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# Evaluating Polynomial, and Gaussian Approaches for Temperature Change Sub Index of Water Quality Index for Smart Environmental Management

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## ABSTRACT

Water temperature plays a crucial role in aquatic ecosystem health, affecting oxygen solubility, organism metabolism, and overall water quality. This study evaluates Linear Interpolation, Polynomial, and Gaussian Models for predicting the Temperature Change Sub-Index in a Water Quality Index (WQI). Using a dataset from NSF-WQI, we propose a new formula based on polynomial and Gaussian approaches, offering improved accuracy over the traditional linear interpolation method for better water quality assessment. The Gaussian Model 1 emerged as the most accurate, with a 97.54% accuracy and a mean error of 2.46, outperforming the Cubic Polynomial and Fourth-Degree Polynomial Models, which achieved accuracies of 94.60% and 96.03%, respectively. The Gaussian Model's ability to capture peak characteristics and symmetrical declines makes it particularly effective for applications requiring precise water quality predictions. The integration of Gaussian Model 1 into IoT-enabled real-time monitoring systems presents significant potential, enabling continuous, accurate predictions critical for managing sensitive aquatic environments. However, the study acknowledges limitations, including the narrow data range that may limit Model generalizability and the complexity of higher-degree polynomial models, which could reduce practical applicability. Future research should focus on validating these Models across more diverse datasets, exploring hybrid Models that combine Gaussian and polynomial strengths, and enhancing computational efficiency to support broader real-time applications.

## 1. INTRODUCTION

Water temperature plays a critical role in aquatic environments [1], influencing the solubility of gases like oxygen [2], metabolic rates of organisms [3], and overall ecosystem health [1]. Fluctuations in temperature, driven by natural factors such as seasonal changes and human activities like industrial discharges, can disrupt species composition and ecosystem balance [4]. As a key parameter in water quality assessments [5], temperature changes can lead to thermal stress [6], migration [7], and even species extinction [8], underscoring the importance of continuous monitoring and adaptive management to maintain ecosystem resilience in the face of climate change [9].

Table 1 summarizes key research studies on the implementation of temperature, given the significant impact of temperature changes on aquatic ecosystems, Water Quality Indexes (WQI) that incorporate temperature are essential tools for assessing these effects [10]. WQIs evaluate how temperature affects critical factors like oxygen solubility and help identify thermal pollution [11]. By monitoring these changes, WQIs provide early warnings of environmental stress [12], guiding conservation efforts to maintain the health and sustainability of aquatic habitats [13]. To quantify the impact of temperature within these indexes, calculating a sub-index is crucial [14].

One common method for calculating the sub-index of water temperature in a WQI is linear interpolation [15]. This method involves mapping the measured temperature values onto a predefined sub-index range, assuming a linear relationship between data points [16]. While linear interpolation is straightforward and easy to apply [17], it has limitations. The primary drawback is its assumption of linearity [18], which may not accurately reflect the complex [19], non-linear nature of temperature impacts on aquatic ecosystems [20]. Additionally, this method is sensitive to outliers, which can distort the accuracy of the sub-index calculation, necessitating the exploration of more sophisticated approaches [21].

To address these limitations, this study proposes exploring Gaussian and polynomial approaches for calculating the sub-index of water temperature changes in WQIs. The Gaussian approach, effective in Modeling bell-shaped data distributions [22], is suited for capturing complex patterns in temperature fluctuations [23]. Meanwhile, the polynomial approach offers flexibility in Modeling non-linear relationships [24], providing a more nuanced understanding of temperature impacts on water quality [25]. Both methods, widely used in statistics for data fitting and prediction [26-28], offer the potential to improve the accuracy of Modeling variable interactions, particularly in environmental science where precise analysis is crucial for decision-making [29].

In summary, this research addresses the limitations of linear Models in calculating the water temperature sub-index in WQIs by exploring Gaussian and polynomial approaches. As part of the second phase of developing the Internet of Things (IoT) Water Quality Index (WQI) [30], focused on data normalization, this study aims to find more accurate formulas for better assessing water quality in IoT-enabled systems.

**Table 1.** Temperature in environmental monitoring

| Researcher | Location | Parameters Used | Research Results |
|---|---|---|---|
| Islam [31] | Jamalpur District, Bangladesh | Temperature, pH, Turbidity | An IoT system for water quality monitoring in five ponds. Only three ponds are suitable for fish farming, and the Random Forest algorithm performed best. |
| Pasika and Gandla [32] | Hyderabad, India | Temperature, pH, Turbidity, Humidity, Water Level | An efficient and cost-effective IoT-based water quality monitoring system for real-time water quality monitoring. |
| Lakshmikantha et al. [33] | Mysuru, India | Temperature, pH, Conductivity, Turbidity | An efficient and cost-effective IoT system for real-time water quality monitoring, tested with three water samples, and data sent to a cloud server for further analysis. |
| Sugiharto et al. [34] | Troso River, Indonesia | Temperature, pH, TDS, Turbidity | A real-time IoT-based water quality monitoring system with high accuracy for temperature (98.54%) and other parameters. |
| Irawan et al. [35] | | Temperature, pH, | Intelligent quality control of shrimp aquaculture based on real-time system and IoT using mobile device. |
| Vasudevan and Baskaran [36] | Tiruchirappalli, India | Temperature, pH, Dissolved Oxygen, Turbidity | A real-time water quality monitoring system using unmanned surface vehicles that improves monitoring efficiency and reduces water pollution. |

## 2. THEORETICAL BACKGROUND

### 2.1 Temperature in environmental monitoring

Temperature is a crucial parameter in environmental monitoring due to its significant impact on various ecosystem processes and life [1]. In this context, temperature is used to assess the physical conditions of the environment [37], such as the atmosphere [38], water [39], and soil [40]. Temperature changes can affect the health of plants [41], animals [42], and humans [43], as well as influence biological and chemical processes within ecosystems [44]. For example, water temperature affects oxygen solubility and aquatic life [2], while air temperature impacts weather patterns and climate. Environmental temperature monitoring is essential for detecting climate changes, such as global warming, which can lead to significant shifts in ecosystems [45].

