Mathematical Modelling of Engineering Problems
Vol. 11, No. 8, August, 2024, pp. 2251-2257
Journal homepage: http://iieta.org/journals/mmep

Approximate Analytic Solution of a Potential Flow Around an Obstacle

Wahiba Delloum*, Brahim Bouderah, Noura Bounab

Laboratory of Pure and Applied Mathematics, Department of Mathematics, Faculty of Mathematics and Computer Science, University of M'sila, M'sila 28000, Algeria

Corresponding Author Email: wahiba.delloum@univ-msila.dz

https://doi.org/10.18280/mmep.110827

Received: 30 January 2024
Revised: 24 April 2024
Accepted: 15 July 2024
Available online: 28 August 2024

Keywords:
free surface, potential flow, Schwartz-Christoffel transformation, streamlines theory

ABSTRACT

The present study aims to solve analytically a free surface flow above a trapezoidal obstacle with angle $\beta = \frac{\pi}{3}$. The flow which forms different angles β with the horizontal plane, is assumed to be steady, irrotational and potential. The fluid is considered as inviscid and incompressible. In addition, the gravity and surface tension effects are not taken into consideration. An approximate analytical solution of the free surface shape problem was successfully determined using the Shwartz-Christoffel conformal mapping transformation technique and the free streamline theory which was introduced by Kirchhoff. The main results were obtained for different values of the inclination angle β between the trapezoidal obstacle and the axis of the struts.

1. INTRODUCTION

It is commonly acknowledged that jet type free surface flows are widely encountered in such diverse industrial and urban applications as engine combustion chambers, jet pumps, as well as reservoirs and dams. This type of flow has been the subject of a large number of theoretical, experimental and numerical studies. For the sake of example, one may cite the flow of a liquid emerging from an orifice of a tank. This kind of problems is generally difficult to solve explicitly due to the nonlinear condition that is imposed on the free boundary of an unknown shape. It was shown that the difficulty in solving such a problem increases with the complexity of the geometry of the flow domain. Its solution also depends on the properties of the fluid and on the flow conditions by Gasmi [1].

During the nineteenth century, the theory of complex variables made it possible to theoretically investigate the two-dimensional free surface flow, provided that the flow domain is polygonal and the fluid is irrotational, incompressible and non-viscous, and the effects of gravity are neglected. These hypotheses indeed allowed using the complex potential theory to solve two-dimensional flow problems within polygonal domains.

Over the past forty years, great attention has been devoted to studying the effect of the obstacle's shape on the two-dimensional free surface flow of fluids. Bernoulli's equation is generally utilized to describe some. Nonlinear problems in which the shape of the free surface is unknown

Furthermore, it was shown that various analytical and numerical methods can be used to determine the shape of the free surface flow for many problems dealing with the potential flow around different obstacle shapes by applying various techniques that are based, for example, on the conformal mapping transformation, the truncation series, and the boundary integral methods. It has indeed been reported that these techniques have been adopted by many authors, such as Forbes and Schwartz [2] and Birkhoff and Zarantoello [3], in order to determine the nonlinear solutions of subcritical and supercritical flows around a semi-circular obstacle. In this context, Gasmi and Mekias [4], Gasmi and Amara [5], Benjamin [6], and Vanden-Broeck [7, 8] considered the problems of flow within a channel with cavities. As for Sekhri et al. [9], they carried out some numerical studies to find the solution of two-dimensional potential flow problems with an immersed triangular obstacle. With regard to Merzougui and Laiadi [10], they investigated the problem of a fluid flowing over a triangular depression. Similarly, Dias and Vanden-Broeck [11] as well as Vanden-Broeck and Killer [12] succeeded in identifying the subcritical and supercritical free-surface solution of flows past a triangular obstacle. Likewise, Hanna et al. [13] adopted a numerical method that is based on a series truncation technique for a super-critical case in order to successfully solve the problem of a super-critical flow over a trapezoidal obstacle. On the other hand, many researchers [7-18] established a method that is based on the Shwartz-Christoffel transformation to solve similar problems. Furthermore, Smith and Lim [19] considered the supercritical flow around an obstacle that has a polygonal symmetrical shape and placed on the bottom of a channel. The conformal mapping and Hilbert's method of a mixed boundary value problem were applied in the upper half-plane in order to determine the nonlinear surface profiles.

The present work focuses on a two-dimensional steady, irrotational and inviscid potential flow within a channel that has an obstacle placed on its bottom. The obstacle has a trapezoidal shape and makes an angle $\beta = \frac{\pi}{3}$ with the horizontal plane. It is important to note that, far upstream, the fluid is uniform, with a constant velocity U and uniform depth

L. The free streamline theory, which is based on the Shwartz-Christoffel transformation and Kirchhoff theory, is then utilized to calculate the shape of the free surface flow of the fluid past a submerged obstacle. The findings indicate that the method used for that purpose can be easily implemented, and is highly effective in providing analytical solutions for these kinds of problems.

