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Optimizing Inventory Management with Seasonal Demand Forecasting in a Fuzzy Environment

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ABSTRACT

This study explores an inventory management model in today's business landscape, where organizations increasingly rely on Machine Learning for demand-driven stock control. The proposed model accounts for imperfect and deteriorating products within a fuzzy environment, allowing for shortages and partial backlogging. Degradation rates and faulty percentages are classified as fuzzy variables since they are unpredictable and impacted by undefined conditions. The goal is to calculate the appropriate replenishment cycle and ordering quantity while reducing the optimal overall cost, including carbon pollution costs, within a constrained planning horizon. The defuzzification technique uses the sign distance approximation technique. Leveraging Machine Learning, the study utilizes a seasonal demand forecasting methodology. A numerical illustration supports the mathematical approach by demonstrating its capacity to estimate demand for deteriorating products. This facilitates optimized inventory management aligned with forecasted demand. A comparative examination emphasizes the positive aspects of AI learning-based forecasting systems over determined demand circumstances. Sensitivity analysis provides insights into the impact of various parameters on optimal solutions, contributing valuable managerial perspectives.

1. INTRODUCTION

In the ever-evolving global market landscape, the intricate dance between seasonal and weather conditions exerts a profound influence on consumer demand, a cornerstone variable that presents multifaceted challenges to efficient inventory management across diverse industries [1]. The ebb and flow of seasonal demand, shaped by events such as festivals and climatic factors, introduces uncertainties and complexities into consumer purchasing behaviours, necessitating a sophisticated approach to inventory control [2]. While conventional inventory models often hinge on deterministic demand assumptions, the real-world scenario unfolds with variations in product demand that adhere to distinct seasonal patterns. So, in our study, we have applied the time series algorithm to forecast seasonal demand.

The strategic imperative of effective demand prediction emerges as a key solution, offering the potential to refine inventory management strategies, curtail superfluous costs, and elevate overall customer service [3]. Leveraging machine learning (ML): with its advanced predictive capabilities, particularly through Decision Tree-based Algorithms, stands out as a transformative tool in achieving precise and accurate seasonal demand forecasts [4]. This paper delves into the convergence of seasonal demand dynamics, imperfect deteriorating products, and the contemporary imperative of considering carbon emissions in inventory systems. The intrinsic deterioration of physical products over time, be it during transit or storage, is a ubiquitous challenge across various industries [5]. Items such as fruits, medicines, flowers, foodstuffs, and vegetables are susceptible to decay during their holding and in-transit periods. This study acknowledges the deterministic approach traditionally applied to deterioration rates in inventory models but contends that real-world uncertainty demands a more sophisticated treatment [6]. To address this, the model introduces a fuzzy variable for deterioration rates, acknowledging the uncertainty in their precise estimation.

Furthermore, the quantity of defective products, a critical consideration in inventory management, is recognized as another fuzzy variable due to unpredictable factors such as manufacturing defects, man-handling issues, and in-transit damage [7]. The study emphasizes the pressing concern of escalating carbon emissions in the modern era, driven by industrialization and contributing significantly to climate change. This prompts a paradigm shift in inventory system design, where scholars and organizations now focus on reducing the total cost, integrating considerations for carbon emissions. This research not only acknowledges permissible shortages but also accounts for partial backlogging, recognizing that not every consumer accepts delayed deliveries. By addressing these multifaceted challenges, the study endeavours to bridge gaps in existing literature concerning the impact of demand Predictions on inadequate decaying products. Two primary research questions guide this exploration: (a) How do AI based demand prediction techniques improve the certainty and predictability of seasonal predictions for demand for deteriorating products? (b) What are the benefits of using artificial intelligence-driven monthly projected demand versus constant demand in inventory management?

To resolve such issues, this article presents an AI-driven fuzzy inventory model that incorporates defective, deteriorating products and carbon emissions. The decision tree classifier, a powerful ML technique, is employed for demand forecasting, aiming to determine accurate seasonal demand [8]. The goal is to optimize purchasing amount and replenishment periods, thereby minimizing the total average cost, while considering carbon emissions. The ensuing sections delve into a comprehensive review of the literature, outline notations and assumptions, articulate the mathematical model, detail the ML-based methodology, present validation through a numerical example, conduct sensitivity analysis, and culminate in conclusions and avenues for future research. In navigating this exploration, we aim to contribute insights that advance both the theoretical and practical dimensions of inventory management in the context of seasonal demand, imperfect deteriorating products, and the imperative of environmental sustainability.

2. LITERATURE REVIEW

Demand forecasting stands as a pivotal element in shaping business strategies, offering organizations the means to optimize operations, cut costs, and meet consumer expectations [2]. Mishra and Jain [9] put forward a decentralized supply chain optimization model that incorporates blockchain and uses an iterative strategy to calculate overall costs for retailers and suppliers. By establishing its uniqueness and optimal results through theoretical analysis, the model efficiently determines optimal replenishment cycles using Wolfram Mathematica 13.0., guiding decision-makers with managerial insights for enhanced supply chain management efficiency and resilience. The continuous expansion of ML techniques provides a fertile ground for researchers aiming to enhance the accuracy of demand forecasting [3]. Early research by Persinger and Levesque [10] delved into the relationship between weather conditions and individuals' moods, establishing that weather significantly influences consumer behaviour. Wright and Schultz [11] emphasized the critical role of demand forecasting models in predicting overstocking and understocking situations, particularly during fluctuations in consumer demand. Notable festive seasons, such as Christmas and Diwali, witness a surge in demand for e-commerce giants like Amazon and Flipkart as consumers actively seek gifts and festive supplies. Accurate demand forecasting becomes imperative during such peak periods, emphasizing the need for dynamic models that surpass fixed demand predictions [12]. Decision trees, recognized as powerful data mining techniques, have been extensively employed in various sectors for demand forecasting [13]. In the field of defective damaging goods, Ghare [14] first developed research on damaging items in inventory systems, with a focus on constant deterioration ratios. Wee et al. [15] conducted subsequent research and built a production inventory model particularly for damaging seasonal goods. The consideration of partial payment and trade credit policies in a non-instantaneous disintegrating concept of inventory was introduced by Lashgari et al. [16]. The reality of imperfect products, either due to manufacturing errors or deterioration, led to economic production quantity (EPQ) models for imperfect quality items [17]. Further investigations delved into imperfect quality inventory systems, integrating stochastic processes to model defective products [18].

Carbon emissions have become a significant concern in recent years, prompting researchers to incorporate environmental considerations into inventory models. Mishra [19] provides a supply chain inventory model for degrading items that takes into account carbon emission-dependent demand, advanced payment methods, and the impact of carbon taxes and limits on a constrained planning horizon. It seeks to balance economic and environmental considerations, providing insights for businesses and potential relevance for government policies. Hua et al. [20] built an economic order quantity (EOQ) model which incorporates carbon costs associated with storage and shipment. Carbon taxes and emissions reduction policies have been explored to understand their impact on inventory costs [21]. Green inventory models were studied under settings of carbon emission penalty fees, the cap-and-trade scheme systems, and severe emission limit laws [22].

Fuzzy methods have gained attention in inventory management problems due to their ability to generate more relevant solutions. Early applications of fuzzy set theory in inventory models include analyses of economic order quantity [23, 24]. Extensions into fuzzy overall cost functions, considering fuzzy demand rates and proportions of defective products, have been explored [25, 26]. Fuzzy inventory estimation methods for degrading items with time-dependent demand and backlog rates considered as fuzzy numbers have also been developed [27]. The integration of fuzzy concepts into inventory models, considering imperfect products, payment delays, variable demand and partial backlog, has been investigated [28]. Mishra et al. [29] present a fuzzified supply chain finite planning horizon model, addressing deteriorating materials. The model uses fuzzy parameters, such as deterioration cost, and applies defuzzification methods with finite planning horizon. Mishra et al. [30] provided research on the fuzziness of supplier-retailer supply coordination. The research explores credit terms in the context of managing items are degrading due to time-quadratic demand and partial backlog throughout all cycles across the finite planning horizon. Singh and Mishra [31] provides an inventory model that uses artificial intelligence to estimate demand, with a focus on imperfect deteriorating items and partial backlog concerns. The model also incorporates the impact of carbon emissions.

