



## Free Vibration Behaviour of Laminated Composite Beam Under Crack Effects: A Combined Numerical and Experimental Approach

Ambica Prasad Mohanty<sup>1</sup>, Priyadarshi Das<sup>2\*</sup>, Sasanka Choudhury<sup>1</sup>, Shishir Kumar Sahu<sup>3</sup>

<sup>1</sup> Department of Mechanical Engineering, Institute of Technical Education and Research, Siksha 'O' Anusandhan Deemed to be University, Bhubaneswar 751030, Odisha, India

<sup>2</sup> Department of Civil Engineering, Institute of Technical Education and Research, Siksha 'O' Anusandhan Deemed to be University, Bhubaneswar 751030, Odisha, India

<sup>3</sup> Department of Civil Engineering, National Institute of Technology Rourkela, Rourkela 769008, Odisha, India

Corresponding Author Email: [priyadarshidas@soa.ac.in](mailto:priyadarshidas@soa.ac.in)

Copyright: ©2024 The authors. This article is published by IETA and is licensed under the CC BY 4.0 license (<http://creativecommons.org/licenses/by/4.0/>).

<https://doi.org/10.18280/acsm.480402>

### ABSTRACT

**Received:** 15 August 2023

**Revised:** 17 June 2024

**Accepted:** 26 June 2024

**Available online:** 30 August 2024

#### Keywords:

*numerical simulation, frequency analysis, crack effect, composite beam, FFT analyzer*

This paper presents a research attempt towards frequency-based crack effect study in laminated composite beams (LCBs) through combined computational and experimental work. The focused objective of the undertaken research is to quantify the crack effects on the frequency behavior of LCBs. Towards achieving the objective, carbon/epoxy LCBs are the subject materials under consideration and to achieve the natural frequencies for the LCBs under free vibration, a combination of numerical simulation and experimental tests are conducted. ABAQUS, a powerful finite element (FE) simulation software is employed to develop precise numerical models representing the LCB's mechanical behavior and response to crack propagation. The free vibration experiments on LCB samples are conducted through a Fast Fourier Transform (FFT) analyzer under controlled crack scenarios aligning with the numerical simulation model. Through the frequency analysis, critical crack depths and positions that significantly influence the dynamic impact are identified. The study also explores the correlation between the computational and experimental results, enhancing the confidence in simulation models. The results from the study can aid in the early detection of damage and the optimization of repair strategies to ensure the structural reliability and safety of LCBs or alike composite components.

## 1. INTRODUCTION

The appreciable strength and stiffness with an excellent tailoring ability towards achieving specific design needs made the laminated composites an automatic choice in diverse applied engineering fields ranging from aerospace to civil and mechanical applications [1]. The widespread utilization of LCBs realized the influence of different static and dynamic loading scenarios that acknowledged as the major cause for crack initiation and propagation. The cracks, if not addressed properly lead to the ultimate collapse of the structure. This attracts many investigators to address the crack effects in structures using different materials and techniques [2-4]. But the complexity involved in crack modelling and simulation presents a real challenging issue. In light of this, analyses in the domain of frequency measurements are proven as a robust and effective non-destructive technique to quantifying impact of cracks in structures. This is evident from the compilation work of addressed literature as reflected in some studies through years [5-7]. Given the significance of vibration analysis, cracked isotropic structures, intact composite beams, plates and shells got wide research attention [8-12]. Despite such practical importance and potential research significance, the research attention towards cracked LCB for failure

analysis is limited and not to the fullest extent.

The eigenfrequencies are evaluated from the structure's stiffness which is greatly influenced by the extent of crack orientation. The crack variation in depth and location continuously changes the dynamics of the structure and thus, the complexity involves in structural discontinuities present a complex geometry. So, the stiffness of the structure can be effectively determined by formulating the flexibility of the cracked element and then made the inverse of it [13-15].

The local compliance-based stiffness evaluation approach was adopted by many investigators towards performing the crack effect studies for composite beams concerning a single crack [16-18]. But with the elevated complexity involved for multi-crack modelling or simulation, reportedly less research attention was paid to composite beams with multiple open cracks [19-21]. One such notable work on multi-crack analysis in the threshold of vibration was found in the work by Kisa [22]. The study adopted finite element method following the work done by Krawczuk and Ostachowicz [23] extending to multiples cracks on a graphite/epoxy beam. Following the work done by Kisa [22] some research works are presented [24-26] for multiple cracks but, the experimental validation to such numerical approaches is still significantly lack as fabricating laminated composites with engineered cracks is a

real challenging task and requires skilled approach. Nonetheless, regardless of the complexity involved, few honest attempts can be found for experimental crack effect study for composite beams under free vibration in open literature [27-29]. In recent years, the vibration-based studies are addressed for crack analysis and their effects on different beam parameters aiming on the functionality of the structure [30-33]. But the optimal result of the frequency studies is yet not achieved which enhances further research scope in the domain of vibration analysis.

The relevant published works in line with the objective of this research are discussed above acknowledged the research progress on dynamic exploration of LCBs and widely expands the scope due to scanty experimental works. Evidently, the comprehensive analysis of free vibrations in multiple engineered cracked composite laminated beams has not yet been fully investigated, where a combined numerical and experimental approach is far from complete. With significant advancements in system configurations, development in instrumentation, that includes accuracy and robustness in tools for signalling data record processing and automated data logging systems, this has become feasible to perform investigational dynamic character testing. This allows for a precise, real-time understanding of the frequency character of structural elements. Though the vibration dependent crack studies are realized for LCBs, limited progress is noticed for achieving the optimal result. Based on this novelty, the present research work attempted to quantify the dynamic behaviour of cracked laminated carbon/epoxy beam through developing a computational simulation model with the experimental validation. The investigation deals with the experimental and computational eigenfrequencies of industry-grade carbon/epoxy LCBs. The frequencies are studied for fiber orientation, boundary conditions and crack orientations (location and depth). The close alignment of simulated results with the empirical results further enhances the robustness and applicability of the model.