The above mentioned method has been successfully utilized by Vanden-Broeck [7, 8], Gasmi [1], Peng and Parker [20], and other researchers to determine the analytical solutions of free surface flow problems.

The problem at hand is formulated in Section 2, while Section 3 describes the method used to solve that problem. As for Section 4, it presents and discusses the results obtained. Finally, some concluding remarks about the problem are drawn, and future research suggestions are presented in Section 5.

2. PROBLEM FORMULATION

The problem of a two-dimensional, incompressible and inviscid flow emerging from a channel with a symmetric trapezoidal-shaped obstacle of width H and infinite length, placed on the bottom, is investigated. It is clearly illustrated in Figure 1. One can easily see that the superficial tension and gravity force are neglected.

The study of the problem is restricted to the half-axis (X'OX) because the flow field considered here is symmetrical with respect to the Y axis. In addition, a Cartesian coordinate system is defined with the X axis along of the streamline *A'B'S'*, as depicted in Figure 2. The far upstream and downstream flow is assumed uniform, with a constant velocity U and a fluid depth that tends to L so that the bottom may be viewed as a horizontal wall AB, with the inclined wall BC. Note that the inclination angle with the (X'OX) axis is given as $\beta = \frac{\pi}{3}$.

It is worth highlighting that the flow under study is irrotational and the fluid is incompressible, which allows defining the complex potential function *f*=(*Z*) in terms of the potential function ϕ and the stream function ψ as follows:

$$f(z) = \phi(x, y) + i\psi(x, y)$$

Since ϕ and ψ are conjugate solutions of Laplace's equation *f*(*z*) is an analytical function of the complex variable *z*=*x*+*iy* within the flow region with a complex conjugate velocity.

$$\frac{df}{dz} = u(x, y) - iv(x, y) = qe^{-i\theta}$$

here, u and v are, respectively, the horizontal and vertical components of the fluid velocity. They are given, in terms of ϕ and ψ, by the following relations:

$$u = \frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$$

$$v = \frac{\partial\phi}{\partial y} = -\frac{\partial\psi}{\partial x}$$

As for q, it is given as $q = \sqrt{u^2 + v^2}$.

Further, the plane *Z* is transformed by the function *f* into an infinite band that is presented as the plane *f*, as illustrated in Figure 3.

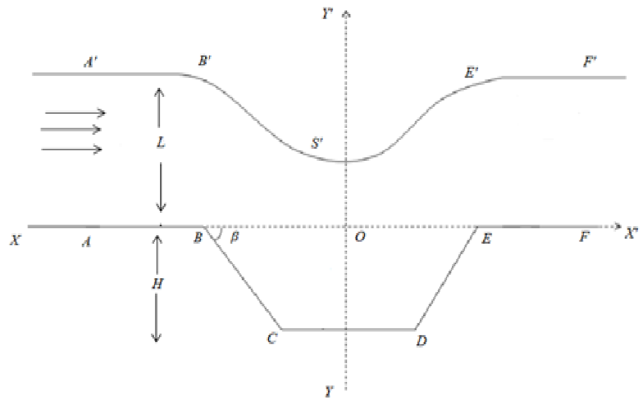

Figure 1. Flow diagram (plane Z)

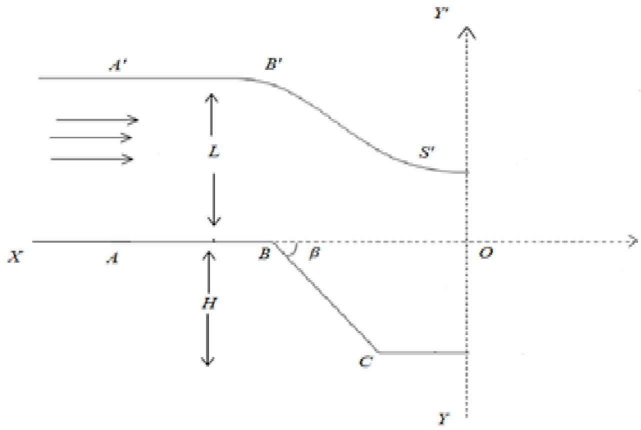

Figure 2. The flow half-domain

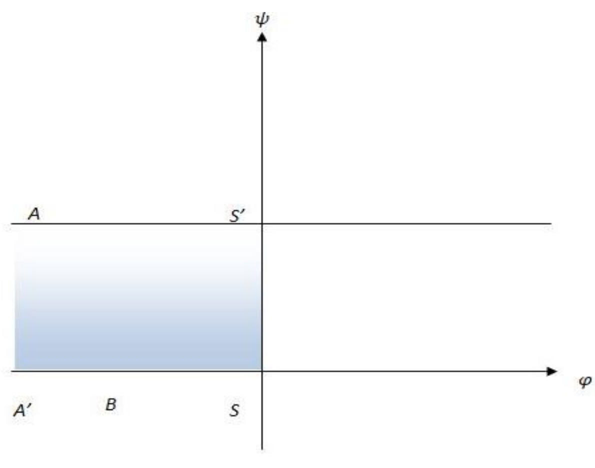

Figure 3. The plane

Finally, the problem considered here consists of determining mathematically the potential velocity ϕ that satisfies the following Laplace equation within the flow region:

$$\frac{\partial\phi^2}{\partial x^2} + \frac{\partial\phi^2}{\partial y^2} = 0 \tag{1}$$

