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Constraint-based Model for Energy Optimization Management of Parallel Pumping Systems with Demand Variability

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ABSTRACT

The research and development of energy optimization methodologies in parallel pumping systems in recent years have aimed to impact operational costs, energy savings, and system reliability. Operational costs are correlated with the number of units operating simultaneously, considering power demand, operating point, and system reliability. Additionally, the optimization strategy must manage the operation of pumping units by regulating the output flow according to process dynamics and the energy tariff structure. In this document, an energy optimization model is presented for parallel pumping systems operating under variable demand conditions. The optimization problem is addressed through an iterative constraint-based analysis model, capable of predicting the number of units that should operate simultaneously and their corresponding speeds during future time intervals. The methodology suggests analyzing system operation indicators as inputs for the prediction model. The effectiveness of the methodological strategy for optimal dispatching of parallel pumping units is verified in a utility sector pumping system. The results obtained demonstrate savings between 20% and 25% in energy costs for system operation, which represents a contribution in the search for a significant use of energy and energy sustainability.

1. INTRODUCTION

The optimization of energy resource utilization is a crucial field of research worldwide, particularly in the context of climate change mitigation and the enhancement of energy efficiency, which represent two significant challenges for research and technological development. Important approaches such as demand planning and management in energy-intensive systems, particularly in sectors such as public services and industry, are essential. The energy optimization of parallel pumping systems requires advancements in operational control so that the process can adapt to demand fluctuations and minimize energy consumption.

Parallel pumping systems (PPS) are part of the Electric Motor Drive Systems (EMDS), a category of energy-intensive systems. These systems consist of an electronic variable frequency drive (VEV) responsible for regulating the electrical power input to the induction electric motor (MI), which in turn supplies mechanical power to the centrifugal pump (BC) through a direct coupling shaft. The pump's function is to convert the received mechanical power from the motor into hydraulic power, which is then transferred to the process fluid. A parallel pumping system comprises multiple units, each contributing to the total output flow. The number of units in simultaneous operation and the speed of each pump are modeled using affinity laws, determining their weighted contribution to the total demand flow [1]. Parallel pumping stations are commonly found in urban centers for the supply of potable water and in the public utilities sector for the transport and sanitation of wastewater. These systems typically operate for over 4000 hours per year, with energy costs accounting for 75% to 80% of the life cycle cost of the system. Therefore, advancements in optimizing energy utilization in these systems play a crucial role in promoting energy and environmental sustainability on a global scale [2-4].

The PPS refers to the component configuration in which two or more pumping units operate collaboratively to meet a variable flow demand, as shown in Figure 1. This configuration is employed across public utility sectors, industrial applications, and commercial sectors such as central air conditioning systems, providing potable water supply, wastewater pumping, and commercial building services. In these systems, the total flow rate and capacity of the system are obtained by summing the contributions of the pumping units in operation [5, 6].

The components of the pumping unit (VEV, MI, and BC) exhibit variations in their energy efficiency, as shown in Figure 2, as a result of changes in the operational conditions of the system, such as the operating point location, the load factor, and the power quality of the input power in the VEV and the induction electric motor [7, 8]. Other factors, including the dynamic characteristics of the process fluid and the type of mechanical coupling between system components, also influence the energy performance of the SBMV [8]. Internationally, agreements have been established to promote the efficient utilization of energy, incorporating regulatory optimization programs with minimum performance standards for SBMV components. These standards encompass efficiency categories, permitted levels of harmonics in power quality, and procedures for the classification and evaluation of energy efficiency within these systems [9-11].

Figure 1. Parallel system of multiple pumping units

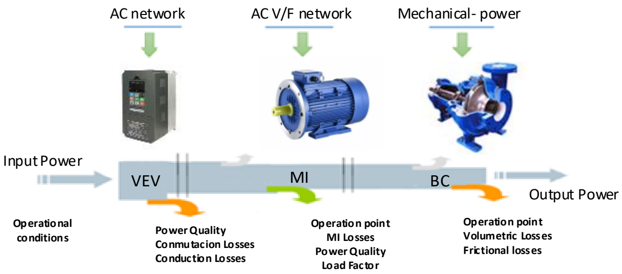

Figure 2. Architecture of a SBMV and power flow

The optimization of energy in SBMV is approached through three methodological strategies: technological substitution, operation control, and demand management [10]. Technological substitution strategies target enhancing the energy efficiency of system components, including the VEV, MI, and BC, by substituting them with more efficient alternatives. These strategies involve the application of conservative design techniques and resizing of the system components [12].

