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Auto-Correlated Multivariate Quality Control for Electronic Products Manufacturing with Decomposition Analysis

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ABSTRACT

Many modern industrial processes involve multiple quality measures, and using individual control charts for each measure can be misleading if these measures are highly related. This paper proposes a new method for statistically controlling electronic products with multiple, interconnected quality characteristics. The method utilizes a combined model: a multivariate autoregressive (MAR) model with neural networks, to handle the presence of both correlation and autocorrelation in the data. The study compares the effectiveness of MCUSUM, MEWMA, and T^2 Hotelling charts in detecting small shifts in the overall process quality. To pinpoint the specific variables causing out-of-control signals in the T^2 Hotelling chart, we introduce a novel decomposition technique. This technique allows us to identify which measures are contributing most to these signals. Additionally, the MCUSUM and MEWMA charts demonstrate excellent performance in detecting small quality changes, leading to faster corrective actions. Overall, these findings suggest that our proposed method can significantly improve the reliability and responsiveness of quality control in electronics manufacturing.

1. INTRODUCTION

The complexity and increasing correlation of product features over time have made the requirements of product inspection more difficult. Statistical process monitoring is a method of quality control since variability is a primary indication of poor quality [1]. In many continuous processes of the electronic device manufacturing business, statistical process monitoring (SPM) typically develops over time to handle auto-correlated, multivariate quality data. Any changes in a process that may be the consequence of unforeseen and uncontrollable factors can be quickly identified by an effective SPM system.

Traditional control charts relied on the idea that the process would be dispersed independently across time, but as electronic device manufacture has advanced, this assumption is no longer valid. Control charts are produced for variables that defy one of the primary assumptions, serial-sample independence, when autocorrelation is present. The average run length (ARL) and stability of the control charts are both impacted by the violating of the presumption of independence [2]. Furthermore, if there are too many false alarms, the process engineer may prefer to completely ignore the control charts or misidentify the true source of variation. This explains why control charts are used on continuous-flow systems very infrequently [3].

Due to the fact that the industry that manufactures electrical products typically bases its product quality on a number of interconnected quality criteria and variables, SPM is also faced with multivariate problems [4]. The statistical characteristics of classic control charts are greatly impacted by interrelated variables, which can also result in a notable rise in the average false alarm rate and a fall in the capacity to identify process changes. Applying univariate control charts to every variable has the potential to mislead in decisions about quality. Consequently, multivariate based quality control techniques needed to take such characteristics into account at the same time.

As electronic products become more intricate and their quality characteristics show stronger connections over time, ensuring proper inspection becomes increasingly difficult. Traditional statistical process monitoring (SPM) systems struggle to keep pace with these complexities in modern manufacturing. One major issue is that traditional control charts assume data is independently distributed, which is often violated due to autocorrelation, leading to instability, and increased false alarms. Additionally, traditional control charts are not designed to handle multiple interrelated quality attributes simultaneously, which can mislead quality decisions and reduce the ability to detect process changes. These challenges have resulted in a low adoption of control charts in continuous-flow systems. This paper aims to address these gaps by proposing advanced SPM techniques that account for autocorrelation and introducing multivariate-based quality control methods to handle interrelated attributes simultaneously. Through these enhancements, we seek to increase the reliability and practical utility of control charts, encouraging their broader adoption in the electronic product manufacturing industry.

Traditional SPM methods, such as Shewhart, CUSUM, and EWMA control charts, assume that process data is independently distributed over time. However, this assumption is often violated in modern manufacturing environments where autocorrelation is prevalent. The violation of the independence assumption leads to several issues with traditional control charts: increased false alarms, decreased sensitivity, and complexity in handling multivariate data. Existing multivariate control charts, such as T^2 Hotelling's, MCUSUM, and MEWMA, address these correlations but are often complex to implement and may not effectively decompose out-of-control signals to identify specific variable contributions.

To address these limitations, our study proposes a novel ANN-based multivariate statistical process control model that integrates a multivariate autoregressive (MAR) approach with neural networks. This model aims to enhance detection sensitivity, improve robustness to autocorrelation, and provide detailed decomposition of out-of-control signals. By leveraging the capabilities of neural networks, our model improves sensitivity to small shifts in the process mean vector, ensuring timely detection of quality issues. The integration of MAR with neural networks effectively manages autocorrelated data, reducing false alarms and improving the stability of average run length (ARL). Additionally, our model offers a detailed decomposition of out-of-control signals, using univariate charts and a decomposition approach to identify specific variable contributions. This feature allows process engineers to pinpoint and address root causes of variations more accurately. By addressing these specific gaps and limitations in current multivariate autocorrelated process monitoring approaches, our study aims to enhance the reliability and practicality of SPM systems in modern manufacturing environments, ultimately improving product quality and manufacturing efficiency.

The present study will address the auto-correlated, multivariate quality control for electronic product manufacturing. We propose a model based on ANN to predict and build the residual based control chart for multivariate data with autocorrelation order p (AR(p)) processes.

The rest of this paper is organized as following. Section 2 discusses relevant literature review, such as SPM of multivariate auto-correlated observations, multivariate control chart, and ANN for multivariate and auto-correlated observations. Section 3 details the research methodology including manufacturing process and research variables. Section 4 illustrates a practical application and discussion. Finally, conclusions are provided in Section 5.

2. LITERATURE REVIEW

2.1 Multivariate auto-correlated control charts

Many industrial processes, both continuous and batch operations, frequently exhibit autocorrelation in their data, leading to ongoing efforts to find solutions [5]. Studies by Loredo et al. [6] and Psarakis and Papaleonida [7] highlight that even minor autocorrelation can significantly disrupt traditional control charts, causing them to signal false alarms more often and miss actual process changes. One solution involves filtering out autocorrelation using time series models and focusing control charts on the resulting uncorrelated residuals. This approach allows standard control charts to function effectively, as demonstrated by Callao and Rius [8] with their AR (1) model-based residual control charts. The concept can be extended beyond single variables. Issam and Mohamed [9] proposed using multivariate autoregressive (MAR) models to handle systems where multiple correlated variables exhibit serial dependence. Their work introduces an MAR control chart specifically designed for these multivariate autocorrelated processes. Their research proposed an MAR control chart for multivariate auto-correlated processes. For an MAR process with m variables, it is denoted by $\boldsymbol{x_t} = (\boldsymbol{x_{1t}}, \boldsymbol{x_{2t}}, \dots, \boldsymbol{x_{mt}})$ as a (m×1) vector.

Psarakis and Papaleonida [7] explained that shifts in the mean or variance of residuals signal changes in the actual process's mean or variance. By plotting residuals on a control chart, shifts in the process can be detected. The principle of residual charts is that, with a correct time series model, residuals become independently and identically distributed random variables, satisfying traditional quality control criteria and enabling the use of standard SPC charts. Therefore, developing an accurate time series model for multivariate autocorrelated data is essential in statistical process control. Although the ARIMA model is widely used for linear time series prediction, it struggles to capture nonlinear patterns.

Autocorrelation in process data is a persistent challenge, prompting researchers to develop solutions. Alwan and Roberts [10] addressed this issue by proposing residual-based control charts, which rely on data where autocorrelation has been removed. Woodall and Faltin [11] investigated the impact of self-correlation on control charts and explored various strategies to manage it. Their work included developing methods like the CUSUM control chart specifically designed for autocorrelated data [12-14]. Additionally, researchers have explored using exponentially weighted moving average (EWMA) control charts for data with autocorrelation [15-18].

A range of multivariate control charts exist in the literature, including T2 Hotelling, multivariate CUSUM (MCUSUM), and multivariate EWMA charts [19-22]. Additionally, researchers have made advancements in control charts to address situations with multivariate data exhibiting autocorrelation and time series effects [23-27]. Notably, Jarrett and Pan [28] proposed separate approaches for independent and autocorrelated processes. They introduced a dedicated multivariate autoregressive (MAR) control chart specifically designed for handling multivariate data with autocorrelation.

