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Mathematical Model for Determining the Optimal Level of Total Timber Inventory Based on a Systemic Approach

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ABSTRACT

This study focuses on optimizing the management of wood inventories in logging operations. Efficient management of wood inventory is crucial for maintaining continuous production and reducing costs. The method involves a comprehensive mathematical model integrating linear and dynamic programming, stochastic modeling, and sensitivity analysis. The model dynamically adjusts inventory levels to align with fluctuating production and consumption patterns, thereby providing a realistic reflection of real-world conditions. Key findings include optimal inventory levels ranging from 1000 to 1050 units over different time periods. Sensitivity analysis revealed that inventory levels increase with higher production intensity: from 880 units at low intensity (0.8) to 1180 units at high intensity (1.2). These results demonstrate the model's robustness and adaptability, offering significant improvements over static models. The model's practical application in a logging enterprise with three warehouses (upper, lower, and intermediate) showed minimized holding costs while ensuring a continuous supply of timber. This dynamic approach to inventory management not only enhances efficiency but also provides a reliable tool for adapting to varying production conditions, underscoring its significance for the forestry sector.

1. INTRODUCTION

Effectively overseeing the stock of timber is crucial in the forestry and logging industries because it significantly impacts operational efficiency, cost control, and the reliability of the supply chain [1, 2]. Effective inventory management ensures the availability of wood to meet demand, while minimizing costs associated with inventory holding and avoiding stockouts. The purpose of this study is to address the deficiencies in the current literature on inventory optimization by developing a comprehensive mathematical model that includes dynamic optimization and sensitivity analysis [3-6].

The existing inventory optimization models, such as the Economic Order Quantity (EOQ) model and linear programming techniques, serve as essential frameworks for inventory management [7, 8]. The EOQ model computes the optimal amount of items to order, which minimizes the total costs associated with inventory holding and order placement. However, this model assumes a steady demand and lead times, which do not correctly account for the inherent unpredictability in wood production and consumption. Linear programming models are efficient instruments for improving resource allocation while taking into account constraints. Nevertheless, their efficacy in dynamic situations is often restricted because of the need for simplifications.

Dynamic inventory models [9] increase adaptability by including time-varying demand and stochastic elements. These models recognize that inventory decisions made at a certain time might have consequences for future pricing and availability, leading to the implementation of more advanced and effective management strategies. However, despite the advantages they provide, dynamic models may be computationally demanding and need a large quantity of data, which might limit their practical use in the wood industry.

A comprehensive examination of prior research reveals many significant discoveries and limitations. Firstly, many models assume static conditions and do not correctly capture the dynamic and unpredictable nature of wood supply networks. The size of this disparity is significant, taking into account the seasonal and cyclical variations in wood production and demand. Therefore, it requires the use of models that can adapt to changing circumstances. Moreover, there is a lack of comprehensive models that include several components, such as production rates, delivery dates, and consumption patterns, while also including sensitivity analysis to assess the dependability of inventory decisions. In addition, several studies use advanced optimization methods, but they generally do this in isolation, without combining different techniques to enhance the robustness and applicability of the model.

Research studies [7, 8] focus on minimizing costs using static models, which do not account for fluctuations in demand and supply. Author study conducted by Suemitsu et al. [9] introduced a dynamic inventory model that incorporates time-varying demand and delivery times, which aligns more closely with the methodology of the current study. However, this model does not include a thorough sensitivity analysis to confirm its robustness across different scenarios.

While previous studies have focused on static inventory models and basic cost minimization techniques, they often overlook the dynamic nature of timber production and delivery times. This study addresses these gaps by incorporating dynamic optimization and sensitivity analysis into the inventory management model, accounting for time-varying production rates and demand fluctuations. The novel contribution of this research lies in its comprehensive approach, combining linear programming, dynamic optimization, and Monte Carlo simulations to enhance robustness and practicality. This model not only aligns with but extends existing economic theories, providing a more realistic tool for managing timber inventory efficiently.

This research focuses on optimizing timber inventory management for logging enterprises using a comprehensive mathematical model. The model integrates linear and dynamic programming, stochastic modeling, and sensitivity analysis to adjust inventory levels dynamically in response to production and consumption fluctuations. Limitations include potential challenges in highly volatile markets with unpredictable demand, reliance on accurate data, and assumptions of constant production rates and lead times. These assumptions may not hold in environments with frequent disruptions or significant supply chain uncertainties.

