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Bayesian Modelling of Tail Risk Using Extreme Value Theory with Application to Currency Exchange

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ABSTRACT

Modelling financial tail risk such as investment or financial risk is important to avoid high financial shocks. This study adopted Bayesian techniques to complement the classical extreme value theory (EVT) models to model the exchange rate risk of Nigeria against the South African ZAR. Hence, this study proposed the Bayesian Generalized Extreme Value (BGEV) model, Bayesian Generalized Pareto distribution (BGPD), Bayesian Gumbel (BG), and classical Generalized Pareto distribution (GPD) to fit the exchange rate returns over one hundred and four observations. The model selection criteria were used to determine the best model, consequently, the model selection criteria were in favour of BGEV model. The Value-at-Risk (VaR) and the Expected Shortfall (ES) were obtained from the estimated parameters. The results show that the Nigeria Naira exchange will experience losses against the ZAR both at 95% quantile and 99% quantile. This study recommends that investors should watch closely before making financial or investment decisions. This study aligns with the sustainable development goals (SDGs), 8.1 (sustainable economic growth), SDG 8 (Promote sustained, inclusive and sustainable economic growth).

1. INTRODUCTION

The exchange rate of Nigeria's Naira against some major foreign currencies has experienced a significant decline in recent times due to economic instability. The volatility of the exchange rate has left businessmen and women no choice but to watch carefully before investing because of the inherent risk associated. For over a decade, the Nigeria Naira has been faced with variability, and Naira exchange rate returns are difficult to predict due to the variability [1]. Trading volume on the foreign currency market each day is measured in trillions of dollars, making it the largest and most liquid financial market worldwide [2]. It operates around the clock across different time zones, with traders from banks, corporations, governments, hedge funds, and private investors all participating. However, the foreign exchange market and investment are without risk. Accurate risk modeling is crucial for traders, investors, banks, and governments to make calculated decisions and effectively manage risks.

Several studies have been conducted on foreign exchange volatility modeling and forecasting, including the Nigerian Naira against currencies such as the USD and the GBP. The majority of these models in the literature are based on time series ARCH-GARCH modeling, including both symmetric (GARCH) and asymmetric GARCH-type models to handle financial time series data. Studies [3-8] utilized both symmetric and asymmetric GARCH models to model volatility and forecast future exchange rates. Srihari et al. [9] applied various ARIMA, ARCH, and GARCH models to predict the return volatility of Indian stocks. The asymmetric GARCH model addresses the skewed nature of financial time series data, as noted in studies [10, 11].

GARCH models can only estimate volatility but not tail risk, and the current study focuses on extreme risk estimation, therefore, we adopt EVT which utilizes probability distribution to model the tail area of the distribution. EVT is used to model the likelihood of a rare huge and damaging events occurring amongst a set of independent random variables of order statistic [12]. There are fundamental probability distributions used in modelling extreme events; the Weibull, Frechet, and Gumbel distribution, and the hybrid of the models gives Generalized Extreme Value distribution (GEVD) which was utilized in reference [13] to model the return level of Apache server. The GEVD approach is based on the block maxima or minima approach, that is, its strength lies in picking observations from maxima or minima from a set of blocks. A substitute for the GEVD is the GPD which uses the peak-over-threshold (POT) approach. The POT picks value from either tail of the distribution based on the threshold value defined by diagnostics tests. Financial time series is associated with non-normal behaviour, usually associated with fat tails. One way to measure an investment risk is the VaR and the Expected shortfall or conditional VaR. Shaik and Padmakumari [14] utilized the Gaussian, historical, exponential weighted moving average value-at-Risk to measure the risk of major banking corporations. The historical and Gaussian Value-at-Risk (VAR) are not sufficient in modelling the risk in the tail area of the distribution [15], hence the need to adopt the EVT based models. The VaR measures how much risk a set of investments might lose in normal market conditions with a given probability. The ES is the average loss of portfolio value given that a loss is occurring.

EVT models are applied across various fields, including engineering, hydrology, and finance. In the realm of financial risk modeling, study [16], which applied EVT models to fit the risk associated with the Chinese Yuan exchange rate, and study [15], which used EVT to model the risk of the Nigerian Naira against major currencies like USD, EUR, and GBP, demonstrate the theory's applicability. Further research, including [17-19], continues to explore related applications.