It is worth recalling that the surface tension and gravity force on the free surface *A'B'C'* are neglected, and therefore Bernoulli's equation gives the following expression:

$$\frac{1}{2}\left(\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2\right)+\frac{P}{\rho}=C^{ste}, \frac{1}{2}q^2+\frac{P}{\rho}=C^{ste} \qquad (2)$$

here, q, P, and ρ are the flow velocity, the pressure on the free surface, and the fluid density, respectively. Also, these quantities are constants on the free surface.

For the sake of simplifying the calculations, it was deemed appropriate to take ϕ=0 at the point S, ψ=LU on the streamline $A'B'S'$ and ψ=0 on the streamline $ABCS$.

It should be mentioned that according to Bounab and Bouderah [21] and Batchelor [22], Vanden-Broeck and Dias [23], and some other researchers, the Bernoulli Eq. (2) yields:

$$q^2=C^{ste}, \text{ on } A'B'S' \qquad (3)$$

Based on the above, the kinematic conditions of the flow region and free surface may be rewritten in the form:

$$\begin{cases}\Delta\phi=0, \text{ interieur of the field}\\ \left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2=C^{ste} \text{ on the surface flow}\\ \frac{\partial\phi}{\partial y}=0, \text{ on the wall } ABCS\end{cases} \qquad (4)$$

3. RESOLUTION OF THE PROBLEM

The free streamline theory, which was initially introduced by Kirchhoff and was based on the hodograph mapping and Schwartz-Christoffel transformation, was utilized. It is noteworthy that this approach has already been adopted by Batchelor [22], Birkhoff and Zarantoello [3], Sekhri et al. [9] and other researchers to determine the exact solutions of problems in the case where the effects of the superficial tension and gravity force are neglected.

Therefore, the complex transformation may be expressed as:

$$\Gamma=\log\left(\frac{Udz}{df}\right)=\log\left(\frac{U}{u-iv}\right)=\log\left(\frac{U}{q}\right)+i\theta \qquad (5)$$

here, $z=x+iy$, q and θ are the fluid speed and the angle between the velocity vector and the horizontal plane, respectively. Consequently, the field occupied by the fluid in the plane Z is transformed into a semi-band in the plane Γ, as shown in Figure 4.

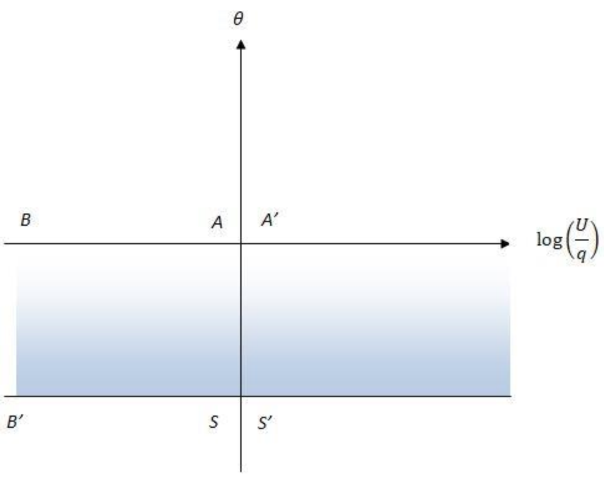

Figure 4. The plane Γ

Figure 5. The plane λ

Moreover, the Schwartz-Christoffel transformation method may be used to transform the half-band plane, as shown in Figure 3, into a lower half-plane that is illustrated in Figure 5.

The hodograph method and Schwartz-Christoffel mapping are used to write the relation below:

$$\frac{d\Gamma}{d\lambda}=A\prod_i(\lambda-\lambda_i)^{\frac{\alpha_i}{\pi}-1}$$

where, λ_i is the coordinate of the plane λ- with respect to the vertex of the polygon, α_i is the corresponding internal angle in the plane Γ, and A is a constant. The previous transformation allows mapping the fluid region in the plane Γ into the upper half λ plane and the fluid region onto the real axis of the λ plane.