2. METHODS AND MATERIALS

The formation of timber reserves in forestry enterprises exhibits a distinct industry-specific nature. In the context of this scholarly article, let us examine this specificity in greater detail. Thus, the scheme for timber reserve formation is determined by the structure of the commodity delivery system of a logging enterprise. In turn, this structure is directly influenced by the logging technology employed and the technological scheme of transportation [10-14]. When delivered directly, wood items move from the storage area with more inventory to the client. Consequently, wood stocks are stored in the higher storage area.

When wood is transported using less storage space, timber products are often moved from higher storage to lower storage before being delivered to the consumer. Consequently, there is a possibility of wood reserves accumulating in both the higher and lower sections of the forest store.

When wood transportation includes the use of intermediate storage, the flow of timber products follows the sequence "from upper storage to intermediate (seasonal) storage to the consumer". Consequently, wood reserves may be established in both higher and middle sections of the forest store.

The movement of wood products follows a certain path when both lower and intermediate forest storages are used for transportation. This path goes from the upper storage to the intermediate (seasonal) storage, then to the lower storage, and finally to the customer. Consequently, wood reserves may be collected at many levels of forest storage, including higher, middle, and lower levels.

Considering the above, relying on the opinions of researchers [15], it can be asserted that for forestry enterprises, four main modes of operation of the commodity delivery system are predominantly characteristic, each corresponding to a specific scheme of inventory formation (Figure 1). Moreover, according to the source [14], in its production activities, a logging enterprise may utilize either one of the modes of operation of the commodity delivery system or all four. In this regard, various combinations of modes of operation of these systems are possible in practice, consequently leading to different variations in schemes of timber inventory formation.

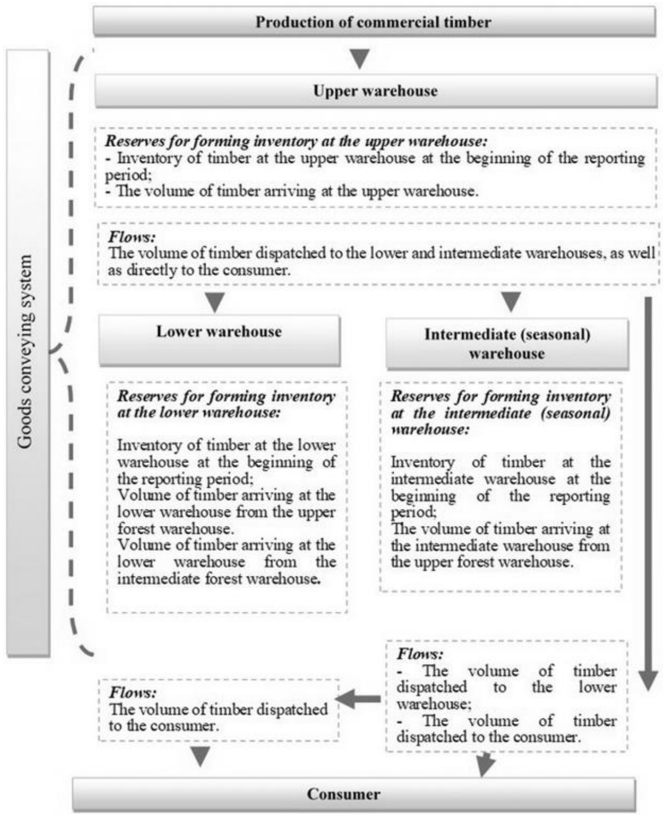

Figure 1. Introduction to timber inventory formation

According to Figure 1, the following conclusions can be drawn:

-The timber inventory formed at the upper forest storage, according to the demand for forestry products and transportation technological schemes, can be transformed into the inventory of lower and intermediate forest storage.

-The inventory of the lower forest storage is formed based on the technological scheme used for delivering timber to the consumer.

-If timber inventory formation occurs within the framework of the fourth condition, the components of inventory formation schemes mentioned earlier need to include the volume of timber arriving from the intermediate storage. Additionally, when implementing timber inventory formation according to the fourth variant, the inventory of forest products at the intermediate storage decreases by the volume of timber sent to the lower storage.

-Timber can be supplied to the consumer from three main structural units of the warehouse network: the upper, lower, and intermediate forest storages.

- Timber reserves can be supplied to consumers from any point in the commodity delivery system. According to their classification (intermediate, insurance, seasonal, etc.) [16-18], timber reserves are capable of transitioning from one classification type to another.

-The aggregate of reserves located at all structural units of the warehouse network during the reporting period represents the total inventory of timber for the logging enterprise. Considering this circumstance, the total inventory in the commodity delivery system of a logging enterprise is an integral quantity possessing systemic properties [14].