## 2. THEORETICAL FORMULATION

With usual conventional geometrical notations for longitudinal (L) and Lateral (width 'b'), carbon/epoxy LCB is illustrated in Figure 1. The crack implications are realized at positions  $X_1, X_2, X_3...X_i$  from the left end, being the crack depth 'a'. The intact sections of the LCB are represented by the composite beam FE proposed by Maiti and Sinha [10], while the cracked sections are modeled using the cracked composite beam FE as described below:

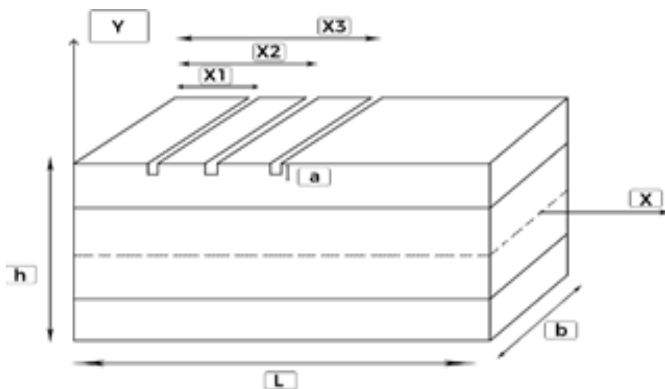


Figure 1. Cracked LCB with geometry

The vibration attributes impacted by cracks can be effectively realized by establishing the cracked beam's local compliance or flexibility matrix. To this end, the flexibility or local compliance may be written as [22, 23]:

$$c_{ij} = \frac{\partial^2}{\partial P_i \partial P_j} \int_A \left( D_1 \sum_{n=1}^6 K_{In}^2 + D_2 \sum_{n=1}^6 K_{IIIn}^2 + D_{12} \sum_{n=1}^6 K_{Ii} \sum_{n=1}^6 K_{IIIn} + D_3 \sum_{n=1}^6 K_{IIIn}^2 \right) dA \quad (1)$$

where, the notations for material parameter (A,  $D_1, D_2, D_3, D_{12}$ ) are expressed as:

$$\begin{aligned} D_1 &= -0.5 \bar{b}_{11} Im \left( \frac{\mu_1 + \mu_2}{\mu_1 \mu_2} \right) \\ D_2 &= 0.5 \bar{b}_{11} Im (\mu_1 + \mu_2) \\ D_{12} &= \bar{b}_{11} Im (\mu_1 \mu_2) K_I K_{II} \\ D_3 &= 0.5 (\bar{b}_{44} \bar{b}_{55})^{\frac{1}{2}} \end{aligned}$$

The SIFs (stress intensity factors)  $K_I, K_{II}$ , and  $K_{III}$  correspond to crack modes I, II, and III, respectively [23]. The term ' $\bar{b}_{ij}$ ' represents the local compliance of the composite material, and the characteristic equation roots are  $\mu_1$  and  $\mu_2$ .

Determining the compliance or flexibility matrix provides an efficient method for defining the stiffness matrix of a beam with crack elements. The total flexibility matrix is considered by combining the contributions from both cracked and intact portions, with the cracked element's length and mass considered negligible. A representative cracked rectangular LCB element is illustrated in Figure 1 and modelled using stress intensity factors from fracture mechanics, allowing the formulation of the local compliance matrix as described in study [26].

$$[C] = \begin{bmatrix} 2D_1 & 1.5D_1 & 0 & 12 \frac{D_1}{h} & 12 \frac{D_1}{b} \\ 1.5D_1 & 4.5D_2 & 0 & 9 \frac{D_{12}}{h} & 9 \frac{D_{12}}{b} \\ 0 & 0 & 4.5D_3 & 0 & 0 \\ 12 \frac{D_1}{h} & 9 \frac{D_{12}}{h} & 0 & 72 \frac{D_1}{h^2} & 36 \frac{D_1}{bh} \\ 12 \frac{D_1}{b} & 9 \frac{D_{12}}{b} & 0 & 36 \frac{D_1}{bh} & 96 \frac{D_1}{b^2} \end{bmatrix} * \frac{\Pi L_i^2}{bh^2} \quad (2)$$

Based on above explanation the flexibility matrix of a composite beam with two degrees of freedom (DOF) can be expressed as:

$$K_C = \begin{bmatrix} [C]^{-1} & -[C]^{-1} \\ -[C]^{-1} & [C]^{-1} \end{bmatrix}_{(6 \times 6)} \quad (3)$$

The above expression can be extended to any DOFs. In ABAQUS, crack simulation is based on inverse of the components of flexibility matrix to derive the stiffness matrix of the laminated beam with cracked element, and subsequently, the stiffness of the cracked composite beam is evaluated.

Now LCB with cracks, the stiffness matrix is

$$[K_{cbe}] = [K_e] - [K_{crack}]. \quad (4)$$

The eigenvalue formulation for LCB with crack is

$$([K] - \omega^2 [M])\{\Delta\} = 0. \quad (5)$$

where,

- [K] = Stiffness of LCB with cracks;
- [M] = Mass matrix of LCB;
- $\omega$  = Eigenfrequency;
- { $\Delta$ } = Vector DOF.

### 3. EIGENFREQUENCY SIMULATION

A simulation using a linear beam model was set up in the ABAQUS software. The process began by creating the geometry of the carbon/epoxy beam and noting elastic properties to its different layers. Once the material properties were assigned, the components were assembled, and the characteristics of cracks were defined. After completing these initial steps, the analysis phase started. This involved setting up boundary conditions, selecting an appropriate mesh size, and simulating the behaviour of cracks. Following the analysis, the next step was post-processing. This phase included conducting modal analysis to determine the eigenfrequencies of the system. The results of this simulation numerical modal frequencies are shown in Figure 2.

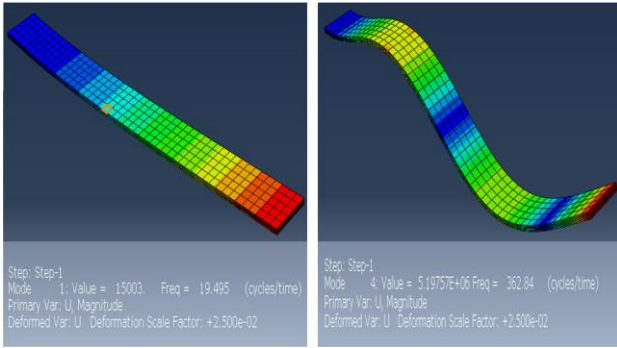


Figure 2. Simulated frequencies on ABAQUS

### 4. EXPERIMENTAL FREE VIBRATION TEST

The experimental work is carried out in three steps as follows:

- LCB sample preparation
- Elastic constant determination from tensile test
- Free vibration experiments

#### 4.1 LCB sample preparation

Industry-grade rolled carbon bi-directional fabrics are engineered through a scissor to produce rectangular carbon fiber sheets towards laying the lamination layers. The consecutive carbon fiber layers are overlapped rigidly by means of slotting an epoxy matrix between the layers. The epoxy matrix is formed by blending 90% of industry-grade epoxy to 10% hardener. Furthermore, the proportion for matrix and fiber is 50:50. The complete procedure for lamination follows in line with previous studies [12]. After the curing of freshly produced laminated plates, the carbon/epoxy LCBs (Figure 3) are produced mechanically using a motorized diamond wheel. The Vernier Calliper measures the carbon/epoxy beam geometries as 250 mm (L), 25 mm (b), and 2.2 mm (h) correspondingly.

