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Advanced Low-Pass Filters for Signal Processing: A Comparative Study on Gaussian, Mittag-Leffler, and Savitzky-Golay Filters

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ABSTRACT

Signal processing plays a crucial role in biomedical applications, facilitating accurate health monitoring and clinical diagnoses. This study presents a comparative analysis of Gaussian, Mittag-Leffler, and Savitzky-Golay filters, evaluating their effectiveness in noise reduction and signal enhancement for electrocardiogram (ECG) signals. These filters offer adjustable parameters, making them adaptable to various applications. Our findings demonstrate that the Savitzky-Golay smoothing filter outperforms the others in smoothing data and computing derivatives of noisy data, despite its limitations in suppressing noise at higher frequencies. On the other hand, the adaptive Gaussian and Mittag-Leffler filters excel in noise reduction but may compromise fine signal details. Through MATLAB simulations and mean squared error (MSE) comparisons as well as Signal to Nosie Ratio (SNR), we evaluate the filters' performance in denoising real-world ECG signals. The results indicate that both the Savitzky-Golay smoothing and Mittag-Leffler filters hold promise for noise reducing in other biomedical signals, such as medical EEG and medical EMG signals. This research serves as a foundational exploration of the application and enhancement of these filters in biomedical signal processing.

1. INTRODUCTION

Biosignals, the physiological and physical measures of human body functions, offer an insightful peek into an individual's physiological, pathophysiological, and emotional states, thus playing an indispensable role in health monitoring and clinical diagnosis. They exemplify the dynamic and complex nature of human physiology, manifesting themselves in myriad forms such as Electrocardiograms (ECGs), Electroencephalograms (EEGs), and Electromyograms (EMGs). Specifically, ECGs, which record the heart's functioning, including its rate and rhythm, provide vital information regarding potential heart disorders like coronary artery narrowing, heart attacks, and irregular heartbeats such as atrial fibrillation [1-6].

The acquisition of biosignals often involves the presence of unwanted components or noise, which can significantly compromise their quality and interpretability. Consequently, signal filtering—the process of removing undesirable elements or features from a signal—becomes extremely significant. Many different fields, including radio, television, image processing, graphics software, radar, recording of sounds, machine control, and music manufacturing, use filters extensively.

However, their utility is perhaps most crucial in biomedical systems, where they enhance the readability and accuracy of outputs from sensors like EEG, EMG, and ECG [7-12].

In this context, Gaussian, Mittag-Leffler, and Savitzky-Golay filters emerge as popular choices in signal smoothing and image processing noise reduction. These filters, while presenting unique advantages, also come with their own set of limitations, varying across different applications. For instance, while the Gaussian and Mittag-Leffler filters excel in noise reduction and offer ease of implementation, they may fail to retain fine image detail and contrast and underperform when dealing with salt-and-pepper noise. Conversely, the Savitzky-Golay smoothing filter demonstrates efficacy in smoothing data and calculating noisy data derivatives but falls short in noise suppression at higher frequencies and is susceptible to artifacts near data range boundaries.

Against this backdrop, our paper seeks to calculate the MSE for Gaussian, Mittag-Leffler, and Savitzky-Golay filters and juxtapose their performances in denoising ECG signals. ECG signals, widely used diagnostic tools for detecting heart-related issues, necessitate superior quality for accurate decision-making and classification. We hypothesize that an integrated approach, combining these filters, could potentially yield a low-pass filter that surpasses individual Gaussian and Mittag-Leffler filters in performance.

Our work builds upon existing literature that has elucidated the properties and applications of Gaussian, Mittag-Leffler, and Savitzky-Golay filters. The Gaussian filter, with its kernel operator representing a bell-shaped filter and mathematical representation based on Gaussian distribution, has proven instrumental in digital signal and image processing [1-4]. Its prowess lies in the ability to effectively remove noise while retaining signal properties, regardless of frequency content variations [5]. More accurate and subtle noise reduction is now possible thanks to adaptive Gaussian filtering approaches like the contrast dependent spread (CDS) filter and the intensity dependent-spread (IDS) model, which have pushed the envelope [6-8].

Conversely, the Mittag-Leffler Filter, initially utilized by Ivo Petráš, has emerged as an innovative alternative to Gaussian filters. This filter, which uses the Mittag-Leffler (ML) function in the probability-density function -PDF, offers more versatility by allowing for curve shape adjustment because of the filter-hidden component.

The Savitzky-Golay (SG) filter, proposed by Savitzky and Golay in 1964, has carved a niche for itself in various fields, specifically for biomedical data filtering. It provides a practical solution for noise reduction by maintaining the original signal shape and, consequently, its information, and by continuing to follow the guidelines of moving average filters [9, 10].

The SG filter has found broad-based acceptance across various applications, including but not limited to electroencephalography and ECG, elastography, near-infrared spectroscopy, functional magnetic resonance imaging, speech enhancement, and eye movement analysis [10].

The criticality of signal processing in the domain of biomedical sciences underscores the necessity for further exploration and understanding of these filters. Each has its unique strengths and trade-offs, and the context of application often dictates the choice of filter. The main goal of this study is to quantify the performances of Gaussian, Mittag-Leffler, and SG filters in denoising ECG signals using MSE as a metric. ECG signals are pivotal in the detection and diagnosis of heart-related abnormalities. Thus, the importance of their quality for precise decision-making and classification cannot be overemphasized.

By comprehensively comparing the efficacy of these filters, we aim to put forth a refined low-pass filter approach that leverages the strengths of each individual filter. The collective benefits may potentially outperform each filter used in isolation, thereby enhancing the accuracy and reliability of ECG signal denoising. We envision that the insights gleaned from this study will cover the way for improvements in biomedical signal processing and catalyze further exploration into two-dimensional extensions of the proposed filters for image processing applications.

2. METHODS

2.1 Gaussian function, Gaussian distribution and Gaussian filter

The Gaussian function and distribution were utilized in our analysis. This mathematical function, originally defined by Carl Friedrich Gauss, is presented as:

$$f(x) = e^{-x^2} \tag{1}$$

and the parametric extension is:

$$f(x) = ae^{-\frac{(x-b)^2}{2c^2}} \tag{2}$$

where a, b, and c are real constants.