Considering this transformation, and in order to achieve a unique mapping, it was decided to take, for example, the points:

$$\text{A: } \Gamma=0\rightarrow\lambda=0$$
$$S: \Gamma=-\frac{i\pi}{3}=-i\beta\rightarrow\lambda=1$$

The following expression may also be considered:

$$\frac{d\Gamma}{d\lambda}=\lambda^{\frac{-1}{2}}(\lambda-1)^{\frac{-1}{2}}\Rightarrow\int d\Gamma=\int\frac{d\lambda}{\lambda^{\frac{1}{2}}(\lambda-1)^{\frac{1}{2}}}$$

In order to calculate the above integral, the new variable $t=\sqrt{1-\lambda}$ in then introduced. Therefore:

$$\lambda-1=t^2, \qquad d\lambda=-2tdt$$

Regarding the transformation Γ, it can be written in the form:

$$\Gamma=\frac{-2M}{i}\int\frac{dt}{\sqrt{1-t^2}}$$

Some simple mathematical operations may then be carried out to get:

$$\Gamma=\frac{-2M}{i}\times arcsin(\sqrt{t})+N$$

Which gives:

$$\Gamma=\frac{-2M}{i}\times arcsin(\sqrt{1-\lambda})+N \qquad (6)$$

where, N and M are two constants to be determined.

For: $\lambda=1\rightarrow\Gamma=-\frac{i\pi}{3}, N=-\frac{i\pi}{3}$

For: $\lambda=0\rightarrow\Gamma=0, M=\frac{1}{3}$

Eq. (6) may therefore be written as:

$$\Gamma = \frac{2i}{3} \times arcsin\left(\sqrt{1-\lambda}\right) - \frac{i\pi}{3} \qquad (7)$$

Eq. (7) can be expressed as:

$$\Gamma = \frac{2}{3} \ln\left(\sqrt{\lambda} + i\sqrt{1-\lambda}\right) - \frac{i\pi}{3} \qquad (8)$$

For the purpose of identifying the transformation that can be used to transform the interior of infinite bands in the plane λ, as shown in Figure 5, into the lower half of the λ plane, the Schwartz-Christoffel transformation is used again, while selecting the same points A and S.

The following equation is then obtained:

$$\frac{df}{d\lambda} = \alpha\lambda^{-1}(\lambda-1)^{-1} \Rightarrow f = \alpha \int \frac{d\lambda}{\lambda\,(\lambda-1)}$$

Then, after some mathematical operations, the relation between λ and f is then found as:

$$f = \alpha \ln\left(\frac{\lambda-1}{\lambda}\right) + b$$

where, α and b are two constants which can be calculated as follow:

$\cdot \lim_{\lambda \to +\infty} f = 0$. Then: $b = 0$

$\cdot \lim_{\lambda \to 0} \lambda \left(\alpha \frac{df}{d\lambda}\right) = \frac{1}{3}$. Then: $\alpha = -\frac{LU}{\pi}$.

$$f = -\frac{LU}{\pi} \ln\left(\frac{\lambda-1}{\lambda}\right) \qquad (9)$$

Afterwards, the relations given below are used:

$$U\frac{dz}{d\lambda} = U\frac{dz}{df}\frac{df}{d\lambda} \qquad (10)$$

$$\frac{df}{d\lambda} = \frac{-LU}{\pi\lambda(\lambda-1)} \qquad (11)$$

On the other hand, transformation Γ may be expressed as:

$$\Gamma = \log\left(\frac{Udz}{df}\right) \qquad (12)$$

Therefore:

$$U\frac{dz}{d\lambda} = exp(\Gamma) = exp\left(\frac{2}{3}\ln\left(\sqrt{\lambda} + i\sqrt{1-\lambda}\right) - \frac{i\pi}{3}\right) = exp\left(-\frac{i\pi}{3}\right) \times \left(\sqrt{\lambda} + i\sqrt{1-\lambda}\right)^{\frac{2}{3}}$$

Next, using relations (10), (11) and (12) allows writing:

$$\frac{dz}{d\lambda} = \frac{LUexp\left(\frac{-i\pi}{3}\right)}{\pi} \frac{\left(\sqrt{\lambda}+i\sqrt{1-\lambda}\right)^{\frac{2}{3}}}{\lambda(\lambda-1)} \qquad (13)$$

Then, the relation (13) is employed to write the following integral:

$$\int dz = \frac{LUexp\left(\frac{-i\pi}{3}\right)}{\pi} \int \frac{1}{\lambda(\lambda-1)} \left(\sqrt{\lambda} + i\sqrt{1-\lambda}\right)^{\frac{2}{3}} d\lambda \qquad (14)$$

Which gives:

$$z = \frac{LUexp\left(\frac{-i\pi}{3}\right)}{\pi} \int \frac{1}{\lambda(\lambda-1)} \left(\sqrt{\lambda} + i\sqrt{1-\lambda}\right)^{\frac{2}{3}} d\lambda \qquad (15)$$

The free surface may therefore be presented parametrically as:

$$\begin{cases} x = real\left(\frac{LUexp\left(\frac{-i\pi}{3}\right)}{\pi} \int \frac{1}{\lambda(\lambda-1)} \left(\sqrt{\lambda} + i\sqrt{1-\lambda}\right)^{\frac{2}{3}} d\lambda\right) \\ y = imag\left(\frac{LUexp\left(\frac{-i\pi}{3}\right)}{\pi} \int \frac{1}{\lambda(\lambda-1)} \left(\sqrt{\lambda} + i\sqrt{1-\lambda}\right)^{\frac{2}{3}} d\lambda\right) \end{cases}$$