2.2 Residual control chart

2.2.1 T^2 Hotelling control chart

The T^2 Hotelling control chart, a more versatile version of the Shewhart chart, was introduced by Harold Hotelling in 1947 to handle multivariate observations. Unlike the Shewhart $\bar{X}$-chart which deals with single variables, the T^2 Hotelling chart can handle multiple variables simultaneously. It comes in two versions: for data grouped by subgroups and for individual observations. The Shewhart X-chart is basic tool for univariate control charts, where it measures process stability against significant changes. It assumes residuals (differences between observed values and expected values) have a zero mean and a standard deviation σ_r. An observation is considered in-control if its residual value falls within control limit defined by a factor λ. The T^2 Hotelling chart uses a different approach. It leverages the Mahalonobis distance [25] to compress residuals from multiple variables into a single value. An observation is considered in control if it satisfies a specific equation as the following Eq. (1). This equation considers factors like the number of observation (n); the number of variables (m); the residual vector of each observation (R_i); and a value from the Fisher distribution $F_{m;n-m;(\alpha)}$. The Fisher distribution is chosen based on a desired risk level (α), which helps determine the expected frequency of false alarms when in-control observations flagged as out-of-control. By analyzing the T^2 statistic value for each observation, the T^2 Hotelling control chart can effectively monitor processes with multiple interrelated variables.

$$T_i^2 = R_i^T \sum\nolimits_R^{-1} R_i \frac{m(n-1)}{n-m} F_{m;n-m;(\alpha)}; \quad \text{for } i = 1,2,\cdots,n \tag{1}$$

2.2.2 Multivariate CUSUM control chart

The CUSUM control chart was developed to overcome a weakness in Shewhart and T2 Hotelling charts. These traditional charts often miss gradual process changes because they only consider the most recent data point [9]. CUSUM charts address this by accumulating the deviations from a target value across residuals of past observations, making them more sensitive to subtle shifts. The most common CUSUM control method is Crosier's chart [29]. This method uses a statistical procedure defined by Eqs. (2) and (3). It starts with $S_0 = 0$, a matrix of zeros representing the initial state. Then, for each observation (i), it calculates a new S_i value based on the previous S_{i-1}, the current residual R_i, a reference value (k) and the estimated residual covariance matrix Σ_R^{-1}. The reference value (k) helps determine how quickly the CUSUM chart reacts to changes. Crosier's chart signals a potential process shift when a statistic called $T_S^2 = S_i^T \sum_R^{-1} S_i$, calculated using S_i and the inverse covariance matrix, exceeds a predefined limit (H). In simpler terms, the CUSUM scheme raises an alarm when the S statistic goes above a certain threshold (H), indicating a possible change in the process.

$$S_i = \begin{cases} 0 & \text{, if } C_i \le k \\ (S_{i-1} + R_i)\left(1 - \dfrac{k}{C_i}\right) & \text{, otherwise} \end{cases} \tag{2}$$

$$C_i = (S_{i-1} + R_i)\Sigma_R^{-1}(S_{i-1} + R_i)^T \tag{3}$$

Crosier's chart signal a shift when $T_S^2 = S_i^T \sum_R^{-1} S_i$ overcomes a predetermined limit H. Thus, if $S_i > H$, the chart indicates a process shift. To achieve the desired in-control run length (RL) characteristic, the parameters k and H must be determined beforehand. In CUSUM procedures, it is standard practice to assume a sample size of one. This simplification is widely adopted as it allows for the continuous monitoring of individual observations, facilitating the prompt detection of small shifts in the process. However, in some cases, it might be beneficial to consider larger sample sizes to account for variations and provide more robust detection capabilities, especially in processes where data is naturally grouped or collected in batches. Adapting the CUSUM procedure to accommodate different sample sizes can enhance its flexibility and effectiveness in various industrial and statistical applications. The multivariate CUSUM (MCUSUM) statistic S_i is designed to detect specific shifts in the process mean vector. This capability enables the identification of changes across multiple variables simultaneously, making MCUSUM particularly useful for monitoring complex processes where interactions between variables may signal deviations from the expected process behavior. By accumulating deviations from the target mean vector over time, the MCUSUM statistic provides a sensitive measure for detecting even small shifts, thus enhancing the ability to maintain quality control and process stability in multivariate settings. Additionally, the MCUSUM approach can be tailored to different types of shifts and can incorporate various weighting schemes to prioritize certain variables or shifts, further improving its applicability and effectiveness in diverse industrial and research environments:

$$S_i = max\{S_{i-1} + a^T R_i - k, 0\} \tag{4}$$

where,

$$a^T = \frac{\delta_r^T \Sigma_r^{-1}}{\sqrt{\delta_r^T \Sigma_r^{-1} \delta_r}}$$

The residual mean vector is denoted as δ_r, while Σ_r represents the variance-covariance matrix. In a MCUSUM scheme, any deviation from the target mean that exceeds k units is aggregated. In this context, k serves as the benchmark value for the scheme. The control scheme signals an out-of-control state when S_i surpasses a specified decision threshold, labeled as H [9].

2.2.3 Multivariate EWMA control chart

While CUSUM charts consider all past measurements equally, EWMA (Exponentially Weighted Moving Average) charts assign weights to recent observations based on their significance in depicting process behavior. A higher value of λ amplifies the impact of the most recent observation [5]. The iterative expression for EWMA statistics is described by Eq. (5),

$$Z_i = (1-\lambda)Z_{i-1} + \lambda R_i; \text{ for } i = 1,2,\cdots,n \tag{5}$$

where, λ is diagonal matrix of value $0 \le \lambda_j \le 1$, $j = 1,2,\cdots,m$. The multivariate EWMA (MEWMA) scheme signals if the

$$T_Z^2 = Z_i^T \sum\nolimits_Z^{-1} Z_i \tag{6}$$

surpasses a predetermined value H, where $H>0$ is chosen to achieve a specified in-control (on-target) ARL_0. The asymptotic form of the covariance matrix is $\Sigma_Z = \left(\frac{\lambda}{2-\lambda}\right)\Sigma_R$ [30].

Reynolds and Lu's study [31] explored the use of AR (1), AR (2), and ARMA (1, 1) models with residual X-charts. They identified a potential limitation: the residual X-chart might not be sensitive enough to detect certain types of process changes, specifically mean shifts. Those research, however, considered only for processes which has small order of p on autoregressive AR (p) model. Whereas real condition sometimes autocorrelation with high order (p>5) are occurred. Besides that, multivariable with high autocorrelation also must considered in one time. This condition usually occurs in the manufacturing industry with mass production and fast flow production. Therefore, the general multivariate autoregressive

(MAR) models should be developed to overcome multivariable and autocorrelation problem on statistical process monitoring using residual based multivariate control chart.

2.3 Applying ANN to SPM of multivariate auto-correlated observations

Artificial neural networks (ANNs) are increasingly used as powerful tools to estimate and forecast process outputs [32]. In particular, the multi-layer perceptron (MLP) is a versatile estimator, working for both classification and prediction tasks. For forecasting problems, a multilayer feed-forward ANN with a continuous output layer is ideal. When dealing with quality data exhibiting autocorrelation through an autoregressive model of order p (AR(p)), the average values for each time period depend on the averages of the previous p periods. Consequently, when using an ANN for forecasting, the input layer feeds the network with quality characteristic data from the past p periods. The output layer then predicts the quality characteristic vector for the target time period. During implementation, the network processes information from the previous p periods through the input layers to generate the forecast vector at the output layer. As highlighted by Arkat et al. [3], the residual vector for each period is simply the difference between the predicted and actual quality characteristic values.

Machine learning has seen growing adoption in statistical process monitoring (SPM) research over the past two decades. This trend reflects the potential of machine learning to detect and diagnose faults in industrial processes and production outcomes. ANNs, specifically, have been used for data analysis in SPM since the 1980s, as demonstrated by Arkat et al. [3]. Research has explored ANN applications in both univariate [33-35] and multivariate control charts [36-40]. Arkat et al. [3] proposed an ANN-based model for forecasting and building residual CUSUM charts for AR (1) multivariate processes. Additionally, Khediri et al. [5] explored using support vector regression to create control charts for monitoring more complex, non-linear, and autocorrelated multivariate processes.

2.4 Summary

Traditional methods often assume independence between observations, which is violated in continuous-flow manufacturing processes due to autocorrelation. This violation leads to increased false alarm rates and reduced average run length (ARL). While multivariate control charts address correlation among variables, they can be complex and computationally intensive, and they may not effectively identify specific variable contributions in out-of-control signals. Our proposed ANN-based model addresses these limitations by integrating a multivariate autoregressive (MAR) approach with neural networks, improving sensitivity to small shifts in the process mean vector and enhancing robustness to autocorrelation. The model offers better detection capabilities, reducing false alarms and improving ARL stability. Additionally, it provides detailed decomposition of out-of-control signals, allowing process engineers to pinpoint and address root causes of variations more effectively. These improvements enhance the reliability and practicality of SPM systems in modern manufacturing, leading to more timely and accurate quality control interventions, ultimately improving product quality and manufacturing efficiency.

In recent years, several studies have advanced the field of SPM, particularly in addressing the limitations of traditional control charts in handling autocorrelated and multivariate data. For instance, Wang and Asrini [41] proposed an enhanced EWMA control chart that incorporates machine learning techniques to better handle autocorrelated data, demonstrating improved sensitivity and reduced false alarm rates. Similarly, Yang and Sutrino [42] developed a hybrid SPM model that combines neural networks with traditional statistical methods to monitor complex manufacturing processes, showing significant improvements in detection capabilities and robustness to data variability.