Gaussian functions are predominantly used in statistics and signal processing, where they represent the probability density function (PDF) of a normally distributed random variable, describe Gaussian filters, and define Gaussian distortions respectively [11] as shown in Figure 1. The Gaussian filter was implemented in our study through MATLAB function "gaussfilt(t,z,sigma)" to apply a Gaussian filter to a time series.

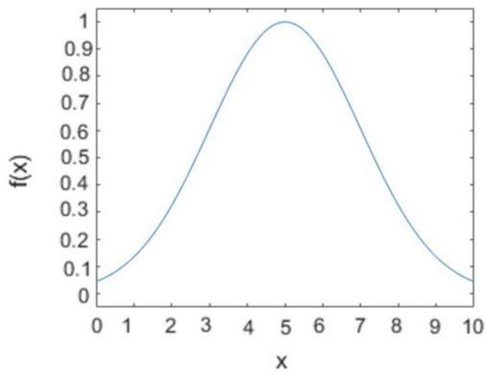


Figure 1. Gaussian function with a =1 and c=2 and b=5

2.2 Mittag-Leffler function, Mittag-Leffler distribution and Mittag-Leffler filter

The Mittag-Leffler function, an integral part of the fractional calculus theory, was also incorporated in our study. Its one-parameter form is defined as:

$$E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + 1)} \ \alpha > 0 \tag{3}$$

where, Γ is a gamma function and $0 < \alpha < 1$. Its two-parameter form is described as the generalized Mittag-Leffler function having two parameters α and β, is described as following power series:

$$E_{\alpha.\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n + \beta)} \tag{4}$$

The Mittag–Leffler filter was introduced to our study as a novel approach, executed using the MATLAB function "ML_filter (t, y, sigma, alpha, beta)" that applies the Mittag-Leffler filter with exponential-type forgetting to a time series.

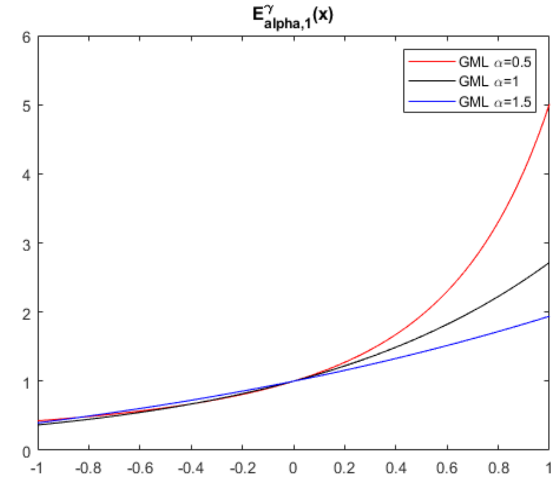
Figure 2(a) displays the graph of the general Mittag-Leffler function where 'x' is plotted on the x-axis and $E^{\gamma}_{alpha\,,0.5}(x)$ is depicted on the y-axis. An enhanced definition of the Mittag–Leffler distribution is provided through the following PDF as per reference [12]:

$$\varphi(x; \sigma, \alpha, \beta) = \frac{1}{\sigma\sqrt{2\pi}} E_{\alpha,\beta}\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \tag{5}$$

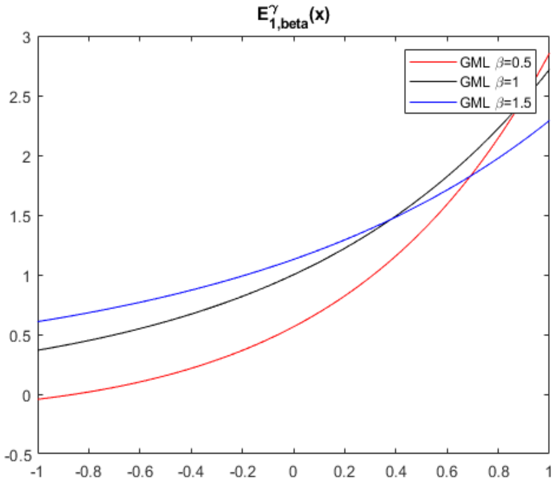
In this equation, σ, α, and β are positive filter parameters bound by the conditions $0 < \alpha < 2$ and $0 < \beta < 2$, while μ represents the mean value of an independent variable x.

Similarly, Figure 2(b) presents the plot for the general Mittag-Leffler function, using 'x' on the x-axis and $E^{\gamma}_{1,beta}(x)$ on the y-axis, shown for different beta values of 0.5,1, and 1.5.

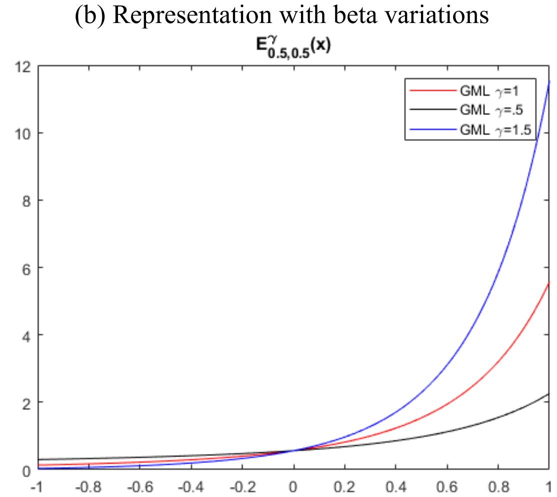
Figure 2(c) presents the graph for the general Mittag Leffler function, where the x-axis represents 'x', and the y-axis depicts $E^{\gamma}_{gamma,0.5}(x)$. This plot is constructed for distinct gamma values of 0.5, 1, and 1.5.



(a) Representation with alpha variations



(b) Representation with beta variations



(c) Representation with gamma variations

Figure 2. Comparative visualizations of the general Mittag-Leffler function

By following the notion of generalizing the exponential function to a two-parameter Mittag-Leffler function, a new generalized filter, referred to as the Mittag-Leffler filter, can be defined as per [13]:

$$y_{MLF}(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} y(t-\tau) E_{\alpha,\beta}\left(-\frac{(\tau-\hat{\tau})^2}{2\sigma^2}\right) d\tau \tag{6}$$

This three-parameter filter has more flexibility than the traditional one, offering more degrees of freedom thanks to the extra adjustable parameters α and β, which allow us to shape the distribution curve.