McNeil and Frey [20] proposed an EVT-based method to estimate VaR, considered an advancement over existing methods due to its effective capture of tail events in distributions. Gencay and Selçuk [21] investigated the performance of VaR models across nine different emerging stock markets, showing that EVT-based VaR estimates provided greater accuracy at higher quantiles compared to competing models. Similarly, Rufino and de Guia [22] utilized EVT to estimate the VaR of portfolios involving foreign exchange exposures of ASEAN+3 countries, highlighting the method's superiority over traditional VaR approaches that assume normality in exchange rate data. Wang et al. [23] introduced a dynamic mixture copula-extreme value theory (DMC-EVT) model for modeling extreme foreign exchange data across 39 currencies, demonstrating its strengths over traditional copula methods.

Despite these advancements, there are limited recent studies in this specific area, particularly those proposing Bayesian EVT models to estimate both the VaR and the ES. Furthermore, while many studies focus on major currencies like USD, GBP, and EUR [24-26], less attention is given to currencies like the Nigerian Naira and the South African Rand (ZAR), despite significant investments and transactions between these countries. Thus, the current study adopts both classical and Bayesian parametric models to analyze the exchange risk of NGN/ZAR.

This research will be of particular interest to academics, financial risk analysts, and investors, providing valuable insights that can enhance financial and investment decision-making processes.

2. MATERIAL AND METHODS

This section provides the procedure for obtaining the parameters of GPD models. The maximum likelihood method is adopted in the study; it involves maximizing the likelihood function, followed by the GEVD.

2.1 GPD

The probability distribution function of GPD is defined by:

$$G_{\xi,\beta} = \begin{cases} 1-\left(1+\xi\dfrac{y}{\beta}\right)^{\frac{-1}{\xi}}, & \xi \neq 0 \\ 1-\left(1+exp\left(-\dfrac{y}{\beta}\right)\right)^{\frac{-1}{\xi}}, & \xi = 0 \end{cases} \tag{1}$$

The ξ is the shape parameter, while the β is the scale parameter. The likelihood function is as follows:

$$L(\xi,\beta;Y_j) = \prod_{j=1}^{N_u}\left[1-\left(1+\xi\frac{y_j}{\beta}\right)^{\frac{-1}{\xi}}\right], \xi \neq 0 \tag{2}$$

$$L(\xi,\beta;Y_j) = \prod_{j=1}^{N_u}\left[1-\left(1+exp\left(-\frac{y_j}{\beta}\right)\right)^{\frac{-1}{\xi}}\right], \xi = 0 \tag{3}$$

The log-likelihood is:

$$L(\xi,\beta;Y_j) = -N_u In\beta + \left(\frac{1}{\xi}-1\right)\sum_{j=1}^{N_u}\left(1-\xi\frac{Y_j}{\beta}\right), \xi \neq 0 \tag{4}$$

$$L(\xi,\beta;Y_j) = -N_u In\beta - \frac{1}{\beta}\sum_{j=1}^{N_u} Y_j, \xi = 0 \tag{5}$$

Taking partial derivative of (5) with respect to ξ and equating to zero, we have:

$$\frac{\partial L}{\partial \xi}(\xi,\beta;Y_j) = \frac{N_u}{\xi}\left(\frac{1}{\xi}-1\right) - \frac{1}{\xi^2}\sum_{j=1}^{N_u} In\left(1-\xi\frac{Y_j}{\beta}\right) - \frac{1}{\xi}\left(\frac{1}{\xi}-1\right)\sum_{j=1}^{N_u}\left(1-\xi\frac{Y_i}{\beta}\right)^{-1} \tag{6}$$

Taking partial derivative of (5) with respect to β, and equating to zero we have:

$$\frac{\partial L}{\partial \beta}(\xi,\beta;Y_j) = \frac{N_u}{\xi\beta} + \frac{1}{\beta}\left(\frac{1}{\xi}-1\right)\sum_{j=1}^{N_u}\left(1-\xi\frac{Y_j}{\beta}\right)^{-1} = 0 \tag{7}$$

The parameter $\hat{\xi}$ and $\hat{\beta}$ can be obtain computationally.