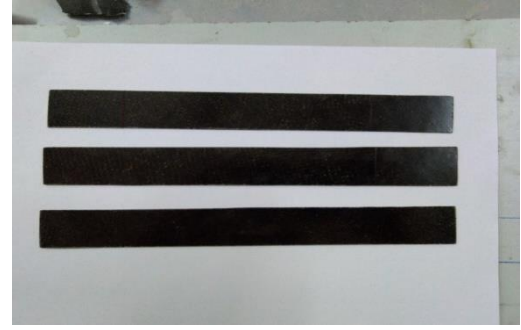


Figure 3. Produced carbon/epoxy LCB samples

#### 4.2 Experimental carbon/epoxy elastic constants

The produced LCB samples are placed under tensile testing by means of a high precession universal testing machine, INSTRON 8862 that follows the guiding principles laid by ASTM-D3039/D3039M-14 [34]. To enhance the accuracy of the Young's modulus measurements, three sets of samples, each comprising three carbon/epoxy specimens, were tested under tensile loading. The average value of these three sets was used as the final measurement. A strain rate of 1 mm/min was applied during the loading process. Since a bi-directional carbon fiber was used, the material constants in mutually transverse directions are equal in measures ( $E_1=E_2$ ). The Poisson's ratio ' $\nu_{12}$ ' and rigidity modulus ' $G_{12}$ ' is estimated in line with some previous literature [12]. The tensile experiment on carbon/epoxy LCB samples through INSTRON 8862 is shown in Figure 4.



Figure 4. Tensile testing using INSTRON 8862

The measure tensile characteristics are presented in Table 1.

Table 1. Modulus of carbon/epoxy beam

Elastic Modulus	Test Results	
Elastic modulus (E)	E <sub>11</sub>	40.96 GPa
	E <sub>22</sub>	40.96 GPa
	E <sub>45</sub>	11.26 GPa
N (Poison ratio)	$\nu_{12}$	0.3
	$\nu_{13}$	0.3
	$\nu_{23}$	0.3
Rigidity modulus (G)	G <sub>12</sub>	3.11 GPa
	G <sub>13</sub>	3.11 GPa
	G <sub>23</sub>	3.11 GPa
Mass density $\rho$	$\rho_m$	1676.19g/m <sup>3</sup>

### 4.3 Experiments under free vibration

Free vibration frequency determination by experimental work is considered as an effective non-destructive analysis tool for dynamic exploration of intact and cracked structures. In this context, a vibration FFT analyzer “B & K” model 3560 is employed which is capable of analyze the smallest excitation generated through the modal impact hammer. The process of experimental work involves the subject mounted accelerometer that receives the excitation initiated by the impact hammer. Towards recording the frequency responses, the accelerometer is mounted on the LCB samples for the certain boundary condition of interest and the modal impact hammer is gently stroked near the accelerometer to generate the responses. The FFT analyzer receives the signal and the frequency responses are displayed for five consecutive strikes. The boundary conditions are achieved by means of a mechanically arranged iron frame. The data for natural frequency are recorded on Pulse platform through FFT analyzer and the coherence during the test shows nearer to unity which shows the credibility of the experimental programme. The experimental arrangement for the research is shown in Figure 5.



Figure 5. Modal testing arrangement set-up

The accelerometer employed in this study is the efficient and robust Brüel & Kjær make model 4507B, commonly referred to as "B&K 4507B." This surface-mounted accelerometer is capable of detecting minute excitation signals. Additionally, it is inbuilt with a Butterworth anti-aliasing filter, that disregards aliasing effects all through the experiment and avoids signal leakage. Consequently, the use of the "B&K 4507B" accelerometer ensures efficient signal transmission to the FFT analyzer, enabling more accurate recording of natural frequencies.

## 5. RESULTS AND DISCUSSIONS

The vibration frequencies of an 8-layer woven roving laminated glass/epoxy beam with transverse open cracks were determined through both numerical and experimental analyses. The results are presented in relation to various parameters and are categorized as follows:

- Validation of simulation model
- Simulation and experimental eigenfrequency results

### 5.1 Validation of simulation model

The accuracy and effectiveness of the current finite element modeling using ABAQUS have been verified against previous research. Vibration frequencies for cracked cantilever composite beams with different angle of ply alignments, obtained via ABAQUS, are related with those reported in study [22] and summarized in Table 2. The outcomes from this

study show good agreement with those from study [22]. The geometry and material specifications are as follows:

$L = 1.00$  m,  $b = 0.025$  m,  $h = 0.005$  m. (crack depth =  $0.2h, 0.4h$  and  $0.6h$  for crack position at  $0.1L$  from fixed end)  $E_m = 2.756$  GPa,  $E_f = 275.6$  GPa,  $\nu = 0.3$ ,  $G_m = 1.036$  GPa,  $G_f = 114.8$  GPa,  $\rho_m = 1600$  Kg/m<sup>3</sup>,  $\rho_m = 1900$  Kg/m<sup>3</sup>.

Table 2. Comparison study for fundamental non-dimensional eigen natural frequency

Fiber Orientation	rcd (a/h)	Ref. [30]	Present Results
15°	0.2	0.9773	0.9625
	0.4	0.9046	0.91
	0.6	0.7526	0.75
30°	0.2	0.9812	0.9733
	0.4	0.9861	0.9093
	0.6	0.7849	0.77
45°	0.2	0.9876	0.9836
	0.4	0.9477	0.9377
	0.6	0.8429	0.83

### 5.2 Simulation and experimental eigenfrequency results

Frequency attributes of cracked LCB in the domain of experimental (FFT) and numerical (ABAQUS) are studied concerning the below mentioned parameters for laminated carbon/epoxy beam with one and two cracks:

- Effect of angle of fiber alignment;
- Effect of boundary condition;
- Effect of crack location;
- Effect of depth of crack.