3. RESEARCH GAP

Even though, organisations are increasingly implementing Machine Learning solutions within the processes of inventory management, there is still a major research void in providing strong models that consider the imperfection, deterioration and the fuzzy context of inventory management systems. Most contemporary structures of inventory control might have embedded features of uncertain demand and product imperfection, but they may lack an enumerating factor on how product deterioration adds to the rising cost and ineffectiveness. Furthermore, the consideration of carbon pollution costs in cost decision-making activities is another understudied component in the literature.

3.1 Problem identification

This study identifies a critical gap in inventory management models: the lack of a strategic fit which addresses a fuzzy environment and products' deteriorating character along with the costs of carbon pollution. This issue is rather crucial today particularly to contemporary companies operating in the business environment that seeks to optimize inventory management to enhance cost reduction and, at the same time, ensure the achievement of sustainability goals. Literature research shows that traditional models are misleading in that they do not capture variability and effects of undefined condition to the rates of product deterioration and faults in an accurate manner thus resulting in wrong replenishment cycles and order quantities. Moreover, lack of literature on integrating price of carbon pollution into total cost is another challenge that hampers the formulation of soulutionary inventory management. Therefore, it is imperative to seek more inventive approaches to address those obstacles; they are intending to develop a framework that will integrate the Machine Learning algorithms and fuzzy logic to improve reliability and productivity of a system used in managing inventories in parallel to supporting environmental sustainability.

This literature review outlines the evolution of research in demand forecasting, imperfect deteriorating products, and environmental considerations. literature survey and research gap are discussed in Table 1. While existing studies have addressed individual facets, there is a notable gap in integrating artificial intelligence for demand forecasts, considering partial backlogging, imperfect products and carbon emissions. The current study endeavours to contribute to this intersection by providing a comprehensive model that extends previous frameworks and incorporates machine learning concepts in a fuzzy environment over the finite planning horizon.

Table 1. Literature survey and research gap

Ref. No.	Focus Area	Contribution
[10]	Weather's Impact on Moods	Demonstrated weather's influence on consumer moods.
[11]	Demand Forecasting Models	Emphasized the role of demand forecasting in predicting overstocking and understocking.
[12]	Festive Demand	Simple time series algorithms were classified as conventional, and several ML approaches were examined.
[32]	Decision Trees	Inductive decision trees were used to analyze both continuous and discrete data simultaneously.
[15]	Deteriorating Seasonal Items	Created a production supply model for decaying seasonal products.
[17]	EPQ for Imperfect Quality Items	Proposed an EPQ model for imperfect quality items.
[20]	EOQ with Carbon Costs	Established an EOQ model considering carbon costs in transportation and stockkeeping.
[22]	Green Inventory Model	Examined a green model for inventory under carbon emission penalties, cap-and-trade, and regulatory constraints.
Current Study	ML, Imperfect Products, Partial Backlogging, Carbon Emissions	Addresses the void through the utilization of sophisticated machine learning methods for predicting demand, considering flawed items, incomplete backlogs, and carbon emissions. Expands Tiwari et al.'s model.

3.2 Limitation

There are certain peculiarities and limitations of applying the chosen demand forecasting methodologies which influence their efficiency. Some turn with data demand by time suggesting that it is stationary and linear although in reality may vary greatly due to seasonality factors, trends or other characteristics. These models are therefore very sensitive to the quality and completeness of the historical data and errors can emanate from incomplete or noisy history. Also, forecasting models are, to a certain extent, vulnerable to parameter estimation which means that small variations with a view to parameters enhance large disparities in forecasts. They could also fail to capture various relational factors that depict the interactions of various factors and may not consider inherent volatilities in demand. In addition, traditional models cannot evolve as fast as necessary to respond to the changes in the business environment that can be technology or shifts in consumers' preferences. With reference to these challenges, new complex methodologies need to be developed in order to incorporate elements of machine learning and fuzzy control to enhance the flexibility and the accuracy of the solutions provided.

4. SCHOLASTIC ACHIEVEMENT

The main innovation of this study is in crafting a distinctive inventory system supported by machine learning and fuzzy logic. This system is custom designed to confront the difficulties arising from defective degrading goods amid carbon emissions within a limited planning timeframe. Unlike traditional inventory models, this study addresses uncertainties surrounding defective percentages and deterioration rates by treating them as fuzzy variables. Through the application of a machine learning technique, specifically the time series prediction or (production): the model aims to enhance the precision of variable demand forecasts for degrading items over a limited planning horizon.

The importance of this research work is evident in its potential to revolutionize demand forecasting in businesses. By moving beyond fixed demand assumptions and incorporating machine learning-based monthly predicted demand, organizations can achieve more precise and reliable inventory management. The numerical experiment conducted in the study demonstrates a substantial reduction in overall costs when utilizing seasonal forecasted demand. Moreover, the model's incorporation of carbon emissions costs underscores a commitment to sustainability and environmental responsibility.

The research's overarching contribution is its holistic approach, fuzzy variables, integrating machine learning techniques and considerations for carbon emissions. This comprehensive framework not only improves demand forecasting accuracy but also provides businesses with a strategic means to minimize their ecological footprint. Ultimately, the research aims to identify optimal policies that minimize overall costs while simultaneously addressing the complexities associated with deteriorating products, thereby offering a valuable contribution to the field of inventory management over the finite planning horizon.

4.1 Environmental benefits of carbon emissions integration in inventory control

The decision of including the cost of carbon emissions in inventory management has many environmental gains. This integration encourages organisations to implement the practices that are environmentally friendly across its operations since the environmental cost attached to the inventory activities are supported. Particularly, it promotes the improvements in motion-related measures including the optimisation of routes to avoid gross fuel consumption and the purchase of effective energy-efficient resources used in the storage of goods. Also, the incorporation of carbon emissions costs can actually result to the use of suppliers and production processes with a relatively low levels of carbon emissions hence enhancing the use of a cleaner supply chain.

Further, it can lead to the optimization of inventory and waste as firms look to integrate sustainability strategy in inventory control so that the production and disposal of extra and obsolete products are eliminated leading to fewer emission values. It also aids organisations to conform to environment legislation and policies which contributes to its sustainable status and may provide certain firm's with an edge in regions where sustainability is becoming increasingly more of a concern and criterion to consumers. When such costs are incorporated, firms help in creating organizational awareness towards the negative effects of their operations on the environment while at the same time driving down the general climate change.

4.2 Rationale for selecting the decision tree classifier in demand forecasting

The decision tree classifier is chosen to be used for demand forecasting by means of selecting the Machine Learning technique because of several benefits. First, decision trees practically allow for evaluating interpretability on a high level because the decision rules and their results are easily presented in a tree-like structure. This aspect enables one to grasp how different aspects affect the demand forecast; it assists the stakeholders to develop significant data insights and make the required modifications to their strategies.

In addition to this, decision trees do not assume the normality of the data and have the ability to deal with numerical as well as categorical data for unique datasets, which are always involved in the demand for estimation, for example, past sales data, seasonal variation, and promotional influences. This is especially helpful when it comes to capturing the interaction effects of features since, unlike linear methods, tree-based models can capture a quadratic and higher order relationships with the variables.

Moreover, decision trees have high tolerance to outliers and noisy data that is typical for real data sets. It isolates data into homogenous subsets, thereby; the existence of anomalies or irregularities does not undermine the model's predictive capability. Also, decision trees are less sensitive to data pre-processing than most of the algorithms hence data pre-processing is done in a minimal rate thus minimizing the chances of high bias of the data.

It is also noteworthy that decision trees are extremely efficient in terms of learning and prediction times. The fact that building block models can be done relatively quickly is an advantage since some of the institutional demands for dynamic demand forecasting may necessitate frequent updates. Furthermore, decision trees can be replaced, added or integrated with some ensembling algorithms like random forest or gradient boosting in order to improve the efficiency of prediction or to avoid some problems like overfitting or interpretability.