Strategies employing operational control focus on enhancing the integrated energy efficiency of the system. This objective is accomplished by adjusting the operating point of the MI and BC through the analysis of operational indicators of the SBMV. These strategies have the potential to yield savings ranging between 12% and 16%. Their applicability depends on factors such as the availability of system operating variables and manufacturer information, including the characteristic curve of the BC [12-14]. The optimization strategies centered on demand management aspire to promote the dissemination and adoption of a culture emphasizing rational and efficient energy usage among users. This approach facilitates reductions in energy consumption and minimizes environmental impact.

In recent years, research on the energy performance of parallel pumping systems (PPS) and optimization models has emphasized the design of optimization strategies that incorporate validation and verification of solution strategies. This entails the development of tools for analyzing system operation indicators that accurately represent the actual operating conditions and process constraints. Such solutions hold significant implications for policies promoting sustainable development and environmental preservation [15].

The document presents a critical analysis of optimization strategies for an SBMV pumping system, with emphasis on the performance of the pumping unit components VEV, MI, and BC. Subsequently, it describes the characteristics of multiple parallel pumping unit systems. The development of a linear optimization model with constraints and operation and performance indicators for managing the operation of pumping units is presented. The model considers indicators such as hourly differential tariff and the integrated energy efficiency of each pumping unit. Finally, validation results of the model in an intensive parallel pumping system in the public services sector are presented. The model's viability is verified through analysis of the energy savings achieved during the testing period.

1.1 Energy optimization strategies for SBMVs

Research focusing on energy efficiency in parallel pumping systems (PPS) serves as a catalyst for the development of methodological strategies and energy optimization models. These tools aim to facilitate decision-making regarding the efficient deployment of the number of pumping units in simultaneous operation based on demand conditions. By doing so, they enhance the prospects for cost-effectiveness, savings, and reduced energy consumption, which are critical factors for ensuring system reliability [16-18].

Olszewski [19] propose an optimization strategy for a multi-unit pumping system to enhance the system's integrated efficiency indicator. Their genetic optimization algorithm generates various operational alternatives for the pumping units. Koor et al. [20] introduces a predictive Levenberg-Marquardt algorithm for parallel pumping system operation, aiming to operate each pumping unit close to the point of highest efficiency (BEP) to minimize energy consumption. Wu et al. [21] develops a Multiple Lagrange optimization model to generate optimal input data for the rotational speed of each pumping unit in combination with proportional flow control valves of the system. Yang and Børsting [22] suggests a mixed-integer nonlinear optimization model (MINLP) with an optimization algorithm that assesses combined operation alternatives for the pump set under static conditions of parallel pumping system operation.

The energy optimization of intensive parallel pumping systems presents significant challenges in the field of applied research, such as the necessity to generate operation and performance indicators contributing to the development of new optimization methodologies. These methodologies should consider variations in hourly energy tariffs, the capability of the model to generate optimization strategies based on past states of the system's operating condition, and the integration of linear models to reduce the complexity of the optimization algorithm.

1.2 Optimization models to achieve energy sustainability

These models varying in complexity are known how, linear and nonlinear optimization models. For the objective function have multi-objective optimization models, models employing computational methods and algorithms for representation.

Additionally, they are used simulations to verify the effectiveness of the model [23, 24]. The constraint modeling approach allows solving complex problems by defining constraints that limit feasible solutions, and then applying solution generation algorithms that comply with all the constraints. Some related algorithms include the constraint propagation algorithm, linear programming, and dynamic programming. In all of them, changing process conditions must be taken into account to achieve long-term goals.

The management of an energy optimization model through constraints entails is an iterative process that leverages historical and operational data to formulate adjustments to the operating condition of the system, thereby ensuring alignment with optimization objectives. The development of a constraint-based optimization model involves the following stages:

a. Definition of design variables; whose states represent the optimal result of the model. Among the relevant variables are the operating speed of each pumping unit, the actual operating point of the system, and the hourly energy tariff ($/kWh).
b. Specification of the objective function; representing the preference criteria to be optimized. In this case, it aims to meet the output flow demand of the pumping system.
c. Selection of constraints; establishing the operating range of the system elements involved in the optimization goal and the location of the actual operating point for each pumping unit.
d. Establishment of the relational dynamic model; between the variables of the objective function and the constraints. In this case, it includes the number of pumping units in operation, the cost of kWh according to the hourly tariff, and the suction well level.
e. Coding of the model; using optimization computational tools.
f. Model resolution; based on input data representing the specific operational condition of the system, the optimization model provides a solution in terms of the values of the design variables.
g. Validation; to verify the solution by comparing it with real operational data with historical data of operation without use of the optimization tool in same period of time.