The implementation of technology in monitoring, such as the use of IoT (Internet of Things) sensors [46], enables real-time and continuous data collection [47]. This technology enhances the efficiency and accuracy of temperature monitoring [48], providing vital data that can be used to build predictive Models and understand temperature change patterns over time [49]. This data is also valuable for assessing environmental risks [50], such as wildfires [51], droughts [52], or floods [53], which are often associated with extreme temperature changes [54]. With this information, scientists, policymakers, and the general public can take more effective and adaptation measures in response to environmental changes [55].

Table 1 summarizes key research studies on the implementation of temperature as a parameter in environmental monitoring. It highlights the researchers, locations of the studies, the parameters used, and the significant findings from each study.

### 2.2 Temperature as parameter Water Quality Index

Temperature is a crucial parameter in the Water Quality Index (WQI) because it directly impacts the physical, chemical, and biological characteristics of water [10]. It affects oxygen solubility [2], chemical reaction rates [56], and the overall health of aquatic ecosystems [1]. Deviations in water temperature can signal pollution or environmental stressors [57], such as industrial discharges or climate change [58, 59]. Integrating temperature into the WQI provides a comprehensive understanding of water quality [60], helping to identify risks and inform management strategies to protect aquatic life and ensure clean water availability [61]. The Table 2 lists of Water Quality Indexes (WQI) that incorporate temperature as parameter. Each index uses temperature to assess various aspects of water quality, ranging from its impact on aquatic ecosystems to compliance with environmental standards.

**Table 2.** Temperature as parameter Water Quality Index

| Water Quality Index (WQI) Name | Description |
|---|---|
| NSF WQI (National Sanitation Foundation) [15] | Uses temperature as one of the indicators to assess overall water quality. |
| Canadian Water Quality Index (CWQI) [62] | Temperature is used to measure its impact on aquatic ecosystems and oxygen solubility. |
| Oregon Water Quality Index [63] | Considers temperature to evaluate the health of rivers and freshwater. |
| Bhargava Water Quality Index [64, 65] | Water temperature is used to determine water quality and its impact on aquatic life. |
| Aquatic Toxicity Index (ATI) [66] | Uses temperature to assess the potential toxicity of water to aquatic organisms. |
| Comprehensive Water Quality Index [67] | Temperature is included as a key parameter to evaluate various aspects of water quality. |
| European Union Water Framework Directive Index [68] | Temperature is used to ensure water quality meets environmental standards set by the European Union. |
| EPA Water Quality Index [69] | Uses temperature as part of the water quality assessment conducted by the U.S. Environmental Protection Agency. |

### 2.3 Temperature change Sub Index (I)

To calculate Temperature Change (ΔT), NSF WQI using linear interpolation between two known data points will use Eq. (1) [15]. This formula allows us to find the value of $y$ at a point $x$ that lies between two data points ($x_0$, $y_0$) and ($x_1$, $y_1$). By utilizing the difference in $y$ $x$ values between these two points, linear interpolation provides the $y$ value that corresponds to the position of $x$ relative to $x_0$ and $x_1$.

$x_0$ and $y_0$ are the first known data points. $x_1$ and $y_1$ are the second known data points. $x$ is the ΔT value that is input, and $I$ is the WQI value calculated through interpolation. These data points are shown in Table 3.

$$Sub\ Index\ (I) = y_0 + \frac{(y_1 - y_0)}{(x_1 - x_0)} . (x - x_0) \tag{1}$$

**Table 3.** Temperature change Sub Index (I)

| Temp. Change (°C) | Sub Index (I) |
|---|---|
| -10 | 56 |
| -7.5 | 63 |
| -5 | 73 |
| -2.5 | 85 |
| -1 | 90 |
| 0 | 93 (max) |
| 1 | 89 |
| 02.05 | 85 |
| 5 | 72 |
| 07.05 | 57 |
| 10 | 44 |
| 12.05 | 36 |
| 15 | 28 |
| 17.05 | 23 |
| 20 | 21 |
| 22.05 | 18 |
| 25 | 15 |
| 27.05.00 | 12 |
| 30 | 10 |

Figure 1 is the curve that illustrates the relationship between Temperature Change (X) and Sub-Index (I) based on Table 3. This curve depicts how the Sub-Index (I) varies with changes in temperature, with the peak occurring at around 0°C temperature change.