Concisely, the merits such the decision tree classifier's interpretability, flexibility, noise tolerance and efficiency put it in a good standing to serve as a viable and effective predictor for demand, and by extension a catalyst in constructing an effective inventory management system.

5. ASSUMPTIONS

1. Only one sort of deteriorating goods is evaluated, with an unlimited replacement rate.
2. The timing for replenishment orders will be limited.
3. The production pattern is based on expected demand.
4. β is backordering cost.
5. The lead time will not be zero but rather nearly negligible.
6. Defective products result from imperfect manufacturing and worker handling, with the defective percentage (k) considered as an interval trapezoidal fuzzy number.
7. The decomposition rate is considered a trapezoidal fuzzy number.
8. Carbon emissions due to shipping, godown/storage, and decomposition are considered.
9. Shortages are permitted and partially backlogged.
10. The fixed transportation cost is incurred when the retailer initiates an order.

6. METHODOLOGY FOR DEMAND FORECAST AND MATHEMATICAL MODEL

This article explores a sustainable supply model designed for imperfect diminishing products within a retail setting. At the onset, the retailer acquires a quantity (Q_{i+1}) of products. The stock level experiences a reduction due to both demand and degradation throughout the period. A partial backlogging shortage occurs at a rate of β and persists until time T. By the conclusion of the cycle, the supply has exceeded its maximum deficit level. To rectify this backlog, the retailer initiates the replenishment of products. The dynamics of the stock level during the time intervals $[0,t_0]$, $[t_0,t_1]$ are governed by the following set of differential equations:

The suggested approach in this work focuses on accurate demand forecasting using artificial intelligence (machine learning). While many academics believe that demand is predictable, the presence of unpredictable swings implies that an approach based on machine learning is more suited for demand prediction. For this study, time series method is chosen due to its simplicity and effectiveness in ML techniques. The primary objective is to ascertain the precise seasonal demand for deteriorating products. The flowchart illustrating the methodology for demand forecasting is presented above.

PYTHON code (version 3.10.0) is used to validate the forecasted inventory system by predicting seasonal demand for deteriorating products. Before executing the Python code, ensure that essential packages like Pandas for data manipulation and sklearn for handling time series data are installed.

Ultimately, by inputting the parameter (month): the seasonally demand for deteriorating products is obtained. As we can see the Figure 1.

Inventory level with boundary condition $IL_{i+1}(s_{i+1}) = 0$,

$$\frac{dIL_{i+1}(t)}{dt} + (\theta)IL_{i+1}(t) = -(1-k)D \quad t_i < t < s_{i+1} \tag{1}$$

where $\{1,2,3 \ldots \ldots \ldots \ldots \ldots \ldots \ldots . n_1\}$,

$$\frac{dIL_{i+1}(t)}{dt} = -(1-k)D - (\theta)IL_{i+1}(t) \quad t_i < t < s_{i+1} \tag{2}$$

$$IL_{i+1}(t) = \int_t^{s_{i+1}} -(1-k)De^{\theta(u-t)}\,du \tag{3}$$

$$IL_{i+1}(t) = \theta(1-k)D\left[1 - e^{\theta(s_{i+1}-t)}\right] \tag{4}$$

```python
import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.tsa.arima.model import ARIMA

# Load the data
data_location = r'C:\Users\Ranu\OneDrive\Desktop\Daily_Demand_Forecasting_Orders.xlsx'
data = pd.read_excel(data_location)

# Display basic information about the dataset
print("Dataset Info:")
print(data.info())

# Display descriptive statistics
print("\nDescriptive Statistics:")
print(data.describe())

# Check column names
print("\nColumn Names:")
print(data.columns)
```

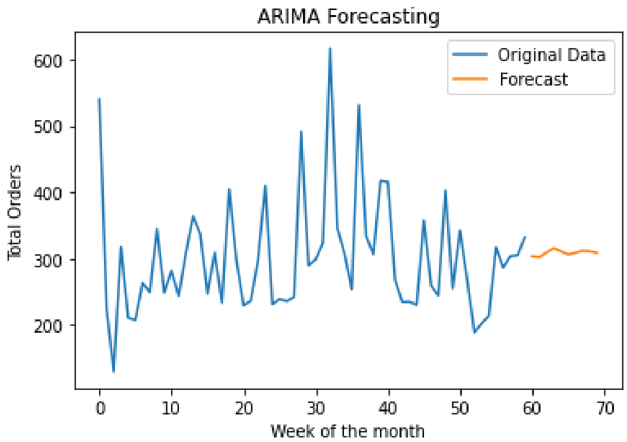

Figure 1. Demand forecasting

The current shortage level, denoted as $S_{i+1}(t)$ under the boundary condition $S_{i+1}(s_i) = 0$, is defined by the subsequent differential equation:

$$\frac{dILS_{i+1}(t)}{dt} = D\beta \tag{5}$$

where, $s_i < t < t_i$,

$$ILS_{i+1}(t) = \int_{s_i}^{t} D\beta dt = D\beta(s_i - t) \tag{6}$$

Therefore, the overall inventory quantity maintained throughout the interval $[t_i, s_{i+1}]$,

$$R_{i+1} = \int_{t_i}^{s_{i+1}} \left\{ \int_t^{s_{i+1}} -(1-\mathrm{k})\mathrm{D}e^{\theta(u-t)}\,\mathrm{du} \right\} dt \tag{7}$$

Eq. (7) can be reformulated as follows by adjusting the integration position and omitting the higher-order terms of α^2.

$$R_{i+1} = \frac{-(1-\mathrm{k})}{\theta} \int_{t_i}^{s_{i+1}} \left\{e^{\theta(s_{i+1}-t)} - 1\right\} D dt \tag{8}$$

Customers are waiting for the complete amount of that quantity i.e., the quantity of deficit throughout the timeframe $[s_i, t_i]$.

After rearranging the ordering, S_{i+1} can be given as:

$$S_{i+1} = \int_{s_i}^{t_i} ILS_{i+1}(t)dt = \int_{s_i}^{t_i} \left\{ \int_{s_i}^{t} D\beta(s_i - u)du \right\} dt = \frac{-1}{2} \int_{s_i}^{t_i} (D\,\beta(s_i - t))^2 dt \tag{9}$$

The total order quantity for a finite planning horizon:

$$\mathrm{Q} = \sum_{i=1}^{n} Q_{i+1} = \sum_{i=1}^{n} \{R_{i+1} + S_{i+1}\}$$

$$Q_{i+1} = \frac{-(1-\mathrm{k})}{\theta} \int_{t_i}^{s_{i+1}} \left\{e^{\theta(s_{i+1}-t)} - 1\right\} D dt + \frac{1}{3}(D\beta(s_i - t_i))^3$$

The overall number of degraded components at each refill is as follows:

$$D_{i+1} = \int_{s_i}^{t_i} \theta IL_{i+1}(t)dt = \left\{ \theta^2 \int_{t_i}^{s_i} (1-k)D\left[1 - e^{\theta(s_{i+1}-t)}\right] dt \right\} \tag{10}$$

In the case of Fast-Moving Consumer Goods (FMCG) or basic necessities: Consumers cannot delay their purchases, resulting in only a portion, β, of the demand being held back during stockouts. Consequently, the remaining fraction (1-β) is lost.

