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On Stochastic Modelling: The Impact of Advertisement on the Consumption - Application on ChatGPT-3

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ABSTRACT

Studying the impact of advertising on consumer consumption of a product is very important because it enables us to understand the relationship between advertising and consumption in order to adopt advertising strategies to increase consumption or sales of a product and in political fields to increase the number of voters for a party, etc. This article aims to model the relationship between advertising and consumption to enable it to be better understood and used. We will use the basic model of diffusion and adoption to find the relationship between advertising and consumption, where adoption will play the role of a mediator in this relationship. We will apply the model to a real dataset on the consumption of an electronic product ChatGPT-3 over a period of 9 months where the model succeeds in real data tests. We will find that advertising directly affects the consumption of the product, and we will use the model to find out the number of adopters needed for a product to maximize the consumption of the target product.

1. INTRODUCTION

Advertisement, in today's world is a very important tool used by companies and businesses to introduce their innovation or product into the world [1], its importance rise in giving the individual who could become a potential consumer of the new product, the necessary information about this latter.

Advertisement plays a significant role to convince people and make them trust the product that the company offers [2], this convincing is based on the information about the product and also to meet the needs of the individuals, this makes people aware of the product, and also plays on their emotions to grab their attention and make them like their product and think they really need it [3].

Therefore, advertisement plays a vital role into convincing people to try to first adopt a product and this will get them into the adoption process that we will explain more later, it has a huge impact on the decision making of an individual and what should he purchase as a product [4], also the marketing strategy used by the company will determine how many people will be consumers at the end [5].

From the above, the following question comes to mind: What is the relationship between advertisements and product consumption? This is an important and pivotal question, as we will answer this main question in this paper.

Where we will investigate the effect of advertising on the consumption of an unknown product, and that by looking for the relationship between adoption and consumption, in the first part we will model the impact of advertisement on the adoption of a product and that by using the fundamental diffusion model, and that by assuming the number of individuals who are exposed to the advertisement, a percentage of them will become adopters of the product.

The number of people who will adopt the product will increase when they are first exposed to the ad until this number reaches its peak and then will drop as an explanation that people will become less interested in the advertisement and it will become familiar for them [5], in our model we will use a function that we call a filter function that will help us to model this situation.

In the second part, we will focus on finding the relationship between adoption and consumption, relying on the assumption that the number of individuals who will adopt the product, a certain percentage of them will eventually consume the product.

In the final part of our work, we will test our assumptions by applying our stochastic model to the Chat-GPT3 product, where we will consider users as consumers and our model will track the number of Chat-GPT3 consumers over a nine-month period and then we will use the model to determine the number of adopters needed to maximize consumption of a product over a period of time.

2. LITERATURE REVIEW

For many years, researchers have taken an interest in determining the effects of advertising on consumption or consumer.

In this literature review, we will explore some of the most

important research papers that have addressed this topic from several aspects, whether types of advertisements or the effects of advertisements on the choices of potential consumers. These studies were published in the past years, starting from 2011 to 2024, to somewhat cover this topic and know its most important aspects.

Many researches have been conducted to clarify how advertising works in purchasing and persuasion, and what can be done about advertising strategies.

The study of the impact of advertising on consumer brand preference was done by Ayanwale et al. [6]. Advertising plays a huge role when it comes to influencing consumer preferences.

This study revealed that well-executed advertising actually attracts consumers to better brand loyalty in a smart way. In this sense, the argument of this paper gains importance, as more effective advertising is supposed to increase brand preference and thus enhance consumer commitment to purchase.

Kelly-Gagnon [7] illustrated how the influence advertising has over individual buying decisions is only one layer within a larger economic context. Advertising is also a stimulant of economic growth. This article highlights by stating that stimulation for consumption with the products will translate into production and as an effect activate the economy in each country. This illustrates an important role of advertising in microeconomic as well as macroeconomic perspective.

Advertisements can be divided according to the advertising medium, where advertising via the Internet is considered the most common type of advertisements, which is the most widespread advertising method today as well, and which plays a great attractive force, as discussed by Srivastava [8], which examines the impact of online advertising, especially through emails, on consumer behavior. Srivastava concludes that online advertising can effectively reach a wide audience and influence their decision to adopt the advertised product, stimulating consumer actions such as clicking and making purchases.

Advertisements play a significant role in influencing the decisions of potential consumers. Kumar and Raju [9] highlighted that advertisements play a crucial role in consumer decision making by effectively communicating information to the target audience. The article suggests a strategy that advertisements should be attractive and concise to attract and retain consumer attention.

In the context of television advertising and its impact on consumption, Hassan [10] compared the effects of television advertising on rural and urban consumers, noting that while both groups are significantly affected by advertising, the impact differs based on demographic factors as well as the target group of the advertisement.

Similarly, Dhaliwal [11] also explored how advertisements lead to affective behaviour among the consumers. He found that the use of emotional and informative appeals in advertisements has a great influence on consumer attitudes as to whether they are willing or not, deciding from the consumption decision-making process that generally if taking care of an advertisement considers it to be emotionally influential.

Sofi et al. [12] presented that commercials or advertisement predicts in a powerful way the consumers consumption behavior, and this relationship gets so strong when advertisements are working in a very proper and in an efficient way and so Sophie, al have shown that advertisements leads to positive results in predicting people purchasing behavior, and

on the same study Sophie, al have found that as long as the advertisements are being shown regularly and frequently to the costumers , the latter gets familiar to the brand or product and it leads them to make a stronger bond with the product, and this influence people purchasing the product which has advertisement on them.