2.3 Savitzky-Golay (SG) smoothing filter

The Savitzky-Golay (SG) smoothing filter, a popular tool for signal smoothening, was utilized in our study. The Savitzky-Golay filters operate by employing a specific polynomial that fits within a signal frame via the least squares method. In this method, the median point of the window is substituted with the corresponding value from the polynomial, thus yielding a smooth output for the signal. The relevant polynomial can be articulated as follows [14]:

$$\rho(r_i) = c_0 + c_1^r + \cdots + c_p^{r^p} \tag{7}$$

In this formulation, 'ρ' refers to the corresponding apparent resistivity data vector, while 'r_i' represents the northern coordinate point of the resistivity map. Constructing a Savitzky-Golay filter involves several initial decisions, including the determination of the filter length 'k', the derivative order n, the polynomial order 'p', and the size of the smoothing window 'N'.

The value of N is chosen as an odd number that satisfies $N \geq p+1$. Upon application of the Savitzky-Golay filter coefficients to the signal, the polynomial is substituted at points defined by $N = N_r + N_l + 1$. In this scenario, N_l and N_r designate the left and right data points, respectively, relative to the current point in the signal.

The estimation of the polynomial coefficients can be performed as follows:

$$Mc = d \tag{8}$$

The N value is selected as an odd number, with $N \geq p+1$. Once the Savitzky-Golay filter coefficients are applied to the signal, the polynomial takes the place of the signal points, where $N = N_r + N_l + 1$. In this context, N_l and N_r represent the points to the left and right of a current signal point, respectively.

Anywhere M can be stated as:

$$M = \begin{vmatrix} 1 & ((k-1)/2) & (-(k-1)/2)^2 & \cdots & (-(k-1)/2)^p \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & -1 & 0 & \cdots & (-1)^p \\ 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & (-1)^p \\ \cdots & \cdots & \cdots & \cdots & ((k-1)/2)^p \end{vmatrix} \tag{9}$$

Where vector of polynomial coefficient 'c' can be denoted as:

$$c = \begin{vmatrix} c_0 \\ c_1 \\ c_2 \\ \cdots \\ c_p \end{vmatrix} \tag{10}$$

'ρ' is the data vector values of size k.

$$\rho = \begin{vmatrix} \rho - (k-1)/2 \\ \rho - (k-2)/2 \\ \cdots \\ \rho_0 \\ \cdots \\ \rho_{(k-1)/2} \end{vmatrix} \tag{11}$$

Using matrix least squares, the vector of polynomial coefficients can be initiate as:

$$c = (M^t M)^{-1} M^t \rho \tag{12}$$

The row values of $(M^t M)^{-1} M^t \rho$ might be combined linearly to represent the polynomial coefficients of 'c'. Since all other polynomial values are zero, the polynomial value at ρ_0 can equal c_0. The Savitzky-Golay at derivative order 0 can be represented as the central row matrix coefficients $(M^t M)^{-1} M^t \rho$.

2.4 MATLAB functions and implementation notes

The discrete-time domain implementation of the Gaussian filter (GF), Mittag-Leffler (ML) filter, and the Savitzky-Golay (SG) smoothing filter was executed through specific MATLAB functions as shown in Figure 3. Given the non-causal nature of the Gaussian and Mittag-Leffler filters, the filter window was symmetric in the time domain, necessitating a truncation for practical implementation. Additionally, in place of an integration process in convolution for these filters, the summation process over all samples was used. Furthermore, for the Mittag-Leffler function, problems of infinity upper sum limit in the definition were circumvented using the integral form of the Mittag-Leffler (ML) function [15-17].

These filters were applied to our data sets in the time series format, as described in the provided MATLAB function headers. The functions utilized were "gaussfilt(t, z, sigma)" for the Gaussian filter and "ML_filter(t, y, sigma, alpha, beta)" for the Mittag-Leffler filter. The implementation details of these functions are available in the Appendix.

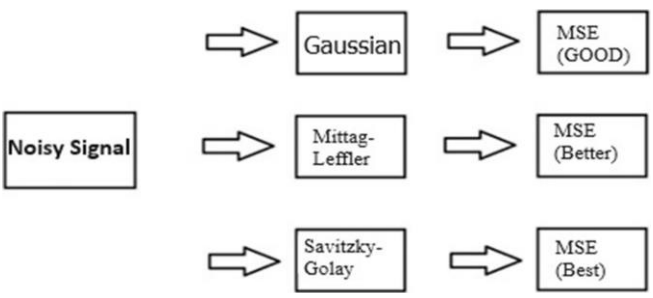
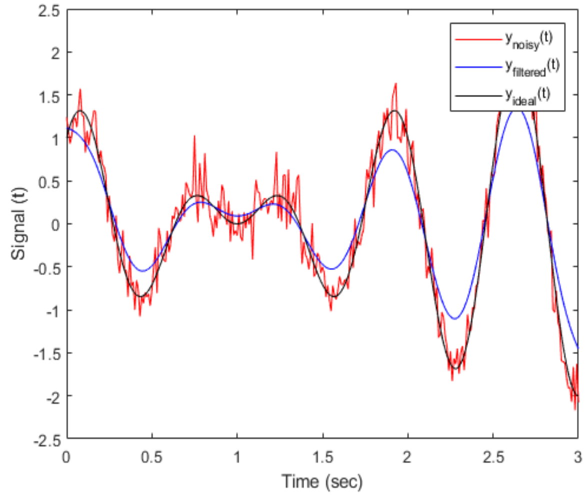


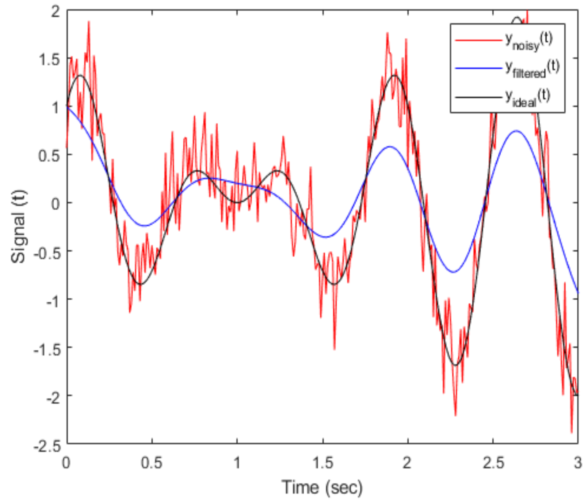
Figure 3. Block diagram representing the Gaussian filter, Mittag–Leffler filter, and Savitzky-Golay smoothing filter, with mean square error assessment