Performing the same calculations, for any angle β such that $0<\beta\leq\pi$, the shape of the free surface can generally be expressed in the following form:

$$z = \frac{LUexp(-i\beta)}{\pi} \int \frac{1}{\lambda(\lambda-1)} \left(\sqrt{\lambda} + i\sqrt{1-\lambda}\right)^{\frac{2\beta}{\pi}} d\lambda \qquad (16)$$

which can be expressed parametrically as:

$$\begin{cases} x = real\left(\frac{LUexp(-i\beta)}{\pi} \int \frac{1}{\lambda(\lambda-1)} \left(\sqrt{\lambda} + i\sqrt{1-\lambda}\right)^{\frac{2\beta}{\pi}} d\lambda\right) \\ y = imag\left(\frac{LUexp(-i\beta)}{\pi} \int \frac{1}{\lambda(\lambda-1)} \left(\sqrt{\lambda} + i\sqrt{1-\lambda}\right)^{\frac{2\beta}{\pi}} d\lambda\right) \end{cases}$$

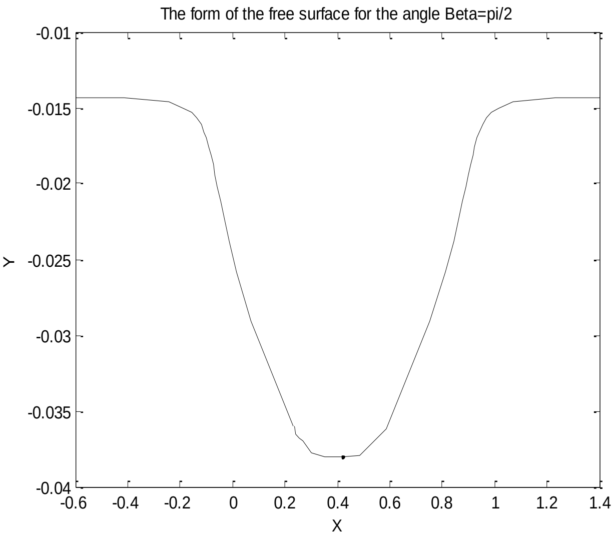

Figure 6. Shape of the free surface for beta=pi/2 ($\beta = \frac{\pi}{2}$)

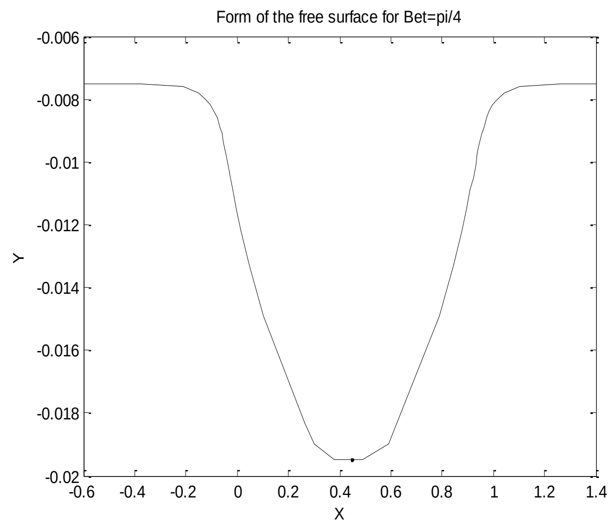

Figure 7. Shape of the free surface for beta=pi/4 ($\beta = \frac{\pi}{4}$)

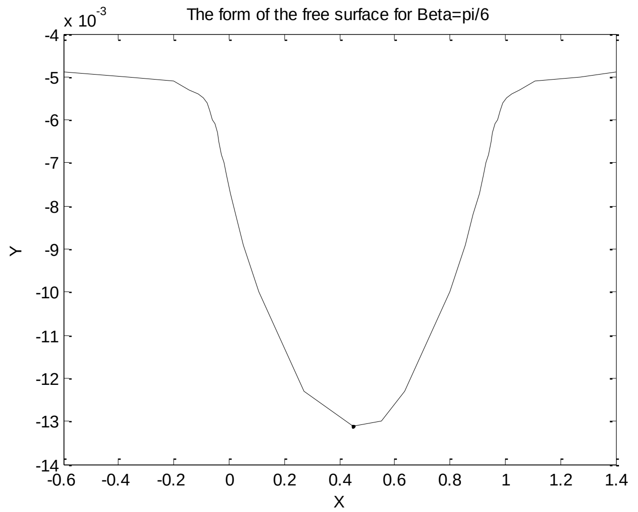

Figure 8. Shape of the free surface for beta=pi/6 ($\beta = \frac{\pi}{6}$)

For the sake of example, the graphical representations of the shape of the free surface flow, for some values of the inclination angle β, are given as Figures 6-8.