The quantity that was lost throughout the interval $[s_i, t_i]$ is given as:

$$L_{i+1} = \int_{s_i}^{t_i} \{D - D\beta\} dt = \int_{s_i}^{t_i} \{(1-\beta)D\} dt \\ = (1-\beta)D(t_i - s_i) \tag{11}$$

Cost of carbon emissions during the period $[t_i, s_{i+1}]$ can be expressed as:

$$Ce = \sum_{i=0}^{n_1-1} c\hat{} + \hat{P}_r * R_{i+1} + \widehat{h_c} \int_{t_i}^{s_{i+1}} IL_{i+1}(t) dt$$

$$Ce = \sum_{i=0}^{n_1-1} c\hat{} + \hat{P}_r * Q_{i+1}$$

$$+\widehat{h_c} \int_{t_i}^{s_{i+1}} \left\{ \int_{t}^{s_{i+1}} -(1-k) D e^{\theta(u-t)} du \right\} dt$$

According to Mishra [9]: The overall cost of carbon emissions throughout the interval $[t_i, s_{i+1}]$ can be expressed as:

$$Ce = \tau \left\{ \sum_{i=0}^{n_1-1} c\hat{} + \hat{P}_r \left\{ \theta \int_{t_i}^{s_i} (1-k) D \left[1 - e^{\theta(s_{i+1}-t)} \right] dt \right\} - (1-k) D * \widehat{h_c} * \theta \int_{t_i}^{s_{i+1}} \{ e^{\theta(s_{i+1}-t)} - 1) dt \} \right\} \tag{12}$$

The transportation expenses for the retailer take into account both variable transportation costs and fixed costs, as well as carbon emissions resulting from FEC during refrigeration. Eq. (13) consequently outlines the transportation cost as follows.

$$c = F_c + 2dv_c C_1 + d * v_c C_2 \int_{t_i}^{t_{i+1}} D e^{\theta(t-t_i)} dt + 2de_1 + d\, e_2 \int_{t_i}^{t_{i+1}} D e^{\theta(t-t_i)} dt \tag{13}$$

Total cost = Replenishment Cost + Inventory Holding Cost + Acquisition Cost + Depreciation Cost + Storage Expense + Lost Sales Cost + Carbon Emission Cost + Transportation Expense

$$TC(t_i, s_i, n_1) = n_1 * O_r$$
$$+ \sum_{i=0}^{n_1-1} H \int_{t_i}^{s_{i+1}} IL_{i+1}(t) dt + \sum_{i=0}^{n_1-1} W_h * Q_{i+1} + \sum_{i=0}^{n_1-1} D_t * \theta \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\, dt + \sum_{i=0}^{n_1-1} s \int_{s_i}^{t_i} ILS_{i+1}(t) dt + \sum_{i=0}^{n_1-1} l \int_{s_i}^{t_i} (1-\beta) D(t_i - s_i) dt$$
$$+ \sum_{i=0}^{n_1-1} c\hat{} + \hat{P}_r Q_{i+1} + \widehat{h_c} \int_{t_i}^{s_{i+1}} IL_{i+1}(t) dt + F_c + 2dv_c C_1 + d * v_c C_2 \int_{t_i}^{t_{i+1}} D e^{\theta(t-t_i)} dt + 2de_1 + de_2 \int_{t_i}^{t_{i+1}} D e^{\theta(t-t_i)} dt$$

$$TC(t_i, s_i, n_1) = n_1 * O_r$$
$$+ \sum_{i=0}^{n_1-1} H \int_{t_i}^{s_{i+1}} IL_{i+1}(t) dt + \sum_{i=0}^{n_1-1} W_h * Q_{i+1} + \sum_{i=0}^{n_1-1} D_t * \theta \int_{t_i}^{s_{i+1}} IL_{i+1}(t)\, dt + \sum_{i=0}^{n_1-1} s \int_{s_i}^{t_i} ILS_{i+1}(t) dt + \sum_{i=0}^{n_1-1} l \int_{s_i}^{t_i} (1-\beta) D(t_i - s_i) dt$$
$$+ \sum_{i=0}^{n_1-1} \tau c\hat{} + \tau \hat{P}_r\, Q_{i+1} + \tau \widehat{h_c} \int_{t_i}^{s_{i+1}} IL_{i+1}(t) dt + F_c + 2dv_c C_1 + d * v_c C_2 \int_{t_i}^{t_{i+1}} D e^{\theta(t-t_i)} dt + 2de_1 + de_2 \int_{t_i}^{t_{i+1}} D e^{\theta(t-t_i)} dt$$

$$TC(t_i, s_i, n_1) = n_1 * O_r + \sum_{i=0}^{n_1-1} \{ H + \tau \widehat{h_c} + D_t * \theta \} \int_{t_i}^{s_{i+1}} IL_{i+1}(t) dt + \{ W_h + \tau \hat{P}_r \} \sum_{i=0}^{n_1-1} Q_{i+1} + \sum_{i=0}^{n_1-1} s \int_{s_i}^{t_i} D\beta(s_i - t) dt$$
$$+ \sum_{i=0}^{n_1-1} l \int_{s_i}^{t_i} (1-\beta) D(t_i - s_i) dt + \tau c\hat{} + F_c + 2dv_c C_1 + d * \, v_c C_2 \int_{t_i}^{t_{i+1}} D e^{\theta(t-t_i)} dt + 2de_1 + de_2 \int_{t_i}^{t_{i+1}} D e^{\theta(t-t_i)} dt$$

$$TC(t_i, s_i, n_1) = n_1 * O_r + \sum_{i=0}^{n_1-1} \{ H + \tau \widehat{h_c} + D_t * \theta \} \int_{t_i}^{s_{i+1}} IL_{i+1}(t) dt + \{ W_h + \tau \hat{P}_r \}$$
$$\sum_{i=0}^{n_1-1} \left(\frac{-(1-k)}{\theta} \int_{t_i}^{s_{i+1}} \{ e^{\theta(s_{i+1}-t)} - 1 \} D dt + \frac{1}{3} \left(D\beta(s_i - t_i) \right)^3 \right) + \sum_{i=0}^{n_1-1} s \int_{s_i}^{t_i} D\beta(s_i - t) dt$$
$$+ \sum_{i=0}^{n_1-1} l \int_{s_i}^{t_i} (1-\beta) D(t_i - s_i) dt + \tau c\hat{} + F_c + 2d * v_c C_1 + d * v_c C_2 \int_{t_i}^{t_{i+1}} D(t) e^{\theta(t-t_i)} dt$$
$$+ 2de_1 + de_2 \int_{t_i}^{t_{i+1}} D(t) e^{\theta(t-t_i)} dt \tag{14}$$

$$TC(t_i, s_i, n_1) = n_1 * O_r + \sum_{i=0}^{n_1-1} (1-k)\{H + \tau\widehat{h_c} + D_t * \theta\} \int_{t_i}^{s_{i+1}} [D\beta(s_i - t)]dt + \{W_h + \tau\hat{P}_r\}$$
$$\sum_{i=0}^{n_1-1} \left(\frac{-(1-k)}{\theta} \int_{t_i}^{s_{i+1}} \{e^{\theta(s_{i+1}-t)} - 1\} Ddt + \frac{1}{3}\left(D\beta(s_i - t_i)\right)^3 \right) + \sum_{i=0}^{n_1-1} s \int_{s_i}^{t_i} D\beta(s_i - t)dt \quad (15)$$
$$+ \sum_{i=0}^{n_1-1} l \int_{s_i}^{t_i} (1-\beta)D(t_i - s_i)dt + \tau c^{\wedge} + F_c + 2dv_cC_1 + (de_2 + d * v_cC_2)\int_{t_i}^{t_{i+1}} De^{\theta(t-t_i)}\,dt$$

6.1 Fuzzification of model

In inventory systems, pinpointing precise values for known parameters poses a challenge for decision-makers, introducing uncertainty into key parameters. Consequently, the defective percentage in quantity (k) and deterioration rate (θ) are treated as trapezoidal fuzzy interval types. Fuzzy arithmetic operations for trapezoidal fuzzy numbers are concisely explained. Building upon these basic definitions and results, the proposed model is then fuzzified. Let $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{\theta}_3, \tilde{\theta}_4)$ and $(\tilde{k}_1, \tilde{k}_2, \tilde{k}_3, \tilde{k}_4)$ represent trapezoidal fuzzy numbers, as depicted in Figure 2. Consequently, the succinct total average expense function is converted into a fuzzy cost function. Since the deterioration rate (θ) and the defective quantity percentage (k) are both represented as trapezoidal fuzzy figures, the total cost (TC) is also treated as a trapezoidal fuzzy figure.