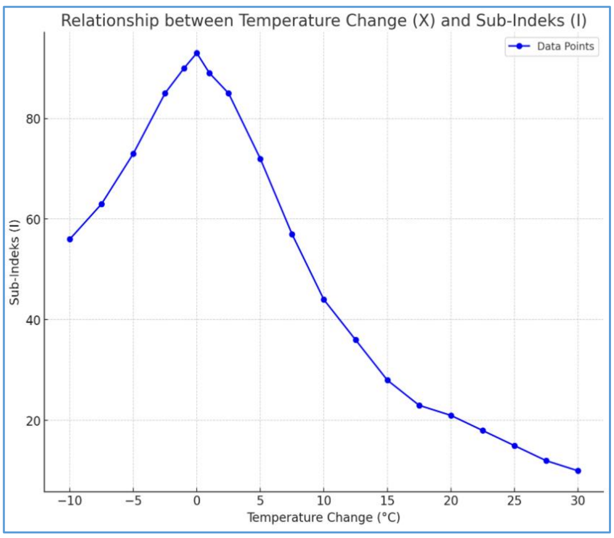


**Figure 1.** Temperature change Sub Index (I)

### 2.4 Polynomial model approach

A polynomial model, with the general form Eq. (2), is a mathematical Model commonly used to analyze and predict non-linear relationships between variables [70], such as environmental temperature and other factors like time or humidity. In temperature analysis, a polynomial allows for the identification of non-linear temperature trends [71], capturing variations that may occur over time. By using least squares regression, the coefficients $a$, $b$ and $c$ can be estimated to minimize prediction errors, making this Model useful for understanding past temperature patterns as well as predicting future temperature changes. This implementation is valuable in applications such as agricultural planning [72], energy management [73], and climate change impact mitigation enabling better decision-making in the face of extreme conditions [74].

$$f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n \tag{2}$$

### 2.5 Gaussian Model approach

Gaussian Model, or normal distribution, is a statistical Model that describes the spread of data around a mean with a bell-shaped symmetric curve, commonly used for analyzing the distribution of environmental [22]. In this context, the Gaussian Model allows for understanding the distribution of daily or annual temperatures, helping to identify anomalies and calculate the probability of extreme temperatures. This Model can also be used to predict shifts in temperature distribution in the future based on historical data and climate change trends, which is highly useful for sectors such as agriculture [75], energy management [76], and healthcare in dealing with dynamic environmental conditions [77].

The Gaussian, or normal, distribution is one of the most fundamental probability distributions in statistics, characterized by a symmetric bell-shaped curve centered around the mean [78]. Mathematically, this distribution is expressed form Eq. (3).

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{3}$$

However, in many practical applications, adjustments to this basic distribution are often necessary to more accurately represent observed data. To achieve this flexibility, several additional parameters are introduced. The parameter $a$ is used to adjust the amplitude or height of the curve's peak, allowing for vertical scaling. The parameter $b$ affects the width of the curve, with larger values of $b$ producing a narrower curve. The peak position of the curve along the horizontal axis can be shifted using the parameter $c$, which replaces the standard mean $\mu$. Finally, the parameter $d$ is added to vertically shift the entire curve, providing an offset that is often required in various data analysis contexts, with these modifications, the Gaussian equation becomes Eq. (4).

$$f(x) = a\,.\,b^{-b\,(x-c)^2} + d \tag{4}$$

## 3. METHODOLOGY

Conducted to Sugiharto et al. [34], his study focuses on the data normalization stage, which is the second phase in the development of the Water Quality Index (WQI) as shown in Figure 2. In this stage, determining the sub-index is crucial because the formula used must provide an accurate and representative picture of water quality. This ensures that the WQI framework and calculation model developed later can be more valid and effective.

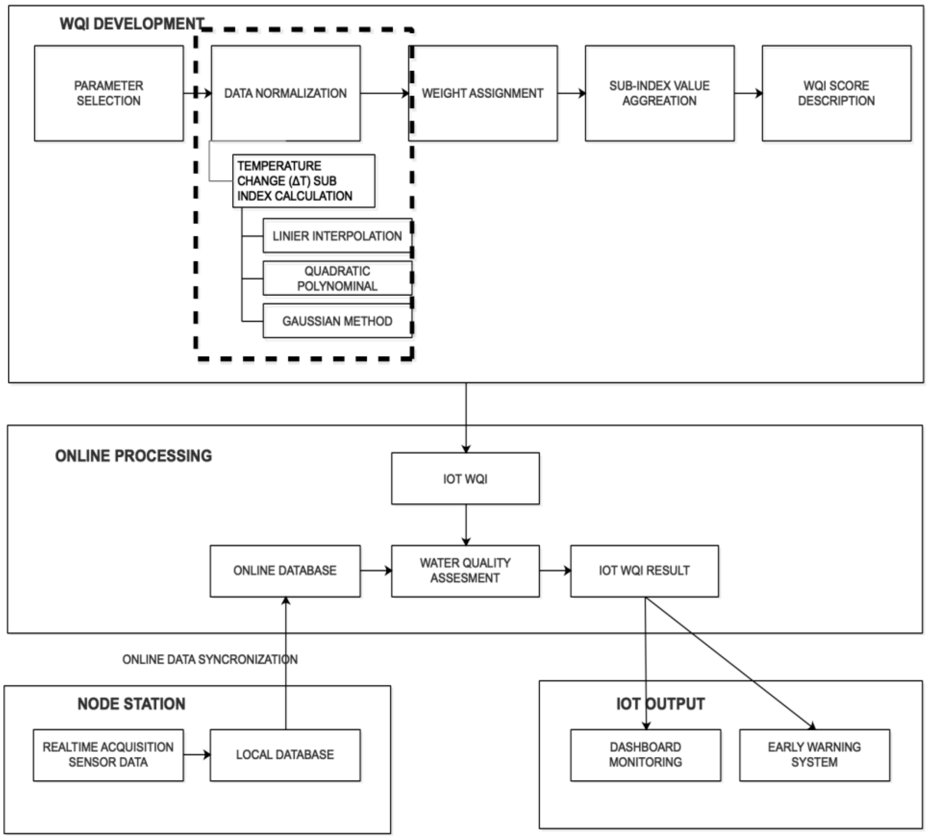


**Figure 2.** WQI development

### 3.1 WQI development

Conducted to Sugiharto et al. [34] previous research, this study continues suggested WQI system incorporates IoT technology and cloud computing to enable real-time collection, processing, storage, and analysis of water quality data. The detailed framework of this system has been outlined in earlier studies [34, 30]. Figure 2 provides a visual representation of the process for gathering data from rivers and conducting water quality assessments.