With advancements in many automation processes, such as electronic component manufacturing, the assumption of independent distribution is frequently violated because the high frequency of sample selection results in observations that are closely related and dependent. It is crucial to understand how to apply and evaluate control charts designed to account for autocorrelation. Residual control charts offer valuable insights into device behavior over time and have effective detection capabilities. However, they do not entirely address the needs for handling autocorrelation and multiple variable observations.

3. METHOD

This study aims to present a residual control chart using MAR model with ANN (MAR-ANN) to solve the SPM problem related to multivariate with auto-correlated observations. Moreover, this study makes diagnostic of out-of-control signal in multivariate control chart using decomposition technique. Comparison of multivariate control chart with univariate one is also conducted. Figure 1 shows the operational procedure of the proposed mothed. Multiple variables are defined as the quality parameter which correlate to each other, and each variable is of time series.

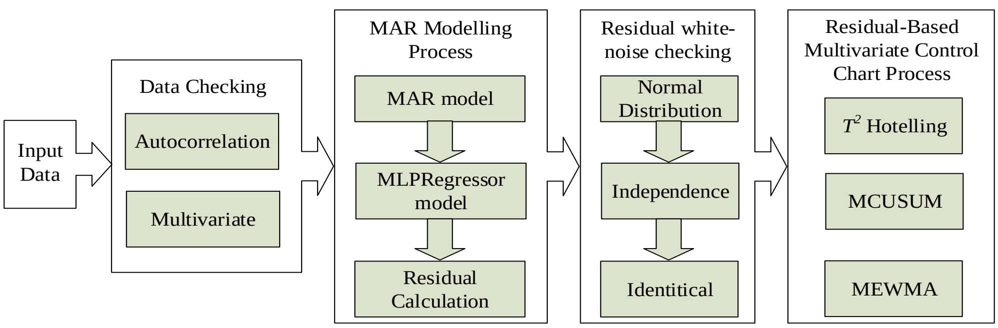


Figure 1. Procedure to build the proposed control chart

Correlation test is employed to know the strength of correlation between variables. In this study, correlation test determines the correlation between quality parameters of a product. If there is a correlation between quality parameters, then the control chart preparation is based on a multivariate control chart approach because it will involve more than one quality parameter in one chart. The hypothesis used in testing the correlation between quality parameters is as follows. To determine whether there is a correlation between quality parameters is based on the p-value.

H_0: $\rho = 0$ or there is no correlation
H_1: $\rho \neq 0$ or there is a correlation

This study tackles autocorrelation in process data using a method developed by Loredo et al. [6]. This method prioritizes residual-based control charts, which have proven more effective than traditional charts in detecting mean shifts when dealing with short-run, autocorrelated data. To identify variables potentially affected by autocorrelation, the study performs an autocorrelation test on each variable. These tests assess whether a variable exhibits a relationship with its own past values over time. A helpful tool for visualizing autocorrelation is the autocorrelation function (ACF) plot. If a variable's ACF plot shows a significant lag, it suggests the presence of autocorrelation. To address autocorrelation, the study employs time series modeling. This modeling process ensures the model's errors (residuals) meet the assumption of white noise, meaning they are uncorrelated with each other. The autocorrelation coefficient, calculated at a specific time lag (*k*), measures the correlation between a variable's value at a given time (*t*) and its value k periods earlier (*t*-*k*). Essentially, it indicates how closely the variable's past values influence its current value. If the autocorrelation plot dips below the 95% confidence interval at a particular lag, it signifies the presence of significant autocorrelation at that time lag.

Figure 1 presents a four-step procedure. First, data are checked for each variable by ACF and correlation between variables by Pearson's correlation. Second, MAR modelling process determines the model considering autocorrelation and multivariate. To estimate MAR, this research proposes an ANN with MLPRegressor approach. Third, residual white-noise checking is conducted to ensure all residual variables can be used for the multivariate control chart. White noise residual checking involves multivariate normality, independence and identical test. Then, fourth step builds a residual-based multivariate control chart where residual is the difference between actual value and estimated value based on the MAR model of each variable.

If the quality characteristics of an autocorrelated process follow an AR (*p*) model, the mean vector for each period depends on the mean vectors from the previous *p* periods. In such cases, the inputs for the desired artificial neural network (ANN) consist of the quality characteristic vectors from the previous p periods, while the output represents the quality characteristic vector to be forecasted for the next period. Before constructing the ANN, the multivariate autoregressive (MAR) model is typically applied to determine the autoregressive order *p* for each factor in the time series. This helps in understanding the dependencies and lagged effects among the quality characteristics over time, ensuring that the ANN model captures the relevant temporal relationships effectively.

This study, following the work of Khediri et al. [5], utilizes time series estimation for a multivariate process using a multivariate autoregressive (MAR) model. The MAR model considers the influence of past values on each variable in the process. Suppose a process with m variables where each variable Y_i for i ranges from 1 to m at specific time t denoted as Y_{it}. The MAR model considers the values of all m variables at p previous time steps to influence the current value (t) of variable Y_i. In other words, Y_{it} is determined by the values of Y_j at times (t-1), (t-2), ..., (t-p) for all j variables (from 1 to m). Eq. (7) summarizes this concept mathematically. It represents Y_{it} as a function of the lagged values of all m variables. The MAR model then estimates this function, denoted by $\hat{f}$, allowing to predict future values (Y_{it}) for each variable using Eq. (8). Eq. (8) essentially replaces the unknown function f with its estimated version ($\hat{f}$) to predict Y_i at time t.

$$Y_{(i=1,2,\ldots,m)t} = f(Y_{1(t-1)},\cdots,Y_{1(t-p)},\cdots,Y_{m(t-1)},\cdots,Y_{m(t-p)}) \quad (7)$$

$$\hat{Y}_{(i=1,2,\ldots,m)t} = \hat{f}(Y_{1(t-1)},\cdots,Y_{1(t-p)},\cdots,Y_{m(t-1)},\cdots,Y_{m(t-p)}) \quad (8)$$

If the estimation is accurately performed, the error term vector is calculated based on Eq. (9). This vector will be used to generate the control chart, which will be time-independent and typically distributed with a mean of zero.

$$\hat{e_t} = Y_{(i=1,2,\cdots,m)t} - \hat{Y}_{(i=1,2,\cdots,m)t} \quad (9)$$

If a shift occurs in the process, it will no longer be accurately described by the function *f*, and consequently, the estimated residual term $\hat{e_t}$ will also be affected and shifted. To determine the residual used for the control chart, this study employs the multivariate autoregressive (MAR) model, as specified in Eq. (10).

$$y_t = c + \emptyset(B)y_t + e_t \quad (10)$$

$$y_t = c + \emptyset_1 y_{t-1} + \emptyset_2 y_{t-2} + \cdots + \emptyset_p y_{t-p} + e_t \quad (11)$$

where,
$y_t = (y_{1,t}, y_{2,t}, \ldots, y_{m,t})'$ is (mx1) vector of variable Y
$c = (c_1, c_2, \ldots, c_m)'$ is (mx1) vector of constant value
$e_t = (e_{1,t}, e_{2,t}, \ldots, e_{m,t})'$ is (mx1) vector residual, with assumption $e_t \sim$ IIDN $(0, \Omega)$ and $var(e_t e_t)$=Ω.
$\emptyset$ =coefficient of MAR model, matrix (mxm)
t=1, 2, …, n
B=backshift operator
m=number of variables
p=order of MAR

In this study, the MAR residual control chart, which involves a number of input and output variables and a fitting technique to find the satisfied residual, is empowered by a multilayer perceptron regressor (MLPRegressor) (Alpaydin, 2010) to obtain good fitting result. MLPRegressor can approximate the nonlinear functions of the input for regression by forming higher-order representations of the input features using intermediate hidden layer.

4. EXPERIMENT RESULT AND DISCUSSIONS

4.1 Manufacturing process

This study applies the MAR-ANN model to electronics product manufacturing processes, encompassing stages like stamping, electroplating, injection molding, assembly, and packaging (depicted in Figure 2). These processes operate on a high-speed continuous production line characterized by multivariate and autocorrelated properties. An automatic optical inspection (AOI) system is integral to the process, facilitating data collection. High-precision instruments, regularly calibrated for accuracy, including digital calipers, micrometers, and AOI systems, are utilized to measure product dimensions and features. Specifically, AOI systems record product feature measurements. The MAR-ANN model proposed in this study aims to enhance defect detection capabilities, leveraging the structured data from AOI and other instruments to improve quality monitoring throughout the manufacturing stages.