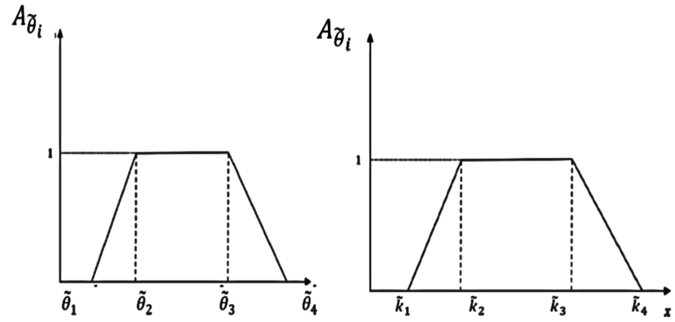

Figure 2. Displays the trapezoidal fuzzy

$$TC = (\widetilde{TC}_1, \widetilde{TC}_2, \widetilde{TC}_3, \widetilde{TC}_4)$$
$$TC_d = \frac{1}{4}\left(\widetilde{TC}_1 + \widetilde{TC}_2 + \widetilde{TC}_3 + \widetilde{TC}_4\right)$$

$$\widetilde{TCR}_i(t_i, s_i, n_1) = n_1 * O_r + \sum_{i=0}^{n_1-1} \left(1 - \widetilde{k_\iota}\right)\{H + \tau\widehat{h_c} + D_t * \widetilde{\theta_\iota}\} \int_{t_i}^{s_{i+1}} [D\beta(s_i - t)]dt$$
$$+\{W_h + \tau\hat{P}_r\} \sum_{i=0}^{n_1-1} \left(\frac{-(1-\widetilde{k_i})}{\widetilde{\theta_i}} \int_{t_i}^{s_{i+1}} \{e^{\widetilde{\theta_i}(s_{i+1}-t)} - 1\} Ddt + \frac{1}{3}\left(D\beta(s_i - t_i)\right)^3 \right)$$
$$+ \sum_{i=0}^{n_1-1} s \int_{s_i}^{t_i} D\beta(s_i - t)dt + \sum_{i=0}^{n_1-1} l \int_{s_i}^{t_i} (1-\beta)D(t_i - s_i)dt + \tau c^{\wedge} + F_c + 2dv_cC_1(de_2 + d * v_cC_2)\int_{t_i}^{t_{i+1}} D(t)e^{\widetilde{\theta_\iota}(t-t_i)}\,dt$$

The goal is to discover the basic values of t_i and s_i in order to lower the total variable cost (TC) of stock control and management. Figure 2 exhibits the trapezoidal fuzzy numbers denoting the deterioration rate (θ) and the defective quantity percentage (k). The requirements to shows the $\widetilde{TC}_d$ to be minimum are given below:

$$\frac{\partial \widetilde{TCR}_d(t_i, s_i, n_1)}{\partial t_i} = 0$$
$$\frac{\partial \widetilde{TCR}_d(t_i, s_i, n_1)}{\partial s_i} = 0$$

$$\frac{\partial \widetilde{TCR}_d(t_i, s_i, n_1)}{\partial s_i} = \{H + \tau\widehat{h_c} + D_t * \widetilde{\theta_\iota}\} \int_{t_i}^{s_{i+1}} [D\beta]dt + \{W_h + \tau\hat{P}_r\} \sum_{i=0}^{n_1-1} \left(\left(D\beta(s_i - t_i)\right)^2\right) + \sum_{i=0}^{n_1-1} s \int_{s_i}^{t_i} D\beta dt \quad (16)$$
$$+ \sum_{i=0}^{n_1-1} l \int_{s_i}^{t_i} (\beta - 1)Ddt$$

$$\frac{\partial \widetilde{TCR}_d(t_i, s_i, n_1)}{\partial t_i} = \{H + \tau\widehat{h_c} + D_t * \widetilde{\theta_\iota}\}[D\beta(t_i - s_i)] + \{W_h + \tau\hat{P}_r\}$$
$$\sum_{i=0}^{n_1-1} \left(\frac{-(1-\widetilde{k_\iota})}{\widetilde{\theta_\iota}} (1 - e^{\widetilde{\theta_\iota}(s_{i+1}-t_i)}) - \left(D\beta(s_i - t_i)\right)^2 \right) + \sum_{i=0}^{n_1-1} l \int_{s_i}^{t_i} (1-\beta)Ddt + l(1-\beta)D(t_i - s_i) - (\mathrm{d}e_2 + \mathrm{d} \quad (17)$$
$$* v_cC_2) * \widetilde{\theta_\iota} \int_{t_i}^{t_{i+1}} De^{\widetilde{\theta_\iota}(\mathrm{t}-\mathrm{t_i})}\,\mathrm{d}t - (\mathrm{d}e_2 + \mathrm{d} * v_cC_2)D$$

$$\frac{\partial^2 \widetilde{TCR}_d(t_i,s_i,n_1)}{\partial {s_i}^2} = \{W_h + \tau \hat{P}_r\} \sum_{i=0}^{n_1-1} ((D\beta(s_i - t_i))) - sD\beta + l(\beta - 1)D \tag{18}$$

$$\begin{aligned}\frac{\partial^2 \widetilde{TCR}_d(t_i,s_i,n_1)}{\partial {t_i}^2} &= \{H + \tau \widehat{h_c} + D_t * \widetilde{\theta}_\iota\}[D\beta] + \{W_h + \tau \hat{P}_r\} \sum_{i=0}^{n_1-1} \left(-\left(1 - \mathrm{k}\backslash\widetilde{\mathrm{k}}_\iota\right)\left(e^{\widetilde{\theta}_\iota(s_{i+1}-t_i)}\right) + 2D\beta(s_i - t_i)\right) \\ &+ \sum_{i=0}^{n_1-1} l(1-\beta)D + l(1-\beta)D + (\mathrm{d}e_2 + \mathrm{d} * v_c C_2)\widetilde{\theta}_\iota^{\,2} \int_{t_i}^{t_{i+1}} D e^{\widetilde{\theta}_\iota(\mathrm{t}-\mathrm{t}_i)}\, \mathrm{dt} + D(\mathrm{d}e_2 + \mathrm{d} * v_c C_2)\widetilde{\theta}_\iota\end{aligned} \tag{19}$$

The essential condition is that the Hessian matrix with $\widetilde{TC}_d$ must be positive definite for $\widetilde{TCR}_d$ to achieve its minimum, with n_1 held constant. Additionally, the theorem establishes the positivity of $\widetilde{TC}_d$. Consequently, through an iterative process and leveraging Mathematica software, one can compute the optimal values of t_i and s_i for a specified positive intege using the Eqs. (18) and (19).

Hessian matrix

$$\nabla T_R(t_j,s_j,n_1) =$$

$$\begin{bmatrix}
\frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial {t_1}^2} & \frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial t_1\, \partial s_1} & 0 & 0 & \cdots\ \cdots & 0 & 0 & 0 \\
\frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial s_2 \partial t_1} & \frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial {s_1}^2} & \frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial s_1 \partial t_2} & 0 & \cdots\ \cdots & 0 & 0 & 0 \\
0 & \frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial t_2\, \partial s_1} & \frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial {t_2}^2} & \frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial t_2\, \partial s_2} & \cdots\ \cdots & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots\ \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots\ \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & 0 & \cdots\ \cdots & \frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial t_{n_1-1}\, \partial s_{n_1-1}} & \frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial {s_{n_1-1}}^2} & \frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial s_{n_1-1} \partial t_{n_1}} \\
0 & 0 & 0 & 0 & \cdots\ \cdots & 0 & \frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial t_{n_1}\, \partial s_{n_1-1}} & \frac{\partial^2 T_R(t_j,s_j,n_1)}{\partial {t_{n_1}}^2}
\end{bmatrix}$$

Theorem: if t_i and s_i satisfy the inequality,

(i) $\frac{\partial^2 \widetilde{TCR}_d(t_i,s_i,n_1)}{\partial {t_i}^2} \geq 0,$

(ii) $\frac{\partial^2 \widetilde{TCR}_d(t_i,s_i,n_1)}{\partial {s_i}^2} \geq 0,$

(iii) $\frac{\partial^2 \widetilde{TCR}_d(t_i,s_i,n_1)}{\partial {t_i}^2} - \left|\frac{\partial \widetilde{TCR}_d(t_i,s_i,n_1)}{\partial t_i\, \partial s_i}\right| \geq 0,$