### 3.2 Temperature change sub-index development

To achieve this objective, this study employs 3 different approaches to calculate the Temperature Change sub-index. The current approach uses linear interpolation is a basic method used to determine the sub-index value based on temperature change. This method assumes that the relationship between ΔT and WQI is linear between two known data points. The sub-index value is calculated by interpolating the measured ΔT value between two reference points in the Table 3 dataset, resulting in an estimated WQI.

In this study, the Polynomial and Gaussian approaches will be used to Model the relationship between temperature change and the Sub Index (I) while considering the curve shape generated by the data in Table 3.

The accuracy of these Models will be compared with the actual data in Table 1. This comparison will determine the extent to which each approach can replicate or approximate the actual Sub Index (I) and assess the effectiveness of these approaches in the context of calculating the Sub Index (I) within the Water Quality Index (WQI).

## 4. RESULT

The first step in developing the equation is selecting the appropriate model. Based on the curve shape, a polynomial model, along with the Gaussian approach, was chosen. After deriving the equation, verification is performed by plotting the results against the original data to assess how well the Model fits the actual data.

### 4.1 Polynomial approach

4.1.1 Curve fitting

The cubic polynomial approach is used for curve fitting on the data points shown in Figure 1. The results of this fitting are presented in Figure 3 to evaluate how well the model fits the existing data, and the equation is Eq. (5). $I$ represents the Sub-Index and $x$ represents the temperature change in degrees Celsius. This equation consists of several key components: the cubic term $0.0084x^3$, which determines the curvature of the graph; the quadratic term $-0.2964\,x^2$, which shapes the parabola of the curve; and the linear term $-0.8492x$, which influences the slope of the line. Additionally, the constant 83.65 serves to vertically shift the curve to better fit the existing data.

$$I = 0.884x^3 - 0.2964x^2 - 0.8492x + 83.65 \tag{5}$$

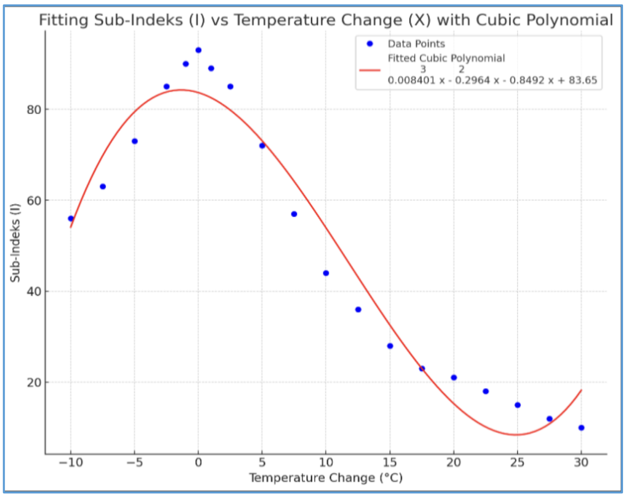


**Figure 3.** Cubic polynomial

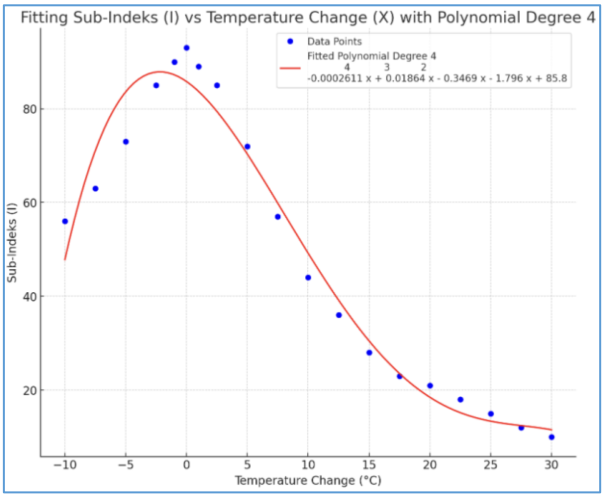


**Figure 4.** Fourth degree polynomial

Tuning was performed in Figure 4 by increasing the polynomial degree, specifically by using a higher-degree polynomial to better capture the curve's details. The equation used in Figure 4 is Eq. (6).

The more accurate fourth-degree polynomial equation where $I$ represents the Sub-Index and $x$ represents the temperature change in degrees Celsius. This equation includes several key components: the fourth-degree term $-0.0002611x^4$, which captures the curve's finer curvature details; the cubic term $0.01864x^3$, which adds more flexibility to the curve; the quadratic term $-0.3469x^2$, which defines the parabolic shape of the curve; and the linear term $-1.796x$, which influences the slope of the line. Additionally, the constant 85.80 serves to vertically shift the curve to better fit the existing data.

$$I = -0.0002611x^4 + 0.01864x^3 - 0.3469x^2 - 1.796x + 85.80 \quad (6)$$

4.1.2 Polynomial model validation

The polynomial model is validated (Figure 5) by comparing the Model's predicted results with the actual data from Table 3. Accuracy is assessed through the percentage error between the Sub-Index (I) produced by the Model and the actual values, and Table 4 evaluates how well the Model represents the data.