#### 5.2.1 Effect of angle of fiber alignment

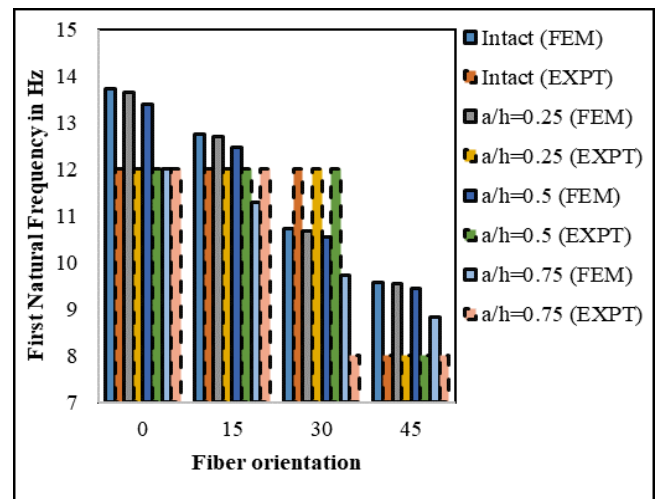


Figure 6. Effect of angle of plies for carbon/epoxy LCB

The variations in the beam vibration frequency upon angle of fiber alignment or orientation are presented in Figure 6 for the first natural frequency of four-layered carbon/epoxy LCB. Four types of composite beams are fabricated with 0°, 15°, 30°, and 45° fiber orientations respectively to check the orientation result on the natural beam vibration frequency of LCB. The experimental frequencies are in line with FE predictions through ABAQUS. The variations in the frequency attributes as a function of fiber alignment are explained for cantilever boundary conditions. Three crack positions at 0.25L, 0.5L and 0.75L from fixed support with varying relative crack depths (a/h) 0.25, 0.5 and 0.75 are defined on the carbon/epoxy LCB.



Material properties are confirmed from Table 1. Figure 6 shows the fundamental frequency of LCB with 0°-fiber orientation reduced by 0.4%, 2.52%, and 14.22% than the intact beam frequency for relative crack depths (rcd) of 0.25, 0.5 and 0.75 respectively. Similar observations are made for 15°, 30° and 45° angles of carbon fibers where the fundamental frequency is reduced in the range of 0.3% to 10.21% with respect to intact LCB for respective (a/h) values. It is evident that the vibration frequencies of LCB with 0°-fiber orientation show maximum reduction subjected to multiple transverse cracks. Thus, 0°-fiber oriented LCBs are used in experimental and numerical analysis for further parametric study in this research.

### 5.2.2 Effect of boundary condition

The effect of multiple transverse cracks on the frequencies of free vibration concerning different boundary conditions are presented in Figure 7, Figure 8 and Figure 9 for Clamped-Free (CF), Simply Supported (SS) and Clamped-Clamped (CC) conditions correspondingly. Within this frame of reference, three different glass/epoxy composite beams are fabricated with 8-layer confirming the depth 2.4 mm with 0°-fiber orientation. The cracks are positioned at 0.25L, 0.5L, and 0.75L with relative crack depths (rcd) of 0.25, 0.5, and 0.75. Analyzing Figure 7, the maximum reduction observed for the CF composite beam was 24.9% for a crack depth of 0.75h. Similar observations are made for CC and SS carbon/epoxy LCB where maximum reduction in frequency magnitude recorded at 0.75h crack depth. Furthermore, lowest, and maximum fundamental frequency recorded for CF and CC beam respectively.

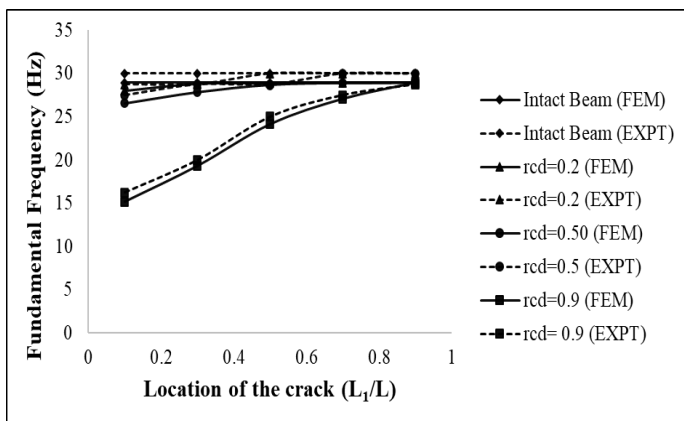


Figure 7. Fundamental frequency for CF carbon/epoxy LCB

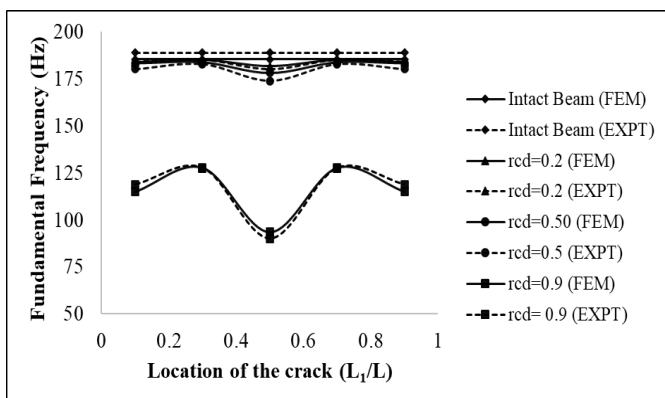


Figure 8. Fundamental Frequency for CC carbon/epoxy LCB

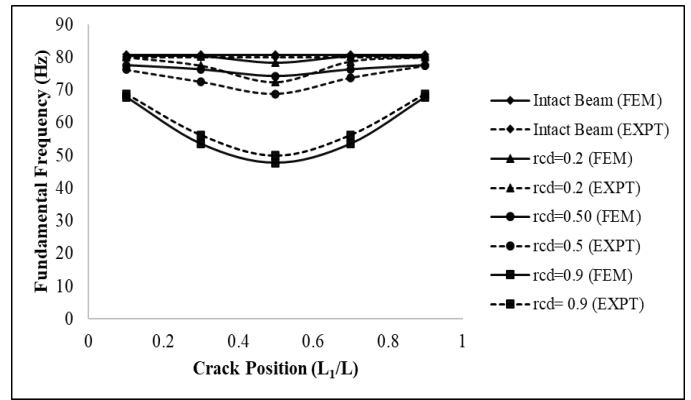


Figure 9. Fundamental Frequency for SS carbon/epoxy LCB

To get a more comprehensive view on the boundary condition effects, the variation in natural frequencies is graphically summarized in Figure 10 considering all boundary conditions taking the crack depths (a/h) equal to 0.25, 0.5 and 0.75 and locations also at 0.25L, 0.5L and 0.75L.