2. DEVELOPMENT OF AN OPTIMIZATION MODEL DUE TO CONSTRAINTS AND OPERATION INDICATORS OF ENERGY-INTENSIVE PROCESSES

Parallel Pumping Systems energy optimization involves the significant use of the energy resource. To do this, the operating conditions of the system components must be analyzed, such as; the power of the equipment, the internal temperature, the load factor (FC), the system control strategy, the topology of the power electronic converters, the harmonic pollution of the electrical network and the efficiency category of the system components are reflected in its operation and performance indicators. Additionally, operation and performance indicators for SBMV provide information on factors such; operating hours, equipment maintenance interventions [17-19].

A sewage pumping station constitutes a branched hydraulic network dedicated to the drainage of wastewater, strategically designed to handle the inflow of wastewater into a wet well via underground pipes. The pumping system facilitates the evacuation of wastewater from the well to the wastewater treatment plant (WWTP), where contaminants undergo removal via biological or physicochemical processes before eventual discharge into the water source.

The overall flow rate from the pumping station is regulated by the operation of parallel pumping units with speed adjustment for each unit. The control law must guarantee the safe level of well operation, as illustrated in Figure 3, for the different time slots, the system operator must be program control actions based on the well level and relying on their experience, determine the number of units that should be in operation and their speed. As depicted in the Figure 3, it shows the relationship of the average flow rate for pumping units 2, 3, and 4 along with their contribution to the overall flow rate of the PPS.

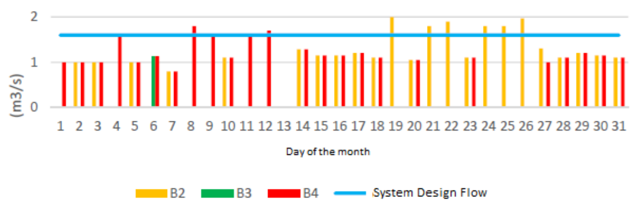

Figure 3. Operation dynamics of a parallel wastewater pumping system

Where B1, B2; B3 are the Pumping Units.

The constraint optimization model is designed to minimize energy consumption while meeting the entire demand requirement. This optimization strategy utilizes operational indicators and measurements of electrical and physical variables within the process as input data. A dynamic programming strategy has been implemented for the operation of each pumping unit to guarantee a reduction in operating costs and enhance operational reliability. The efficacy of the optimization model has been validated in a sewage pumping station comprising five parallel pumping units, each equipped with a centrifugal pump driven by a three-phase induction motor controlled by an electronic variable speed drive.

2.1 Development of the optimization model for the wastewater pumping system

Below is an example of use of the developed energy optimization methodology with the optimization model with constraints and performance indicators for a pumping system in the public utility sector, specifically for the pumping of wastewater, is presented. The system consists of five pumping units with an individual capacity of 1.6 m^3/s, each driven by a 1000 HP three-phase induction motor and controlled by a scalar V/F electronic drive.

The overall material balance for the wastewater pumping station, Figure 4 comprises four components at the entry to the wet well (A) rainwater, wastewater, sludge, and solid waste (C). At the system's exit (B), there is a flow of pumped wastewater and residual sludge and solids that are not pumped and must be periodically removed from the well. This necessitates considering the evacuation of well material as a non-linear system.

The energy optimization strategy analyzes a set of potential scenarios, each offering options for efficient dispatching of pumping units. To achieve this, the model incorporates five indicators to support the selection of the optimal scenario. The first indicator compares the current flow rate at the pump to the inflow rate into the well. The second indicator estimates the final level that the well should attain within the analysis time period and the number of units in operation. The third indicator evaluates the cost of kilowatt-hour relative to the time slot. The fourth indicator assesses the pump's power capacity. Lastly, the fifth indicator considers the trend of the well's entry level between the previous hour and the current hour to generate the recommended dispatching strategy.