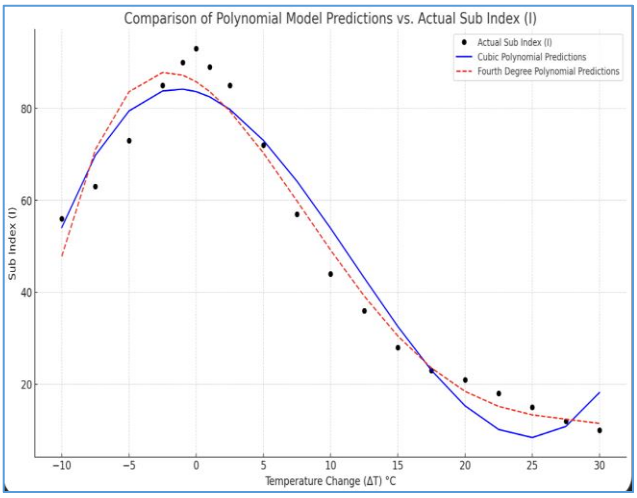


**Figure 5.** Polynomial model comparison

**Table 4.** Polynomial model validation

| ΔT (°C) | (I) | Cubic Polynomial | Fourth Degree Polynomial I | Cubic Polynomial Error | Fourth Degree Polynomial Error |
|---|---|---|---|---|---|
| -10 | 56 | 54.10 | 47.82 | 1.90 | 8.18 |
| -7.5 | 63 | 69.80 | 71.07 | 6.80 | 8.07 |
| -5 | 73 | 79.44 | 83.61 | 6.44 | 10.61 |
| -2.5 | 85 | 83.79 | 87.82 | 1.21 | 2.82 |
| -1 | 90 | 84.19 | 87.23 | 5.81 | 2.77 |
| 0 | 93 | 83.65 | 85.80 | 9.35 | 7.20 |
| 1 | 89 | 82.51 | 83.68 | 6.49 | 5.32 |
| 2.5 | 85 | 79.81 | 79.42 | 5.19 | 5.58 |
| 5 | 72 | 73.04 | 70.31 | 1.04 | 1.69 |
| 7.5 | 57 | 64.15 | 59.85 | 7.15 | 2.85 |
| 10 | 44 | 53.92 | 49.18 | 9.92 | 5.18 |
| 12.5 | 36 | 43.13 | 39.18 | 7.13 | 3.18 |
| 15 | 28 | 32.57 | 30.50 | 4.57 | 2.50 |
| 17.5 | 23 | 23.04 | 23.54 | 0.04 | 0.54 |
| 20 | 21 | 15.31 | 18.46 | 5.69 | 2.54 |
| 22.5 | 18 | 10.17 | 15.18 | 7.83 | 2.82 |
| 25 | 15 | 8.42 | 13.35 | 6.58 | 1.65 |
| 27.5 | 12 | 10.84 | 12.39 | 1.16 | 0.39 |
| 30 | 10 | 18.214 | 11.49 | 8.21 | 1.50 |
| | | **Average Error** | | **5.40** | **3.97** |

### 4.2 Gaussian approach

4.2.1 Curve fitting

Gaussian curve fitting is particularly effective in capturing curves with a distinct peak and symmetrical decline on both sides. This study explores the use of the Gaussian approach to determine whether it can yield more accurate results in modeling the data. The Gaussian fitting is shown in Figure 6, with the resulting equation being Eq. (7).

$$I = 74.18.e^{-0.0077(x+0.475)^2} + 14.54 \quad (7)$$

Tuning was performed by increasing the coefficient that controls the height of the curve's peak and narrowing the coefficient that controls the curve's width. Eq. (8) is modifying the parameters $a$ (which controls the peak height) and $b$ (which controls the curve's width) in the Gaussian equation. The curve now has a higher peak and a narrower width, The Gaussian tuning is shown in Figure 7, with the resulting equation being Eq. (8).