Wang and Gambaro [13] presented the relationship between advertising and consumption, explains the more direct relationship between advertising spending and levels of consumption. Higher ad spending is often correlated with increased consumer spending, Wang and Gambaro [13] explained a close connection between how much companies are willing to invest in advertising and what consumers have available as capital. This research supports the argument and is rooted in that advertising drives consumer behavior as well economic vibrancy, hence emphasizing how important advertisement plays a role among companies. However, to paint an advertising picture of the coin is understood to its consumption part.

Chavan and Deo [14] have worked on the impact of advertisement on the consumption behavior and purchasing ,according to their study ,that has led to the important result that is when the advertisement is efficient and works properly ,it increases the awareness of the customer about the product and also it increases his perception on the quality of the product on sale, and it leads also to brand awareness, and can change the preferences of a consumer and thus advertisement may lead to higher and big demand.

The authors have shown also that there is a big relation between the performance and volume of advertising on the volume of sales of a certain product and have found that by statistical analysis of survey and sales data, in addition to these results in their paper the authors have found that the maintaining of the interest of the consumers and improving the market, it demand commercials that are innovative and that targets a specific population of consumers, in this way the companies and businesses can increase their total sales using commercials to get to a new kind of customers that will certainly maximizes their revenue.

Goel and Dewan [15] took a contemporary view by deciding to explore the influence of social media on consumer adoption (the process or decision made earlier) of the advertised product in their article Influence of Social-Media Marketing upon Consumer Decision-making. Their research implies that social networking sites are an effective tool for advertisers to expose customers and increase loyalty by engaging with interactive content.

In the same context, Gupta [16] have reached the results, he had explored and determined the effects of advertisements on the purchasing behavior of the Shmitten advertisement where Surat city premises were utilized in his study, where he has targeted 100 respondents who had seen the Shmitten ad and he has found out that advertisement has a positive impact on consumer purchasing behavior.

Gupta [16] has shown that when people get exposed to an ad and they get aware of the brand or name of the product or the information about it, and when people have made a positive idea from the brand and advertisement on it and they make a good image about it then the brand and its reputation gets in their minds and so it affects their purchasing behavior. Also, in his research Gupta have studied which types of advertisements are most influential over people's purchasing behavior and he has found that the most influencing type of advertisement is television advertisement where his study

gathered a number of people and out of 127 people, 107 people agreed on the fact that TV ads are the most influential than any other type of advertisement.

The study of Gupta [16] has also discovered that for the methods of online advertising people believe that social media advertisements are the most effective and influential than any other method for online advertising, and so to summarize, Gupta has shown the impact of advertisement on consumer electronics purchasing behavior, and so customers are influenced by advertisement and their buying behavior.

Returning to online advertising, Saranya [17] goes on record with a comprehensive study namely; The Impact of Online Advertising on Consumer Purchasing Behavior asserting how digital marketing formulates consumer purchase behaviors. Main implication of the study shows that online advertising can be a persuasive tool to promote consumer behavior by inducing higher product awareness and influencing preferences. With digital, you can fish in the ocean guaranteeing broad reach and higher audience diversity for improved product awareness.

These papers might suggest or the concerned authorities who commissioned them may feel a part of some variation in high-definition and quality pictures accompanied by narration, placing interactivity opportunities to have more inclination towards interest from consumers making possible for desired friendly image creations leading to brand preference generation due exposure. Interactive ads with quizzes or clickable objects, as well visually appealing adverts grab more attention and increase engagement rates which in turn create meaningful experience that directly affects how consumers feel about the brand. One of the biggest factors in successful online advertising comes down to personalization, ads that cater specifically to what people like and how they behave. With the help of data analytics, companies can offer an ad experience tailored to each visitor translating into higher conversion rates.

While platforms like Facebook and Instagram rely on user-generated content, influencer endorsements etc., for credibility to the advertising campaign (as they are more sensitive towards what is shown in their feeds: supply-and-demand), search engines like Google can target consumers who actively looking online so that ads would be helpful instead of being disruptive.

Good online advertising also creates closer bonds with a brand, encouraging consumers to engage through likes and shares of the material by promoting their interaction towards the creation of communities capable of being loyalty, good online advertising supports the existence of a brand by maintaining it on screens all over the digital district, thus ensuring that a considered decision to buy there is far less likely not get driven - and when someone buys your product more than once.

Effective ads of the type that draw attention, engage consumers and build awareness are more likely to serve as a stimulus for purchase behavior, help guide shoppers through their shopping journey and convert prospects into customers.

The convenience of shopping online and the persuasive nature of ad content simplifying the process often result in more sales which means higher revenues to companies ,this article gives you a great understanding of how consumer behavior is changing in the world and emphasis on strategic, engaging & personalized advertising content to grab consumers attention for purchasing ,as landscape of digital marketing constantly matures, advertising strategies should adapt to ensure the investment is used properly across all online platforms for a more effective and efficient advertising creation which improves consumer engagement whether it be an improved brand name recognition or provide value in terms sales [17].

One of the Latest works in the year, was done by Chatterjee [18]. Chatterjee [18] has demonstrated the importance of advertisement on persuading customers and how the seller could use the tool of advertisement which is the art used by the seller's and businesses to turn the attention of customers into a profit in sale according to what Chatterjee [18] said if the businesses reach and get in to the heart of consumers and that by giving priority to their needs , advertisement is a great tool to do so , and thus it will increase the demand and volume sales of the company or business by a larger scale, and to influence consumers. Marketer's and people in control of the company used various social media platforms, and other types of commercials to reach their financial goals.

