqrt{2}t\mathrm{Sin}(x+y) - \frac{1}{3}\sqrt{2}t^3\mathrm{Sin}(x+y) + \frac{t^5\mathrm{Sin}(x+y)}{15\sqrt{2}} - \frac{t^7\mathrm{Sin}(x+y)}{315\sqrt{2}} \ldots$$

The exact solution can be acquired by $u(x,y,t) = \lim_{n\to\infty} u_n = \mathrm{Cos}(\sqrt{2}t - x - y)$.

Example 4. Let us examine the one-dimensional nonlinear wave equation.

$$u_{tt} + u_{yy} = uu_{xx} + 2 - \left(\frac{x^2+y^2+t^2}{2}\right) \tag{29}$$

with the given initial conditions

$$u(x,y,0) = \frac{x^2+y^2}{2},\ u_t(x,y,0) = 0.$$

To resolve Eq. (29) utilizing TAM and the given initial conditions, we can express it as follows

$$L(u) = u_{tt}(x,y,t), \quad N(u) = u(x,y,t)u_{xx}(x,y,t) - u_{yy}(x,y,t), \quad g(x,y,t) = 2 - \left(\frac{x^2+y^2+t^2}{2}\right) \tag{30}$$

The initial problem is

$$L(u_0) = 2 - 2(x^2+t^2), \quad \text{with } u_0(x,y,0) = \frac{x^2+y^2}{2},\ u_{0_t}(x,y,0) = 0 \tag{31}$$

The following issues will be resolved using the generalized iterative formula

$$L(u_{n+1}) + N(u_n) = g(x,y,t), \quad u_{n+1}(x,y,0) = \frac{x^2+y^2}{2},\ u_{n+1_t}(x,y,0) = 0$$

By solving the Eq. (31), we get

$$u_0 = t^2 - \frac{t^4}{24} + \frac{x^2}{2} - \frac{t^2x^2}{4} + \frac{y^2}{2} - \frac{t^2y^2}{4},$$

The first iteration $u_1(x,y,t)$ can be obtained by solving

$$u_{1tt} = u_0(x,y,t)\, u_{0_{xx}}(x,y,t) - u_{0_{yy}}(x,y,t) + 2 - \left(\frac{x^2+y^2+t^2}{2}\right) \quad \text{with } u_1(x,y,0) = \frac{x^2+y^2}{2},\ u_{1_t}(x,y,0) = 0$$

The solution will be

$$u_1 = \frac{t^2}{2} + \frac{t^4}{12} - \frac{13t^6}{720} + \frac{t^8}{2688} + \frac{x^2}{2} - \frac{t^4x^2}{24} + \frac{t^6x^2}{240} + \frac{y^2}{2} - \frac{t^4y^2}{24} + \frac{t^6y^2}{240},$$

Applying the same process for u_2, we have

$$u_{2tt} = u_1(x,y,t)\, u_{1_{xx}}(x,y,t) - u_{1_{yy}}(x,y,t) + 2 - \left(\frac{x^2+y^2+t^2}{2}\right) \quad \text{with } u_2(x,y,0) = \frac{x^2+y^2}{2},\ u_{2_t}(x,y,0) = 0$$

By solving this equation, we get

$$u_2 = \frac{t^2}{2} + \frac{t^6}{180} - \frac{7t^8}{5760} - \frac{97t^{10}}{3628800} + \frac{19t^{12}}{1140480} - \frac{439t^{14}}{440294400} + \frac{t^{16}}{77414400} + \frac{x^2}{2} - \frac{t^6x^2}{360} + \frac{t^8x^2}{6720} + \frac{t^{10}x^2}{25920} - \frac{t^{12}x^2}{190080} + \frac{t^{14}x^2}{5241600} + \frac{y^2}{2} - \frac{t^6y^2}{360} + \frac{t^8y^2}{6720} + \frac{t^{10}y^2}{25920} - \frac{t^{12}y^2}{190080} + \frac{t^{14}y^2}{5241600}.$$

The approximations will continue till $n = 5$, for brevity it's not stated.