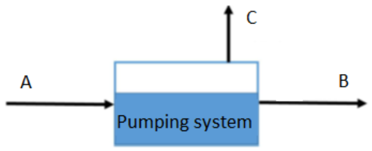

Figure 4. Material balance for the pumping system

2.1.1 Objective function specification

The objective function aims to minimize the cost of electrical energy demanded by each pump during the system's operating period, with the operating period defined as a duration of 60 minutes, to do this, the model must generate dispatch alternatives with the smallest number of pumps in simultaneous operation in each time slot.

$$\boldsymbol{minz = Cost\ kWh * \left(\sum_{i=1}^{n} KVPump_{(i)} tPump_{(i)} \right)} \quad (1)$$

where: **z**-is the number of pumps that can be activated simultaneously, **Cost kWh**: is the cost of kWh according to the hourly rate, **KVPump**: depends on the equation of the characteristic curve Kilowatt-Flow (kWh/m^3), **tPump** is the operating time of pump (i).

2.1.2 Selection of operation indicators

The design variables and operational indicators of the system; cost of kWh, liquid level in the well, rate of variation of the inflow, condition of the wastewater treatment plant (PTAR), shift hour, previous operating condition, time period, as shown in Figure 5, which will be related to the control of the well's outflow. The selection of the operation indicators for the optimization model was conditioned on the online measurement data of process variables, available at the pumping station.

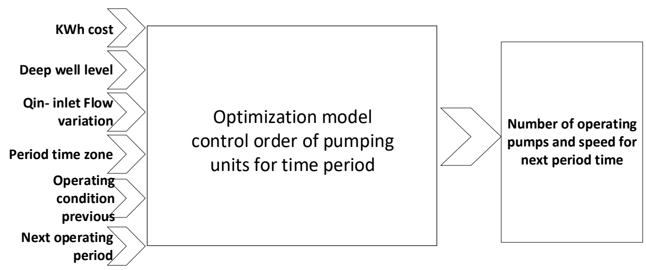

Figure 5. Representation of the optimization model

2.1.3 Specification of restrictions

For the optimization model, a set of restrictions is defined to be considered in the analysis of dispatch alternatives. The set of restrictions and their limit values are listed below:

- The maximum number of simultaneously operating units is 3.

$$\boldsymbol{1 \geq \sum_{i=1}^{3} OperationPump(i) \leq 3} \quad (2)$$

where: ***OperationPump*** Refers to the number of pumping units in simultaneous operation, with a restriction to a maximum of 3 units.

- The minimum level of the deep well is 2.2 m, and the maximum is 5.2 m.
- A flow rate of 1.6m^3/s is assumed for 100% of the speed with a lower limit of 40% of the speed corresponding to 0.6m^3/s.

$$\boldsymbol{0.6 \geq \sum_{i=1}^{3} Qout(i) * OperPump(i) \leq 1.6} \quad (3)$$

where: **Qout** Refers to simultaneous operation, with a restriction to a maximum of 3 units, **OperPump** is the pump unit in operation.

- Dynamic relational input-output model: In this, it is considered that the cross-sectional area of the well is variable and depends on the time of day and the rate of variation between the inflow and outflow.

$$\boldsymbol{Qin = Vout(i) - Vin + Qout} \quad (4)$$

where, **Qin** is the estimated inflow rate, Vout is the final level of the wet well, Vin is the initial level of the wet well, and **Qout** is the flow to be pumped in the next hour.