If $\xi > 0$, it is heavy tailed distribution, it also means that the GPD model is the version of Pareto distribution which is applicable in modeling huge loss. If $\xi < 0$ it is light tail, the higher the shape parameter, the higher the derived return. The GPD approach involved modeling excess distribution over a high threshold, u to estimate the value of x distribution function F_u for the random variable X.

$$F_u(y) = P(X-u \leq y | X > u), \ 0 \leq y \leq x_F - u \tag{8}$$

The hybrid of Frechet, Gumbel and Weibull give the GEVD. The cumulative density function is as follows:

$$H(x|\xi,\sigma,\mu) = \begin{cases} exp-\left(1+\xi\left(\dfrac{x-\mu}{\sigma}\right)\right)^{\frac{-1}{\xi}}, & \xi \neq 0 \\ exp-\left(exp\left(-\dfrac{x-\mu}{\sigma}\right)\right)^{\frac{-1}{\xi}}, & \xi = 0 \end{cases} \tag{9}$$

The parameters of GEV include the location paramter (μ_n), the scale paramter (σ_n), and the shape parameter (ξ_n). The minimum distribution can be expressed as:

$$F(x)(= \begin{cases} 1-exp[-(1+\xi x)]^{\frac{-1}{\xi}} & \xi \neq 0 \\ 1-exp[-\exp(x)] & \xi = 0 \end{cases} \tag{10}$$

where, $-\infty < x < \infty$, when $\xi = 0$

$x < -1$ for $\xi < 0$

$x > -1$ for $\xi > 0$

If $\xi > 0$, is the heavy-tailed Fréchet case, ξ=0, the light-tailed Gumbel case; and ξ <0, the short-tailed negative-Weibull case.

2.2 VaR

The VaR for NGR/ZAR exchange rate returns, helps to assess the potential losses (or gains) in the value of NGN relative to the ZAR. There is historical VaR and Gaussian VAR. The extreme VaR based on GPD is as follows:

$$\widehat{VaR}_\alpha = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}\alpha\right)^{-\hat{\xi}} - 1\right) \tag{11}$$

where, u is the threshold value, n is the number of observations, N_u is the number of tail observations, $\hat{\xi}$ and $\hat{\beta}$ are estimated parameters. The VaR concentrates on the distribution quantile and neglects extreme losses beyond the VaR level because VaR only detect the lower limit of the worst losses [27].

To address this constraint, ES is introduced as follows:

$$\widehat{ES}_\alpha = \frac{\widehat{VaR}_\alpha}{1-\hat{\xi}} + \frac{\hat{\beta} - \hat{\xi}N_u}{1-\hat{\xi}} \tag{12}$$

The VaR and ES based on GEV is as follows:

$$VaR_\alpha(X) = \begin{cases} -\mu_n - \dfrac{\sigma_n}{\xi_n}\{[-In(\alpha)^{-\xi_n} - 1]\} if\ \xi \neq 0 \\ -\mu_n + \sigma In(-In\alpha) \qquad if \xi = 0 \end{cases} \tag{13}$$

$$ES_\alpha(X) = \begin{cases} -\mu_n - \dfrac{\sigma_n}{\alpha\xi_n}\{[1-\xi, -In\alpha) - \alpha]\}\ if\ \xi \neq 0 \\ -\mu_n + \dfrac{\sigma_n}{\alpha\xi_n}\big(\alpha - \alpha In(-In\alpha)\big), \quad if\ \xi = 0 \end{cases} \tag{14}$$

where, α is the quantile.

2.3 Bayesian inference

Bayesian technique requires combining the likelihood $L(X|\theta)$ with the prior distribution $\pi(\theta)$, to obtain a posterior distribution $\pi(\theta|X)$ using Bayes rule. Since the underlying distributions are continuous distributions, the posterior distribution can be expressed in Eq. (15).

$$\pi(\theta) = \frac{L\big(X|\theta\ \pi(\theta)\big)}{\int L\big(X|\theta\ \pi(\theta)\big)d\theta} \tag{15}$$

2.3.1 Prior distribution

The parameters of GEV distributions are μ, σ, and ξ, as provided in section 2.2. We use Gamma (a, b) prior for the scale parameter $\sigma > 0$. The μ and ξ can be either positive or negative with a Normal prior specified as $N(\mu_0, \sigma_\mu^2)$ and $N(\xi_0, \sigma_\xi^2)$ respectively. Details on the parametrization can be found in references [28, 29].