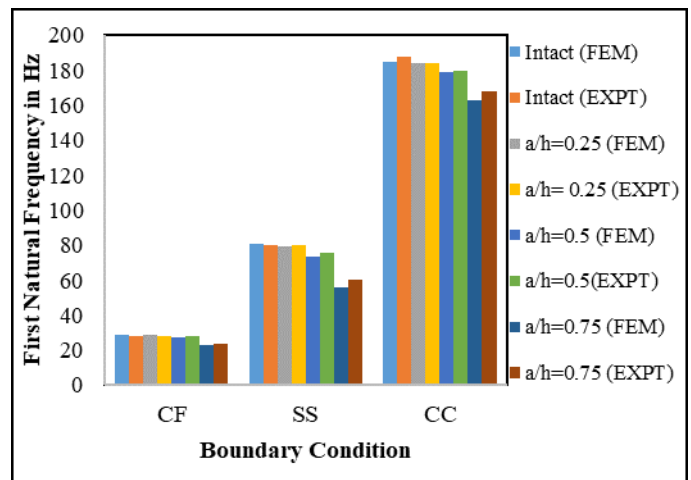


Figure 10. Shifting in fundamental frequency of vibration of LCB due to different boundary conditions with 0°-fiber orientation

The study presented pertaining boundary condition concludes that among three boundary conditions that are considered for parametric study, CC beam exhibits higher magnitude for free vibration frequency and it is vital to apprehend the knowledge towards crack effect on natural frequency at higher magnitudes. Thus, the ensuing sections dealing with the critical crack position and crack depth is carried out considering the CC boundary condition.

### 5.2.3 Effect of crack location

To study the effect of crack location, CC carbon/epoxy put for free vibration testing and modal simulation. In this respect the crack positions are defined from 0.1L to 0.9L with 0.1L interval between two consecutive crack locations. The critical crack locations are thus identified through a sequential approach.

#### (a) Effect of single crack

In relation to the positions of the cracks, the calculated natural fundamental frequencies are displayed in Figure 11 for fixed (CC) boundary conditions for an 8-layer carbon/epoxy LCB. The depth of crack kept constant at 0.375h for study. To

record the single crack effect, the crack location is varied for 9-different positions as mentioned above, that is from 0.1L to 0.9L to this respect, carbon/epoxy LCBs are prepared for each respective crack location. The crack signatures are implied with equal capacity as reflected in modal simulation cracked beam model in ABAQUS domain. It is noteworthy to be mentioned that from Figure 11, the crack sited at 0.5L results greatest drop in the first mode of the LCB frequency concerning intact carbon/epoxy beam. For the vibration frequencies, the computational and experimental results indicate a drop of 4.44% in the fundamental LCB frequency compared to the that of an uncracked LCB.

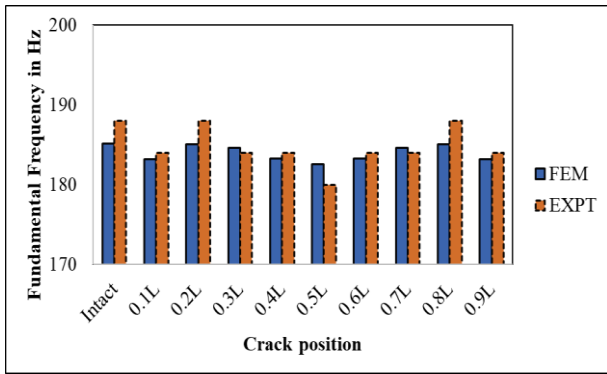


Figure 11. Effect of a single crack

(b) Effect of double crack

Figure 12 reveals the variations in first natural frequency because of the spot of a single crack concerning CC supported edge. Previously it was found that for one engineered crack, 0.5L crack position is the critical position. Thus, moving to the two cracks on the beam, the analysis is progressed by keeping the crack location at 0.5L is persistent and another crack is shifted at different positions. The second crack is varied from 0.1L to 0.9L leaving out 0.5L locations correspondingly. For this instance, the determined drop in the vibration frequencies of the LCB is observed at positions 0.5L and 0.1L from the clamped edge. At such places, the greatest decrease in fundamental frequency is noted as 6.8% than intact composite beam. Following this trend the analysis can be stretched to multiple cracks in a sequential approach.

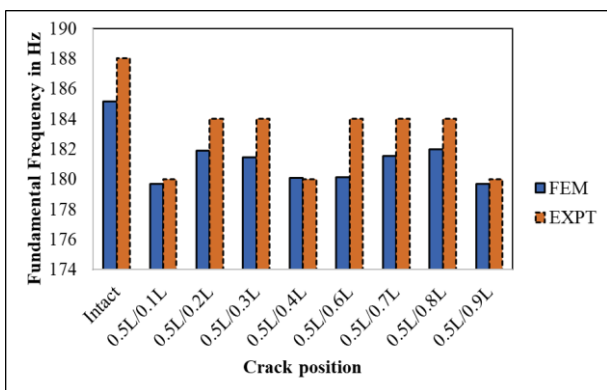


Figure 12. Effect of two cracks

5.2.4 Effect of crack depth

To study the effect of crack depth, CC carbon/epoxy put for modal analysis. In this respect the critical positions of crack for single and two cracks are defined on the carbon/epoxy LCB as concluded from the preceding analysis. Both for

modal simulation and free vibration experiments, the crack depths are engineered at 0.125h, 0.25h, 0.375h, 0.5h and 0.625h keeping 0°-fiber orientation for all the cases.