The convergence analysis of the suggested iterative procedure allows us to determine the values β_i for the problem in Eq. (29). Thus, we determine the terms of the series $\sum_{i=0}^{\infty} v_i(x,t)$ in the above Eq. (11), and proceed to deduce them in the following manner:

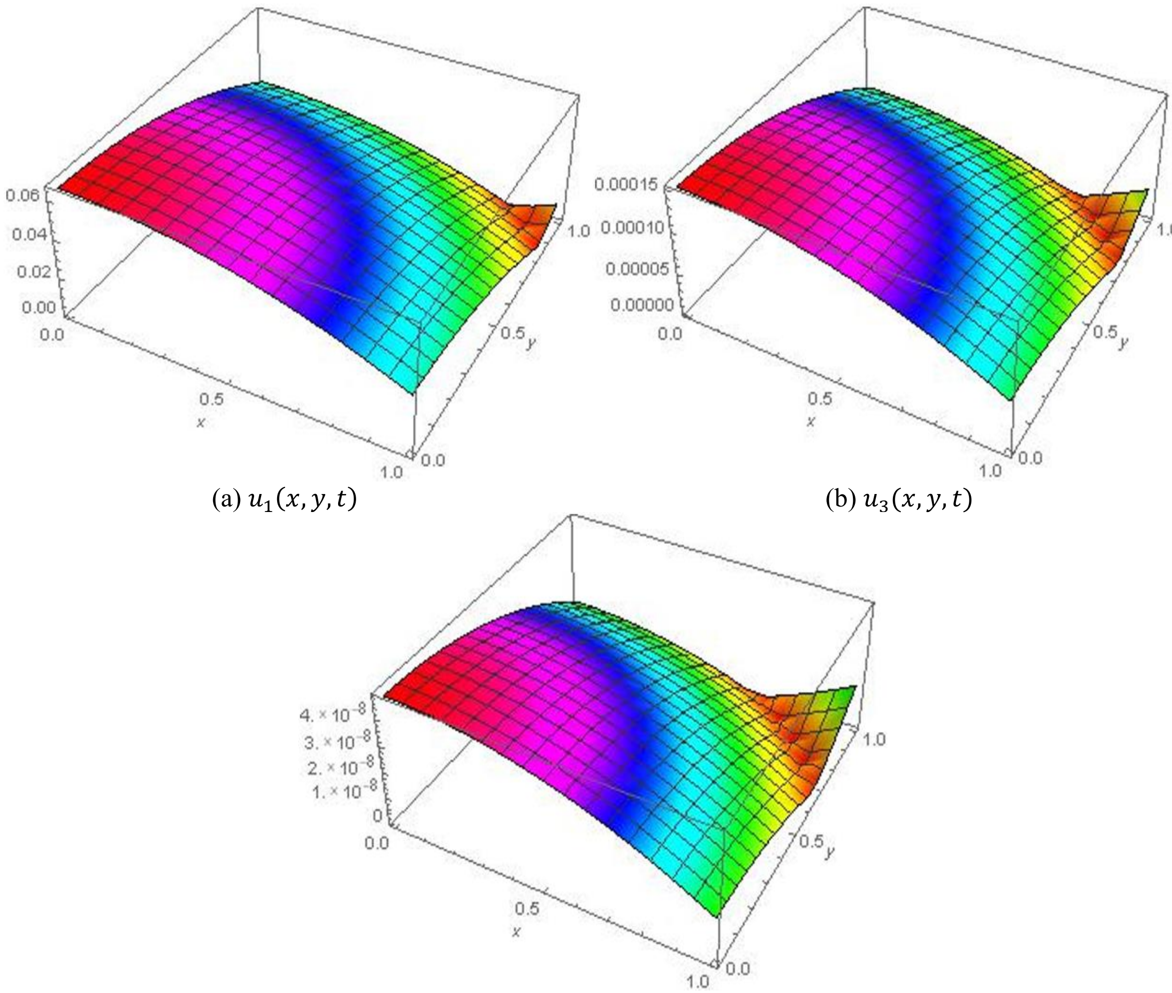
$$\beta_0 = \frac{\|v_1\|}{\|v_0\|} = 0.825234 < 1$$

$$\beta_1 = \frac{\|v_2\|}{\|v_1\|} = 0.512054 < 1$$
$$\beta_2 = \frac{\|v_3\|}{\|v_2\|} = 0.211578 < 1$$
$$\beta_3 = \frac{\|v_4\|}{\|v_3\|} = 0.109301 < 1$$
$$\beta_4 = \frac{\|v_5\|}{\|v_4\|} = 0.0977504 < 1$$

where, the β_i values for $i \geq 0$ and for each (x, y, t): $x, y \in R$, $0 < x, y, t \leq 1$ and are all smaller than 1. Therefore, the suggested iterative approach guarantees convergence.