4. RESULTS AND DISCUSSION

The Schwartz-Christoffel transformation and the theory of aerodynamic lines introduced by Kirchhoff were used to find the shape of the free surface of a two-dimensional flow of an ideal fluid in a channel with a symmetrical trapezoidal obstacle at the bottom that forms an angle $\beta = \frac{\pi}{3}$ with the horizontal axis (X'OX). In the present case, the effects of the gravity force and surface tension are neglected.

Furthermore, an approximate analytical solution was successfully determined and is explicitly expressed by the relation (15). Figure 9 clearly depicts the shape of the free surface flow.

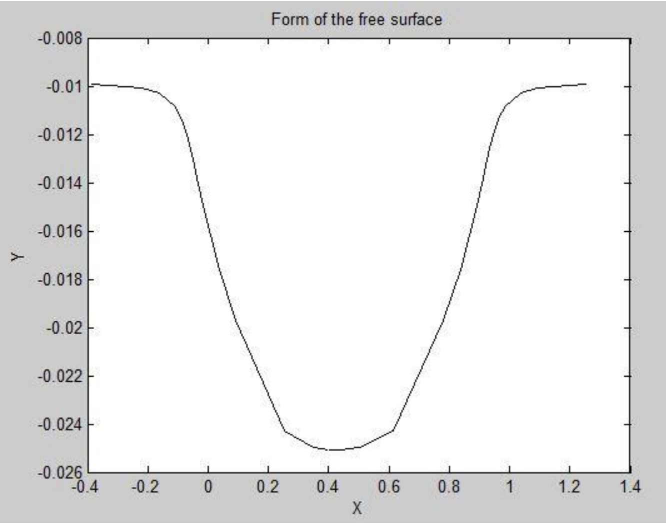

Figure 9. Shape of the free surface flow (exact solution)

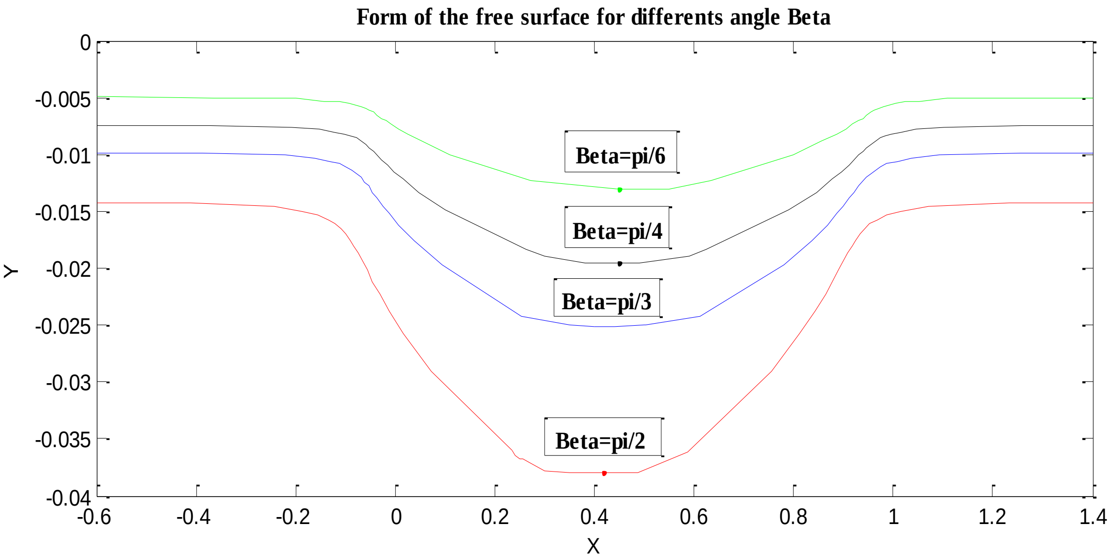

Figure 10. Shapes of the free surface for different beta angles

In addition, it was deemed appropriate to find a solution for each inclination angle β, such that $0<\beta\leq\pi$, in order to confirm the effect of this inclination angle on the free surface profile, as expressed in relation (16). Moreover, the free surface depths, when angle β takes various values within the above interval, are illustrated in Figure 6 for $\beta = \frac{\pi}{2}$, Figure 7 for $\beta = \frac{\pi}{4}$, and Figure 8 for $\beta = \frac{\pi}{6}$.

Likewise, Figure 10 presents the minimum free surface profile that corresponds to $\beta = \frac{\pi}{6}$. This minimum is equal to 0.0131. Figure 11 shows that the free surface depth increases as the angle β grows, as is presented in Table 1.

It is worth mentioning that when the Froude number is taken into account, a numerical solution can be determined using the series truncation method. In this regard, Hanna et al. [13] found out that the Froude number, the height of the trapezoidal obstacle, the width of the upper part of the obstacle and the inclination angle do have an impact on the free surface profile.