$$I = 77.e^{-0.0153(x+0.475)^2} + 14.54 \quad (8)$$

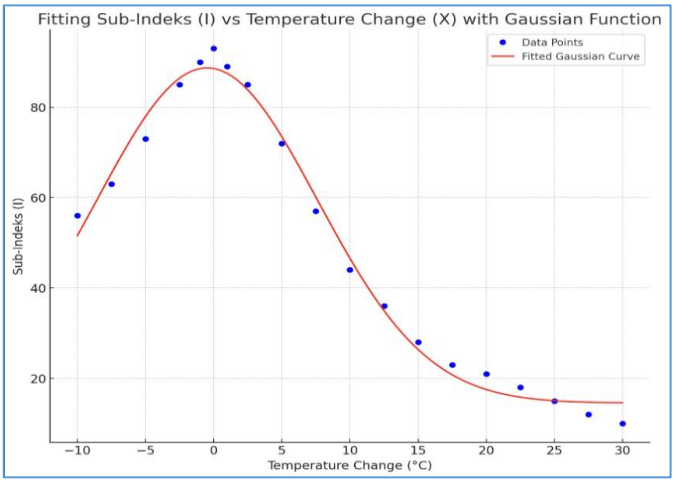


**Figure 6.** Gaussian Model 1

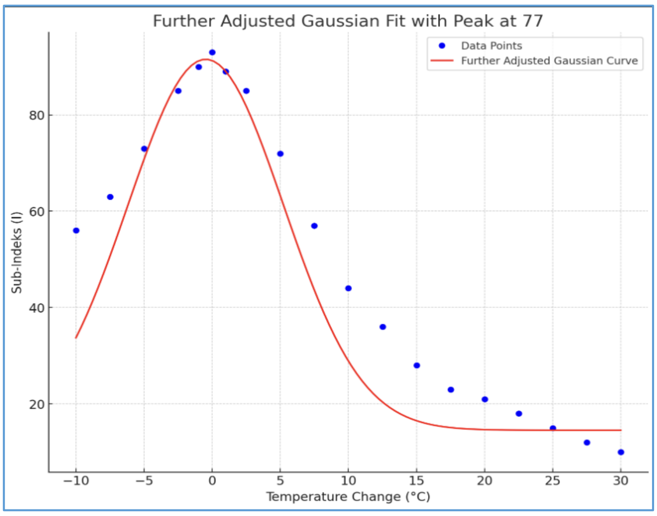


**Figure 7.** Gaussian Model 2

Further tuning was performed by widening the curve through lowering the coefficient $b$, so the Gaussian curve now fits the data points more closely. Eq. (9) represents the equation after tuning, where the curve has been adjusted to a peak height of 78.3, as shown in Figure 8.

$$I = 78.3.e^{-0.0077(x+0.475)^2} + 14.54 \quad (9)$$

Further tuning was performed by adjusting $d$ so that the peak value at $x$=0 is Sub-Index (I)=93. This tuning is expected to provide more accurate results. Eq. (10) represents the equation after adjusting $d$, where the curve has been modified as shown in Figure 9.

$$I = 74.47.e^{-0.0115(x+0.125)^2} + 18.54 \quad (10)$$

4.2.2 Gaussian approach validation

The Gaussian approach model is validated by comparing the Model's predicted results with the actual data from Table 3. Accuracy is assessed through the percentage error between the Sub-Index (I) produced by the Model and the actual values, Table 5 displayed the predictions from each Gaussian Model alongside the actual Sub-Index (I) values in a table.

Table 6 calculates the error for each Gaussian Model and compares it with the actual Sub-Index (I) data, including the mean error for each Gaussian Model. Figure 10 illustrate the accuracy each Model.