Chatterjee [18] has used an online survey for collecting data from customers, where he has chosen a sample consisting of 100 consumers where 65 are males and 35 are females, and his collecting of the data was obtained by using the interview method to get answers from these number of people and that by giving them a questionnaire that is containing of 14 questions, Chatterjee [18] obtained the following results:

In the first part of his study and to find the influence of advertisement towards customers, and in this part, he has found that 34% of the consumers had a larger influence and so advertisement in all its types has a huge impact and influence according to them, and only 4% they say no it does not influence them.

In the second part in his paper, Chatterjee [18] looked at the part which focuses on the preference of the people in advertisement, whether consumers look at advertisements for product price or information or to see the brand that would encourage them to buy the product, in this study he wanted to know what are the parameters that plays on people's mind to look at a certain advertisement, and in this study he has found that 40% the majority of the costumers have price influence and only 10% are influenced by celebrity performed ad.

In the third part of his work, Chatterjee [18] wanted to know about online advertising if they are seen and people made attention to them or not, in this part his study has shown that 37% of consumers pays attention to online advertisement and 7% who don't, and there is 56% of costumers that are indifferent between paying attention or not who sometimes pays attention to online advertisements.

To sum up, Chatterjee [18] has found that 34% of consumers are highly influenced by advertisement and a majority of 40% of these consumers observe the price information in advertisement followed by discount related information.

Chatterjee [18] has made it to the conclusion that if the businesses and companies have used the advertisements in an efficient manner, this would lead them to find potential customers that would buy the product and this would lead them into attaining higher financial success.

Through this review of the literature, we note that the relationship between advertisements and consumption of a particular product is a strong relationship, as advertisements play an important role in influencing consumer decisions and convincing them to consume. We will explain this relationship in this article and model it mathematically.

3. METHODOLOGY

Before going into the details of our work, we will need some definitions, without which the understanding will not be complete, and which you will learn about below.

3.1 Adaptation

3.1.1 Definition
Adaptation is when the individual begins to use the product continuously for a certain period of time.

3.1.2 The adoption process
According to Rogers, the adoption process can be presented in five basic stages as shown in Figure 1, where this process is considered a learning process [19]
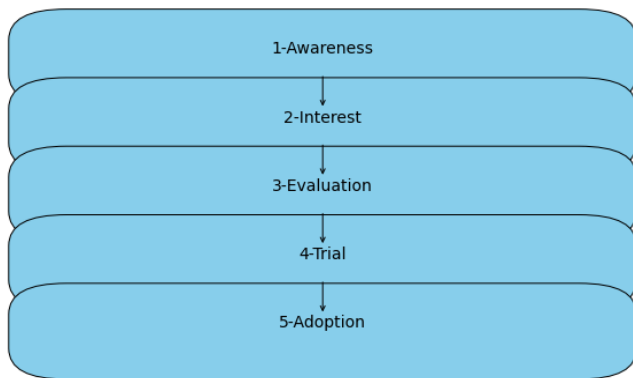

Figure 1. The adoption process

1. Awareness: the individual becomes aware of the existence of the product or the innovation, but does not have information about it.
2. Interest: the individual seeks information on product or innovation.
3. Evaluation: the individual mentally evolves the attributes of the product or innovation and wonders if he should try it.
4. Trial: the individual tries the product or innovation in a short period of time.
5. Adoption: the individual decides to continue using the product or innovation.

It should be noted that an individual's adoption of a product does not mean that he has become a consumer of the product.

We will give the difference between adoption and consumption in the following paragraph in Section 3.1.3. In the process of adopting a product, as shown here, it is the initial stage, meaning that the individual goes through the 4 stages that we mentioned to reach the adoption stage. The most important stage is stage 1, where the individual is aware of the existence of the product. This comes through and this is the pivotal role of advertising, where advertising contributes directly and fundamentally to the individual's awareness of the existence of the product. The second most important stages are stages 3 and 4, the evaluation stage, where the individual forms an image of a product and whether the product is worth trying. Then comes the experimentation stage, which is a pivotal stage here, where the individual tries the product for the first time and learns about the benefits of this product and its importance and whether it matches what was announced in the product's advertisement, because advertising is the initial

spark that enters the individual into this process.

3.1.3 The difference between consumption and adoption
Adoption depends on the initial decision to start using the product, and adoption is a process that must be achieved in all stages in order to reach it. As for consumption, it depends on the ongoing use or actual use of the product and the exhaustion of the value of the product.

3.2 Fundamental model of diffusion

To model the relationship between advertising and consumption, we first need to model the relationship between advertising and product adoption. To model this relationship, we need the basic diffusion model, which is the basis for all models based on the differential equation that aim to study diffusion, whether it is influence, information, or other things.

We will need this model to model the effect of advertising on the diffusion of information.

It is written in the form of the following differential equation:

$$\frac{dN}{dt} = h(t)(\bar{N} - N(t)) \tag{1}$$

$h(t)$ represents the diffusion coefficient, which is limited between 0 and 1 and can be constant or change over time.

$\bar{N}$ represents the total number of adopters of the social system or the total population sample. $N(t)$ represents the cumulative number of adopters. At time t, that is, the number of individuals who started using the product or information. Information here means whether the name of the product, its features, benefits, disadvantages, etc.