Similarly, Sekhri et al. [4] investigated the problem of a two-dimensional potential flow past a submerged triangular obstacle. They first found an analytical solution to the problem by using the method proposed in our paper. Then, they determined a numerical solution by adopting the series truncation method while taking into account the effect of surface tension.

On the other hand, Smith and Lim [19], Hanna et al. [13], Bounif and Gasmi [24], Alwatban and Othman [25], Toison and Hureau [26], as well as other researchers, employed the Hilbert transform method and the perturbation technique in the case of large Froude numbers, for flows over triangular and trapezoidal obstacles. The results obtained were used to determine the shape of the free surface flow.

In the end, it can be asserted that the results of the present study on conformal mapping are in good agreement with those obtained by means of numerical approaches such as the series truncation method and the perturbation technique.

Table 1. The free surface depth versus angle β

The Beta Inclination	The Free Surface Depth
Pi/6	0.013
Pi/4	0.0195
Pi/3	0.025
Pi/2	0.038

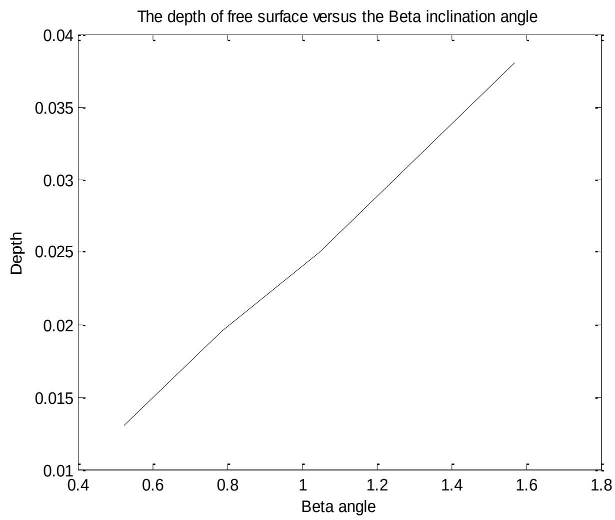

Figure 11. Free surface depth versus inclination angle beta

5. CONCLUSIONS

Until now, researchers have made a lot of effort to prove the existence and unity of the exact solution. Note that the simultaneous presence of a nonlinear term (a non-local term) and a diffusion term makes the Navier and Stokes equations more complicated. They therefore resorted to solving nonlinear equations. Once the solution was found, they then neglected the nonlinear term in order to find the exact solution.

The streamline method was applied in the present work in view of identifying the exact solution to the problem of finding the shape of the free surface flow. The adopted method is based on the hodograph method and the Schwartz-Christoffel transformation technique. The transformations utilized here are considered as fundamental approaches in fluid mechanics, as they make studies of difficult problems in the field of fluids mechanic simpler and easier to solve. They can be used to successfully transform complex flow domains into simpler ones. They can also help to simplify all calculations in order to obtain the exact solution to the problem of finding the shape of a free surface flow. It should be highlighted that the gravity and superficial tension are neglected in this work. These two conditions first made the calculations simpler as all non-linear terms disappeared from the equations used, and second, they contributed to finding the exact solution easily.

This study adopted the free streamline theory, which is based on the Kirchhoff and Hodograph transformations, for the purpose of finding an approximate analytical solution for different values of inclination angle β when a symmetric trapezoidal obstacle is used.

In our opinion, and according to the results obtained in this work, it can be concluded that the calculation approach used in this paper is highly efficient as it provides adequate solutions to this type of free surface flow problems. Moreover, this method may also be utilized to compare the exact solutions to the approximate solutions found in the case where gravity is taken into account, when the Froude number tends towards ∞, and/or in the case where the superficial tension is taken into consideration, when the Weber number tends towards ∞.

In a forthcoming work on this topic, we intend to present and study related problems dealing with the field of potential flows over different forms of obstacles, while considering new boundary conditions and adopting other numerical methods, such as the finite-volume method, in order to find approximate solutions and shapes of free surface flows.

REFERENCES

[1] Gasmi, A. (2014). Two-dimensional cavitating flow past an oblique plate in a channel. Journal of Computational and Applied Mathematics, 259: 828-834. https://doi.org/10.1016/j.cam.2013.07.035

[2] Forbes, L.K., Schwartz, L.W. (1982). Free- Surface flow over a semicircular obstruction. Journal of Fluids Mechanics, (114): 299-314. https://doi.org/10.1017/S0022112082000160

[3] Birkhoff, G., Zarantoello, E.H. (1957). Jets, Wakes and Cavities. New York. https://doi.org/10.1017/S0022112058210100

[4] Gasmi, A., Mekias, H. (2003). The effect of surface tension on the contraction coefficient of a jet. Journal of Physics A: Mathematical and General, 36(3): 851-862. https://doi.org/10.1088/0305-4470/36/3/318

[5] Gasmi, A., Amara, A. (2018). Free-surface profile of a jet flow in U-shaped channel without gravity effects. Advanced Studies in Contemporary Mathematics, 28(3): 393-400. https://doi.org/10.17777/ascm2018.28.3.393