The parameters of the GPD are ξ and β, so, if $\xi = 0$, we use a prior of $\pi(1/\beta) \sim$ Gamma (a, b). Let $v = 1/\sigma$, then the gamma prior becomes:

$$v^{n+a-1} exp(-(b + n\bar{y})v) \tag{16}$$

If $\xi \neq 0$, we use a normal prior $N(\mu_\xi, \sigma_\xi^2)$ for the parameter ξ, and for β we use Gamma (a, b). Castellanos and Cabras [30] presented a joint Jeffery's prior distribution for the vector (ξ, β) as follows:

$$p(\xi,\beta) \propto \beta^{-1}(1+\xi)^{-1}(1+2\xi)^{-1/2} \quad \xi > -0.5, \quad \beta > 0 \tag{17}$$

2.3.2 Posterior distribution

The likelihood of GPD distribution combining with the prior gives a posterior distribution. If we consider the case of $\xi \neq 0$, combining Eq. (2) with Eq. (17) we have:

$$\pi(\theta|Y) = \prod_{j=1}^{N_u}\left[1 - \left(1 + \xi\frac{y_j}{\beta}\right)^{\frac{-1}{\xi}}\right] \times \beta^{-1}(1+\xi)^{-1}(1+2\xi)^{-1/2} \tag{18}$$

The solution is difficult to obtain, therefore we adopted a popular sampler namely, the MCMC Metropolis-Hasting's algorithm. Adesina et al. [31] provided a detailed procedure for conducting MCMC Metropolis-Hastings algorithm. The method estimates the posterior mean, the posterior median, and their respective confidence intervals. Model selection criteria such as the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Deviance Information Criterion (DIC) were used to identify the best model. For the GEVD, the block maxima/minima approach, 10 blocks were specified, and each block contains approximately 100 observations. 5000 grid was specified for all the models, to allow model training and hyperparameters tuning and select the best performed model. For GPD, the mean excess plot was made to choose the threshold value, as required in the Peak-over-threshold approach, unlike the GEVD approach that uses blocks.

2.4 Data and data description

The data on the exchange rate of the Nigerian Naira against the South African Rand (ZAR) was sourced from the Central Bank of Nigeria website at https://www.cbn.gov.ng/rates/exchratebycurrency.asp. This dataset covers the period from January 2, 2024, to May 6, 2024, and includes a total of 1047 observations. If the exchange rate of ₦1 to R1 on any given day is denoted as E_t, then the 1-day exchange rate returns (r_t) can be represented by the following relationship:

$$r_t = \frac{E_t}{E_{t-i}}, i = 1,2 \ldots \tag{19}$$

Exchange rate returns refer to the changes in the value of one currency relative to another over a certain period, the lag is determined by stationary, hence the Eq. (19) will be specified based on the number of lags i. The analyses were obtained using software by R Core Team [32] along with "fEtremes" package [33], and "MCMC4Extremes" [34], provided in refernce [35].

3. RESULTS

3.1 Descriptive statistics

In this session, results obtained are presented and discussed, first of such is the descriptive statistics in Table 1.

Table 1. Descriptive Statistics of Nigeria Naira exchange South Africa ZAR

	Mean	SD	Kurt.	Skew.	Min.	Max
NSA	30.57	13.42	5.94	2.47	14.02	87.08
SAN	0.03	0.01	0.53	-0.85	0.01	0.07
RSAN	1.00	0.06	315.52	12.56	0.40	2.47