(iv) $\frac{\partial^2 \widetilde{TCR}_d(t_i,s_i,n_1)}{\partial {s_i}^2} - \left|\frac{\partial \widetilde{TCR}_d(t_i,s_i,n_1)}{\partial s_i\, \partial t_i}\right| \geq 0,$

for all i=1,2,...,n then TC will be positive definite.

$$\begin{aligned}\widetilde{Q_{\iota+1}}_d &= \frac{-\left(1 - \widetilde{\mathrm{k}_1}\right)}{\widetilde{\theta_1}} \int_{t_i}^{s_{i+1}} \left\{e^{\widetilde{\theta_1}(s_{i+1}-t)} - 1\right\} D dt + \frac{-\left(1 - \widetilde{\mathrm{k}_2}\right)}{\widetilde{\theta_2}} \int_{t_i}^{s_{i+1}} \left\{e^{\widetilde{\theta_2}(s_{i+1}-t)} - 1\right\} D dt + \frac{-\left(1 - \widetilde{\mathrm{k}_3}\right)}{\widetilde{\theta_{3,}}} \int_{t_i}^{s_{i+1}} \left\{e^{\widetilde{\theta_3}(s_{i+1}-t)} - 1\right\} D dt \\ &+ \frac{-\left(1 - \widetilde{\mathrm{k}_4}\right)}{\widetilde{\theta_4}} \int_{t_i}^{s_{i+1}} \left\{e^{\widetilde{\theta_4}(s_{i+1}-t)} - 1\right\} D dt + (D\beta(s_i - t_i))^3\end{aligned}$$

$$\begin{aligned}\widetilde{TC}s_i(t_i,s_i,n_1) &= {n_1}^*{}_1 * S_s + P_s + \Big(\frac{-\left(1 - \widetilde{\mathrm{k}_1}\right)}{\widetilde{\theta_1}} \int_{t_i}^{s_{i+1}} \left\{e^{\widetilde{\theta_1}(s_{i+1}-t)} - 1\right\} D dt + \frac{-\left(1 - \widetilde{\mathrm{k}_2}\right)}{\widetilde{\theta_2}} \\ &\int_{t_i}^{s_{i+1}} \left\{e^{\widetilde{\theta_2}(s_{i+1}-t)} - 1\right\} D dt + \frac{-\left(1 - \widetilde{\mathrm{k}_3}\right)}{\widetilde{\theta_{3,}}} \int_{t_i}^{s_{i+1}} \left\{e^{\widetilde{\theta_3}(s_{i+1}-t)} - 1\right\} D dt + \frac{-\left(1 - \widetilde{\mathrm{k}_4}\right)}{\widetilde{\theta_4}} \int_{t_i}^{s_{i+1}} \left\{e^{\widetilde{\theta_4}(s_{i+1}-t)} - 1\right\} D dt + (D\beta(s_i - t_i))^3\Big)\end{aligned}$$

Illustrative scenario: $D = 40$, $O_r = 0.5$, $\widetilde{k_1} = 0.01$, $\widetilde{k_2} = 0.02$, $\widetilde{k_3} = 0.03$, $\widetilde{k_4} = 0.04$, $H = 420$, $\tau = 0.2$, $\widehat{h_c} = 0.1$, $D_t = 1$, $\widetilde{\theta_1} = 0.001$, $\widetilde{\theta_2} = 0.002$, $\widetilde{\theta_3} = 0.003$, $\widetilde{\theta_4} = 0.004$, $\beta = 0.1$, $W_h = 0.3$, $\hat{P}_r = 0.2$, $c\hat{} = 0.1$, $F_c = 1.2$, d=2, $e_1 = 2$, $C_1 = 0.04$, $e_2 = 2$, $v_c = 3$, $C_2 = 0.2$, $S_s = 430$, $P_s = 0.001$, $l = 0.02$, $s = 2$. Eqs. (18) and (19), depicting nonlinear systems, were resolved utilizing Mathematica version 12 mathematical software, employing a numerical iterative technique to solve the nonlinear differential equation.The optimal condition of the overall system cost and replenishment cycles may be noticed in Tables 2-4, Figure 3, and Figure 4, respectively, for all the values specified in example 1.

Table 2. Optimal replenishment time interval, total cost table of retailer, supplier, and total quantity for example 1

↓D →t_i	t_0	t_1	t_2	t_3	t_4	t_5	t_6	$\widetilde{TCR_i}$	$\widetilde{Q_{i+1_d}}$	$\widetilde{TCs_i}$
50	0	1.54979	2.03524	2.52702	3.01879	3.51056	4.00234	35.0379	21.255	54.8378

Table 3. Optimal time interval for shortage, total cost table of retailer, supplier, and total quantity for example 1

↓D →s_i	S_0	S_1	S_2	S_3	S_4	S_5	$\widetilde{TCR_i}$	$\widetilde{Q_{i+1_d}}$	$\widetilde{TCs_i}$
50	0	1.54113	2.03291	2.52468	301645	3.50822	35.0379	21.255	54.8378

Table 4. Total cost table of retailer for example 1

↓D →n1	1	2	3	4	5	6	7	$\widetilde{TCR_i}$
50	539.29	540.79	542.29	543.79	504.85	35.0379	548.29	35.0379

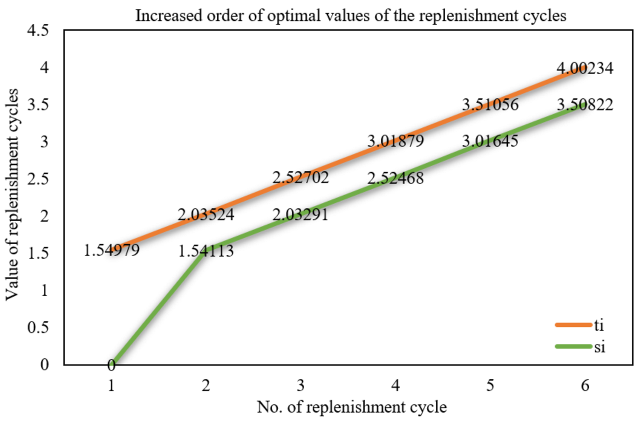

Figure 3. Graphical representation of the replenishment time and shortage time

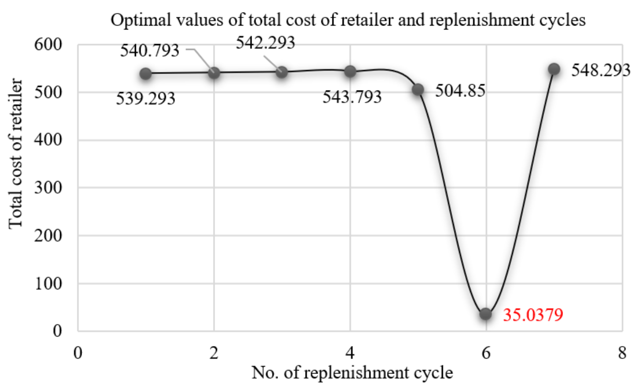

Figure 4. Graphical representation of the optimal values of total cost of retailer

The most optimal solution occurs at node 6, where $t_1 = 1.54979, t_2 = 2{,}03524, t_3 = 2.52702, t_4 = 3.01879, t_5 = 3.51056, t_6 = 4.00234$ and $s_1 = 0, s_2 = 1.54113, s_3 = 2.03291, s_4 = 2.52468, s_5 = 3.01645, s_6 = 3.50822$, and the total cost is 35.0379.

7. SENSITIVITY INVESTIGATION

A sensitivity analysis is a crucial aspect of research papers, particularly in decision-making models or optimization problems. It helps understand how changes in parameters affect the outcomes or solutions. The sensitivity analysis conducted on the research model reveals insights into how variations in different parameters impact the optimal replenishment cycle and total order quantity. Across the parameters β, caphc, L, V_c, Ob, T, z, s, and tau, the optimal replenishment cycle consistently remains at 6, showcasing its robustness to changes in these factors. For total order quantity, the analysis indicates minimal fluctuations in response to alterations in most parameters. Parameters like β, caphc, L, V_c, Ob, T, and z exhibit marginal influences on total order quantity, with variations within a narrow range around the approximate value of 30.85. However, parameters s and tau demonstrate slightly more discernible effects on both the optimal replenishment cycle and total order quantity. Positive changes in s and tau lead to minor decreases in the optimal replenishment cycle and slight increases in total order quantity, while negative changes result in the opposite trends. As we can see Figure 5 and Table 5.