The frequency changes in 8-layered carbon/epoxy LCB natural frequency regarding the red under the clamped edges at both ends are illustrated in Figure 13 and Figure 14. The figures present a good agreement between modal simulation and modal experimental results. The frequency-based analysis is done for crack positions at 0.5L for one crack and (0.5L,0.1L) for two cracks. The frequency behaviour upon depth variation illustrated in Figure 13 shows that for a rcd of 0.125 to 0.375 the fundamental frequency varies from 0.2% to 1.4% whereas it significantly reduced for crack depth exceeding 0.5h comparing to frequency of intact LCB. For a crack depth of 0.625h the percentage reduction in fundamental frequency recorded as 8.63% for fixed laminated beam with a single crack located at 0.5L. Similar observations are made for double crack fixed LCB from Figure 14 where the bigger drop in first frequency is observed for a relative depth of crack 0.625 as 12.63%. The higher mode frequencies are also exhibiting an alike trend where the shifting in frequency attributes for crack depths 0.125h to 0.375h is not-significant but, with rcd increases, substantial decrease in frequencies of vibration is recorded. This elicit a conclusion that though increasing crack depth dynamically affects the LCB, but up to 50% crack depth the LCB still can be addressed for its functionality and when it exceeds 0.5h the failure may happen rapidly and thus, needs to be replaced.

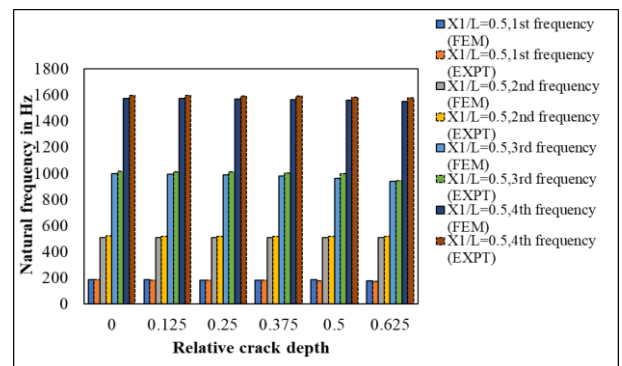


Figure 13. Effect of single crack concerning crack depth

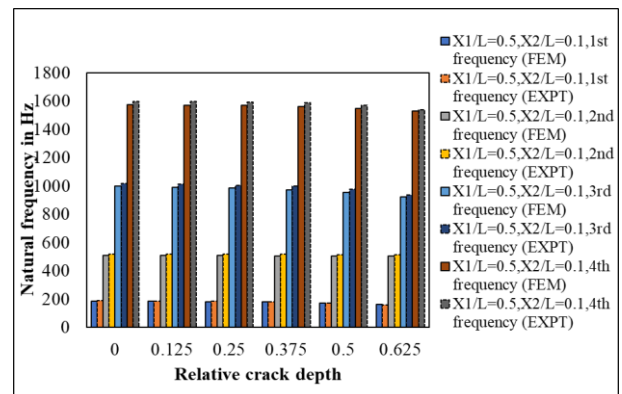


Figure 14. Effect of double cracks concerning crack depth

5.2.5 A statistical analysis

Quantifying the differences between numerical (FE) and experimental (EX) results will provide an insightful view to the robustness and accuracy of the research work undertaken. To this end, a statistical analysis is performed for cantilever

carbon/epoxy LCB as presented in Table 3.

**Table 3.** Statistical study for fundamental frequency of cracked cantilever carbon/epoxy LCB

L1/L	Fundamental Frequency							
	Intact		rcd=0.2		rcd=0.5		rcd=0.9	
	FE	EX	FE	EX	FE	EX	FE	EX
0.1	28.95	30	29.0	28.7	26.6	27.5	15.0524	16.2
0.3	28.95	30	29.8	28.7	28.7	28.75	19.1667	20
0.5	28.95	30	29.9	30	29.6	28.75	25	25
0.7	28.95	30	29.9	30	28.1	30	28.0374	27.5
0.9	28.95	30	29.9	30	29.9	30	29.985	28.7

The correlation between experimental and numerical analysis is crucial for a comprehensive understanding of crack behavior in laminated composite beams under free vibration. The present research approach necessitates a strong correlation between experimental and numerical analyses. Experimental analysis provides real-world validation through physical testing, while numerical analysis offers controlled, extensive parametric studies using computational models. The developed numerical model, based on the finite element method (FEM) and local compliance modelling, has been calibrated against a comprehensive set of experimental data. The model accurately incorporates the material properties, fiber orientations, boundary conditions, and crack characteristics observed in the experiments. The combined numerical and experimental analysis showed a high degree of agreement, with deviations within acceptable limits. For instance, the discrepancies between the numerical predictions and experimental observations were generally within 5% for most configurations. The numerical model captured the trends in frequency variation with changes in fiber orientation, consistent with experimental observations. Different boundary conditions, such as clamped-clamped and clamped-free, showed a close resemblance between numerical and experimental frequencies. Further, the impact of crack location and depth on natural frequencies was accurately predicted by the numerical model, aligning well with experimental results.

Methodological limitations, such as precision in crack fabrication, material inhomogeneity, and boundary condition control, may impact the repeatability and reliability of the research findings. Variations in crack dimensions and orientations can lead to discrepancies between experimental and numerical results, affecting validation accuracy. Inconsistent boundary conditions further alter vibration responses, complicating result comparability. To mitigate these limitations, advanced manufacturing techniques like laser cutting for precision crack fabrication, and non-destructive evaluation methods such as X-ray CT to ensure consistency are some potential strategies that may be proposed. Comprehensive parameter studies can strengthen experimental-numerical correlations and isolate variable effects, improving the reliability of the research findings. These strategies can address the methodological challenges effectively and advancing our understanding of these complex behaviors of cracked LCB.