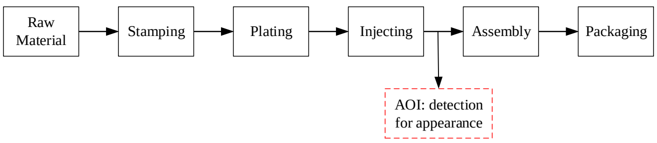


Figure 2. Manufacturing process of the product under investigation

Table 1. Product feature and variable notation

Feature	Variable Notation
Excess metal material is present at the terminal.	Y1_1
Excess colloids appear in the hold-down of the metal.	Y1_2
Excess metal material is found on the plastic body.	Y2_1
Excess plastic material is present on the product's edge.	Y2_2
Overflow occurs on both sides.	Y3
The root is overflowed.	Y4

In this study, the automatic optical inspection (AOI) system detects key defect types such as "overflowed", "extra-materials", and "metal debris". These defects are identified based on numerical specifications such as length, width, and area measured by the AOI system. Each product is characterized by six features, as detailed in Table 1, which describes each variable. The data collection process involves measuring products in batches, with each batch comprising 100 units. For the MAR-ANN control chart analysis, this study collects samples from 300 such batches. This structured approach ensures that a comprehensive dataset is used to develop and validate the MAR-ANN model for effective quality control in the manufacturing process.

4.2 Residual-based multivariate control by MAR-ANN model

The proposed MAR-ANN model is implemented according to Figure 1.

4.2.1 Data checking

The MAR (multivariate autoregressive) model assumes that each time series in the system influences others, allowing predictions based on past values of all series involved. Granger's causality test is a method used to assess these dependency relationships by testing whether past values of one series help predict another. In the study, Table 2 presents the results of Granger's causality test for all possible combinations of time series in a given dataset, storing the corresponding p-values in an output matrix. A p-value less than the 5% significance level indicates a significant causal relationship, where the series in the column influences the series in the row. For example, a p-value of 0.0000 in (row 1, column 2) suggests that Y1_2 (column) causes Y1_1 (row). Conversely, a p-value of 0.000 in (row 2, column 1) indicates that Y1_2 (row) causes Y1_1 (column). Therefore, Table 2 demonstrates that there are significant correlations among the variables overall. Specifically, it can be concluded that variables Y1_1 and Y1_2; Y1_1 and Y2_2; Y1_1 and Y3; Y2_1 and Y2_2; Y2_1 and Y4; Y2_2 and Y3 are correlated based on the p-values obtained from Granger's causality test. These findings help validate the interconnectedness assumed by the MAR model in your analysis.

ACF test of each variable is shown in Figure 3, showing that almost all variables have lags over than the red likelihood limit (95%), which means every variable has significant autocorrelation. Vector autoregressive model found that the optimal lag to define order *p* is 16 (p=16). This order was chosen based on the minimum Akaike Information Criterion (AIC) value.

4.2.2 MAR modeling process-constructing residuals control chart using ANN model

This approach tackles the challenges of analyzing complex, autocorrelated, and multidimensional quality data by combining a MAR model with an ANN. The MAR model excels at capturing the data's temporal nature and the interconnectedness between variables. The ANN, on the other hand, is adept at learning non-linear relationships and becoming more sensitive to subtle changes in the average values of the entire quality measurement process. To train and validate the effectiveness of this MAR-ANN model, we utilized a dataset gathered from a continuous-flow electronic product manufacturing line. The data was meticulously divided into training (70%), validation (15%), and testing (15%) sets. This methodical split ensures the model's robustness and generalizability. The training set serves as the foundation for model fitting, while the validation set allows for fine-tuning crucial hyperparameters that influence the ANN's performance. Finally, the test set provides an unbiased assessment of the model's overall accuracy. Selecting the optimal hyperparameters is paramount for maximizing the ANN's effectiveness. We employed a grid search technique to identify the ideal configuration, encompassing factors like the number of hidden layers, the number of neurons within each layer, the learning rate, and parameters for controlling overfitting. This grid search was meticulously conducted using cross-validation on the training data. Ultimately, the combination of hyperparameters that yielded the best performance on the validation set was chosen for the final model.

Eq. (12) represents an autoregressive process with 6 variables and order p=16.

$$Y_{(i=1,2,\ldots,6)t} = f\,(Y_{1(t-1)},\cdots,Y_{1(t-16)},\cdots,Y_{6(t-1)},\cdots,Y_{6(t-16)}) \quad (12)$$

Estimation of the process using ANN provides $\hat{f}$ which allows to predict $Y_{(i=1,2,\ldots,m)t}$ as Eq. (13).

$$\hat{Y}_{(i=1,2,\ldots,6)t} = \hat{f}\,(Y_{1(t-1)},\cdots,Y_{1(t-16)},\cdots,Y_{6(t-1)},\cdots,Y_{6(t-16)}) \quad (13)$$

Table 2. Pearson's correlation test between variables

	Y1_1_x	Y1_2_x	Y2_1_x	Y2_2_x	Y3_x	Y4_x
Y1_1_y	1	**0.000***	**0.003***	0.170	**0.003***	0.071
Y1_2_y	**0.000***	1	**0.000***	0.155	0.813	**0.000***
Y2_1_y	0.145	**0.000***	1	**0.027***	0.101	**0.000***
Y2_2_y	**0.000***	0.2957	0.282	1	**0.000***	**0.027***
Y3_y	**0.031***	0.7830	0.094	**0.000***	1	0.281
Y4_y	0.085	**0.000***	0.381	0.101	0.115	1

Note: (*) at 5% significance level

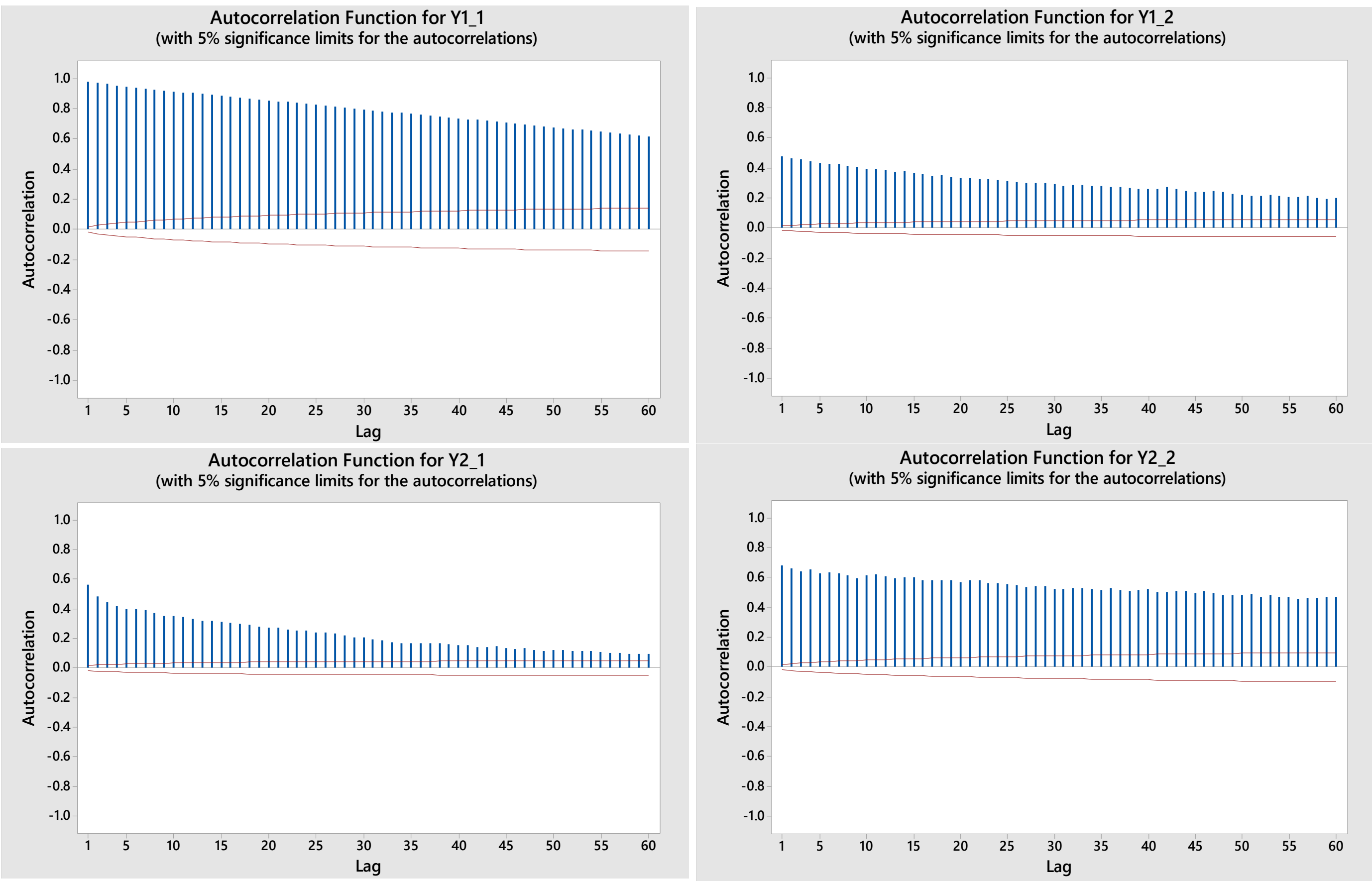
The specific ANN model that used to figure out the autoregressive model with 6 variables and order *p*=16 in this case is MLPRegressor. The model, trained on 14,000 observations from an electronic product manufacturing process, includes a single hidden layer with 50 neurons and uses the 'relu' activation function. Key hyperparameters include an alpha of 0.0001 for regularization, a learning rate of 0.001, and the Adam optimizer. The model underwent up to 1000 iterations, with early stopping disabled, and employed cross-validation (70% training, 15% validation, 15% test split). These details ensure the model's reproducibility and highlight the robustness and appropriateness of our approach. Using the MLPRegressor which involved multiple outputs, the selected optimal model that used in this study is shown as following code:

```
# Simple MLPRegressor with ReLU activation and Adam optimizer
regressor = MLPRegressor(
    activation='relu'; # Non-linear activation
    solver='adam',# Efficient optimizer
    hidden_layer_sizes=(50,);# Single hidden layer with 50 neurons
    learning_rate_init=0.001; # Initial learning rate
    max_iter=1000#Maximum training iterations)
```