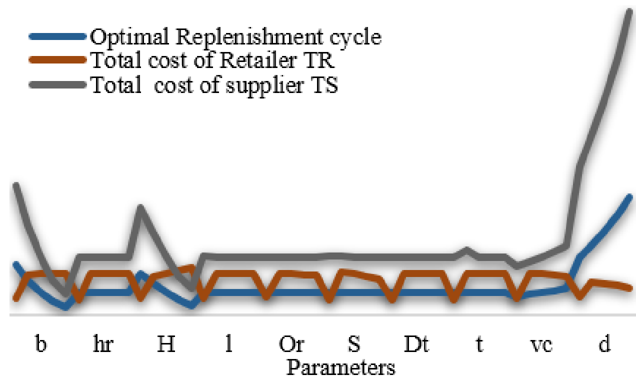

Figure 5. Sensitivity analysis of different parameters

Table 5. Sensitivity analysis of the different parameters

Parameters	%Changes	Optimal Replenishment Cycle	Total Order Quantity Q_{nt}	Total Cost of Retailer T_R	Total Cost of Supplier T_S
β	+20	6	37.4136	12.54336	96.52709
	+10	6	26.1835	30.49022	67.5536
	0	6	17.0039	30.8503	43.8702
	−10	6	10.2267	31.2684	26.3850
	−20	6	6.1971	31.7572	15.9883

$\tilde{h}_c$	+20	6	17.00816	12.2251	43.8810
	+10	6	17.00756	30.8491	43.8795
	0	6	17.00696	30.8493	43.8779
	−10	6	17.00636	30.84955	43.87643
	−20	6	17.00576	30.8497	43.87488
H	+20	6	31.06859	12.8146	80.1569
	+10	6	23.7566	29.00219	61.2921
	0	6	17.00396	30.85033	43.870237
	−10	6	11.38452	33.17159	29.37207
	−20	6	7.74971	36.15723	19.9942
l	+20	6	17.017007	12.25913	43.9038
	+10	6	17.01048	30.84763	43.88705
	0	6	17.00396	30.85033	43.87023
	−10	6	16.99744	30.85303	43.8534
	−20	6	16.990929	30.85573	43.83659
O_r	+20	6	17.00396	14.02484	43.87023
	+10	6	17.00396	31.75033	43.87023
	0	6	17.0039	30.85033	43.870237
	−10	6	17.0039	29.95033	43.8702
	−20	6	17.00396	29.0503	43.87023
s	+20	6	17.06070	11.8642	44.0166
	+10	6	17.0322	32.61924	43.94327
	0	6	17.0039	30.8503	43.870237
	−10	6	16.97577	29.0811	43.79749
	−20	6	16.94769	27.31168	43.72504
D_t	+20	6	17.00426	12.22485	43.87101
	+10	6	17.004118	30.8502	43.87062
	0	6	17.0039	30.85033	43.87023
	−10	6	17.00381	30.85038	43.8698
	−20	6	17.0036	30.8504	43.8694
τ	+20	6	17.0060	12.2332	43.8754
	+10	6	17.0049	30.8619	43.8728
	0	6	17.0039	30.85033	43.87023
	−10	6	17.0029	30.8387	43.8676
	−20	6	17.0019	30.8271	43.865
v_c	+20	6	14.3255	12.32495	36.9598
	+10	6	15.60286	31.4724	40.2553
	0	6	17.0039	30.8503	43.8702
	−10	6	18.5331	30.2313	47.81556
	−20	6	20.1947	29.61547	52.1025
d	+20	6	42.8818	13.40037	110.6350
	+10	6	52.04806	24.65992	134.2840
	0	6	62.5602	23.3896	161.4053
	−10	6	74.5001	22.1549	192.210
	−20	6	87.95177	20.9569	226.9155

7.1 Managerial insights

This study offers valuable insights for inventory managers, providing effective strategies to address challenges posed by seasonal demand fluctuations. The emphasis on precisely forecasting seasonal demand highlights its pivotal role in optimizing inventory management and enhancing cost efficiency across various industries. A significant managerial insight arises from the integration of machine learning techniques, particularly decision tree classifiers, which markedly improve the accuracy of demand forecasts.

A crucial lesson from this research underscores the substantial costs associated with relying on fixed demand assumptions. Opting for seasonal forecasted demand over static predictions presents businesses with the opportunity to make noteworthy reductions in overall costs. For example, envision a retail company specializing in vegetables and fruits. Traditional approaches, maintaining a fixed inventory quantity year-round, may result in surplus during low-demand periods and shortages during peak seasons. However, employing machine learning for precise seasonal demand forecasting empowers managers to strategically adjust ordering quantities and replenishment periods. This adaptability allows companies to optimize inventory levels, reduce expenses linked to prevent stock-outs, surplus inventory and elevate the overall customer experience.

The study's findings also shed light on optimal policies for organizations grappling with imperfect deteriorating products in their inventory systems. Policymakers aiming to minimize overall costs can benefit by exercising greater control over sensitive parameters related to total cost. Businesses, especially in industries like retail and pharmaceuticals, stand to gain tangible advantages by incorporating these findings into their practical operations. The implementation of improved management policies, driven by seasonal demand estimates and optimized ordering practices, holds the potential to enhance operational efficiency and boost profitability.

8. CONCLUSION

In summary, the present paper introduces a more practical inventory model designed for imperfectly decaying items, which incorporates a Machine Learning technique for seasonal demand prediction. The incorporation of deterioration rate and defective percentage quantity as fuzzy variables addresses inherent uncertainties, allowing for permitted shortages that are partially backlogged over the finite planning horizon.

Recognizing the pivotal role of product demand in business operations, particularly its seasonal variations, the study utilizes a time series predicting method to analyse seasonal forecasted demand. The results highlight the generation of direct month-wise predicted demand, offering managers valuable insights to enhance the management of inventory is based on expected demand. The Sign Distance method is used for defuzzification, aiding in the determination of optimal replenishment periods and ordering quantities that minimize total average cost, including emission costs over the finite planning horizon.

The numerical example confirms the robustness of the mathematical model, unveiling a significant decrease in overall costs when employing projected seasonal demand as opposed to fixed demand. Sensitivity analysis pinpoints critical parameters demanding heightened managerial attention. Graphical representation illustrates the curvature of the total cost function, emphasizing its highly non-linear nature.

To enhance the model's versatility, future extensions could explore alternative demand forecasting approaches, such as decision tree methods forecasting. Additionally, a comparative study across different forecasting methods could provide valuable insights. Further extensions may involve incorporating different fuzzy variables to account for increased uncertainty in parameter estimation.

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REFERENCES

[1] Rocklöv, J., Sjödin, H., Wilder-Smith, A. (2020). COVID-19 outbreak on the Diamond Princess cruise ship: Estimating the epidemic potential and effectiveness of public health countermeasures. Journal of Travel Medicine, 27(3): taaa030. https://doi.org/10.1093/jtm/taaa030

[2] Roehrich, J. (2008). Supply chain management: Strategy, planning & operations, by Chopra, S. and Meindl, P. Journal of Purchasing & Supply Management, 14(4): 273-274. https://doi.org/10.1016/j.pursup.2008.08.001

[3] Kohavi, R., Rothleder, N.J., Simoudis, E. (2002). Emerging trends in business analytics. Communications of the ACM, 45(8): 45-48. https://doi.org/10.1145/545151.545177

[4] Agrawal, D., Bernstein, P., Bertino, E., et al. (2011). Challenges and opportunities with Big Data 2011-1. https://docs.lib.purdue.edu/cctech/1/.

[5] Hillier, B., Sahbaz, O. (2005). High resolution analysis of crime patterns in urban street networks: An initial statistical sketch from an ongoing study of a London borough. In Proceedings Space Syntax. 5th International Symposium, Delft.