## 6. CONCLUSIONS

The integrated modal simulation with experimental work for free vibration analysis of cracked LCB has yielded valuable insights to the dynamic behavior in the presence of

cracks. The results elicited from the combined numerical and experimental work offer observations that shed light on complex interplay between crack presence, structural integrity and vibration characteristics. In this context, the conclusions drawn for this research work are highlighted below:

- The modal simulation technique offers a powerful tool for predicting the effects of cracks on the dynamic response of laminated composite beams.
- Complimenting the simulation, the experimental work validates the numerical simulation and provides the real-world insights.
- The simulation model offered the results in line with empirical outcomes further demonstrates its robustness and effectiveness under dynamic conditions. Moreover, its alignment with the experimental work affirms its predictive accuracy and further expands the scope for its wide applicability in the threshold of vibration.
- The experimental data offered a comprehensive view on the beam's dynamic response under controlled conditions, enabling the direct assessment of the impact of cracks on its vibration characteristics.
- On implication of transverse cracks, the laminated carbon composite beam fabricated with 0° ply-orientation records the bigger drop in frequency attributes for all modes.
- The increase in cracks reduce the LCB frequencies given any angle of ply-orientation.
- Due to edge restraint, significant reduction in natural frequencies is observed for any particular depth of crack.
- The theoretical contributions of this study lie in the development of a robust finite element model incorporating local compliance for accurately predicting the free vibration behavior of cracked laminated composite beams. Practically, the validated numerical model offers a reliable tool for engineers to assess structural integrity and optimize design in composite material applications, thereby enhancing safety and performance in various engineering fields.

The outlined conclusions elicited from the research undertaken in the present study, it can be predicted that the vibration frequencies are greatly influenced by angle fiber alignment and crack depth with conditions of restraining at edges. Along these lines, cracks are responsible for the variations in frequencies of vibration of the carbon composite beam. Ultimately, the holistic approach of combining modal simulation with experimental validation advances the knowledge of cracked composite beam dynamics, driving innovation in structural engineering practices and ensuring the continued safety and reliability of these critical components.

Future work could explore the implementation of more sophisticated crack modelling techniques, such as extended finite element methods (XFEM), which can better capture the initiation and propagation of multiple cracks. Investigating the free vibration behavior of cracked beams in different composite materials, such as hybrid composites and fiber metal laminates could provide insights into the material-specific responses and improve the generality of the findings. Considering the effects of various environmental and operational conditions, such as temperature, humidity, and loading frequency, on the vibration behavior of cracked composite beams could lead to more comprehensive understanding and applicability of the models. Integrating the numerical models with real-time structural health monitoring systems could provide practical benefits in terms of early crack detection and proactive maintenance strategies.

## REFERENCES

- [1] Hsissou, R., Seghiri, R., Benzekri, Z., Hilali, M., Rafik, M., Elharfi, A. (2021). Polymer composite materials: A comprehensive review. *Composite Structures*, 262: 113640. <https://doi.org/10.1016/j.compstruct.2021.113640>
- [2] Jassim, Z.A., Ali, N.N., Mustapha, F., Jalil, N.A. (2013). A review on the vibration analysis for a damage occurrence of a cantilever beam. *Engineering Failure Analysis*, 31: 442-461. <https://doi.org/10.1016/j.engfailanal.2013.02.016>
- [3] Ramdoun S., Serier B., Bouafia F., Fekirini H. (2017). Numerical analysis of crack behavior subjected to residual stresses in the metal matrix composites. *Revue des Composites et des Matériaux Avancés*, 27(3-4): 335-356. <https://doi.org/10.3166/rcma.2017.00024>
- [4] Zergot, S., Moussaoui, M., Hachi, B.E. (2022). Evolution of crack propagation rate in notched specimens using XFEM method under bending load condition. *Annales de Chimie-Science des Matériaux*, 46(3): 155-162. <https://doi.org/10.18280/acsm.460307>
- [5] Sabnavis, G., Kirk, R.G., Kasarda, M., Quinn, D. (2004). Cracked shaft detection and diagnostics: A literature review. *Shock and Vibration Digest*, 36(4): 287. <https://doi.org/10.1177/0583102404045439>
- [6] Parhi, D.R.K., Kumar, D.A. (2009). Analysis of methodologies applied for diagnosis of fault in vibrating structures. *International Journal of Vehicle Noise and Vibration*, 5(4): 271-286. <https://doi.org/10.1504/IJVNV.2009.031131>
- [7] Imran, M., Khan, R., Rafiullah, S. (2019). Vibration analysis of cracked composite laminated plate: A review. *Mehran University Research Journal of Engineering & Technology*, 38(3): 705-716. <https://doi.org/10.3316/informit.503002763931695>
- [8] Kisa, M., Gurel, M.A. (2007). Free vibration analysis of uniform and stepped cracked beams with circular cross sections. *International Journal of Engineering Science*, 45(2-8): 364-380. <https://doi.org/10.1016/j.ijengsci.2007.03.014>
- [9] Rajasekaran, S., Khaniki, H.B. (2018). Free vibration analysis of bi-directional functionally graded single/multi-cracked beams. *International Journal of Mechanical Sciences*, 144: 341-356. <https://doi.org/10.1016/j.ijmecsci.2018.06.004>
- [10] Maiti, D.K., Sinha, P.K. (1994). Bending and free vibration analysis of shear deformable laminated composite beams by finite element method. *Composite Structures*, 29(4): 421-431. [https://doi.org/10.1016/0263-8223\(94\)90111-2](https://doi.org/10.1016/0263-8223(94)90111-2)
- [11] Abdelmoula, F., Refassi, K., Bouamama, M., Elmeiche, A. (2021). Modal analysis of FSW plate considering the residual stresses effect. *Annales de Chimie - Science des Matériaux*, 45(1): 75-82. <https://doi.org/10.18280/acsm>
- [12] Biswal, M., Sahu, S.K., Asha, A.V. (2017). Dynamic stability of woven fiber laminated composite shallow shells in hygrothermal environment. *International Journal of Structural Stability and Dynamics*, 17(08): 1750084. <https://doi.org/10.1142/S0219455417500845>
- [13] Faella, C., Martinelli, E., Nigro, E. (2002). Steel and concrete composite beams with flexible shear connection: "Exact" analytical expression of the stiffness matrix and applications. *Computers & Structures*, 80(11): 1001-1009. [https://doi.org/10.1016/S0045-7949\(02\)00038-X](https://doi.org/10.1016/S0045-7949(02)00038-X)
- [14] Daneshmehr, A.R., Nateghi, A., Inman, D.J. (2013). Free vibration analysis of cracked composite beams subjected to coupled bending-torsion loads based on a first order shear deformation theory. *Applied Mathematical Modelling*, 37(24): 10074-10091. <https://doi.org/10.1016/j.apm.2013.05.062>
- [15] Jena, P.C., Parhi, D.R., Pohit, G. (2016). Dynamic study of composite cracked beam by changing the angle of bidirectional fibres. *Iranian Journal of Science and Technology, Transactions A: Science*, 40: 27-37. <https://doi.org/10.1007/s40995-016-0006-y>
- [16] Kim, S.S., Kim, J.H. (2003). Rotating composite beam with a breathing crack. *Composite Structures*, 60(1): 83-90. [https://doi.org/10.1016/S0263-8223\(02\)00291-X](https://doi.org/10.1016/S0263-8223(02)00291-X)
- [17] Kim, K., Choe, K., Kim, S., Wang, Q. (2019). A modeling method for vibration analysis of cracked laminated composite beam of uniform rectangular cross-section with arbitrary boundary condition. *Composite Structures*, 208: 127-140. <https://doi.org/10.1016/j.compstruct.2018.10.006>
- [18] Zhang, C., Shao, H., Liang, S., Geng, Y., Chen, N., Gu, Z., Jiang, J. (2021). Single edge crack effect on mechanical property of polyimide flexible composite envelope. *Composite Structures*, 274: 114340. <https://doi.org/10.1016/j.compstruct.2021.114340>
- [19] Hajianmaleki, M., Qatu, M.S. (2013). Vibrations of straight and curved composite beams: A review. *Composite Structures*, 100: 218-232. <https://doi.org/10.1016/j.compstruct.2013.01.001>
- [20] Topac, O.T., Gozluclu, B., Gurses, E., Coker, D. (2017). Experimental and computational study of the damage process in CFRP composite beams under low-velocity impact. *Composites Part A: Applied Science and Manufacturing*, 92: 167-182. <https://doi.org/10.1016/j.compositesa.2016.06.023>
- [21] Kahya, V., Karaca, S., Okur, F.Y., Altunışık, A.C., Aslan, M. (2021). Damage localization in laminated composite beams with multiple edge cracks based on vibration measurements. *Iranian Journal of Science and Technology, Transactions of Civil Engineering*, 45: 75-87. <https://doi.org/10.1007/s40996-020-00393-x>
- [22] Kisa, M. (2004). Free vibration analysis of a cantilever composite beam with multiple cracks. *Composites Science and Technology*, 64(9): 1391-1402. <https://doi.org/10.1016/j.compscitech.2003.11.002>
- [23] Krawczuk, M., Ostachowicz, W.M. (1995). Modelling and vibration analysis of a cantilever composite beam with a transverse open crack. *Journal of Sound and Vibration*, 183(1): 69-89. <https://doi.org/10.1006/jsvi.1995.0239>
- [24] Maghsoodi, A., Ghadami, A., Mirdamadi, H.R. (2013). Multiple-crack damage detection in multi-step beams by a novel local flexibility-based damage index. *Journal of Sound and Vibration*, 332(2): 294-305. <https://doi.org/10.1016/j.jsv.2012.09.002>
- [25] Sha, G., Radzieński, M., Cao, M., Ostachowicz, W. (2019). A novel method for single and multiple damage detection in beams using relative natural frequency changes. *Mechanical Systems and Signal Processing*, 132: 335-352. <https://doi.org/10.1016/j.ymssp.2019.06.027>