The R-square value of 91.4% indicates that the MLPRegressor model explains a substantial portion, 91.4%, of the variance in the data, highlighting a strong fit for the MAR-ANN model to the observed values. To verify the white noise assumption of the residuals, the residuals are computed by subtracting the predicted values from the actual observations for each Y variable. Subsequent checks include ensuring the residuals have a mean close to zero, exhibit no significant autocorrelation through plots or tests like the Durbin-Watson test, and demonstrate constant variance (homoscedasticity) across different values. Meeting these criteria indicates that any remaining patterns in the data are likely due to random noise, validating the accuracy and reliability of the model's predictions.

Moreover, in terms of white noise checking, residuals should follow multivariate normal distributions with mean of zero and variance equal to one, and the residuals are free of auto-correlation effects. The following step is a checking of white noise assumptions of residuals. Firstly, multivariate normal distribution checking in this study is done by the Henze-Zirkler test [43]. According to this test, we found that the p-value is equal to 0.150, which means all of residual variables have already followed multivariate normal distributions with significance value at 5%.

Next, independence assumption checking for residuals is conducted by ACF, as shown in Figure 4. The lags of residual variables Y1_1; Y1_2; Y3 and Y4 are lower than the red likelihood limit (95%) and the autocorrelation value is around zero. Meanwhile, the lags are over than the red likelihood limit (95%) and the autocorrelation value is lower than 0.4 for residual variables Y2_1 and Y2_2, which means the correlation is weak so the effect can be ignored. Therefore, it can be concluded that all residual variables are free of the autocorrelation effect. Therefore, those residual variables can be used to create multivariate control chart.



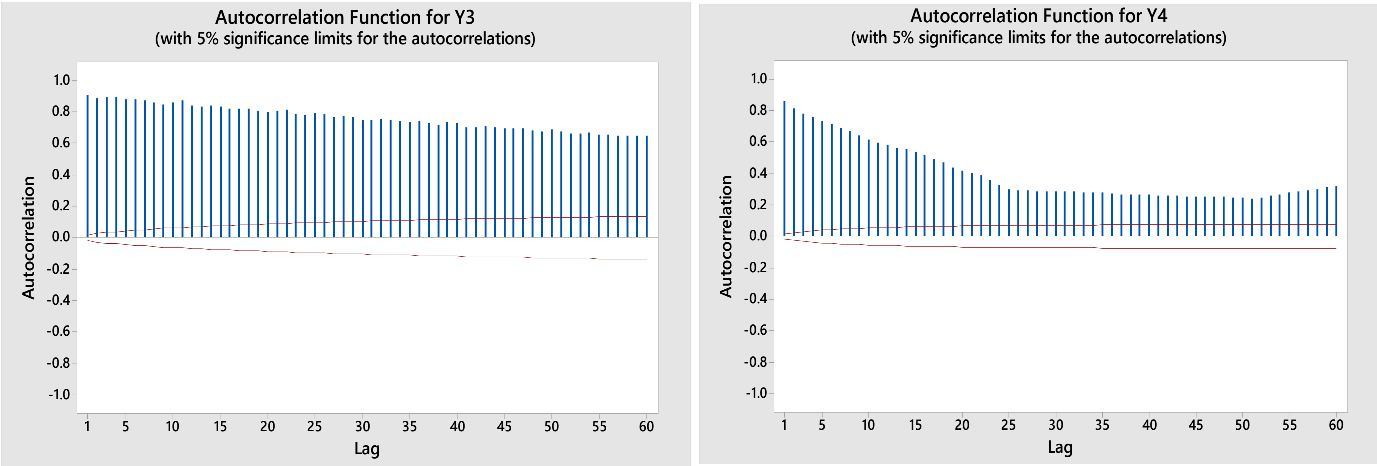


Figure 3. Autocorrelation test of each variable

Table 3. Descriptive of residual model of each variable

Residual of Variable	Mean	Variance
Y1_1	-0.001	0.0234
Y1_2	0.081	0.3685
Y2_1	-0.037	0.2193
Y2_2	0.024	0.148
Y3	-0.002	0.0004
Y4	0.052	0.627

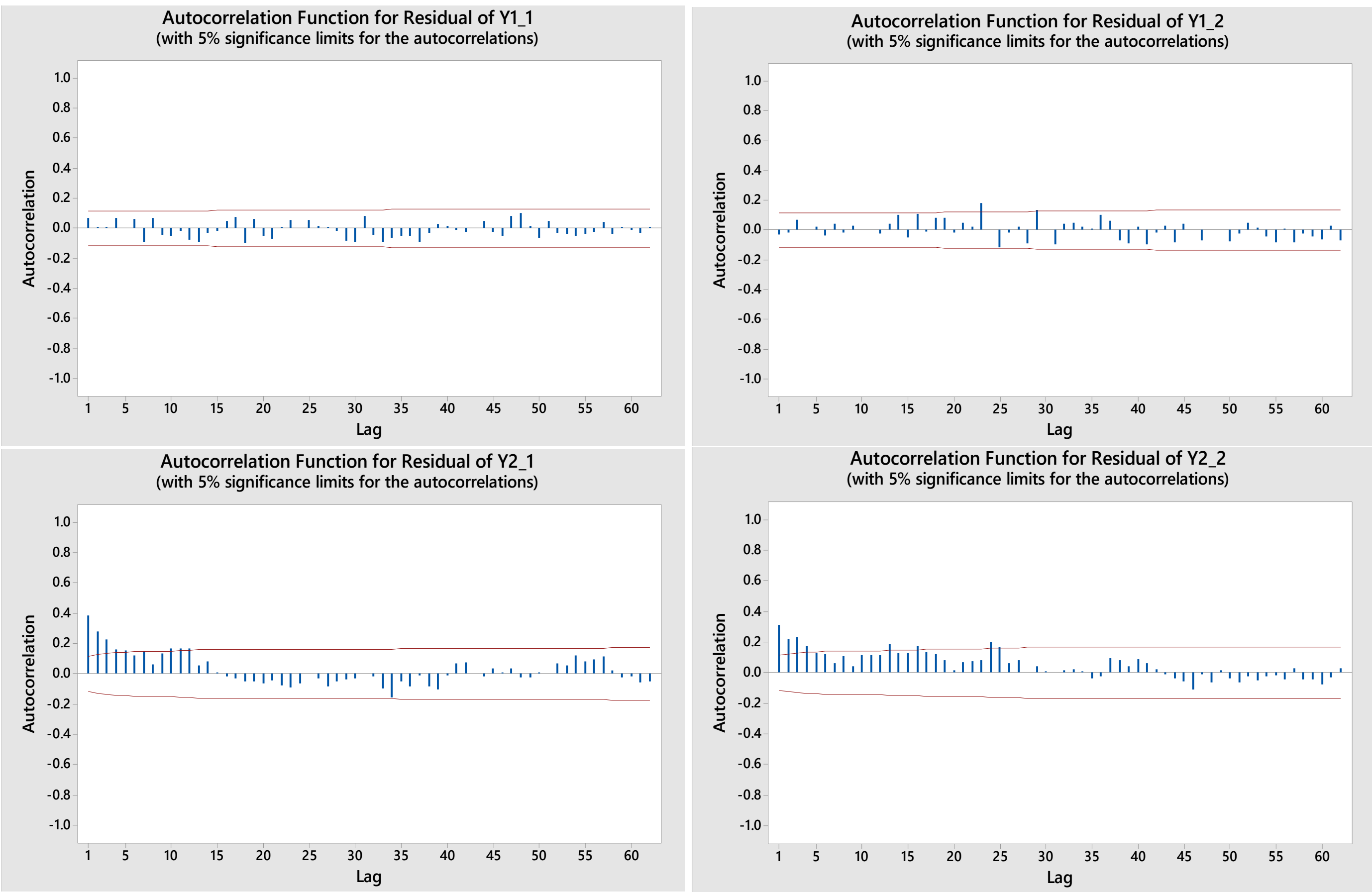
Third step is residual white noise checking. Table 3 shows that the mean and variance of each residual variable almost near zero. Therefore, all residual variables have already satisfied the white noise assumption. The application of residual data satisfied the assumption of normality distribution and absence of autocorrelation effects.