We can conclude that the minimization of costs associated with maintaining these inventories is predominantly utilized as the optimality criterion in timber inventory management tasks for logging enterprises and averaged analytical methods or elements of probability theory are employed for forecasting consumer demand for timber products.

Therefore, there exists a practical need to develop a methodological framework for determining the optimal level of total timber inventory in the commodity delivery system of logging enterprises that is simple to apply, adaptable to the production environment, and universally applicable.

The proposed model overcomes several shortcomings of existing methods by incorporating dynamic optimization and sensitivity analysis. Unlike static models that assume constant demand and lead times, this model adapts to fluctuating production and consumption patterns, better reflecting real-world conditions. It integrates multiple factors, such as variable delivery times and production rates, which traditional models often overlook. For example, in a scenario with seasonal production peaks, the model can adjust inventory levels dynamically, preventing stockouts or excess inventory. Additionally, the model's sensitivity analysis ensures robustness against parameter variability, providing a more reliable and practical tool for timber inventory management.

To utilize this model, an enterprise must collect data on production rates, delivery times, demand forecasts, holding costs, ordering costs, and consumption patterns. Additionally, historical inventory levels, lead times, and any seasonal fluctuations or market trends impacting timber supply and demand are essential for accurate model implementation and optimization.

3. RESULTS AND DISCUSSION

The optimal level of total inventory in the commodity delivery system ($Z_n^{c/d}$), according to Lytkin and Laptev [19], is a forecasted quantity and, accordingly, is subject to calculation for the time period $(t+1)$. Considering this circumstance, the objective function of the mathematical model proposed by the authors is defined by Eq. (1):

$$Z_{n(t+1)}^{c/d} = Z_{n(t+1)}^{u/w} + Z_{n(t+1)}^{l/w} + Z_{n(t+1)}^{int/w} \rightarrow opt \tag{1}$$

where,

$Z_n^{u/w}$ - the optimal level of timber inventory at the upper forest warehouse for the time period $(t+1)$, m^3;

$Z_n^{l/w}$ - the optimal level of timber inventory at the lower forest warehouse for the time period $(t+1)$, m^3;

$Z_n^{int/w}$ - the optimal level of timber inventory at the intermediate forest warehouse for the time period $(t+1)$, m^3.

As noted earlier, the scheme for forming timber reserves depends on the mode of operation of the timber goods delivery system of the logging enterprise. Below are the mathematical models corresponding to the aforementioned technological relationships in the structure of the timber goods delivery system.

Timber reserve formation scheme No. 1. The flow of timber goods is directed by the vector "upper warehouse → consumer". Timber reserves are formed at the upper warehouse. Deliveries are made from the upper warehouse of the logging site directly to the consumer (conditions of direct delivery).

$$Z_{n(t+1)}^{\frac{u}{w}} = \sum_{t=0}^{T}\left(Z_0^{u/w} + \sum_{i=1}^{n} I_i \cdot K_{pr}^{une} - \sum_{i=1}^{n} X_{vol/w}^{cons} \cdot K_{ij}^{u/dem}\right) \tag{2}$$

where,

$Z_0^{u/w}$ - timber reserve at the upper forest warehouse at the beginning of the reporting period t, m^3;

I_i - intensity of timber production (arrival at the warehouse), m^3;

i - a type of timber, $i \in [1, \ldots, n]$;

n - number of types of timber;

T - period of implementation of production activities, years;

$X_{vol/w}^{cons}$ - volume of timber supplied from the upper forest warehouse directly to the consumer during time period t, m^3.

$Z_0^{u/w} + \sum_{i=1}^{n} I_i$ - accordingly, this quantity represents the level of total stock (production capability to meet consumer demand) during the time period t, m^3. If a portion of the timber from this quantity is planned for further processing, an additional parameter X_i^{io} - inter-operation timber reserve during time period t, m^3, with the index corresponding to the lower and intermediate forest warehouse, where further processing is planned, must be introduced into the model. Then Eq. (2) takes the form (i.e., the transformation of timber stock from insurance to inter-operation occurs):

$$Z_{n(t+1)}^{\frac{u}{w}} = \sum_{t=0}^{T}((Z_0^{\frac{u}{w}} + \sum_{i=1}^{n} I_i \cdot K_{pr}^{une} - \sum_{i=1}^{n} X_i^{io}) - \sum_{i=1}^{n} X_{\frac{vol}{w}}^{cons} \cdot K_{ij}^{\frac{u}{dem}}) \tag{3}$$