The second column (NSA), in Table 1 is the summary statistics of the equivalent of 1 ZAR to Naira, and the third column (SAN) is the equivalent of 1 Naira to ZAR. While the fourth column (RSAN), is the return of SAN, that is the return of 1 Naira to ZAR. The mean of NSA, SAN, and RSAN is 30.57711, 0.03651, and 1.000401 respectively indicating that on average, ZAR appreciates over Naira over the observed period. The standard deviation (SD) indicates exchange rate swings and potential losses. The SD for NSA, SAN, and RSAN are 13.41868, 0.009469, and 0.062856, and somewhat high which indicates that there is a potential high loss of Naira against the ZAR. The skewness for SAN is (-0.8526) which shows that the highest long tail to the right, suggests an upward trend in the exchange rate of Naira to Rand. NSA (2.4672), and RSAN (12.5592) show that the Naira to ZAR exhibited the highest long tail to the left, which implies frequent small increases and few large decreases in the exchange rate. The kurtosis of NSA is (5.94390), that of SAN (0.53196), and RSAN (315.518), indicating a peaked distribution than the normal distribution (leptokurtic). The minimum for NSA, SAN, and RSAN is 14.0199, 0.011484, and 0.398422 while the maximum is 87.0778, 0.071327, and 2.474173 respectively.

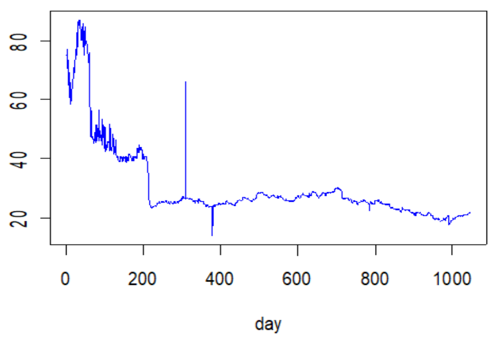

Figure 1. Exchange rate trend of one South African Rand to Nigerian Naira

Figure 1 shows the plot of one South African ZAR to Naira over the period observed. The figure shows that in recent times there is a high foreign exchange risk of the Naira against the ZAR. Figure 2 shows the exchange rate of one Nigeria Naira to ZAR.

Figure 2 presents a time series plot of the exchange rate from one Nigerian Naira to South African Rand over a period of one hundred and forty-seven days. This figure essentially replicates Figure 1, as the data is the reciprocal of that shown in Figure 1. This relationship explains the negative skew observed in Figure 2.

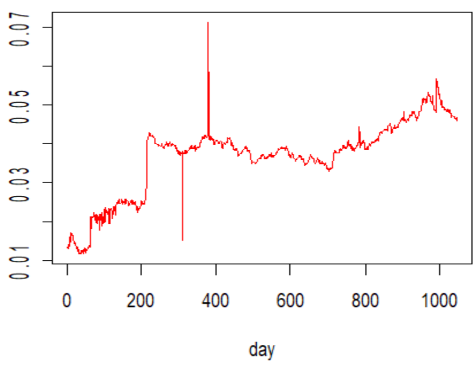

Figure 2. Exchange rate of one Nigeria Naira to ZAR

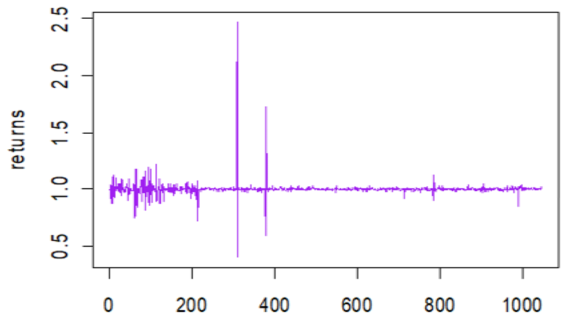

Figure 3. Exchange rate one South African ZAR to Naira

Figure 3 shows the exchange rate one South African ZAR to Nigeria Naira based on equation (19) at lag 1. The Augmented Dickey-Fuller = -10.013, p-value = 0.01, and stationary at lag 10. The null hypothesis of non-stationarity was rejected, and it shows that the data is stationary at lag 10. The resulting equation from Eq. (19) is:

$$r_t = \frac{E_t}{E_{t-10}} \quad (20)$$

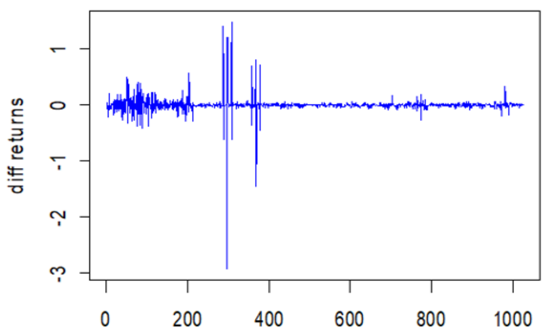

Figure 4. Exchange rate returns of ZAR to Naira

Table 2. Normality tests of Returns (H_o: Normal): significance at 1%

Test	Test Value	p-Value
Cramer-von Mises	40.651	7.37e-10
Anderson-Darling	202.06	2.2e-16