All quality characteristics are monitored simultaneously. Table 4 illustrates the correlation among the six residual variables.

Multivariate T^2 Hotelling control chart

Multivariate T^2 Hotelling control chart is constructed. Multivariate T^2 Hotelling control chart for original data with upper control limit (UCL) value equal to 29.2 is illustrates in Figure 5(a) showing that there were 50 instances where the mechanism failed, which shown by some points are out of control. There are oscillating and it fails out at points particularly after sample 157th. This behavior comes from the dependence of measurements over the time on original data.

Instead, by using residual data, the number out-of-control samples decreases down to 9 samples. Multivariate T^2 Hoteling control chart using residual data is more stable than multivariate T^2 Hottelling control chart using original data, as shown in Figure 5(b).



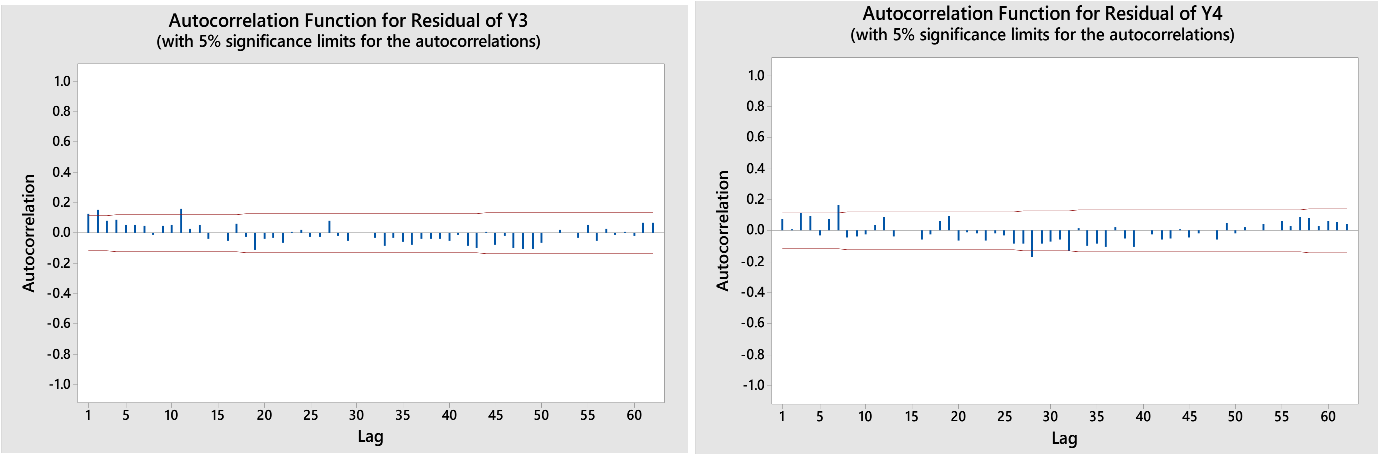
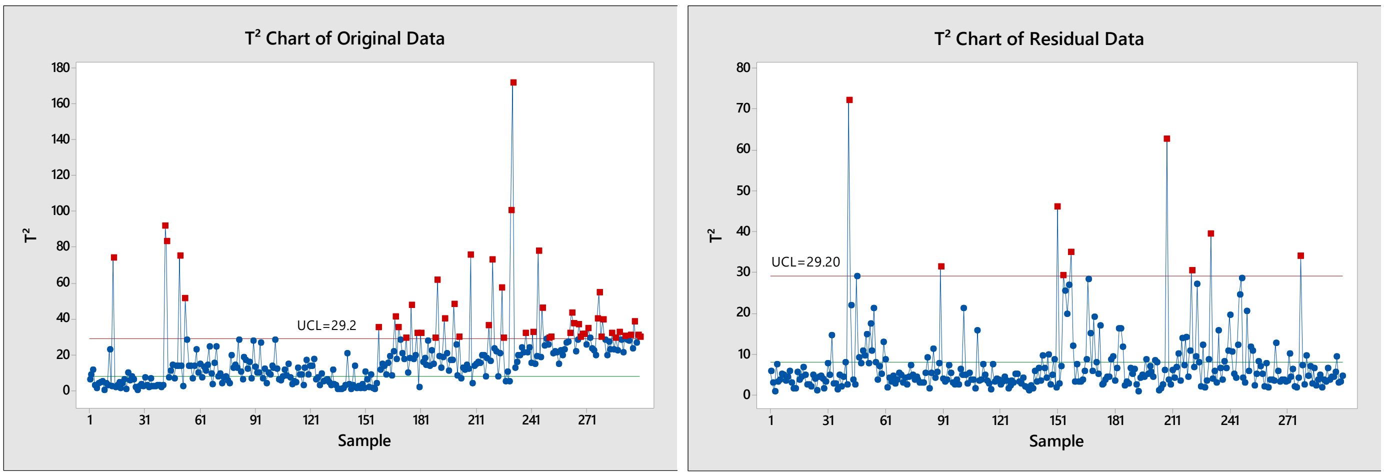


Figure 4. Autocorrelation test of each residual variable

Table 4. Correlation test between residual variables

Characteristics	Variables	Residual of Y1_1	Residual of Y1_2	Residual of Y2_1	Residual of Y2_2	Residual of Y3
Residual of Y1_2	Correlation	0.046				
	P-Value	0.431				
Residual of Y2_1	Correlation	-0.092	0.064			
	P-Value	0.111	0.266			
Residual of Y2_2	Correlation	0.105	0.112	-0.019		
	P-Value	0.069	0.052	0.743		
Residual of Y3	Correlation	0.189	0.087	0.035	0.064	
	P-Value	**0.001***	0.133	0.542	0.268	
Residual of Y4	Correlation	0.272	0.033	0.131	-0.014	0.144
	P-Value	**0.000***	0.571	**0.023***	0.808	**0.013***

Note: (*) at 5% significance level



(a) by original data　　(b) by residual data

Figure 5. T^2-Hoteling multivariate control chart

Table 5. Decomposed T^2 Hotteling value

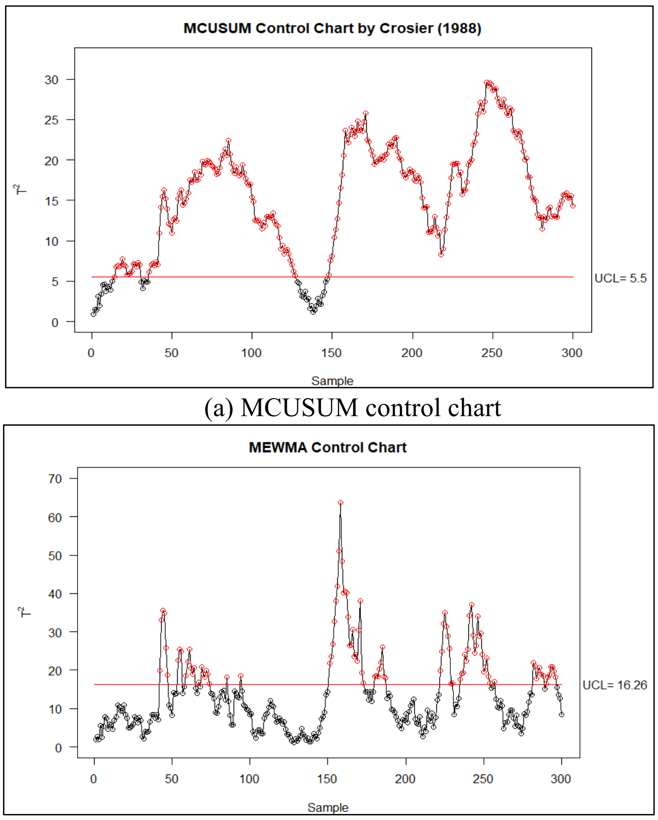
Sample	Variables					
	Y1_1	Y1_2	Y2_1	Y2_2	Y3	Y4
42	11.44	0.922	3.919	0.756	41.316	3.766
90	14.47	0.187	0.007	4.659	0.518	4.742
151	0.033	0.726	0.312	0.115	1.314	39.02
154	10.73	0.390	15.44	2.142	0.145	0.559
158	1.450	0.622	7.821	0.239	0.044	19.47
208	5.918	8.376	0.967	40.04	1.127	1.883
221	0.066	0.101	4.608	26.74	1.552	0.113
231	3.693	24.50	0.721	2.862	7.022	0.302
278	21.09	1.052	0.119	0.005	2.198	3.815