To evaluate the accuracy of the projected solution for the given example, an absolute error is computed using $Absr_n$ where $w = \frac{x^2+y^2+t^2}{2}$ is the exact solution. Figure 2 shows a three-dimensional graph showing the absolute error of the approximate solution given by the suggested techniques, showing how well these techniques work. Also, as the number of iterations grows increases up, the mistakes go down, which makes the approximation answer more accurate.

Additionally, Table 2 displays ($Absr_n$) obtined by TAM for the iterations with n={1, 3, 5}. The reduction of error is readily apparent with an increase in the number of repetitions, particularly when t = 1.



(a) $u_1(x, y, t)$ (b) $u_3(x, y, t)$

(c) $u_5(x, y, t)$

Figure 2. Three-dimensional plotted graph for the $Absr_n$ at n=(1, 3, 5) of example 4, where t=1

Table 2. The $Absr_n$ values for example 4 computed using the TAM method over three iterations with t=1

(x, y)	Abs. Errs. for u_1	Abs. Errs. for u_3	Abs. Errs. for u_5
0	0.0656498	0.000151279	4.79946×10^{-8}
0.1	0.048998	0.000149373	4.73387×10^{-8}
0.2	0.0626498	0.000143655	4.53709×10^{-8}
0.3	0.0588998	0.000134126	4.20912×10^{-8}
0.4	0.0536498	0.000120785	3.74996×10^{-8}
0.5	0.0468998	0.000103632	3.15961×10^{-8}
0.6	0.0386498	0.0000826668	2.43807×10^{-8}
0.7	0.0288998	0.0000578903	1.58535×10^{-8}
0.8	0.0176498	0.000029302	6.01436×10^{-9}
0.9	0.0048998	0.00000309807	5.13665×10^{-9}
1	0.0093502	0.0000393099	1.75995×10^{-8}

Example 5. Consider the 3D linear acoustic wave equation, given as [37]

$$u_{tt} = V^2\left(u_{xx} + u_{yy} + u_{zz}\right) \tag{32}$$

with the initial condition

$$u(x, y, z, 0) = \cos(x)\, sin(y)\, cos(z),\ u_t(x, y, z, 0) = 0.$$

Eq. (32) will be solved using the approach of steps that was suggested, $V = 1$ [37]

$$u_{0tt} = 0, \text{ with } u_0(x, y, z, 0) = \cos(x) sin(y) \cos(z), \quad u_{0t}(x, y, z, 0) = 0.$$

Then,

$$u_0 = \cos(x)\cos(z)\sin(y)$$

The first iteration $u_1(x, y, z, t)$ can be get by solving

$$u_{1tt} = V^2\left(u_{0_{xx}} + u_{0_{yy}} + u_{0_{zz}}\right), \text{ with } u_1(x, y, z, 0) = \cos(x)\, sin(y)\, cos(z), u_{1t}(x, y, z, 0) = 0$$

We get,

$$u_1 = \cos(x)\cos(z)\sin(y) - \frac{3}{2}t^2\cos(x)\cos(z)\sin(y)$$

The iteration $u_2(x, y, z, t)$ can be obtained by solving this problem:

$$u_{2tt} = V^2\left(u_{1_{xx}} + u_{2_{yy}} + u_{3_{zz}}\right), \text{ with: } u_2(x, y, z, 0) = \cos(x) sin(y) cos(z), u_{2t}(x, y, z, 0) = 0,$$