Figure 4 shows the lag 10 differenced exchange rate one South African ZAR to Nigeria Naira recommended by Augmented-Dickey Fuller test as provided in Eq (20). We present the normality test in Table 2.

Table 2 shows that the data of the returns is not normally distributed by rejecting the null hypothesis of normality, hence the need for adopting a suitable model to fit the data such as the extreme value model.

3.2 Tail risk modelling

This section contains exchange rate risk modeling. The threshold value is needed to fit the GPD model, is determined by the mean excess plot.

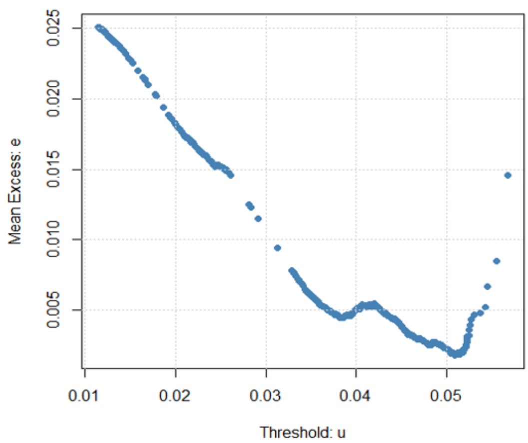

Figure 5. Mean excess plot

The point where there is a break from the main curve, in some instances could be a point where there is a sharp deviation on the curve and can be chosen from either tail of the curve. From the mean excess plot in Figure 5, we choose value of 0.055 from the upper tail to fit the GPD model. Table 3 contains the parameter estimates of the distributions considered in this study.

Table 3. Parameter estimates

Model	Parameter	Post Mean	Lower C.I	Upper C.I
BGEV	Location ($\hat{\mu}$)	0.04000	0.03325	0.04759
	Scale ($\hat{\sigma}$)	0.03429	0.02631	0.04419
	Shape ($\hat{\xi}$)	0.82159	0.63936	1.02155
	AIC	-268.190*		
	BIC	-253.651*		
	DIC	-269.350*		
BG	Scale ($\hat{\sigma}$)	0.06446	0.04394	0.08533
	Shape ($\hat{\xi}$)	0.10027	0.08301	0.12036
	AIC	-100.100		
	BIC	-85.5612		
	DIC	-102.053		
BGPD	Scale ($\hat{\beta}$)	0.08050	0.05177	0.12808
	Shape ($\hat{\xi}$)	0.657381	0.30082	1.05710
	AIC	-129.390		
	BIC	-116.120		
	DIC	-131.551		
GPD	Scale ($\hat{\beta}$)	0.07933	0.01236	0.06044
	Shape ($\hat{\xi}$)	0.63227	0.23451	1.26549
	AIC	-131.274		
	BIC	-126.079		

The BGEV in Table 3 stands for Bayesian GEV, the BG is Bayesian Gumbel distribution, and BGPD is the Bayesian GPD. The posterior mean, the 95% Confidence interval for lower and upper is provided. The best model was decided based on selection criteria such as AIC, BIC, and the DIC. The lowest AIC, BIC, and DIC were asterisked (*). From Table 3, it can be deduced that the BGEV is the best model to estimate the VaR and ES. The location, shape, and scale of the BGEV model are 0.04000849, 0.03429452, 0.82159415. The shape parameter, $\xi > 0$ is of heavy-tailed Fréchet case, the location $\hat{\mu} > 0$ shows that the distribution is more to the right, and the scale $\hat{\sigma} > 0$ shows that the distribution is widely spread, since the larger $\hat{\sigma}$, the more the spread of the distribution. The spread of BGEV is wider than that of BG model all other models in Table 3. The shape parameter (ξ=0.63227), $\xi > 0$, it implies that it is heavy tailed distribution. The GPD is applicable in modeling huge losses, high shape parameter β, imply high derived return. The BGPD would have performed better than the BGEV based on model selection criteria if the threshold was taken from the lower tail (0.032), with AIC, BIC, and DIC of -504.2564 -488.1609 and -507.1196 respectively, β =0.0224357, ξ =1.0684673, but returned unrealistic Expected shortfall values such as -236, and since the interest is the risk modelling, we stick to the threshold value of the upper tail. Another important plot is the tail estimate plot presented in Figure 6.