Decomposition is a valuable diagnostic technique for identifying out-of-control signals in multivariate control charts, particularly in T^2 Hotelling charts. It breaks down the T^2 statistic into components that represent the contribution of each individual variable to the out-of-control signal. This approach involves estimating values d_i for each variable and focusing on those variables where d_i) values are relatively large. Table 5 typically displays the variables with the highest d_i values, indicating which variables contribute most significantly to the out-of-control signals detected in the multivariate T^2 Hotelling control chart. This helps pinpoint specific factors or characteristics that may need attention or correction in the manufacturing or process control environment. Y1_2, Y2_1, and Y3 are responsible for out-of-control signals on sample 231st, 154th, and 42nd, respectively. Y1_1 is responsible for samples 90th and 278th. Y2_2 is responsible for samples 208th and 221st. Y4 is responsible for samples 151st and 158th.

MCUSUM and MEWMA control chart

The study utilized RStudio (version 2020) and the MSQC package to create MCUSUM and MEWMA control charts. These charts were designed to identify subtle changes in the process by setting a specific false alarm rate (chosen to be 5%). The configuration also included a reference value (k=0.5) and a decision limit (h=5.5). The MCUSUM chart, analyzing the data's residuals, proved to be significantly more adept at detecting small shifts in the overall process mean compared to the standard T^2 Hotelling chart. This improved sensitivity is evident in Figure 6(a), where the MCUSUM chart detects the first process change much earlier (sample 14) compared to the T^2 chart (which only signals at sample 42) under identical test conditions (number of variables: p=6, subgroup size: n=1, and desired false alarm rate: ARL_0=200).

Conversely, Figure 6(b) illustrates that the MEWMA control chart exhibits comparable sensitivity to the multivariate T^2 Hotelling control chart in detecting process changes. This comparison highlights the effectiveness of MCUSUM and MEWMA charts in differentiating their capabilities in sensitivity relative to traditional T^2 Hotelling control charts in process monitoring and quality control scenarios.



(a) MCUSUM control chart

(b) MEWMA control chart

Figure 6. Multivariate control charts using residuals data

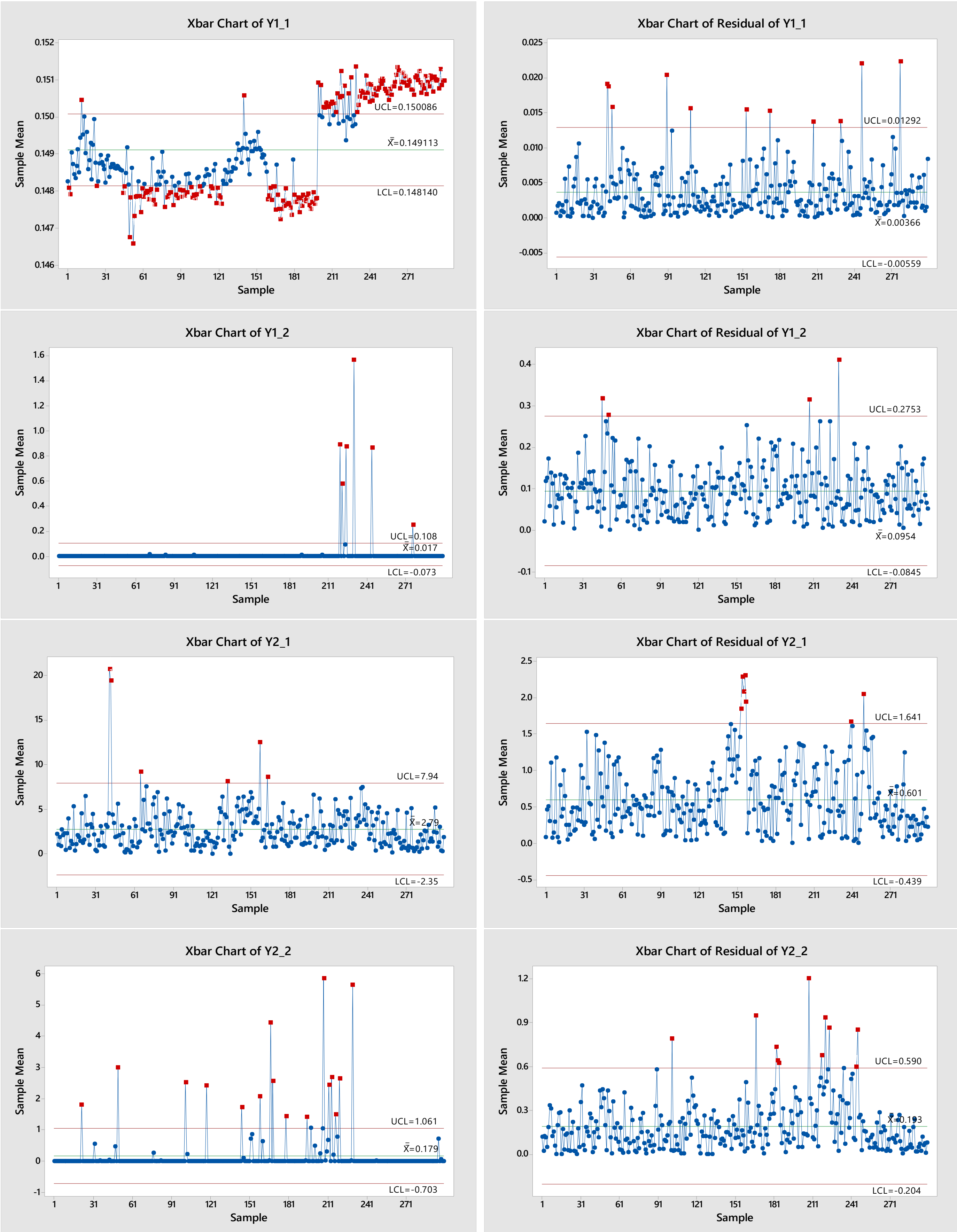
Implementing T^2 Hotteling control chart, MCUSUM chart and MEWMA chart for the obtained residual data can overcome multivariate autocorrelated data effectively. When compared with MCUSUM and MEWMA, T^2 Hotteling has better performance in detecting small shifts in the process. Meanwhile, the MCUSUM residual chart and the MEWMA residual chart show a large shift from the average. In addition, T^2 Hotteling also shows stable shifts around the average. However, the MCUSUM chart and the MEWMA chart show oscillations in shifts and even show a trend. This shows that the T^2 Hotteling residual chart has better performance than the MCUSUM and MEWMA residual control charts.