3.3 Filter function

To measure the impact of advertising on the spread of information or its adoption, i.e., product adoption, we will create a new function that we will call the filter function. This function gives the coefficient of adoption of the advertised product, i.e., the percentage of individuals who adopt the information or product during each moment from the beginning of their exposure to the advertisement.
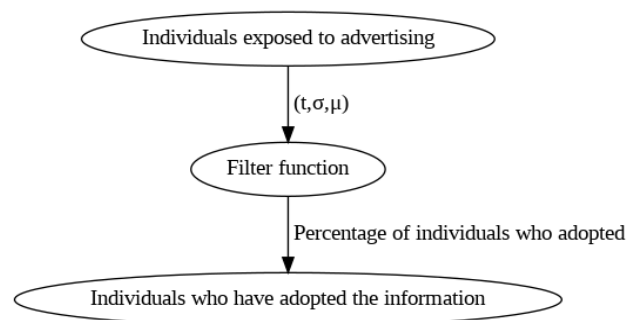

Figure 2. Filter function diagram

We define the filter function as follows:

$$\mathbb{R} \rightarrow [0,1]$$
$$t \rightarrow h(t)$$

where, $h(t) = \psi \exp\left(\frac{-(t-\mu)^2}{2\sigma^2}\right)$. So, σ and ψ are constants with

$\psi = \exp\left(\frac{-\mu}{2\sigma^2}\right)$.

We called this function the filter function because it plays the role of filters. This function gives the percentage of individuals who were exposed to the advertisements and may adopt the product in the future (Figure 2).

3.4 Modeling the impact of advertising on the adoption of a product

The process of adopting a product or information is a difficult process and requires a lot of time. The human ability to remember and make decisions is restricted [20].

The role of advertising is to make this process simple, give the individual sufficient information to make a decision, and make the agent interested in adopting the product. However, this interest does not last long. After a certain period of time, the individual loses his interest in the product, and this period ranges from 4 to 8 weeks [20].

Each advertisement targets a certain percentage of population. This category represents a percentage β of the target population sample, where β is between 0 and 1.

When we assume that the advertisement reaches the peak of its impact after a period of time, we will symbolize it as μ.
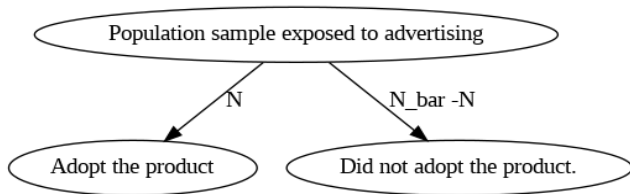

Figure 3. Flowchart of the adoption process

Based on the adoption process that we mentioned previously before the advertisement, there is no impact of the ad. During the beginning stage of the advertisement, a number of the population sample $\bar{N}$ is exposed to an advertisement. After this sample is exposed to an advertisement, it is divided into a part affected by the advertisement, that is, the part that began to adopt the product or information we symbolize it with N, and a part that was not affected by the advertisement, that is, the part that did not adopt it (Figure 3).

We will call this stage, the filtration stage. After this stage, only the individuals who were affected by the advertisement will remain, and we will call this stage the post-filter stage (Figure 4).
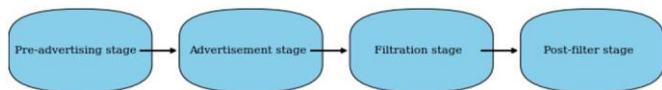

Figure 4. Stages of advertising influence

To model the evolution of the number of people who adopt the product over time, we will use the basic diffusion model.

$$\frac{dN}{dt} = h(t)(\bar{N} - N(t))$$

where, we will replace $h(t)$ with the filter function in the basic diffusion model, where the filter function plays the role of the product adoption factor over time, and thus we obtain a model of individuals who adopted the information over time as

follows:

$$\frac{dN}{dt} = (\bar{N} - N(t))\psi\exp\left(\frac{-(t-\mu)^2}{2\sigma^2}\right) \quad (2)$$

$\psi = \exp\left(\frac{-\mu}{2\sigma^2}\right)$ represents the target percentage of advertisement, μ represent the time during which the advertising effect reaches its peak, and we assume that $\sigma = \frac{\mu}{2}$ to simplify.

Therefore, the final model will be as follows:

$$\frac{dN}{dt} = (\bar{N} - N(t))\,\psi\exp\left(-\frac{2t^2 - 4\mu t + 4\mu^2}{\mu^2}\right) \quad (3)$$

where, N represents the number of people who adopted the product and $\bar{N}$ (we assume it is constant) represents the number of people who were exposed to the advertisement

3.5 Modeling relationship between the impact of advertising and consumption

Advertising is an investment for the company and affects the credibility of the company and the recognition of its adopters as well as the consumption of its products [21].

The more balanced the advertising strategy is, the longer the impact of the advertisement will be and vice versa. The more intense and fiercer the advertisements are, the less their long-term impact will be [19].

That is, the more individuals adopt the product or information at moment t, the more the product will be consumed.

After the individual enters the adoption process, there are two possibilities, as we mentioned previously, in the adoption process, whether the individual adopts the product or vice versa after going through the product adoption process and we assume that the individual adopts the advertised product, there are also two possibilities, whether the individual becomes a consumer of the product or not (see Section 3.1.3), that is, he only adopts the product and does not become a consumer, see Figure 5.
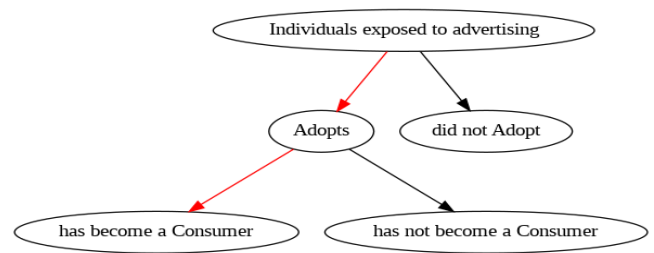

Figure 5. A diagram of how an individual becomes a consumer

Thus, the mediating factor between the effect of advertising and consumption is the adoption factor, as we assume that this relationship is a logarithmic relationship, and this hypothesis is based on the article in reference [22], as this article shows that the relationship between advertisements and their effect on consumption is a logarithmic relationship. To model this relationship, we will symbolize the number of consumers of a product at moment t with C_α, and C_0 is the number of consumers at moment t_0, and we will symbolize N_0 as the initial population who adopted the product at moment t_0, and

thus this relationship can be expressed as follows, assuming that it is a direct relationship, i.e., it is written in the form:

$$C_\alpha = C_0 + \alpha Ln(\frac{N(t)}{N_0}) \qquad (4)$$

α represents the sensitivity of consumption to a logarithmic change in the number of consumers, or more precisely, α represents the strength of the response of consumption to a relative change in the number of consumers.