Then

$$\begin{aligned} u_2 &= \cos(x)\cos(z)\sin(y) - \frac{3}{2}t^2\cos(x)\cos(z)\sin(y) + \frac{3}{8}t^4\cos(x)\cos(z)\sin(y), \\ u_5 &= \cos(x)\cos(z)\sin(y) - \frac{3}{2}t^2\cos(x)\cos(z)\sin(y) + \frac{3}{8}t^4\cos(x)\cos(z)\sin(y) - \frac{3}{80}t^6\cos(x)\cos(z)\sin(y) \\ &\quad + \frac{9t^8\cos(x)\cos(z)\sin(y)}{4480} - \frac{3t^{10}\cos(x)\cos(z)\sin(y)}{44800} \end{aligned}$$

When the solution converges to the exact solution, then

$$\begin{aligned} u(x, y, z, t) &= \cos\left(\sqrt{3}t\right)\cos(x) sin(y) cos(z) \\ &= \cos(x)\cos(z)\sin(y) - \frac{3}{2}t^2\cos(x)\cos(z)\sin(y) + \frac{3}{8}t^4\cos(x)\cos(z)\sin(y) - \frac{3}{80}t^6\cos(x)\cos(z)\sin(y) \\ &\quad + \frac{9t^8\cos(x)\cos(z)\sin(y)}{4480} - \frac{3t^{10}\cos(x)\cos(z)\sin(y)}{44800} + \cdots \end{aligned}$$

The exact solution can be found by $u(x, y, z, t) = \lim_{n\to\infty} u_n = \cos(\sqrt{3}t)\cos(x) sin(y) cos(z)$.

Example 6. The equation provided is a three-dimensional nonlinear acoustic wave equation.

$$\begin{aligned} u_{tt}(x, y, z, t) &= u(x, y, z, t)\, u_{xx}(x, y, z, t) + u_{yy}(x, y, z, t) \\ &\quad + 2x\, u_{zz}(x, y, z, t) - 4x - 2(x^2 + y^2 + z^2 + t^2) \end{aligned} \tag{33}$$

with the initial conditions

$$u(x, y, z, 0) = x^2 + y^2 + z^2, u_t(x, y, z, 0) = 0.$$

When we apply the TAM to solve Eq. (33) with the given initial conditions, the following outcomes are obtained

$$\begin{aligned} L(u) &= u_{tt}(x, y, z, t), \\ N(u) &= u(x, y, z, t)u_{xx}(x, y, z, t) + u_{yy}(x, y, z, t) + u_{zz}(x, y, z, t) - 2 - 2(x^2 + y^2 + z^2 + t^2) \end{aligned}$$

The initial problem is

$$\begin{gathered} L(u_0) = -4x - 2(x^2 + y^2 + z^2 + t^2) \\ \text{with } u_0(x, y, z, 0) = x^2 + y^2 + z^2, \\ u_{0_t}(x, y, z, 0) = 0 \end{gathered} \tag{34}$$

The generalized iterative method can be used to get the next problems

$$\begin{gathered} L(u_{n+1}) = N(y_n) + g \\ \text{with } u_{n+1}(x, y, z, 0) = x^2 + y^2 + z^2, \\ u_{n+1_t}(x, y, z, 0) = 0 \end{gathered}$$

By solving the Eq. (34), one can get

$$u_0 = -\frac{t^4}{6} - 2t^2x + x^2 - t^2x^2 + y^2 - t^2y^2 + z^2 - t^2z^2$$

The first iteration $u_1(x, y, z, t)$ can be found by solving

$$\begin{aligned} u_0 = u_{1tt}(x, y, z, t) &= u_0(x, y, z, t)u_{0_{xx}}(x, y, z, t) + u_{0_{yy}}(x, y, z, t) + 2xu_{0_{zz}}(x, y, z, t) - 4x \\ &\quad - 2(x^2 + y^2 + z^2 + t^2) \end{aligned}$$
$$\text{with } u_1(x, y, z, 0) = x^2 + y^2 + z^2 \ldots, u_{1_t}(x, y, z, 0) = 0$$

The solution will be

$$\begin{aligned} u_1 &= t^2 - \frac{t^4}{3} - \frac{t^6}{90} + \frac{t^8}{168} - \frac{2t^4x}{3} + \frac{2t^{64}x}{15} + x^2 - \frac{t^4x^2}{3} + \frac{t^6x^2}{15} \\ &\quad + y^2 - \frac{t^4y^2}{3} + \frac{t^6y^2}{15} + z^2 - \frac{t^4z^2}{3} + \frac{t^6z^2}{15}, \end{aligned}$$