3. RESULTS

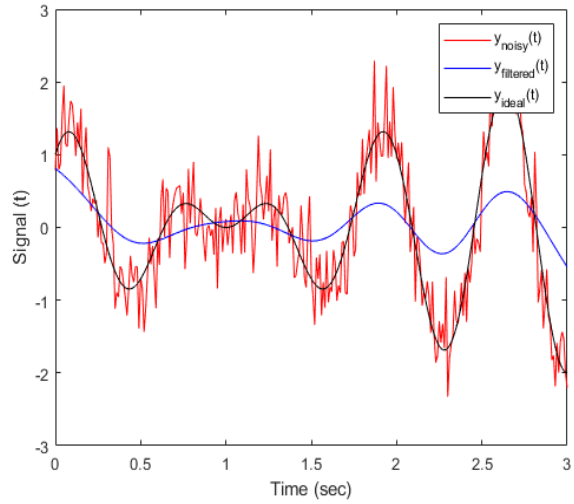
Figure 4 provides a schematic representation of the three signal filters under consideration in this research: the Gaussian, the Mittag-Leffler, and the Savitzky-Golay smoothing filters, with the graph depicting their respective mean square error calculations. The primary intent of these filtering mechanisms is to elucidate the inherent signal $y_T(t)$ from the contaminated, noise-ridden signal $y_N(t)$. In our study we assume only normal white noise in the ECG signal and the three filters were applied to remove this particular noise.



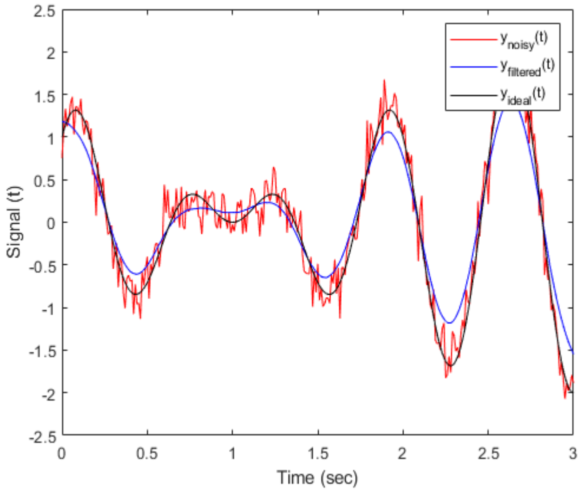
(a) Gaussian filtered signal vs. noisy and ideal signal with σ=0.1



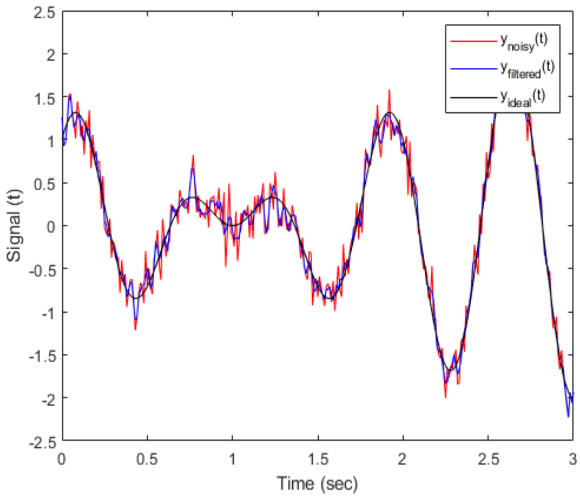
(b) Gaussian filtered signal vs. noisy and ideal signal with σ=0.15



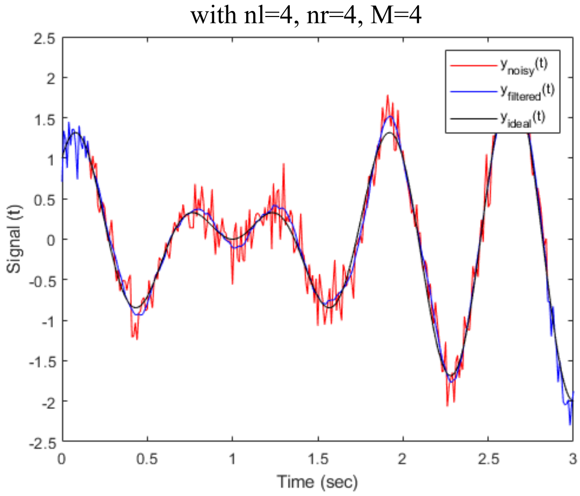
(c) Mittag-Leffler filtered signal vs. noisy and ideal signal with σ=0.2, α=1, and β=1



(d) Mittag-Leffler filtered signal vs. noisy and ideal signal with σ=0.1, α=0.95, and β=0.9



(e) Savitzky-Golay filtered signal vs. noisy and ideal signal with nl=4, nr=4, M=4



(f) Savitzky-Golay filtered signal vs. noisy and ideal signal with nl=16, nr=16, M=4

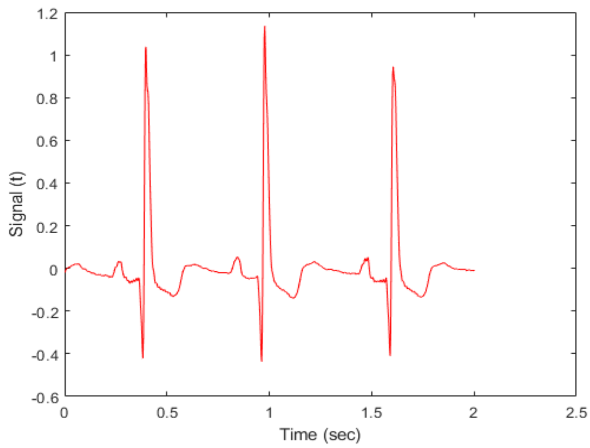
Figure 4. Comparative analysis of Gaussian, Mittag-Leffler, and Savitzky-Golay filters on a noisy signal

This investigation was bifurcated into two primary sections. In the first segment, a prototypical signal was subjected to stochastic noise of normal distribution. Subsequently, this 'noisy' signal was processed via the Gaussian, Mittag–Leffler, and smoothing filters to produce the filtered signal. Figures 4(a)-(f) demonstrate the raw noisy signal, the subsequent filtered signal, and the optimal signal for each filter, under varying operational parameters.