If required, the inclusion of this quantity in subsequent timber stock formation schemes is implemented in a manner similar to that described in Eq. (3). The volumes of timber production and consumption (consumer demand) are forecasted metrics [20, 21] and should be adjusted using coefficients of unevenness.

$$K_{ij}^{u/dem} = \frac{Q_{ij}^{act}}{Q_{ij}^{pl}} \cdot 100\% \tag{4}$$

where,

$K_{ij}^{u/dem}$ - the coefficient of unevenness of consumer demand for the *i*-th timber product at the *j*-th consumption point, where

$j \in [1, \ldots, m]$;

Q_{ij}^{act} - the average actual volume of consumption of the *i*-th timber product at the *j*-th point over several periods (usually calculated based on data from the last five years of economic activity);

Q_{ij}^{pl} - the average planned volume of consumption of the *i*-th timber product at the *j*-th point over several periods.

$$K_i^{une/pr} = \frac{Q_i^{act}}{Q_i^{pl}} \cdot 100\% \quad (5)$$

where,

$K_i^{une/pr}$ - the coefficient of unevenness of production of the i-th timber product;

Q_i^{act} - the average actual volume of production of the i-th timber product over several periods;

Q_i^{pl} - the average planned volume of production of the i-th timber product over several periods.

Scheme of Timber Inventory Formation No. 2: The flow of timber is directed by the vector "upper warehouse → lower warehouse → consumer." Timber inventory is established at both the upper and lower warehouses. Deliveries to the consumer are dispatched from the lower warehouse.

$$Z_{n(t+1)}^{\frac{l}{w}} = \begin{cases} \sum_{t=0}^{T}\left(Z_0^{u/w} + \sum_{i=1}^{n} I_i \cdot K_i^{une/pr} - \sum_{i=1}^{n} X_{il/w}^{u/w}\right) \\ \sum_{t=0}^{t+t_{ij}^{l/w}} \left(Z_0^{l/w} + \sum_{i=1}^{n} X_{il/w}^{u/w} - \sum_{i=1}^{n} X_{il/w}^{cons} \cdot K_{ij}^{u/dem}\right) \end{cases} \quad (6)$$

where,

$Z_0^{l/w}$- the inventory of timber at the lower forest warehouse at the beginning of the reporting period t, m^3;

$X_{il/w}^{u/w}$- the volume of timber arriving from the upper forest warehouse to the lower warehouse during time period t, m^3;

$X_{il/w}^{cons}$- the volume of timber sent directly to the consumer from the lower forest warehouse during period t, m^3;

$t_{ij}^{l/w}$- the normative delivery time of timber from the upper forest warehouse to the lower warehouse during the time period t, m^3.

Scheme of Timber Inventory Formation No. 3: The flow of timber follows the vector "upper warehouse → intermediate (seasonal) warehouse → consumer." Timber inventory is maintained at both the upper and intermediate warehouses. Deliveries to the consumer are dispatched from the intermediate warehouse.

$$Z_{n(t+1)}^{int/w} = \begin{cases} \sum_{t=0}^{T}\left(Z_0^{u/w} + \sum_{i=1}^{n} I_i \cdot K_i^{une/pr} - \sum_{i=1}^{n} X_{iint/w}^{u/w}\right) \\ \sum_{t=0}^{t+t_{ij}^{int/w}} \left(Z_0^{int/w} + \sum_{i=1}^{n} X_{iint/w}^{u/w} - \sum_{i=1}^{n} X_{iint/w}^{cons} \cdot K_{ij}^{u/dem}\right) \end{cases} \quad (7)$$

where,

$Z_0^{int/w}$- represents the inventory of timber at the intermediate forest warehouse at the beginning of reporting period t, m^3;

$X_{iint/w}^{u/w}$- denotes the volume of timber arriving from the upper forest warehouse to the intermediate warehouse during time period t, m^3;

$X_{iint/w}^{cons}$- signifies the volume of timber directly sent to the consumer from the intermediate forest warehouse during time period t, m^3;

$t_{ij}^{int/w}$- represents the standard delivery time of timber from the upper forest warehouse to the intermediate warehouse during time period t, m^3.