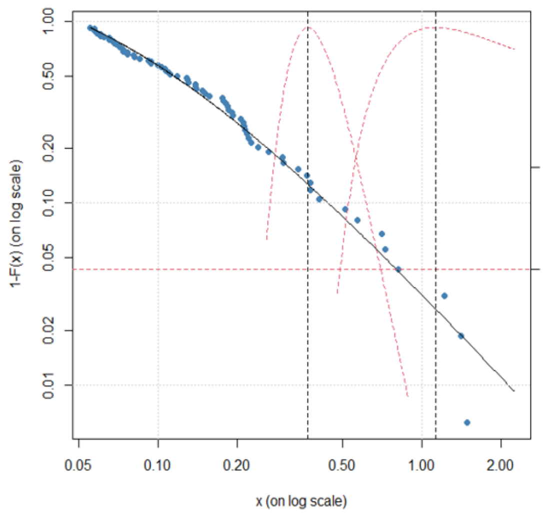

Figure 6. Tail estimate plot

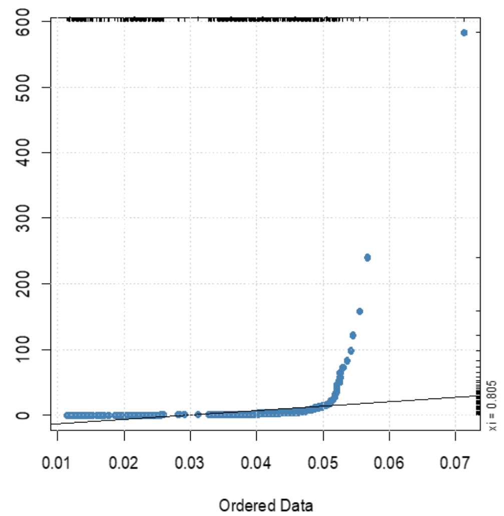

Figure 7. Exploratory QQ plot

A smooth curve is observed in the tail plot in Figure 6, which shows that the GPD model suitably fits the data with most of the points aligning on the curve. The Exploratory Quantile-Quantile plot is presented in Figure 7, number of observations above the threshold was 75.

Most of the points fall on the curve in the exploratory Q-Q plot. If further shows that the parametric model (GPD) is suitable to fit the data at the chosen threshold. Figure 8 is the histogram of the parameters of BGEV.