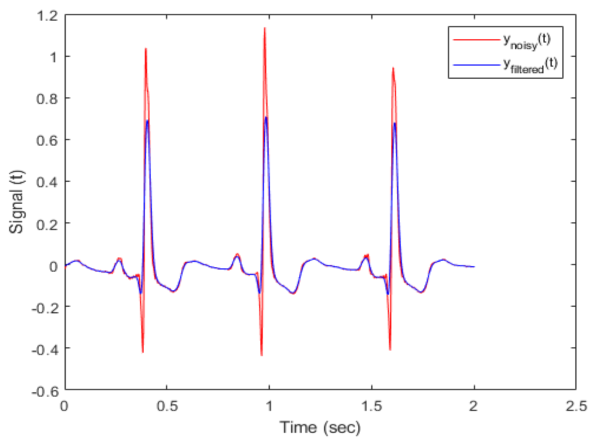
Table 1 provides a summarized view of these results, showcasing that, based on the mean squared errors (MSE) derived from the test signals, the Savitzky-Golay smoothing filter outperforms the others, followed by the Mittag–Leffler filter, with the Gaussian filter manifesting the least efficacious results. also, the Signal to Nosie Ratio was calculated and shown in the Table 1. The results shows that SNR is changeable for the three filters and depend on the adjustable parameters. However, it can be noted that SNR is better for Savitzky-Golay Filter then the Mittag-Leffler then the Gaussian filters.

Table 1. Comparative analysis of MSE and SNR value in dB for three filters the Mittag–Leffler filter, the Gaussian filter, and the smoothing filter and parameter sets in simulated signals $y_1(t)$

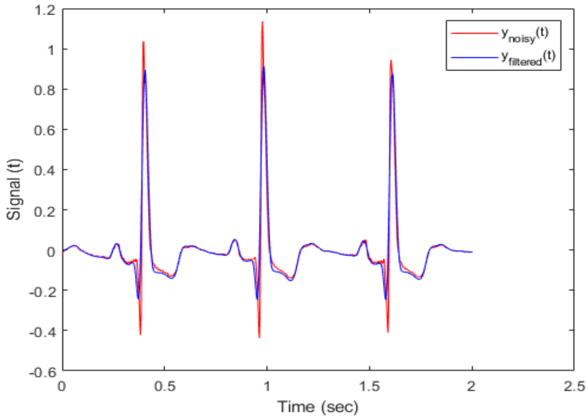
Figure 4	Filter Type	Filter Parameters	MSE Value	SNR Value in dB
(a)	Gaussian	σ=0.1	0.0815	8.7169
(b)	Gaussian	σ=0.15	0.2675	4.2187
(c)	Mittag–Leffler	σ=0.1, α=1, β=1	0.0841	8.6453
(d)	Mittag–Leffler	σ=0.1, α=0.95, β=0.9	0.0486	9.5736
(e)	Savitzky-Golay	nl=4, nr=4, M=4	0.0167	16.3803
(f)	Savitzky-Golay	nl=16, nr=16, M=4	0.0078	14.3581



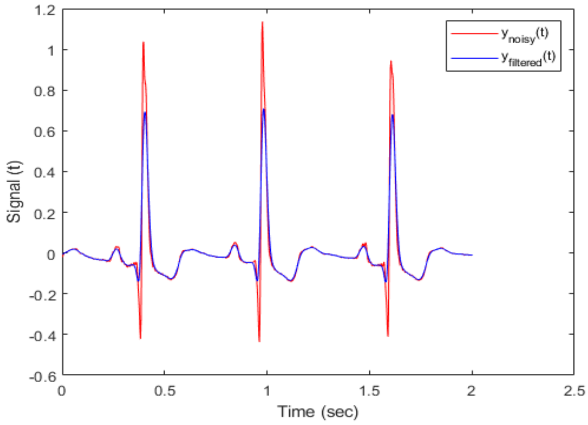
(a) Real ECG signal with noise within the timeframe of 0 to 2 seconds



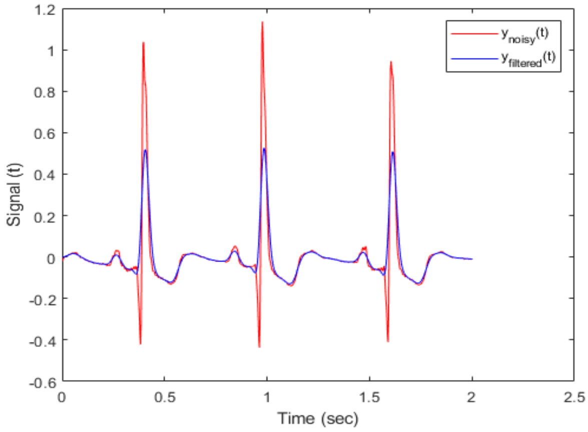
(b) Noisy ECG signal and equivalent filtered signal using the Gaussian variant of the Mittag-Leffler filter (σ=0.01, α=1, and β=1)



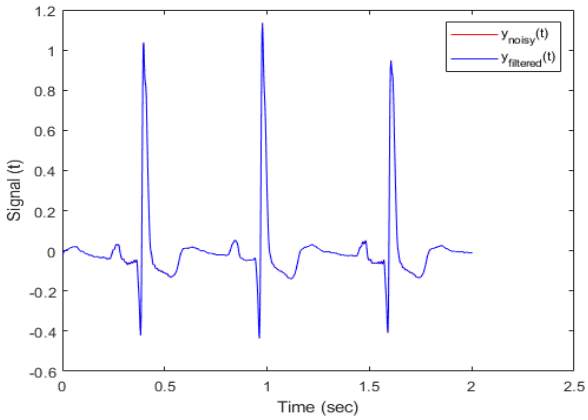
(c) Noisy ECG signal and corresponding filtered signal using the Mittag–Leffler filter (Parameters: σ=0.01, α=1.20, and β=1)