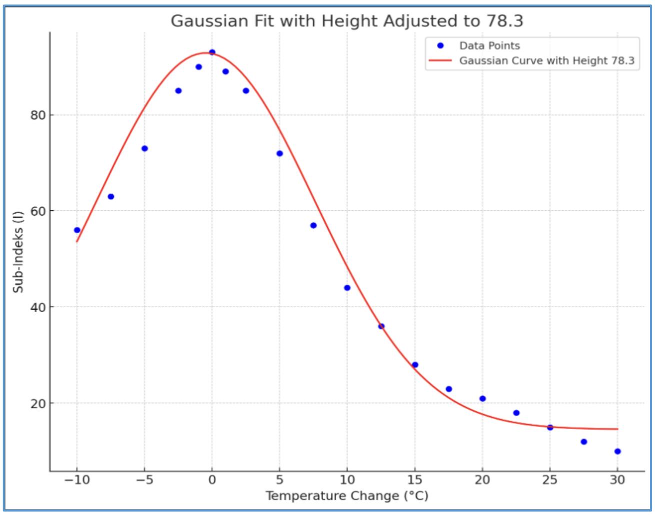


**Figure 8.** Gaussian Model 3

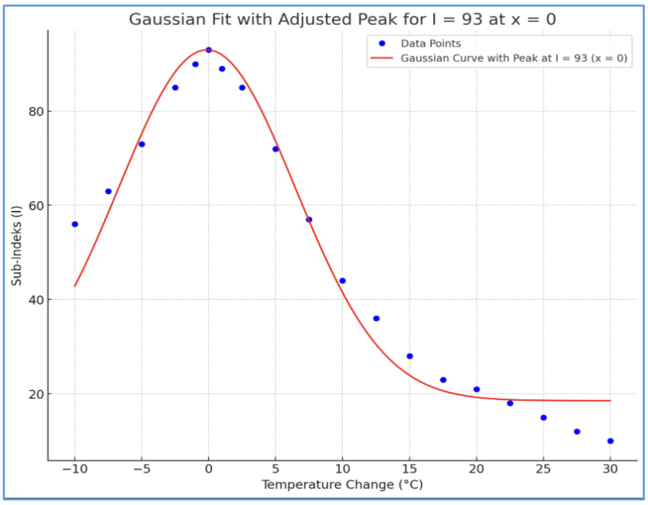


**Figure 9.** Gaussian Model 4

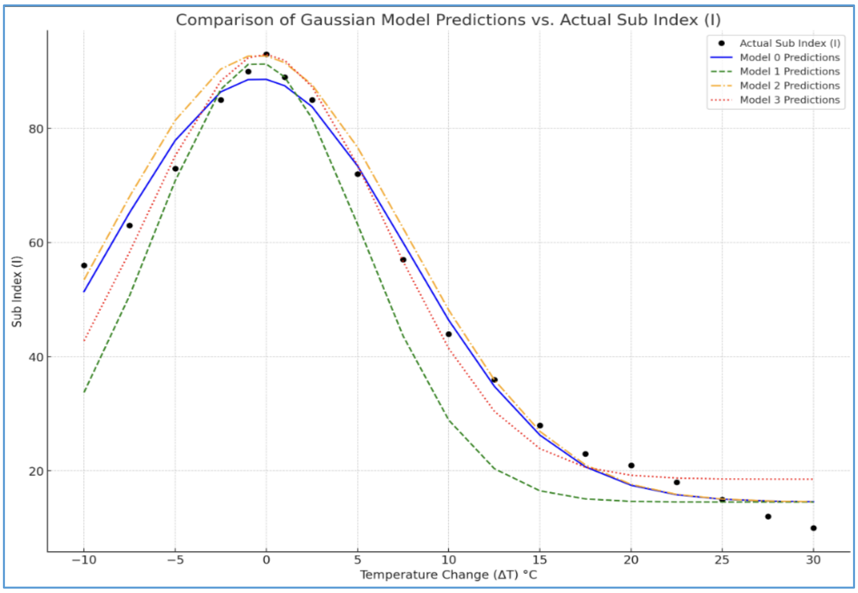


**Figure 10.** Gaussian Model comparison

**Table 5.** Prediction result Gaussian Models

| ΔT (°C) | Actual Sub Index (I) | Gaussian Model 1 | Gaussian Model 2 | Gaussian Model 3 | Gaussian Model 4 |
|---|---|---|---|---|---|
| -10 | 56 | 51.43 | 33.76 | 53.48 | 42.80 |
| -7.5 | 63 | 65.27 | 50.73 | 68.09 | 58.38 |
| -5 | 73 | 77.90 | 70.83 | 81.42 | 75.20 |
| -2.5 | 85 | 86.41 | 86.86 | 90.41 | 88.33 |
| -1 | 90 | 88.56 | 91.22 | 92.67 | 92.36 |
| 0 | 93 | 88.59 | 91.27 | 92.70 | 93.00 |
| 1 | 89 | 87.49 | 89.02 | 91.54 | 91.93 |
| 2.5 | 85 | 83.83 | 81.79 | 87.68 | 87.34 |
| 5 | 72 | 73.43 | 63.22 | 76.70 | 73.60 |
| 7.5 | 57 | 60.00 | 43.64 | 62.52 | 56.70 |
| 10 | 44 | 46.41 | 28.91 | 48.18 | 41.45 |
| 12.5 | 36 | 34.83 | 20.40 | 35.96 | 30.45 |
| 15 | 28 | 26.27 | 16.51 | 26.93 | 23.90 |
| 17.5 | 23 | 20.70 | 15.09 | 21.05 | 20.63 |
| 20 | 21 | 17.48 | 14.67 | 17.64 | 19.25 |
| 22.5 | 18 | 15.81 | 14.56 | 15.88 | 18.75 |
| 25 | 15 | 15.04 | 14.54 | 15.07 | 18.59 |
| 27.5 | 12 | 14.72 | 14.54 | 14.73 | 18.55 |
| 30 | 10 | 14.60 | 14.54 | 14.60 | 18.54 |

**Table 6.** Mean error Gaussian Models

| ΔT (°C) | Actual Sub Index (I) | Gaussian Model 1 | Gaussian Model 2 | Gaussian Model 3 | Gaussian Model 4 |
|---|---|---|---|---|---|
| -10 | 56 | 4.57 | 22.24 | 2.52 | 13.20 |
| -7.5 | 63 | 2.27 | 12.27 | 5.09 | 4.62 |
| -5 | 73 | 4.90 | 2.17 | 8.42 | 2.20 |
| -2.5 | 85 | 1.41 | 1.86 | 5.41 | 3.33 |
| -1 | 90 | 1.44 | 1.22 | 2.67 | 2.36 |
| 0 | 93 | 4.41 | 1.73 | 0.30 | 0.00 |
| 1 | 89 | 1.51 | 0.02 | 2.54 | 2.93 |
| 2.5 | 85 | 1.17 | 3.21 | 2.68 | 2.34 |
| 5 | 72 | 1.43 | 8.78 | 4.70 | 1.60 |
| 7.5 | 57 | 3.00 | 13.36 | 5.52 | 0.30 |
| 10 | 44 | 2.41 | 15.09 | 4.18 | 2.55 |
| 12.5 | 36 | 1.17 | 15.60 | 0.04 | 5.55 |
| 15 | 28 | 1.73 | 11.49 | 1.07 | 4.10 |
| 17.5 | 23 | 2.30 | 7.91 | 1.95 | 2.37 |
| 20 | 21 | 3.52 | 6.33 | 3.36 | 1.75 |
| 22.5 | 18 | 2.19 | 3.44 | 2.12 | 0.75 |
| 25 | 15 | 0.04 | 0.46 | 0.07 | 3.59 |
| 27.5 | 12 | 2.72 | 2.54 | 2.73 | 6.55 |
| 30 | 10 | 4.60 | 4.54 | 4.60 | 8.54 |
| | **Mean Error** | 2.46 | 7.07 | 3.16 | 3.61 |

## 5. DISCUSSION

### 5.1 Model accuracy comparison

To perform the model accuracy comparison analysis, Table 7 shows the mean error of each Model that has been calculated previously. This mean error serves as the primary indicator to evaluate how well each Model is able to predict the Sub-Index (I) based on temperature change data. The results show that Gaussian Model 1 has the highest accuracy with 97.54%.

**Table 7.** Model accuracy comparison

| Model | Model | Mean Error | Accuracy |
|---|---|---|---|
| Gaussian Model | Gaussian Model 1 | **2.46** | **97.54** |
| | Gaussian Model 2 | 7.07 | 92.93 |
| | Gaussian Model 3 | 3.16 | 96.84 |
| | Gaussian Model 4 | 3.61 | 96.39 |
| Polynomial | Cubic Polynomial | 5.40 | 94.60 |
| | Fourth-Degree Polynomial | **3.97** | **96.03** |

### 5.2 Comparison of linear interpolation and Gaussian Model 1

The comparison was made by calculating linear interpolation between two known data points, using ΔT values between the existing temperature change values in the table, instead of using the exact data points. Table 8 shows the comparison between the linear interpolation results for points between the given data points (excluding the exact data points) and the predictions from Gaussian Model 1. From this validation, the Gaussian Model achieved an accuracy of 97.87%.