Applying the same process for u_2 as stated below

$$\begin{gathered} u_{2tt}(x, y, z, t) = u_1(x, y, z, t)u_{1_{xx}}(x, y, z, t) + u_{1_{yy}}(x, y, z, t) + 2xu_{1_{zz}}(x, y, z, t) - 4x - 2(x^2 + y^2 + z^2 + t^2) \text{ with } u_2(x, y, z, 0) = x^2 + y^2 + z^2, \\ u_{2_t}(x, y, z, 0) = 0. \end{gathered}$$

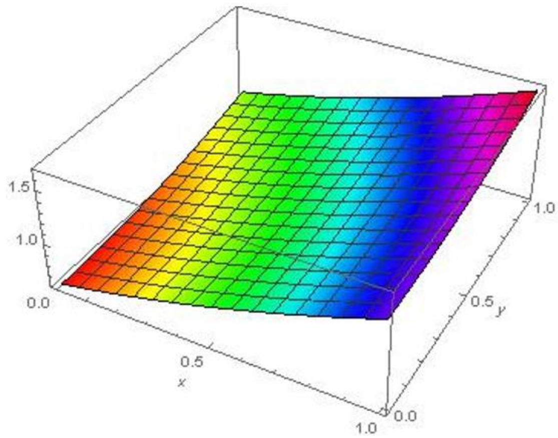
We solve this problem Eq. (35) and then we have

$$u_2 = t^2 - \frac{2t^6}{45} - \frac{5t^8}{504} + \frac{463t^{10}}{113400} - \frac{t^{12}}{3564} - \frac{103t^{14}}{3439800} + \frac{t^{16}}{302400} - \frac{4t^6x}{45} + \frac{t^8x}{105} + \frac{2t^{10}x}{405} - \frac{2t^{12}x}{1485} + \frac{2t^{14}x}{20475} + x^2 - \frac{2t^6x^2}{45} + \frac{t^8x^2}{210} + \frac{t^{10}x^2}{405} - \frac{t^{12}x^2}{1485} + \frac{t^{14}x^2}{20475} + y^2 - \frac{2t^6y^2}{45} + \frac{t^8y^2}{210} + \frac{t^{10}y^2}{405} - \frac{t^{12}y^2}{1485} + \frac{t^{14}y^2}{20475} + z^2 - \frac{2t^6z^2}{45} + \frac{t^8z^2}{210} + \frac{t^{10}z^2}{405} - \frac{t^{12}z^2}{1485} + \frac{t^{14}z^2}{20475}$$

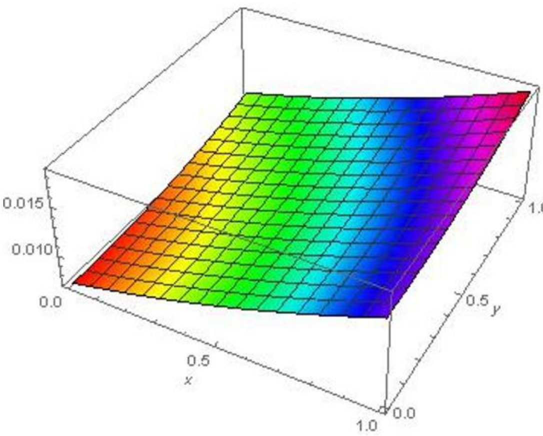
We continue to find approximations until *n*=5, which we will not list for brevity.

To demonstrate convergence, we calculate the values of β_i for the issue outlined in Eq. (33). Therefore, by substituting the terms of the series $\sum_{i=0}^{\infty} v_i(x,y,z,t)$ given in Eq. (11), we get