We will derive $C_\alpha(t)$ with respect to time to obtain a model for calculating the change in consumption over time, so we obtain:

$$\frac{dC_\alpha}{dt} = \alpha \frac{1}{N(t)} \frac{dN(t)}{dt} \qquad (5)$$

By replacing $\frac{dN(t)}{dt}$ with its form in Eq. (3), we obtain the form:

$$\frac{dC_\alpha}{dt} = \alpha \frac{1}{N(t)} (\bar{N} - N(t)) \psi exp(-\frac{2t^2 - 4\mu t + 4\mu^2}{\mu^2})$$

To simplify the model in terms of writing, we put $\xi(t) = \frac{\psi}{N(t)} (\bar{N} - N(t))$, and thus the model becomes as follows:

$$\frac{dC_\alpha}{dt} = \alpha \xi(t) \exp(-\frac{2t^2 - 4\mu t + 4\mu^2}{\mu^2}) \qquad (6)$$

In Model (6), we do not take noise into account. To make the model more accurate and realistic, we will add $\epsilon(t)$ to take randomness into the model.

$\epsilon(t)$ is random noise, where:

$$\epsilon(t) \sim \mathcal{N}(0, \lambda^2) \qquad (7)$$

λ represents the standard deviation of the noise.

The final model representing the change in consumption becomes as follows:

$$\frac{dC_\alpha}{dt} = \alpha \xi(t) \exp(-\frac{2t^2 - 4\mu t + 4\mu^2}{\mu^2}) + \epsilon(t) \qquad (8)$$

To calculate the value of α, we will use linear regression, through Eq. (4), we get:

$$C_\alpha = C_0 + Ln(\frac{N(t)}{N_0})$$

Let us take

$$x_i = Ln(\frac{N(t_i)}{N_0}); \ y_i = C_\alpha(t_i) - C_0$$

So

$$y_i = \alpha x_i$$

Using the linear regression rule to estimate α, we obtain the estimated α:

$$\hat{\alpha} = \frac{n \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_i \sum_{1}^{n} y_i}{n \sum_{i=1}^{n} x_i y_i (\sum_{i=1}^{n} x_i)^2}$$

where, n represents the number of data points.

In this part, all the images were created using the Python programming language, and specifically we used the graphic library, as we chose to create the images using the digraph tool found in this library, due to the speed of working with this tool and also the ease of use.

4. SIMULATIONS AND RESULTS

4.1 Data description

In this part, we will apply the consumption model to real data related to the consumption of an electronic product. This data is related to the number of visitors to the ChatGPT-3 website for a period of nine months, starting from the beginning of its announcement. Here, we will know that an individual's visit to the Chat-GPT website indicates that he has consumed the product, and the product here is ChatGPT-3.

The data we have is only related to consumption at moment t, where this data consists of two columns, the column for the time period and the column for the number of consumers, where we assume that the individual's visit to the site indicates that he has consumed the product, which is ChatGPT-3. We collected this data from the similarweb.com website [23], which specializes in collecting information about pages and websites and providing statistics on the number of visitors and other information related to the site targeted for study. The Table 1 below represents the data that we will work on to simulate the model on it.

Table 1. Data

Time Period	Visits Count
November 2022	152700000
December 2022	266000000
January 2023	616000000
February 2023	1000000000
March 2023	1600000000
April 2023	1800000000
May 2023	1800000000
June 2023	1600000000
July 2023	1500000000

The Figure 6 below shows the number of individuals who consumed ChatGPT-3 during the 9 months between 2022 and 2023, assuming that visiting the ChatGPT-3 product website is considered consumption of it.
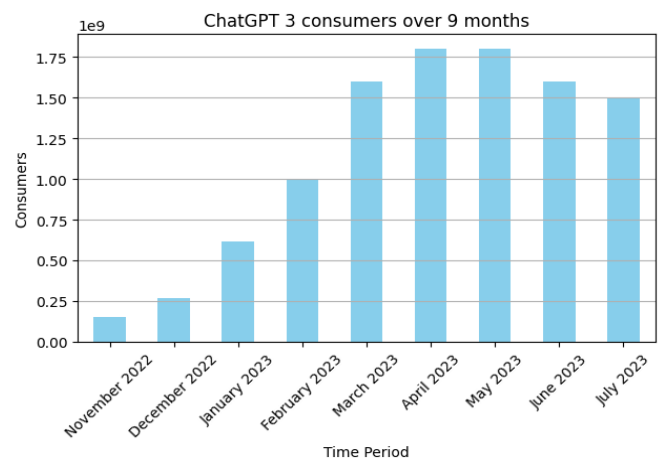

Figure 6. Data graph

The tools used in this part for drawing and simulating whether data or the model are as follows, where we will rely on the Python programming language to draw curves and also to deal with data and compare it with the model in the following paragraphs, where we will use the following libraries: NumPy to deal with mathematical functions and others, matplotlib to draw curves and represent data, and pandas to deal with data and retrieve it, display it, and perform operations on it.