**Table 8.** Comparison of linear interpolation and Gaussian Model 1

| ΔT (°C) | Interpolated Sub Index (I) | Gaussian Model 1 Prediction | Error | Accuracy |
|---|---|---|---|---|
| -8.75 | 59.5 | 58.32 | 1.18 | 98.82 |
| -6.25 | 68 | 71.92 | 3.92 | 96.08 |
| -3.75 | 79 | 82.84 | 3.84 | 96.16 |
| -1.75 | 87.5 | 87.80 | 0.30 | 99.70 |
| -0.5 | 91.5 | 88.72 | 2.78 | 97.22 |
| 0.5 | 91 | 88.18 | 2.82 | 97.18 |
| 1.75 | 87 | 85.95 | 1.05 | 98.95 |
| 3.75 | 78.5 | 79.19 | 0.69 | 99.31 |
| 6.25 | 64.5 | 66.91 | 2.41 | 97.59 |
| 8.75 | 50.5 | 53.06 | 2.56 | 97.44 |
| 11.25 | 40 | 40.28 | 0.28 | 99.72 |
| 13.75 | 32 | 30.16 | 1.84 | 98.16 |
| 16.25 | 25.5 | 23.15 | 2.35 | 97.65 |
| 18.75 | 22 | 18.85 | 3.15 | 96.85 |
| 21.25 | 19.5 | 16.50 | 3.00 | 97.00 |
| 23.75 | 16.5 | 15.35 | 1.15 | 98.85 |
| 26.25 | 13.5 | 14.84 | 1.34 | 98.66 |
| 28.75 | 11 | 14.64 | 3.64 | 96.36 |
| | **Average** | | **2.13** | **97.87** |

### 5.3 Comparison analysis

Based on the model accuracy comparison in Table 7 and comparison of linear interpolation and gaussian Model 1 in Table 8, it is evident that the Gaussian Model, particularly Gaussian Model 1, shows the best performance with the highest accuracy of 97.54% and the lowest mean error of 2.46. The advantage of Gaussian Model 1 lies in its ability to capture specific peak characteristics and symmetrical decline on both sides of the curve, which is challenging for polynomial models. On the other hand, the Cubic Polynomial Model demonstrates fairly good results with an accuracy of 94.60% and a mean error of 5.40. However, this Model is less effective in capturing extreme temperature variations, especially in regions with sharper temperature changes, where the Gaussian Model excels. The Fourth-Degree Polynomial, although more complex than the cubic polynomial, provides an accuracy of 96.03% with a mean error of 3.97. However, the increased complexity of this Model does not always correlate with a significant increase in accuracy compared to Gaussian Model 1, which consistently shows superior performance in predicting the Sub-Index (I) based on temperature change data.

### 5.4 Practical implication

The higher accuracy of Gaussian Model 1 makes it particularly suitable for applications that require highly precise Temperature Change Sub-Index (I) predictions, such as water quality monitoring in highly sensitive environments where even minor deviations could have significant consequences. In these contexts, the Model's ability to accurately capture peak and symmetrical declines is critical. However, in scenarios where computational efficiency and simplicity are prioritized, especially when minor reductions in accuracy are acceptable, polynomial models might be preferred. Their simpler structure can lead to faster computations, which is advantageous in systems with limited processing power or where rapid analysis is required. Thus, the choice between Gaussian and polynomial models should consider both the specific accuracy needs and the computational constraints of the application

### 5.5 Limitation, potential and future work

A key limitation of this study is the limited data range, which may restrict the Models' generalizability, as their accuracy could decline when applied to broader or different temperature ranges. The increased complexity of Gaussian and high-degree polynomial models, while improving accuracy, may also reduce interpretability and practicality in certain applications. Despite these challenges, there is significant potential to enhance these Models by incorporating additional variables like pH levels or dissolved oxygen, making them more comprehensive across diverse environmental conditions. Future work should validate these Models with varied datasets to ensure generalizability and explore hybrid Models that combine Gaussian and polynomial strengths for greater flexibility and accuracy.

Conducted to Sugiharto et al. [34] previous research, with implementing these Models in real-time water quality monitoring systems could enable continuous prediction of the Sub-Index (I), allowing for timely responses to environmental changes. Furthermore, optimizing computational efficiency would make these models suitable for devices with limited processing power without compromising accuracy.

## 6. CONCLUSIONS

This study demonstrates that Gaussian Model 1 provides the highest accuracy for predicting the Sub-Index (I) based on temperature changes ($\Delta T$), with an accuracy of 97.54% and a mean error of 2.46. Its ability to capture peak characteristics and symmetrical declines makes it particularly suitable for sensitive water quality monitoring applications. In contrast, the Cubic Polynomial and Fourth-Degree Polynomial Models, while offering reasonable accuracy of 94.60% and 96.03% respectively, fall short in scenarios with extreme temperature variations. The integration of Gaussian Model 1 into IoT-enabled systems offers significant potential for real-time water quality monitoring, enabling timely responses to environmental changes and ensuring the protection of aquatic ecosystems.

However, the study acknowledges limitations, particularly the restricted data range, which may limit the generalizability of these Models. The increased complexity of Gaussian and high-degree polynomial models, while improving accuracy, could reduce interpretability and practicality in certain contexts. Future research should focus on validating these Models with diverse datasets and broader environmental conditions to enhance their applicability. Additionally, incorporating other water quality parameters and optimizing computational efficiency could further improve these Models, making them more suitable for real-time monitoring systems.

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