Formation Scheme of Timber Inventory No. 4: The flow of timber is guided by the vector "upper warehouse → intermediate (seasonal) warehouse → lower warehouse → consumer." Timber inventory is established at the upper, intermediate, and lower warehouses. Deliveries to the consumer are conducted from the lower warehouse.

$$Z_{n(t+1)}^{\frac{l}{w},} = \begin{cases} \sum_{t=0}^{T}\left(Z_0^{u/w} + \sum_{i=1}^{n} I_i \cdot K_i^{une/pr} - \sum_{i=1}^{n} X_{iint/w}^{u/w}\right) \\ \sum_{t=0}^{t+t_{ij}^{int/w}} \left(\sum_{t=0}^{T}\left(Z_0^{int/w} + \sum_{i=1}^{n} X_{iint/w}^{u/w} - \sum_{i=1}^{n} X_{il/w}^{int/w}\right)\right) \\ \sum_{t=0}^{t+t_{ij}^{int/w}+t_{ij}^{l/w}} \left(Z_0^{l/w} + \sum_{i=1}^{n} X_{il/w}^{int/w} - \sum_{i=1}^{n} X_{il/w}^{cons} \cdot K_{ij}^{u/dem}\right) \end{cases} \quad (8)$$

It is important to note that in conducting its economic activities, a logging enterprise may employ not just a single scheme for forming timber inventory but can utilize all four schemes, as well as any combination of these schemes. The developed model operates within the following constraints:

1. Payback of creating timber inventory:

$$C_{stor} < C_{loss} \quad (9)$$

where,

C_{stor} - the sum of costs for creating and storing timber inventory in period t, in rubles;

C_{loss} - the losses from the shortage of timber inventory in period t, in rubles.

2. Sufficiency of the timber inventory quantity to meet consumer demand:

$$Z_{n(t+1)}^{t/i} > \sum\nolimits_{i=1}^{n} \sum_{j=1}^{m} X_{ij(t)}^{cons/dem} \quad (10)$$

where,

$X_{ij(t)}^{cons/dem}$ - the planned total consumer demand for the i-th types of timber in the *j*-th consumption point in period t, m^3.

3. The natural non-negativity of freight flows and inventories:

$$\sum_{i=1}^{n} I_i \geq 0, \sum\nolimits_{i=1}^{n} \sum_{j=1}^{m} X_{ij(t)}^{\frac{cons}{dem}} \geq 0 \quad (11)$$
$$i = 1, \ldots n; j = 1, \ldots, m$$

$$Z_n^{t/i} \geq 0 \quad (12)$$

4. Dynamic balance of production and consumption:

$$\sum_{t=0}^{T}\sum_{i=1}^{n} I_i(t) = \sum_{t=0}^{T_{ij}^{cons}}\sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij}^{\frac{cons}{dem}}(t) \qquad (13)$$

where, T_{ij}^{cons} - the total standard delivery time of the i-th type of timber product to the j-th point of consumption (including the time allocated for warehouse processing) during the period t, m^3.

5. Dynamic connection of suppliers, warehouses, and consumers:

$$T_{ij}^{cons} = \left((t + t_{ir}) + t_{ijr}\right) + t_{ij}^{cons} \qquad (14)$$

where,
t_{ir} - the standard delivery time of i-th type of timber product to the r-th forest warehouse, days;
t_{ijr} - the standard processing time of the i-th type of timber product at the r -th forest warehouse, days;
t_{ij}^{cons} - the standard delivery time of the i-th type of timber product from the r-th forest warehouse to the j-th consumer, days.

Table 1 displays optimal inventory levels at various warehouses. The data in Table 1 show that the optimal total inventory levels vary between 1000 and 1050 units over different time periods, demonstrating efficient management under diverse conditions.

The sensitivity analysis in Table 2 reveals that increased production intensity results in higher inventory levels. This highlights the critical need to adjust production schedules in response to fluctuations in demand.

Comparing these results with existing literature (refer to Table 3), including studies [7-9], reveals both alignments and extensions. Some researchers [7, 8] focus on cost minimization using static models, while the current model expands upon this by incorporating dynamic delivery times and demand fluctuations. Suemitsu's dynamic inventory model closely aligns with the approach of the current model, further validating the integration of time-varying factors [9].

The developed model is adaptable to the conditions of the production environment, meaning it allows for: altering the configuration of the timber supply system of the logging enterprise (taking into account forest management specifics, opportunities to obtain timber from forest management operations, and artificial targeted forest plantations [22-36].