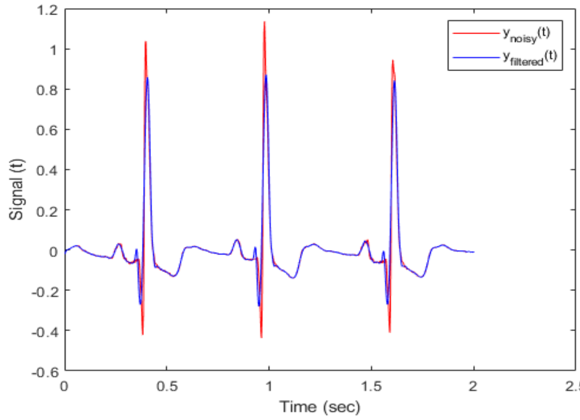
(d) Noisy ECG signal and equivalent filtered signal using the Gaussian filter (Parameter: σ=0.01)



(e) Noisy ECG signal and equivalent filtered signal using the Gaussian filter (Parameter: σ=0.015)



(f) Noisy ECG signal and equivalent filtered signal using the Savitzky-Golay smoothing filter (Parameters: nl=4, nr=4, M=4)



(g) Noisy ECG signal and equivalent filtered signal using the Savitzky-Golay smoothing filter (Parameters: nl=16, nr=16, M=4)

Figure 5. Comparative analysis of noisy and filtered real ECG signals: A study using Gaussian filter, Mittag-Leffler filter, and Savitzky-Golay smoothing filter with varied parameters

In the second section, authentic ECG signals were deployed for analysis. The ECG signal was taken from PhysioNet website (https://physionet.org/content/?topic=ecg). This collection contains 50 ECG records, each lasting 30 minutes and having 648,000 sampling points overall. The records are verified at a frequency of 360 Hz. For this experiment, three QRS were selected as a compromise, and about 2 sec in length. In general, the adjustable parameters for each filter were selected arbitrary for this paper. And many filters can be taken by changing those parameters. However, for the Gaussian filter the adjustable parameter sigma that control the smoothness of the filter with high sigma we will have more smoothness (σ =0.01 and 0.015), for the Mittag-Leffler filter the alpha and beta parameter control the shape of the filters the result of Mittag-Leffler with different alpha and beta may result sin , cos or exp filter or other type of functions (σ=0.01, α=1.2, β=1), for Savitzky-Golay filter the number of points to the left of the locus point and the number of points to the right of the locus point also, the degree of the least squares polynomial, those parameter control the smoothness of the filter (nl=4, nr=4, M=4).

Figure 5(a)-(g) depicts both the noisy ECG signals. and their filtered counterparts for each respective filter, across different operational parameters.

Table 2 consolidates this data and offers a comparative view of the presentation of these three filters, with the mean squared errors (MSE) functioning as the core evaluation metric. Consistent with the synthetic signal results, the Savitzky-Golay smoothing filter exhibits superior performance, followed by the Mittag–Leffler filter, and the Gaussian filter lagging. also, the Signal to Nosie Ratio was calculated and shown in the Table 2. The results shows that SNR is changeable for the three filters and depend on the adjustable parameters. However, it can be noted that SNR is better for Savitzky-Golay Filter then the Mittag-Leffler then the Gaussian filters. The response time for the Gaussian filters was 0.038083 seconds. For the Mittag-Leffler filter was 0.143777 seconds. And for the Savitzky-Golay Filter was 0.004479 seconds. Whish shows that the Savitzky-Golay Filter is the best.

The trade-offs between the different filters in terms of computational efficiency and real-time processing capabilities can be descried as follow the advantage of Savitzky-Golay filters is that they preserve the area, location, and breadth of peaks, which may be helpful for some types of analysis. However, their computation time is related to window width.

The Gaussian filter has a number of advantages over other types of windows, such as rectangular or triangular windows.

Smoothness: The Gaussian filter produces a smoother signal than other types of windows. This is because the Gaussian function is a bell-shaped curve, which means that it gives more weight to the signal that are closest to the center of the window. This helps to reduce noise and artifacts in the signal.

Robustness: The Gaussian filter is more robust to noise than other types of windows. This is because the Gaussian function is a smooth function, which means that it is not as sensitive to outliers as other types of functions. This helps to ensure that the edges in the signal are not blurred by noise.

Efficiency: The Gaussian filter is relatively efficient to compute. This is because the Gaussian function (GF) is a simple function, which means that it can be computed quickly. This makes the Gaussian filter a virtuous choice for real-time applications.

For the Mittag-Leffler (ML) filter is a novel lowpass filter that reduce the noise by selecting different alpha and beta parameters but we should optimize those parameters for better noise removal.

The hypothesis test for the confidence interval for the ECG signals in this study was computed for the Gaussian, Mittag-Leffler, and Savitzky-Golay filters, respectively. μ=M±Z(sM) is the estimate formula, where M is the sample mean Z=Z statistic specified by the confidence level sM is the standard error = √(s2/n), and Sample Mean (M) for the Gaussian is 0.0053. Number of Samples (n): 50 We have a 95% confidence level in the standard deviation (s) of 0.01, which indicates that the population mean (μ) mendacities between 0.002528 and 0.008072. Regarding the Mittag-Leffler filtration Mean (M) of Sample: 0.0035 Number of Samples (n): 50 We have 95% confidence that the population mean (μ) mendacities between 0.000728 and 0.006272, with a standard deviation (s) of 0.01. Sample Mean (M) for the Savitzky-Golay filter: 0.0039 Number of Samples (n): 50 0.001 is the standard deviation (s). The population mean (μ) cascades between 0.003623 and 0.004177, with a 95% confidence level.