- [26] Das, P., Sahu, S.K. (2020). Experimental and numerical study on free vibration of cracked woven fiber glass/epoxy composite beam. *Materials Today: Proceedings*, 33: 5505-5510. <https://doi.org/10.1016/j.matpr.2020.03.320>
- [27] Ghoneam, S.M. (1995). Dynamic analysis of open cracked laminated composite beams. *Composite Structures*, 32(1-4): 3-11. [https://doi.org/10.1016/0263-8223\(95\)00023-2](https://doi.org/10.1016/0263-8223(95)00023-2)
- [28] Toygar, M.E., Kiral, Z., Sayman, O., Arman, Y., Özen, M. (2013). Effect of interface crack on lateral buckling behavior and free vibration response of a sandwich composite beam. *Journal of Composite Materials*, 47(15): 1843-1851. <https://doi.org/10.1177/0021998312451611>
- [29] Das, M.T., Yilmaz, A. (2018). Experimental modal analysis of curved composite beam with transverse open crack. *Journal of Sound and Vibration*, 436: 155-164. <https://doi.org/10.1016/j.jsv.2018.09.021>
- [30] Loya, J.A., Aranda-Ruiz, J., Zaera, R. (2022). Natural frequencies of vibration in cracked Timoshenko beams within an elastic medium. *Theoretical and Applied Fracture Mechanics*, 118: 103257. <https://doi.org/10.1016/j.tafmec.2022.103257>
- [31] Taima, M.S., El-Sayed, T.A., Shehab, M.B., Farghaly, S.H., Hand, R.J. (2023). Vibration analysis of cracked beam based on Reddy beam theory by finite element method. *Journal of Vibration and Control*, 29(19-20): 4589-4606. <https://doi.org/10.1177/10775463221122122>
- [32] Pal, R., Chaudhury, M., Dewangan, H.C., Hirwani, C.K., Kumar, V., Panda, S.K. (2023). Numerical frequency prediction of combined damaged laminated panel (delamination around cut-out) and experimental validation. *Journal of Vibration Engineering & Technologies*, 11(8): 4241-4251. <https://doi.org/10.1007/s42417-022-00812-5>
- [33] Akkasali, N.K., Biswas, S., Panda, S.K. (2024). Damage (delamination and crack) effect on frequency and strain energy release rate (SERR) in adhesively bonded multi-material single lap joint—An experimental verification. *Acta Mechanica*, 235(2): 1271-1290. <https://doi.org/10.1007/s00707-023-03807-9>
- [34] ASTM D3039/D3039M-08. (2008). Standard test method for tensile properties of polymer matrix composite materials. ASTM International. [https://doi.org/10.1520/D3039\\_D3039M-08](https://doi.org/10.1520/D3039_D3039M-08)

## NOMENCLATURE

L	Length of LCB
b	Width of LCB
h	Thickness of LCB
X <sub>1</sub> , X <sub>2</sub> , X <sub>3</sub> ,...	Position of crack
a	Depth of crack
C <sub>ij</sub>	Local compliance
D <sub>1</sub> , D <sub>2</sub> , D <sub>12</sub>	material parameters coefficients
K <sub>i</sub> , K <sub>II</sub> , K <sub>III</sub>	Stress intensity factors
[K]	Stiffness matrix
[M]	Mass matrix
ω	Eigenfrequency
E	Elastic modulus
G	Rigidity modulus

## Greek symbols

ν	Poisson's ratio
ρ	Mass density in kg/m <sup>3</sup>