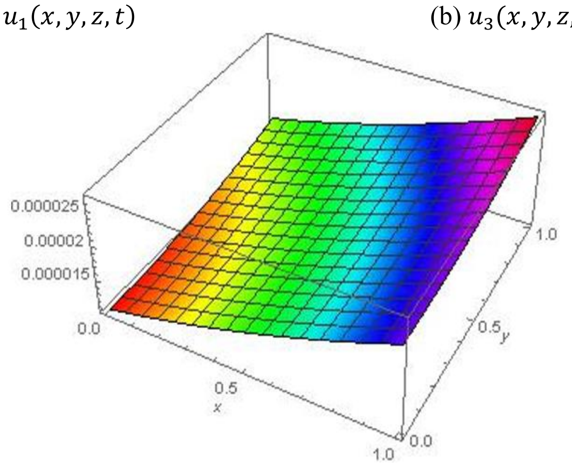
$$\beta_0 = \frac{\|v_1\|}{\|v_0\|} = 0.475491 < 1$$
$$\beta_1 = \frac{\|v_2\|}{\|v_1\|} = 0.0105437 < 1$$
$$\beta_2 = \frac{\|v_3\|}{\|v_2\|} = 0.0042245 < 1$$
$$\beta_3 = \frac{\|v_4\|}{\|v_3\|} = 0.00226105 < 1$$
$$\beta_4 = \frac{\|v_5\|}{\|v_4\|} = 0.00140942 < 1$$



(a) $u_1(x,y,z,t)$



(b) $u_3(x,y,z,t)$



(c) $u_5(x,y,z,t)$

Figure 3. Three-dimensional plotted graph for the $Absr_n$ at $n = (1,3,5)$ of example 6, where $t = 1$

Table 3. The $Absr_n$ values for example 6 computed using the TAM method over three iterations with *t*=1

(*x*, *y*, *z*)	Abs. Errs. for u_1	Abs. Errs. for u_3	Abs. Errs. for u_5
0	0.338492	0.00407229	0.00000630417
0.1	0.399825	0.00473579	0.00000724843
0.2	0.477159	0.00557239	0.00000843902
0.3	0.570492	0.00658208	0.00000987593
0.4	0.679825	0.00776486	0.0000115592
0.5	0.805159	0.00912073	0.0000134887
0.6	0.946492	0.0106497	0.0000156646
0.7	1.10383	0.0123517	0.0000180869
0.8	1.27716	0.0142269	0.0000207554
0.9	1.46649	0.0162751	0.0000236703
1	1.67183	0.0184964	0.0000268315

The β_i values for $i \geq 0$ and $\forall(x, y, z, t)$: $x, y, z \in R^3$, $0 < x, y, z, t \leq 1$ are all smaller than 1. Therefore, the suggested iterative method satisfies convergence. To evaluate the estimated solutions accuracy for this example by using $Absr_n$ of the approximate solution, when the exact solution is $w = x^2 + y^2 + z^2 + t^2$.

Figures 3(a)-(c) display the three-dimensional plot of the absolute value of the approximate solution generated using the TAM, by increasing the iterations, the resolution of the approximate solution will be increased. Furthermore, Table 3 presents the absolute error values derived from TAM for the iterations with *n*={1, 3, 5}. The diminution of mistakes is clearly evident with an increase in the frequency of repetitions, especially when *t*=1.

6. CONCLUSION

In this study, we have employed a dependable semi-analytic technique known as TAM for the solution of the acoustic wave equations. We demonstrate the efficacy of the suggested approach by using multiple test cases. Which include applications of non-linear acoustic wave equations in one-dimensional, two-dimensional, and three-dimensional. All examples were successfully solved, and exact solutions of linear equations and approximate solutions of nonlinear equations were obtained. The TAM approach is simple to implement, and its application does not require intricate constraints for the nonlinear terms, unlike certain current methods such as the ADM and VIM. This method's programming is efficient and cost-effective regarding computer processing, eliminating the need for laborious evaluations. We conducted a comparison between the precise solutions and their approximations. by the absolute error. Through the convergence test and the graphs shown that demonstrated the effectiveness and precision of this employed technique.

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NOMENCLATURE

x, y, z	the independent variable
t	time
u	unknown function
g	known function
L	linear operator
N	non-linear operator
B(.)	boundary operator
v_i	iterations of convergent
S_k	solution of the given problem
$E_n(x,t)$	maximum error
$Absr_n$	absolute error

Greek symbols

β_i	values of converge
β	$\max\{\beta_i, i = 0,1,\dots,n\}$
θ	angle parameter
π	pi

Subscripts

V	velocity of sound
u_n	approximate solution
w	exact solution