Table 2. Comparative analysis of MSE and SNR Value in dB values for ECG signals processed with different filters and parameters

Subfigure	Filter Type	Filter Parameters	MSE Value	SNR Value in dB
Figure 5(a)	Mittag–Leffler Filter	σ=0.01, α=1, β=1	0.0053	8.0443
Figure 5(b)	Mittag–Leffler Filter	σ=0.01, α=1.2, β=1	0.0035	9.7668
Figure 5(c)	Gaussian Filter	σ=0.01	0.0053	8.0443
Figure 5(d)	Gaussian Filter	σ=0.015	0.0103	5.1181
Figure 5(e)	Savitzky-Golay Smoothing Filter	nl=4, nr=4, M=4	3.2038×10^{-6}	40.1950
Figure 5(f)	Savitzky-Golay Smoothing Filter	nl=16, nr=16, M=4	0.0039	9.3569

3.1 Performance analysis

For evaluating the quality of the noise reduction effect, the literature currently in publication often considers the signal-to-noise ratio and mean square error [18]. This article uses the following defined evaluation markers in order to compare the proposed algorithm with the existing method and assess its effectiveness in reducing noise. The mean square error is defined as follows:

$$MSE = \frac{1}{N}\sum_{i=1}^{i=N}[x(i) - \hat{x}(i)]^2 \tag{13}$$

The definition of the signal-to-noise ratio is as follows:

$$SNR = 10 * log_{10}\left(\frac{\sum_{i=1}^{i=N}[x(i)]^2}{\sum_{i=1}^{i=N}\left[x(i) - \widehat{x(i)}\right]^2}\right) \tag{14}$$

4. DISCUSSION

In this study, we conducted an empirical comparative analysis of Gaussian, Mittag-Leffler, and Savitzky-Golay smoothing filters using both synthetic and authentic ECG signals. The ECG signal was taken from physionet website (https://physionet.org/content/?topic=ecg). The results highlight the effectiveness of the Savitzky-Golay smoothing filter, demonstrating its value in intricate ECG signal processing applications [19]. The Mittag-Leffler filter, with its variable hidden parameters, outperforms the Gaussian filter, providing greater flexibility in accommodating signal idiosyncrasies.

The Gaussian distribution, commonly known as the normal distribution, is widely used in image and signal processing to minimize noise or smooth out details [20]. It is a bell-shaped curve characterized by an equal number of observations above and below the mean value. Understanding the mean, median, and mode is essential to grasp the Gaussian distribution's concept. The mean represents the calculated average of all values, the median is the value falling in the middle of the distribution, and the mode is the most frequently observed value.

Gaussian noise in digital photos mainly arises during the capture process, resulting from factors such as poor lighting, high temperatures, or transmission [21]. When an image or signal is smoothed using a Gaussian filter, fine-scaled borders and features may become blurred due to the suppression of high frequencies. Traditional spatial filtering methods for noise reduction include mean (convolution), median, and Gaussian smoothing.

The Gaussian filter blurs the intended area while simultaneously reducing noise at higher frequencies. It shares similarities with mean filters in terms of uniformly weighted averaging. These filters are effective for both noise reduction and edge blurring. In digital image and signal processing, Gaussian filters are implemented as matrices that traverse each pixel of the selected area. Various applications of Gaussian image processing include noise reduction in low-light photographs, removal of bright pixels, edge smoothing, and reduction of blurriness [22]. The level of smoothing can be specified to achieve the desired outcome, and the Gaussian distribution exhibits rational symmetry.

The Savitzky-Golay filter is a type of digital filter that smoothes data to improve accuracy while preserving the underlying signal trend. This is accomplished by fitting sequential selections of neighboring data points with low-degree polynomials using the linear least squares approach [23]. When data points are equally spaced, a statistical answer to the least-squares equations may be calculated. This solution consists of a set of "convolution coefficients" that apply to all data subsets and provide estimates of the averaged signal or its derivatives at the Centre of each subset.

Initially introduced by Abraham Savitzky and Marcel J. E. Golay in 1964, the technique gained popularity due to its mathematical soundness. The convolution coefficients have been refined over time, and the method has been extended to handle 2D and 3D data [24]. Savitzky-Golay smoothing filters are influenced by different forms of noise. In this study, we examined the impact of varying the Gaussian filter kernel values on filter performance by filtering test ECG signals. The kernel values, determined by sigma, ranged from 0.01 to 0.015. The denoising responsiveness varied depending on the kernel, with ECG signal quality and MSE used to evaluate the filter's performance. Our results demonstrated that kernel values between 0.01 and 0.015 yielded good results in filtering test ECG signals contaminated with common noise. It was observed that specific kernel values were more effective in handling typical noise, indicating the importance of selecting appropriate values based on the ECG signals and denoising requirements.

The Mittag-Leffler filter introduces three parameters that affect the shape of the filter curve. The MSE of the filtered signal was evaluated for different parameter combinations. As the alpha parameter decreased, the MSE reduced, indicating improved filtering performance. The filter's forgetting factor contributed to its adaptability and allowed for customization to specific signal characteristics [25-27].

Comparing the performance of the filters, the Savitzky-Golay smoothing filters preserved the height and original shape of the ECG signals, as depicted in the graphical tracings. The Savitzky-Golay filter demonstrated effective noise reduction and signal preservation within the positive axes. The MSE values for the ECG signals with Savitzky-Golay smoothing filters (nl=4, nr=4, M=4) and (nl=16, nr=16, M=4) were 3.2038×10^{-6} and 0.0039, respectively.

While our study establishes the superiority of the Savitzky-Golay smoothing filter and the potential of the Mittag-Leffler filter, further refinement is needed. We identified opportunities for improvement, such as enhancing the smoothing of F-waves in atrial fibrillation segments. Exploring alternative filter parameters or denoising methodologies, such as the Butterworth or moving average filter, may help overcome these challenges. Additionally, the impact of noise variance on the efficacy of Savitzky-Golay smoothing filters underscores the need for further investigation and the potential for future research in ECG denoising.

Our comparative study provides valuable insights into the performance and adaptability of Gaussian, Mittag-Leffler, and Savitzky-Golay filters for signal processing, specifically in the context of ECG signals (Table 3). The Savitzky-Golay smoothing filter outperforms the others in noise reduction and signal preservation, while the Mittag-Leffler filter offers flexibility with its adjustable parameters. The Gaussian filter remains a viable option but may compromise fine signal details. These findings contribute to the foundational exploration of advanced low-pass filters in biomedical signal processing, offering promising avenues for further research and application in various biomedical signals beyond ECG, such as EEG and EMG signals.

Features of Gaussian Filtering: They are linear low pass filters, rotationally symmetric (perform the same in both directions), computationally inexpensive (big filters are implemented using small 1D filters), and highly successful at eliminating Gaussian noise. σ determines the degree of smoothing (higher σ for more extensive smoothing).

For Mittag-Leffler filters there are many possibilities to create a filter by changing alfa and beta parameters so we should optimize the filters by playing with alpha and beta parameters.