4.2 Simulate the model and compare it with the data

In this section, we will simulate the $\frac{dC_\alpha}{dt}$ model alongside the data for ChatGPT-3 consumption over a period of 9 months. The image below represents the simulation of the model alongside the data, where the yellow curve represents the development of ChatGPT-3 consumption over time, and the blue dots represent the real data (see Figure 7).

Where we got the parameter λ by observing the real data and the linear regression.
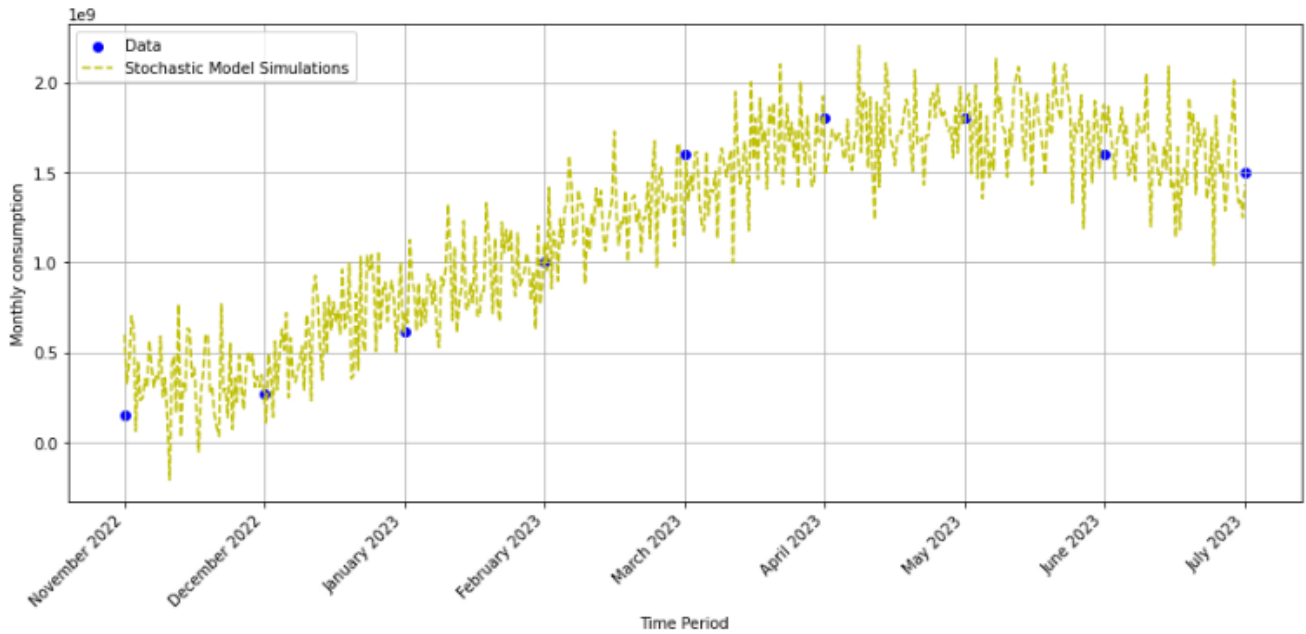

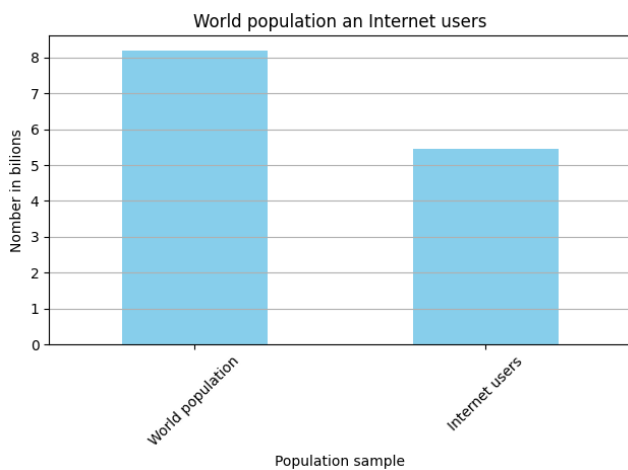**Figure 7.** Comparing the stochastic model with the real data with λ=0.20

Figure 8. World population sample and number of Internet users

The parameter β represents the target sample for the advertisement made by OpenAI. We assume that the company targets all individuals with access to the Internet in the world. This sample represents 67.1% of the world's population, or 5.44 billion Internet users (see Figure 8), according to the website statista.com, which specializes in collecting this data [24].

So β=67.1%. and through observation, the value of α is estimated at 0.73×10^9, where we simulated the model several times to obtain alpha, and we will take μ=6, six months, which

is the period in which consumption reached its peak, and we observe this through the data.

The data in our hands does not contain information about the people who adopted ChatGPT-3 or information before consumption. To know the relative change in the individuals who adopted the product or information at the moment t, we will use relationship (5) to obtain the relationship between the relative change in the number of adopters and consumption.

$$\frac{1}{N(t)}\frac{dN(t)}{dt} = \frac{1}{\alpha}\frac{dC_\alpha}{dt} \tag{9}$$

Knowing consumption enables us to know the number of adopters over time and vice versa.

We notice that the number of individuals who adopt the product is greater than the number of individuals who consume the product. This is very logical, as at the beginning, after the hypothesis is exposed to an advertisement, we assume that the individual has entered the process of adopting the product.