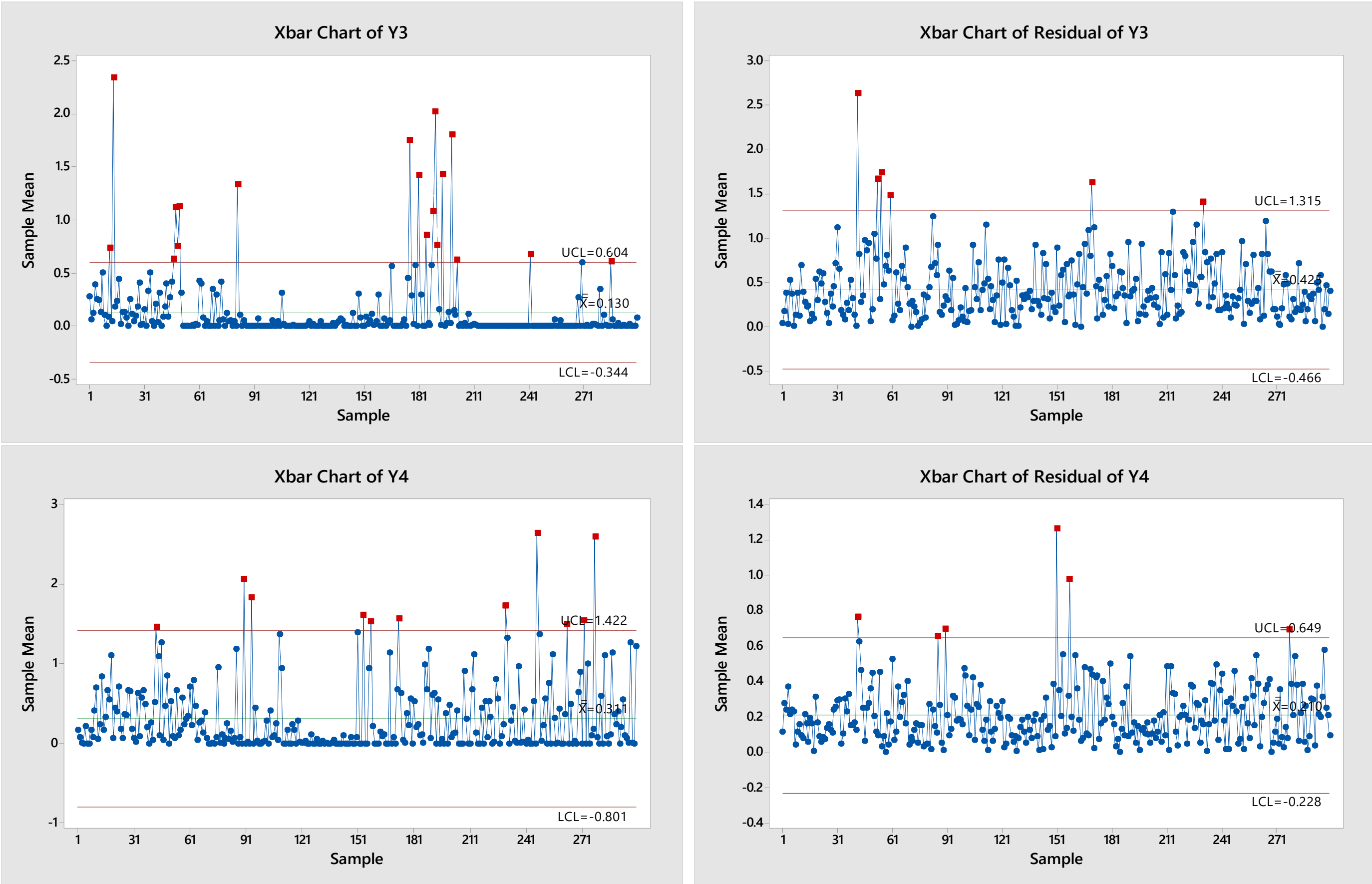
4.3 Comparison univariate control chart between original and residual data

In this section, Figure 7 presents a comparison between univariate control charts based on original data and residual data. The findings illustrate that the residual control chart outperforms the original data-based chart, particularly when the original data exhibit significant time series effects indicated by high autocorrelation. Specifically, Figure 7 demonstrates that control charts based on the original data exhibit more instances of out-of-control signals compared to those based on residual data. This indicates that using residuals, which account for the modeled effects and reduce autocorrelation, leads to improved performance in detecting deviations from the expected process behavior. Thus, employing residual-based control charts can enhance the accuracy and reliability of quality control measures in manufacturing or other monitored processes. Using three times of standard deviation from the center line rules, based on original data control chart, there are 181 samples, 6 samples, 6 samples, 16 samples, 18 samples, and 11 samples are out-of-control for each variable Y1_1, Y1_2, Y2_1, Y2_2, Y3 and Y4; respectively. Otherwise, using the same rules for testing, based on the residual data control chart, number of samples out of control decreasing into 11 samples, 4 samples, 7 samples, 11 samples, 6 samples and 6 samples for each variable Y1_1, Y1_2, Y2_1, Y2_2, Y3 and Y4; respectively. Even though variable Y2_1 has increasing number of samples out of control in residual data control chat, but there five consecutive points of out-of-control samples are getting large shift from the centerline.

The overall impression of process stability shown by control charts using residual data are rather different than was obtained from the control charts based on the original data. Otherwise, univariate control chart by original data for each variable, the pattern also shows the trend and large shift from the average. It might cause by the autocorrelation effects that happened on original data. As shown by the Figure 3, we see that the autocorrelation effect of the original data is very high which shown by the significant lag.

To address the practical implications of our proposed model in a real-world manufacturing context, we emphasize several key benefits. Firstly, the MAR-ANN model enhances detection sensitivity, enabling the early identification of potential quality issues. This allows for prompt corrective actions, reducing the incidence of defective products and minimizing rework, ultimately enhancing overall product quality. Additionally, the model effectively handles autocorrelation, a common challenge in traditional control charts, ensuring more reliable monitoring and reducing false alarms. This reliability is crucial for maintaining consistent product quality in continuous-flow manufacturing processes. Furthermore, our model is scalable and adaptable to various manufacturing processes with complex, multivariate, and autocorrelated data, making it suitable for diverse industries, from electronics to automotive. The implementation can be seamlessly integrated with existing manufacturing execution systems (MES) and statistical process control (SPC) software, allowing manufacturers to leverage advanced analytics without overhauling their current systems. This integration, coupled with the model's ability to lower operational costs by improving detection of process deviations and reducing false alarms, highlights its economic impact. Additionally, successful implementation requires comprehensive training for operators and engineers, demonstrating the model's benefits in improving process control and reducing false alarms. By promoting a culture of continuous improvement, manufacturers can use insights gained from the model to refine their processes continuously and maintain a competitive edge.





(a) (b)

Figure 7. Univariate control chart: (a) $\bar{x}$ control chart of original data; (b) $\bar{x}$ control chart of residual data

5. CONCLUSION

This case study highlights the importance of checking for autocorrelation and analyzing time series data before using control charts. Autocorrelation, where observations are dependent on previous ones, can negatively impact control chart performance. It can lead to more false alarms, as seen in charts built with raw data. Furthermore, in multivariate control charts, autocorrelation can mask the true relationships between variables. Additionally, dependence between variables can affect the performance of even univariate control charts. Therefore, using multivariate control charts is generally recommended.

To address the issue of autocorrelation in multivariate data, the paper suggests employing a combined approach: an ANN-based MAR (multivariate autoregressive) model. This approach aims to mitigate the effects of autocorrelation and improve the effectiveness of control charts. While univariate control charts are simpler to implement, they can be misleading when dealing with correlated variables. In such cases, the T^2 Hotelling control chart with decomposition techniques is a better option. This method helps identify which specific variables are contributing to out-of-control signals. The residual control charts, derived from the ANN-based model, perform significantly better in detecting mean shifts. This improvement is particularly evident in terms of sensitivity, where residual control charts showed a higher capability in identifying small process changes compared to traditional control charts. By emphasizing the practical benefits of residual control charts in handling autocorrelated multivariate data, our study contributes to the field by showcasing an effective solution for improved process monitoring and quality control. This approach offers valuable insights and practical implications for practitioners aiming to enhance their process control systems.

Based on the analysis of the case study, it is evident that in scenarios where detecting small changes in process parameters is critical, MCUSUM and MEWMA charts offer advantages over the T^2 Hotelling control chart. These alternative control charts exhibit superior run length performance and greater sensitivity in detecting minor shifts in the process's mean vector. This heightened sensitivity enables quicker response and action in maintaining process quality and efficiency. Given that the data in this study were derived from product specifications measured by an AOI system, which operates on numerical values, an intriguing area for future research involves exploring the application of P control charts. P control charts are pertinent for monitoring defect proportions and could provide valuable insights when applied to control charts using image data as input. Analyzing and interpreting control charts with image data presents a promising avenue to enhance quality control methodologies, particularly in sectors reliant on visual inspection and image-based measurements. This potential research direction could further advance understanding and implementation of robust quality control strategies in manufacturing and related industries.

In future studies, we aim to investigate the optimal selection and sensitivity of ANN models for handling multivariate time series data in industrial processes. This will involve conducting comprehensive experiments to compare various

ANN architectures (e.g., MLP, RNN, LSTM, GRU) and configurations (e.g., number of layers, neurons per layer, activation functions) to identify the most suitable models. Additionally, we will perform a sensitivity analysis by systematically varying key ANN parameters such as learning rate, hidden layer sizes, and regularization terms, as well as introducing variations in data quality like noise and missing data. By evaluating the impact of these changes on performance metrics, we intend to determine the robustness and stability of the ANN models and identify the configurations that produce the most representative and reliable residuals for process monitoring. This research will provide deeper insights into optimizing ANN models, enhancing the effectiveness of residual control charts, and offering valuable guidance for practitioners and researchers in improving industrial process monitoring and quality control.

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