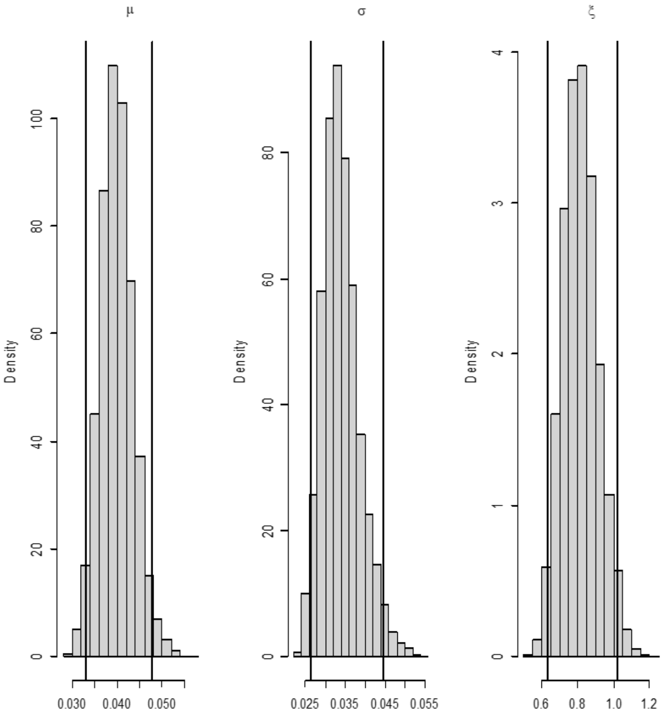

Figure 8. Histogram of the BGEV

Figure 8 shows the parameters estimates of the location $(\hat{\mu})$, scale $(\hat{\sigma})$, and the shape $(\hat{\xi})$, it can be observed that the plot adequately represents the estimates provided in Table 3. The value at risk and expected shortfall is presented in Table 4.

Table 4. GPD value-at-risk and expected shortfall

Model	Prob	VaR	ES
GPD	95%	0.04961	0.05340
	99%	0.05299	0.05920
BGEV	95%	0.00103	0.06978
	99%	0.03879	0.06854

The 95% quantile VaR of the GPD model is 0.04961, meaning that the NGN/ZAR VaR returns is 4.96%. This means that on a particular day of 20 days, there is a 95% chance of losing at least 4.96% of NGN against the ZAR based on the exchange rate of May 06, 2024 (₦75.15). If the losses happened, the average return (or loss) expected in the worst 5% of cases is 5.34%. The 99% quantile VaR of the GPD model is 0.05299, meaning that the NGN/ZAR VaR returns is 5.3%. This means that under normal market conditions, on a particular day of 100 days, there is a 99% chance of losing at least 5.3% of NGN against the ZAR. If the losses happened, the average return (or loss) that would be expected in the worst 1% of cases is 0.0592 (5.9%).

The 95% quantile VaR of the BGEV model is 0.00103, meaning that the NGN/ZAR exchange rate VaR returns over is 0.10%. This means that on a particular day of 20 days, there is a 95% chance of losing at least 0.10% of NGN against the ZAR. The average return (or loss) expected in the worst 5% of cases is 6.9%. The 99% quantile VaR of the BGEV model is 0.03, meaning that the NGN/ZAR exchange rate VaR returns is 3.8%. This means that on a particular day of 100 days, there is a 99% chance of losing at least 3.8% of NGN against the ZAR. The average return (or loss) expected in the worst 1% of cases is 0.06854 (6.85%).

The VaR and ES help investors and financial managers understand the potential downside risk in their currency positions and make informed decisions based on their risk tolerance. This helps investors or risk managers gauge the potential severity of extreme losses and prepare for them accordingly. This study agrees with reference [36] who mentioned that Nigeria's currency is in chaos and gave various reasons why it is so and identified the need to change the foreign exchange regime. So, this study helps businesses and individuals to be aware of the risk they are exposed to concerning the NGN/ZAR exchange rate.

4. CONCLUSION

This study applied various models to fit the exchange rate time series data of the Nigeria Naira and the South African ZAR. Various diagnostics checks were made such as stationary tests, normality tests, threshold plots, and tail plots. Classical GPD model, and Bayesian models EVT models such as the Bayesian GEV, Bayesian GPD, and Bayesian Gumbel were fitted to the data and estimate the model parameters accordingly. The best model was selected based on model selection criterion, hence the BGEV was found to be most suitable to measure the tail risk. The Value-at-Risk and expected shortfall were computed at different quantiles to determine the exchange tail rate risk associated with NGN/ZAR. The results showed that the Nigeria Naira will fall significantly against the ZAR under normal market conditions and measures should be taken to mitigate against the losses. This study recommends that investors and risk managers should adopt the result of this study for business and investment decisions. Also, the Nigerian government should implement policies that will strengthen the Naira in the foreign exchange market so that the currency will be less volatile. The Nigerian government should implement policies such as reducing reliance on imported goods and reviving Nigerian industries. Future work may consider applying the Bayesian EVT models to estimate the exchange rate risks of some major world currencies. This study aligns with the sustainable development goals (SDGs), 8.1 (sustainable economic growth), SDG 8 "to strengthen the capacity of domestic financial institutions".

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