After the adoption process, the individual makes the decision whether to consume the product (refer to the definition of consumption and see the difference between it and adoption at the beginning of the article) or to stop, i.e., adopt the product only. This explains that the number of individuals who adopt the product is greater than the number of individuals who consume it, as in Figure 9. We notice that the number of adopters is greater than the number of consumers, as the red curve represents the number of adopters of the product.
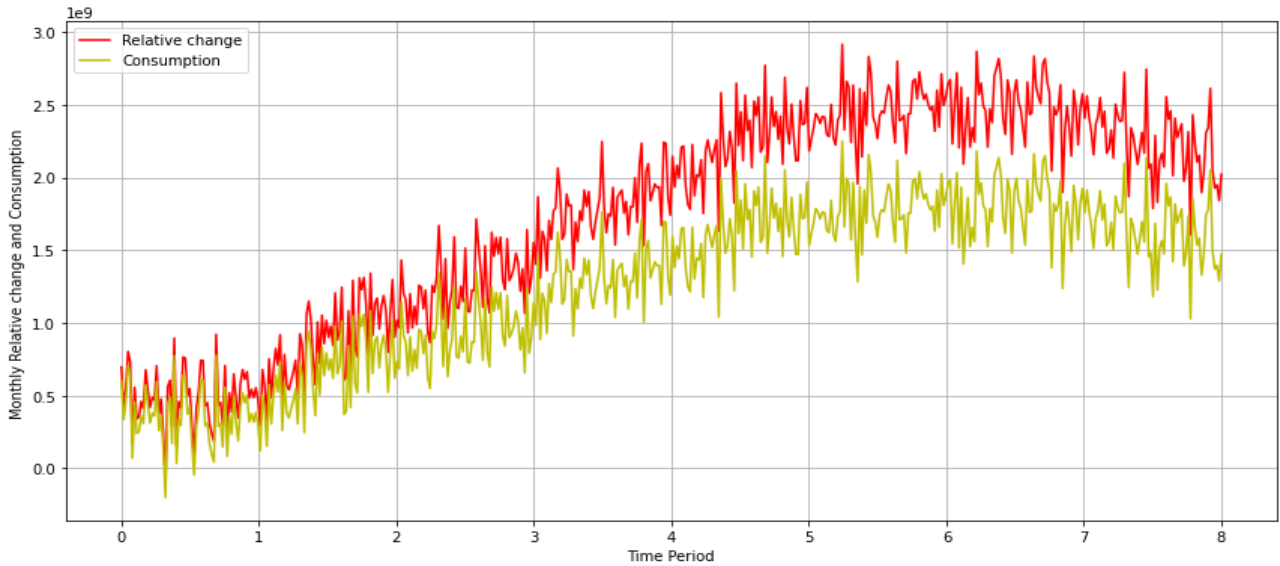
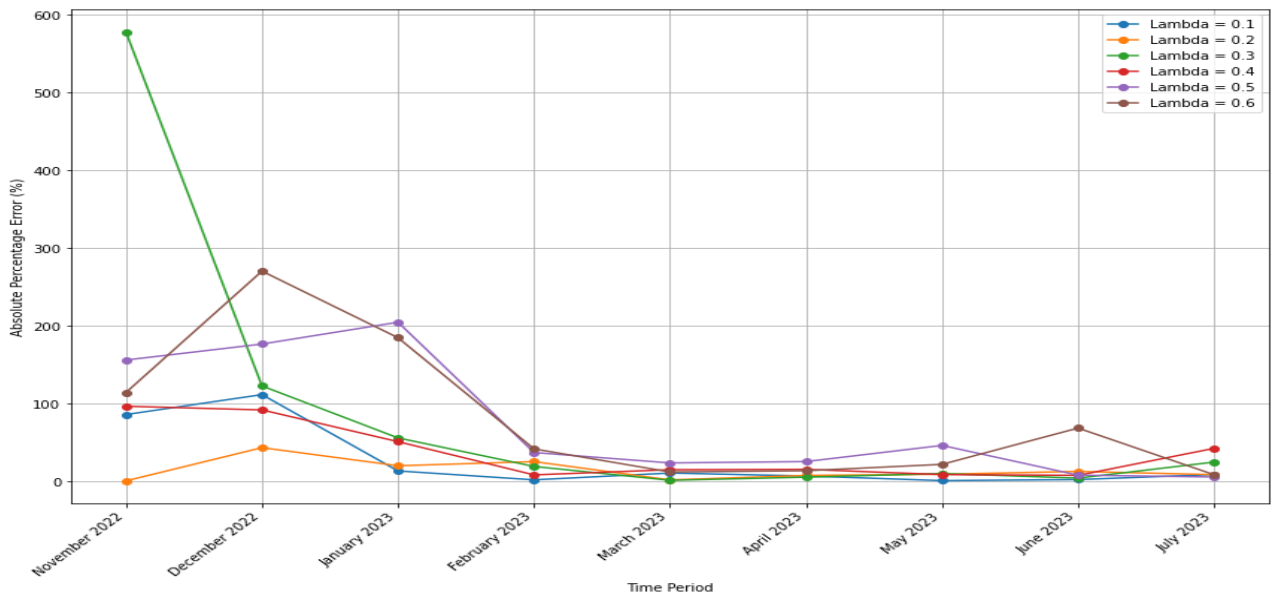**Figure 9.** Relative change and consumption

Figure 10. Simulating the absolute percentage errors

4.2.1 Measuring the effectiveness of the consumption model

In the simulation, we notice that the model is close to the real data. To find out the effectiveness of the model, we will calculate the absolute percentage errors, knowing that the absolute percentage error is also related to λ.