Table 3. Evaluation of this work with parallel scientific works stated in the literature analyzing the impact of using different types of filters the Gaussian, Mittag-Leffler, and Savitzky-Golay filters

Ref.	Target of Study	Method	SNR	MSE
[28]	to enhance the mode-mixing drop among near IMF scales for the purpose of improving the noise-filtering performance. The ECG signal was generated using both standard ECG templates that were taken from the Arrhythmia ECG database and the simulator, and the noise source was Gaussian white noise. The filter performance indicator was the mean square error (MSE) among the original ECG and the rebuilt ECG.	Gaussian noise filtering	Not provided	0.71
[14]	The Mittag-Leffler filter, a unique variation of the Gaussian filter, is introduced. The probability-density function's (pdf) Mittag-Leffler (ML) function is used by this new filter. This type of Mittag-Leffler distribution is employed in the filter's convolution kernel. Because of the filter-forgetting factor, the filter's three parameters might change the shape of the curve.	Mittag-Leffler noise filtering	Not provided	121.2894
[29]	The two parameters that determine the performance of the S-G filter are polynomial degree and frame size. This research examines the influence of varying degree of polynomial and frame size.	Savitzky-Golay noise filtering	1.4	Not provided
[30]	MATLAB has been used to analyses the Savitzky-Golay (S-G) filter for ECG de-noising utilizing Daubechies wavelets. Using an S-G filter of polynomial order 9, noisy ECG signals downloaded from physionet.org under the MIT-BIH arrhythmia database were de-noised to data frames of length 21 displayed in both the time and frequency domains. The filter's performance was quantitatively evaluated under the parameters of SNR, MSE, and signal-to-interference ratio (SIR).	Savitzky-Golay noise filtering	32.78	0.0001

The fairly weedy conquest of some high frequencies (poor stopband suppression) and artefacts when employing polynomial fits for the first and last points are the drawbacks of the Savitzky-Golay filters. Finally, to the best of our knowledge, there are no attempts to combine these filters and no outcomes of such integrations for the last 10 years and no studies show the efficiency of such combination. Using this study, we can reduce calculations and use Savitzky-Golay Filter to remove normal noise with low MSE, and good SNR. however, we can use integrated approach to use two or more filters to reduce the noise to lowest MSE and good SNR. but in our study, we use filters separately. Choosing a suitable theoretical framework for our research is a crucial and essential procedure that every research must undertake. A thorough and careful evaluation of your topic, goal, importance, and research questions is necessary before choosing a theoretical framework. To ensure that your theoretical framework can support your work and direct your selection of research design and data analysis, it is essential that all four constructs-the problem, purpose, significance, and research questions-are closely matched and intricately woven.

5. CONCLUSION

In this study, we planned a comprehensive approach to low-pass filtering employing three distinct filters—Gaussian, Mittag–Leffler, and Savitzky-Golay smoothing—for signal processing applications. Each of these filters presents various adjustable parameters, offering more flexibility than traditional Gaussian filters. Among these, the Savitzky-Golay smoothing filter emerged as the superior choice.

In the context of specific filter parameters, the Gaussian filter can be viewed as a particular instantiation of the innovative Mittag–Leffler filter. We utilized MATLAB functions of the proposed filters for our analysis. Our simulation example illustrated the distinct advantages of the Savitzky-Golay smoothing filter over both the Mittag–Leffler and Gaussian filters, demonstrated through a comparative analysis of MSE and SNR values.

Furthermore, our empirical analysis on real-world ECG signals substantiates the theoretical outcomes, asserting their utility in a crucial application area such as ECG-signal noising reduction. The newly introduced Savitzky-Golay smoothing and Mittag–Leffler filters open new avenues for potential applications in the denoising of other biomedical signals, including medical EEG and medical EMG.

The methodologies, tools, and techniques outlined and utilized in this study enable the further extension of the proposed filters to two-dimensional realms, opening up prospects for applications of image-processing. Thus, this study provides a solid foundation for future explorations in diverse signal processing scenarios.

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APPENDIX

```
Gaussian filter
function [zfilt]=gaussfilt(t,z,sigma)
%Apply a Gaussian filter to a time series
%  Inputs: t=independent variable, z=data at points t, and
%       sigma=standard deviation of Gaussian filter to be applied.
%  Outputs: zfilt=filtered data.
%
%  written by James Conder. Aug 22, 2013
%  Sep 04, 2014: Convolution for uniformly spaced time time vector (faster)
%  Mar 20, 2018: Damped edge effect of conv (hat tip to Aaron Close)
Mittag-Leffler filter
function [y_filt]=ML_filter(t, y, sigma, alpha, beta)
%
% function [y_filt]=ML_filter(t, y, sigma, alpha, beta)
%
%  Mittag-Leffler filter with exponential-type forgetting
%  Inputs: t=independent variable
%        y=noisy data to be filtered at the points t
%     sigma=standard deviation
%  alpha, beta=parameters of the Mittag-Leffler function
%  Output: y_filt=filtered data given in variable y
%
% Acknowledgements: This software was created by Technical
% University of Kosice under Army Research Office (ARO) Award
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% agencies: VEGA 1/0365/19, APVV-14-0892, and APVV-18-0526.
%
% Author: Ivo Petras, Technical University of Kosice, Slovakia
Savitzky-Golay smoothing filter
function g=savGol (f, nl, nr, M)
% SAVGOL SavGol smoothes the data in the vector f by means of a
%       Savitzky-Golay smoothing filter.
%
%       g=savGol (f, nl, nr, M)
%       Input: f: noisy data
%       nl: number of points to the left of the locus point
%       nr: number of points to the right of the locus point
%       M: degree of the least squares polynomial
%
%       Output: g: smoothed data
%
%       Example:
%       g=savGol(f,16,16,4)
%
%        In many tenders one is measuring a variable that is both
%      slowly changing and tarnished by random noise. Then it is often
%       wanted to put on a smoothing filter to the measured data in
%         order to renovate the underlying smooth function. We assume
%      that the noise is independent of the observed variable and that
%         the noise follows a normal distribution with zero mean and given
%       variation.
%
%       W. H. Press and S. A. Teukolsky,
%       Savitzky-Golay Smoothing Filters,
%       Computers in Physics, 4 (1990), pp. 669-672.
```