[6] Benjamin, T.B. (1956). On the flow in channels when rigid obstacles are placed in the stream. Journal of Fluid Mechanics, 1(2): 227-248. https://doi.org/10.1017/S0022112056000147

[7] Vanden-Broeck, J.M. (1986). Flow under a gate. Physics of Fluids, 29(10): 3148-3151. https://doi.org/10.1063/1.865966

[8] Vanden-Broeck, J.M. (1983). The influence of surface tension on cavitating flow past a curved obstacle. Journal of Fluid Mechanics, 133: 255-264. https://doi.org/10.1017/S0022112083001901

[9] Sekhri, H., Guechi, F., Mekias, H., (2016). A Waveless free surface flow past a submerged triangular obstacle in presence of surface tension. Electronic Journal of Differential Equations, 190: 1-8. http://ejde.math.txstate.edu.

[10] Merzougui, A., Laiadi, L. (2014). Free surface flow over triangular depression. TWMS Journal of Applied and

Engineering Mathematics, 4(1): 67-73. https://hdl.handle.net/11729/2518.
[11] Dias, F., Vanden-Broeck, J.M. (1988). Flows over rectangular weirs. The Physics of Fluids, 31(8): 2071-2076. https://doi.org/10.1063/1.866659
[12] Vanden-Broeck, J.M., Keller, J.B. (1987). Weir flows. Journal of Fluid Mechanics, 176: 283-293. https://doi.org/10.1017/S0022112087000673
[13] Hanna, S.N., Abdel-Malek, M.N., Abd-El-Malek, M.B. (1996). Super-critical free-surface flow over a trapezoidal obstacle. Journal of Computational and Applied Mathematics, 66(1-2): 279-291. https://doi.org/10.1016/0377-0427(95)00160-3
[14] Vanden-Broeck, J.M. (1987). Bubbles rising in a tube and jets falling from a nozzle. Physics of Fluids, 27: 1090-1093. https://doi.org /10.1063/1.864754
[15] Tuck, E.O. (1987). Efflux from a slit in a vertical wall. Journal of Fluid Mechanics, 176: 253-264. https://doi.org/10.1017/S002211208700065X
[16] Mekias, H., Vanden-Broeck, J.M. (1989). Supercritical free-surface flow with a stagnation point due to a submerged source. Physics of Fluids A: Fluid Dynamics, 1(10): 1694-1697. https://doi.org/10.1063/1.857534
[17] Bloor, M.I.G. (1978). Large amplitude surface waves. Journal of Fluid Mechanics, 84: 167-179. https://doi.org/10.1017/S0022112078000099
[18] Elcrat, A.R., Trefethen, L.N. (1986). Classical free-streamline flow over a polygonal obstacle. Journal of Computational and Applied Mathematics, 14(1-2): 251-265. https://doi.org/10.1016/0377-0427(86)90142-1
[19] Smith, A.C., Lim, T.H. (1985). Symmetric supercritical free surface flow over a polygonal obstacle. International Journal of Engineering Science, 23(3): 289-306. https://doi.org/10.1016/0020-7225(85)90049-7
[20] Peng, W., Parker, D.F. (1997). An ideal fluid jet impinging on an uneven wall. Journal of Fluid Mechanics, 333: 231-255. https://doi.org/10.1017/S0022112096004399
[21] Bounab, N., Bouderah, B. (2013). Computing solution of the flow due to a nozzle. International Journal of Computer Applications, 67(13): 14-18. http://doi.org/10.5120/11455-7060
[22] Batchelor, G.K. (2000). An Introduction to Fluid Dynamics. Cambridge University Press. https://doi.org/10.1017/CBO9780511800955
[23] Vanden-Broeck, J.M., Dias, F. (1991). Nonlinear free-surface flows past a submerged inclined flat plate. Physics of Fluids A, 3(12): 2995-3000. https://doi.org/10.1063/1.857842
[24] Bounif, M.M., Gasmi, A. (2021). First order perturbation approach for the free surface flow over a step with large Weber number. INCAS Bulletin, 13(2): 11-19. https://dx.doi.org/10.13111/2066-8201.2021.13.2.2
[25] Alwatban, A., Othman, H. (2023). Numerical analysis of turbulent air flow dynamics in a rectangular channel with perforated nozzle-shaped vertical baffles. International Journal of Heat and Technology, 41(6): 1407-1416. https://doi.org/10.18280/ijht.410602
[26] Toison, F., Hureau, J. (2001). Potential flow issuing from a two-dimensional curved nozzle. Physics of Fluids, 13(3): 793-798. https://doi.org/10.1063/1.1344896

NOMENCLATURE

Greek symbols

ϕ	Potential function
ψ	Stream function
Γ	Complex transformation
ρ	Density
ϕ	Potential function

Subscripts

p	Pressure
H	Width of field
L	Depth of the fluid