We take y_i data represents the moment t_i and y represents the model at moment t_i:

$$\widehat{y_i} = \alpha\,\xi(t_i)\exp\left(-2\frac{t_i^2 - 2\mu t_i + 2\mu^2}{\mu^2}\right) + \epsilon_{i,t}$$

Therefore, Absolute Percentage Error Functions (APEF) for the model are written as follows:

$$APEF_\lambda = \left|\frac{y_i - \widehat{y_i}}{y_i}\right| \times 100$$

$$APEF_\lambda = \left|\frac{y_i - \alpha\,\xi(t_i)\exp\left(-2\frac{t_i^2 - 2\mu t_i + 2\mu^2}{\mu^2}\right)\epsilon_{i,t}}{y_i}\right| \times 100 \qquad (10)$$

In the Figure 10, we notice that the model's error rate in prediction changes according to the λ parameter, and this is a pointer because λ affects the random part in the model. To reach a lower error rate, λ must be estimated after each period, but we take λ constant over time by simplifying the model.

We can reduce the error rate in the model by applying the model to a data set specific to the product to be studied to know and estimate the parameters specific to the targeted product, as each product has its own set of parameters that must be estimated by simulating the model on data specific to the product to be studied.

5. MAXIMIZE THE NUMBER OF CONSUMERS

The importance of the model lies in the fact that it links the relationship between the impact of advertisements on consumption in another way, the impact of advertisements on individuals' decisions to consume a product or not.

The model can be used to maximize the number of

consumers, i.e., to solve the following problem:

$$C_\alpha = C_0 + \alpha Ln\left(\frac{N(t)}{N_0}\right) \geq H \qquad (11)$$

where, H represents the number of consumers to be reached over time.

After solving the equation, we obtain the following condition:

$$N(t) \geq N_0 e^{\frac{H-C_0}{\alpha}} \qquad (12)$$

Therefore, to reach the number H of consumers, condition $N(t) \geq N_0 e^{\frac{H-C_0}{\alpha}}$ must be met, i.e., the company or selling entity must target a specific number of adopters of the guarantee to reach the number H of consumers, which can be calculated using inequality (12) above.

Accordingly, by using the model, we can develop strategies to maximize the consumption of a product by focusing on the number of adopters, as adoption affects consumption in a direct way. After the adoption, process comes the consumption process directly, and advertising plays the role of a motivator to make individuals enter the adoption process. Therefore, we can say that in order to maximize consumption, adoption must be maximized.

6. CONCLUSIONS

The relationship between advertising and consumption is a complex relationship, where in this article we modeled the relationship between advertising and adoption on the one hand and between adoption and consumption on the other hand, where adoption plays the role of a mediator in this relationship, i.e. the relationship between advertising and consumption. Thus, we reached the relationship between advertising and consumption based on the basic model of basic diffusion and the logarithmic relationship between adoption and consumption. To make the model take into account the random factors that can affect consumption, we introduced the randomness factor by adding a random variable that takes the natural law so that the model that models the relationship between consumption and advertising becomes more realistic and more accurate.

After that, we tested the model on real data for the consumption of an electronic product, which is ChatGPT-3, and we considered that the consumption of the product in this case is that individuals will visit the site a certain number of times. Then we used the model to develop a strategy to maximize the number of consumers by knowing the number of adopters required for consumption to reach its maximum value.

The importance of modeling the relationship between advertising and product consumption is in developing effective strategies to target a larger number of consumers and also developing systematic and more influential advertisements, as well as predicting the consumption of the consumer product and developing more accurate marketing plans.

However, the model needs to be developed by researchers and include factors such as frequent advertising of the product, which is very important, as well as a competitive factor and how it affects consumption, etc., so that the model becomes more accurate and more practical, and this will be part of our next work.

In conclusion, studying the relationship between advertising a product and its adoption and consumption is an important and useful issue for companies and others, and many psychological, social, economic, political and other factors intervene in it, which makes it complex.

7. PERSPECTIVES

The model in our hands

$$\frac{dC_\alpha}{dt} = \alpha \frac{1}{N(t)} \frac{dN(t)}{dt} + \epsilon(t)$$

$$\frac{dN}{dt} = (\bar{N} - N(t)) \psi \exp\left(-\frac{2t^2 - 4\mu t + 4\mu^2}{\mu^2}\right)$$

It is considered a simple model through which we can predict and study the relationship between adoption and product consumption, as product advertising is considered the starting spark for the individual to enter the adoption process in order to pass through several stages until reaching the consumption stage that we discussed in this paper.

Like any model, there is room for development and modification to reach the model with a high level of accuracy and effectiveness. One of the most important things is to find λ and determine it according to the product and the duration of the months, if that is possible.

This model can also be generalized to become valid for predicting the value of digital currencies and precious metals such as gold and others, by following the approach that we followed to find the relationship between the value of currencies, for example, to find the relationship between the price of gold at the moment t and how it affects the rise in the value of the dollar at the moment $t + 1$ if it is this relationship is logarithmic, so the model can work in this case.

The model can also be good for studying the relationship between some types of diseases and the factors affecting the spread of the disease, as we require that the relationship between the factor and the disease be a logarithmic relationship, and the function h in the basic model of spread can be replaced with an appropriate function, and to clarify the idea, we assume that we have data for COVID infection. COVID 19 through the data, we noticed, for example, that the virus spreads more in cold climates than in hot climates, and that the relationship between the number of infections with the disease and temperature is a logarithmic relationship. We can follow the same approach that we presented above to find the effect of temperature on the number of infections with this virus.

In the end, the model can be developed according to its intended purpose, and researchers can use the approach we followed to find the relationship between influence and influence with the aim of exploring strategies whether to increase influence or reduce influence.

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