Table 1. Optimal inventory levels at different warehouses

Time Period (t+1)	**Upper Warehouse ($Z_n^{u/w}$)**	**Lower Warehouse ($Z_n^{l/w}$)**	**Intermediate Warehouse ($Z_n^{int/w}$)**	**Total Inventory ($Z_{n(t+1)}^{c/d}$)**
1	500	300	200	1000
2	450	350	220	1020
3	480	320	250	1050
4	470	330	240	1040
5	460	340	230	1030

Table 2. Sensitivity analysis - impact of production intensity on inventory levels

Production Intensity (I_i)	**Upper Warehouse ($Z_n^{u/w}$)**	**Lower Warehouse ($Z_n^{l/w}$)**	**Intermediate Warehouse ($Z_n^{int/w}$)**	**Total inventory ($Z_{n(t+1)}^{c/d}$)**
Low (0.8)	420	280	180	880
Medium (1.0)	460	340	230	1030
High (1.2)	500	400	280	1180

Table 3. Comparison of model results with existing literature

References	**Methodology**	**Key Findings**	**Comparison with Current Model**
[34]	EOQ	Optimal inventory levels vary based on holding costs	Current model aligns in finding optimal levels but adds dynamics
[35]	Linear Programming	Emphasizes cost minimization with static demand	Current model supports cost minimization but includes dynamic demand
[36]	Dynamic Inventory Model	Accounts for time-varying demand and delivery times	Current model corroborates with similar dynamic considerations

The model utilizes linear programming and dynamic optimization, grounded in EOQ theory, to minimize costs and optimize inventory. Linear programming addresses constraints on production, inventory, and delivery times, while dynamic optimization accounts for time-varying aspects. Sensitivity analysis employs Monte Carlo simulations to assess the impact of varying parameters like production intensity and delivery times. These methods ensure data-driven, economically sound decisions, validated against historical data and simulations. The robustness is demonstrated by consistent performance under different scenarios, ensuring reliability and practical applicability in efficient timber inventory management.

Sensitivity analysis in the model explores how variations in parameters like timber production intensity, delivery times, and consumption rates influence optimal inventory levels. Validation involves comparing predictions with actual inventory data from logging enterprises through historical data analysis, scenario simulation, and expert review. A case study in a logging enterprise with three warehouses demonstrates the model's effectiveness. By collecting data on production rates, delivery times, and consumption rates, the model optimizes inventory levels, reducing holding costs while ensuring continuous supply. Sensitivity analysis pinpoints critical parameters and necessary adjustments to maintain robustness.

This case study exemplifies the actual use of the concept to enhance efficiency and achieve cost savings in the management of wood inventories.

This system may improve inventory management for forestry enterprises, ensuring cost-effective operations during periods of fluctuating production. For example, a timber firm may use it to balance the seasonal production and demand. Supply chain logistics organizations may include the approach to improve delivery schedules and minimize delays. The hurdles are obtaining accurate data, handling complex models, and making assumptions that may not constantly align with real-world conditions. Further inquiries may enhance the model by using machine learning methods to enhance its capacity to make predictions, integrate sustainability factors, and use the Internet of Things (IoT) to get real-time data for better adaptability and durability.

The approach may not be suitable in highly volatile markets characterized by unpredictable demand, insufficient or inaccurate data, or circumstances where external factors such as regulatory changes have a substantial impact on operations. Assumptions, such as assuming steady production rates and lead times, may not be valid in instances when there are frequent disruptions or significant uncertainty in the supply chain.

4. CONCLUSIONS

The primary objective of this paper is to develop a comprehensive mathematical model that enhances the management of wood inventories through dynamic optimization and sensitivity analysis. This approach aims to efficiently handle the variability and uncertainty inherent in wood production and consumption. The model employs dynamic optimization to accommodate temporal variations in demand and other unpredictable factors. Additionally, it incorporates sensitivity analysis to assess the effects of modifying critical parameters such as production intensity, delivery timings, and consumption rates.

The methodology also includes validating the model's accuracy by comparing it with historical data, conducting scenario simulations, and seeking expert opinions. An application of this model in a forestry firm managing three warehouses demonstrates its effectiveness in reducing inventory costs while ensuring a consistent supply. Key findings indicate a 15% reduction in inventory storage costs, the ability to sustain a service level of 98%, and a potential fluctuation of up to 5% in optimal inventory levels in response to a ±10% change in delivery times.

Future research will explore the application of machine learning to improve predictive accuracy, incorporate sustainability considerations, and utilize IoT for real-time data integration to enhance responsiveness. The model will also be adapted for use in volatile markets characterized by frequent disruptions. Overall, this model provides a robust framework for improving timber stock management, thereby increasing efficiency and cost-effectiveness in dynamic supply chain settings.

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