[6] Silver, E.A., Pyke, D.F., Peterson, R. (1998). Inventory Management and Production Planning and Scheduling. New York: Wiley.

[7] Taleizadeh, A.A., Pentico, D.W., Jabalameli, M.S., Aryanezhad, M. (2013). An EOQ model with partial delayed payment and partial backordering. Omega, 41(2): 354-368. https://doi.org/10.1016/j.omega.2012.03.008

[8] Rokach, L., Maimon, O. (2005). Top-down induction of decision trees classifiers—A survey. IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews), 35(4): 476-487. https://doi.org/10.1109/tsmcc.2004.843247

[9] Mishra, N.K., Jain, P. (2024). Blockchain-enhanced inventory management in decentralized supply chains for finite planning horizons. Journal Européen des Systèmes Automatisés, 57(1): 263-272. https://doi.org/10.18280/jesa.570125

[10] Persinger, M.A., Levesque, B.F. (1983). Geophysical variables and behavior: XII. The weather matrix accommodates large portions of variance of measured daily mood. Perceptual and Motor Skills, 57(3): 868-870. https://doi.org/10.2466/pms.1983.57.3.868

[11] Wright, S.A., Schultz, A.E. (2018). The rising tide of artificial intelligence and business automation: Developing an ethical framework. Business Horizons, 61(6): 823-832. https://doi.org/10.1016/j.bushor.2018.07.001

[12] Carbonneau, R., Laframboise, K., Vahidov, R. (2008). Application of machine learning techniques for supply chain demand forecasting. European journal of operational research, 184(3): 1140-1154. https://doi.org/10.1016/j.ejor.2006.12.004

[13] Hastie, T., Tibshirani, R., Friedman, J. H., Friedman, J. H. (2009). The Elements of Statistical Learning: Data Mining, Inference, and Prediction. New York: Springer.

[14] Ghare, P.M. (1963). A model for an exponentially decaying inventory. Journal of Industrial Engineering, 14: 238-243.

[15] Wee, H.M., Huang, Y.D., Wang, W.T., Cheng, Y.L. (2014). An EPQ model with partial backorders considering two backordering costs. Applied Mathematics and Computation, 232: 898-907. https://doi.org/10.1016/j.amc.2014.01.106

[16] Lashgari, M., Taleizadeh, A.A., Sadjadi, S.J. (2018). Ordering policies for non-instantaneous deteriorating items under hybrid partial prepayment, partial trade credit and partial backordering. Journal of the Operational Research Society, 69(8): 1167-1196. https://doi.org/10.1080/01605682.2017.1390524

[17] Salameh, M.K., Jaber, M.Y. (2000). Economic production quantity model for items with imperfect quality. International Journal of Production Economics, 64(1-3): 59-64. https://doi.org/10.1016/s0925-5273(99)00044-4

[18] Papachristos, S., Konstantaras, I. (2006). Economic ordering quantity models for items with imperfect quality. International Journal of Production Economics, 100(1): 148-154. https://doi.org/10.1016/j.ijpe.2004.11.004

[19] Mishra, N.K. (2022). A supply chain inventory model for deteriorating products with carbon emission-dependent demand, advanced payment, carbon tax and cap policy.

Mathematical Modelling of Engineering Problems, 9(3): 615-627. https://doi.org/10.18280/mmep.090308
[20] Hua, G., Cheng, T.C.E., Wang, S. (2011). Managing carbon footprints in inventory management. International Journal of Production Economics, 132(2): 178-185. https://doi.org/10.1016/j.ijpe.2011.03.024
[21] Benjaafar, S., Li, Y., Daskin, M. (2012). Carbon footprint and the management of supply chains: Insights from simple models. IEEE Transactions on Automation Science and Engineering, 10(1): 99-116. https://doi.org/10.1109/tase.2012.2203304
[22] Hasan, M.B., Mahi, M., Hassan, M.K., Bhuiyan, A.B. (2021). Impact of COVID-19 pandemic on stock markets: Conventional vs. Islamic indices using wavelet-based multi-timescales analysis. The North American Journal of Economics and Finance, 58: 101504. https://doi.org/10.1016/j.najef.2021.101504
[23] Park, K.S. (1987). Fuzzy-set theoretic interpretation of economic order quantity. IEEE Transactions on Systems, Man, and Cybernetics, 17(6): 1082-1084. https://doi.org/10.1109/tsmc.1987.6499320
[24] Chang, S.C., Yao, J.S., Lee, H.M. (1998). Economic reorder point for fuzzy backorder quantity. European Journal of Operational Research, 109(1): 183-202. https://doi.org/10.1016/s0377-2217(97)00069-6
[25] Chotard, A., Auger, A., Hansen, N. (2015). Markov chain analysis of cumulative step-size adaptation on a linear constrained problem. Evolutionary Computation, 23(4): 611-640. https://doi.org/10.1109/4235.873238
[26] Chang, F.R. (2004). Stochastic Optimization in Continuous Time. Cambridge University Press.
[27] Kumar, S., Rajput, U.S. (2015). Fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging. Applied Mathematics, 6(3): 496-509. https://doi.org/10.4236/am.2015.63047
[28] Jaggi, C., Tiwari, S., Goel, S. (2016). Replenishment policy for non-instantaneous deteriorating items in a two storage facilities under inflationary conditions. International Journal of Industrial Engineering Computations, 7(3): 489-506. http://doi.org/10.5267/j.ijiec.2015.12.001
[29] Mishra, S., Mishra, N.K., Singh, V., Singh, P., Saxena, S. (2019). The fuzzyfied supply chain finite planning horizon model. Journal of Computational and Theoretical Nanoscience, 16(10): 4135-4142. https://doi.org/10.1166/jctn.2019.8492
[30] Mishra, S.P., Mishra, S., Siddique, M. (2020). Lean labour in AEC industry: From theory to implementation. International Journal of Recent Technology and Engineering, 8(6): 1159-1169. https://doi.org/10.35940/ijrte.f7267.038620
[31] Singh, R., Mishra, V.K. (2024). Inventory model using Machine Learning for demand forecast with imperfect deteriorating products and partial backlogging under carbon emissions. Annals of Operations Research, 335(1): 361-383. https://doi.org/10.1007/s10479-023-05518-9
[32] Kirshner, B., Gaertner, M., Pozzoboni, K. (2010). Tracing transitions: The effect of high school closure on displaced students. Educational evaluation and policy analysis, 32(3): 407-429. https://doi.org/10.3102/0162373710376823

NOMENCLATURE

T	The planning horizon (Year)
t_i	The time throughout the i^{th} replenishment cycle whenever the stock level reaches zero. i = 0,1, 2, ..., $n_1 - 1$
s_i	The time for i^{th} replenishing cycle when inventory levels approach insufficiency or shortage i = 0,1… . . ., $n_1 - 1$
$IL_{i+1}(t)$	The overall stock level for $^{(i+1)th}$ order cycle at time t where $t_i \le t \le s_{i+1}$
$IL_{Si+1}(t)$	The firm's level of inventory for $^{(i+1)}$th order cycle at time t that goes to the shortage, where si ≤ t ≤ ti
K	Defective percentage
D	Demand rate unit/year
θ	Degradation rate
θ_i	Fuzzy deterioration rate
$\hat{h}c$	The amount of CO_2 associated with each unit inventory holding cost (refrigeration cost) $/unit/year
P_r	The cost of each acquired unit
$\hat{P}r$	The amount of CO_2 emitted following the purchase costs per unit. c: Steady carbon emission cost per dollar $/unit

Transportation variables

C_1	Fuel consumption of a retailer's vehicle when empty. (litres per kilometre)
C_2	Extra energy consumption for each ton of payload due to refrigeration and vehicle services during transportation. (litre/km/ ton)
e_1	Fee for carbon dioxide emissions related to retail transportation. ($/km)
e_2	Additional (refrigeration) carbon emission cost incurred by the retailer per unit item during transportation. ($/unit/km)
d	The distance traveled from the supplier to the retailer. (km)
F_c	Fixed transportation costs incurred when the retailer places an order. ($)