3. IMPLEMENTATION OF THE OPTIMIZATION MODEL

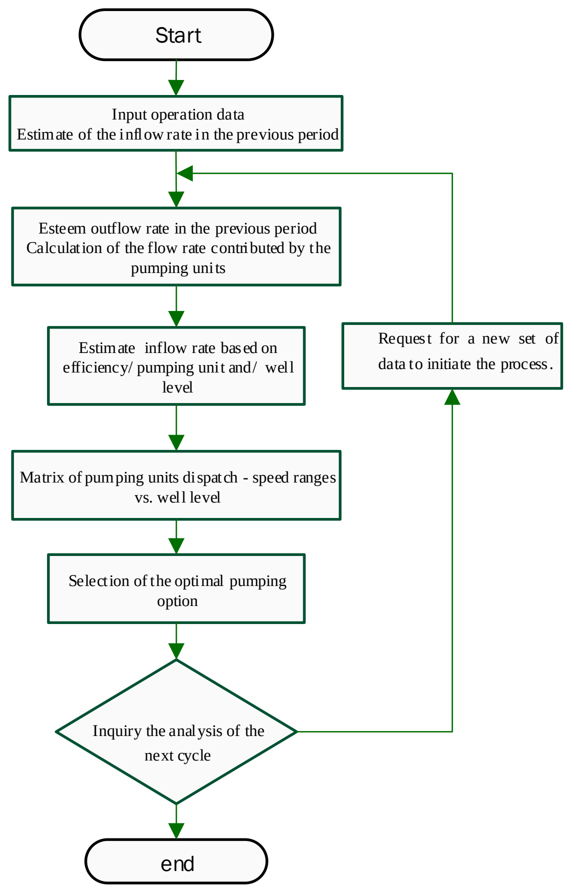

Figure 6. Optimization strategy flowchart

The optimization tool was developed using MATLAB software, with user interaction integrated into LabVIEW Software. These components were combined into a code module and a block diagram (Math-script). The tool generates the necessary parameters for efficient operation strategy, including the number of active pumping units, speed setpoints, and estimated values for management indicators for the upcoming time period (60 minutes). These indicators encompass the following: i) Percentage of pump flow evacuation compared to the design point. ii) Predicted future behavior of the wet well level. iii) Fluctuations in the suction well level at the WWTP. iv) Energy analysis categorized by time slots, Figure 6 presents the flowchart of the implemented analysis and prediction model.

3.1 Test and results

The optimization strategy model underwent initial testing using historical operational data from the pumping station, enabling us to validate that the conventional system operation and assumed control decisions, in certain instances, resulted in mechanical overstrain on the equipment, particularly the pumps, along with energy inefficiencies. In the subsequent verification scenario, the optimization tool was employed to aid decision making during the 24 hours of operational control shifts, yielding satisfactory operating outcomes.

The field verification of the energy optimization model for the parallel pumping system involved implementing the model over an 8-hour operational period on a designated day. The algorithm provided the operator with recommendations for operating the pumping units in 1-hour intervals. Table 1 presents the data measured in the operating system of pumps 1 and 2 of the station. Following this, the results were compared with historical data from a typical day of operation, without the assistance of the optimization tool.

Table 1. Energy and economic results of the field verification of the optimization model in 6 hours

Period Time	Well Level [m]	Flow Pump 1 [m^3/s]	Speed Pump 1 [RPM]	Flow Pump 2 [m^3/s]	Speed Pump 2 [RPM]	Energy Period [kWh]	Cost of Energy Model [$USD]	History /Cost /Energy [$USD]
7	2.6	1.5	470	0	0	288	16.4	28.7
8	3.8	1.5	470	0	0	288	17.6	30.9
9	3.5	1	450	1	450	452	29.5	32.9
10	3.8	0	0	1.6	550	427	29.5	43.2
11	2.6	1.2	450	1.2	450	452	32.1	43.1
12	1.6	1.1	450	1.1	450	452	34.1	45.8
Total for the period						**2359**	**159.4**	**224.6**

At the end of the first two shifts on a specific day, and following the recommendations of the optimization tool, an energy savings of up to 65% was achieved. Additionally, an economic analysis is conducted, comparing historical energy costs for the period of the year and time slot, verifying the effectiveness of the optimization tool, which can achieve savings of up to 29 % on the energy bill.

The algorithm that provides an estimate of the pumping system's behavior offers a series of suggestions to vary the drainage speed of the pumps. The energy indicators used by the tool support decision-making. It is important to note that the operator retains the freedom to make decisions regarding water evacuation.

4. DISCUSSION

Methods of optimization by constraints yield satisfactory results in terms of energy savings for intensive energy systems. Similarly, implementation is feasible because the analysis base can focus on historical information on the system's operation and some real-time data on the operating condition of the process. On the other hand, uncertainty in the optimization results arises because the options for efficient system operation rely on hypothetical information, such as system design parameters like the point of maximum efficiency, which may be altered by the increase in friction head over time.

The tests conducted on the optimization tool show that daily energy savings exceeding 5% are achieved. It is evident that a tuning process for the parameters estimated by the model, such as the inflow rate to the well, is required. Additionally, operational parameters such as the variation in the density of incoming wastewater, whose behavior is related to the time slot, should be considered, affecting the pump speed estimation.

The algorithm that provides an estimate of the pumping system's behavior offers a series of suggestions to vary the drainage speed of the pump (i). The energy indicators used by the tool support decision-making. It is important to note that the operator retains the freedom to make decisions regarding water evacuation.

5. CONCLUSIONS

A new energy optimization model was developed to operate a parallel pumping station, incorporating analysis criteria that directly impact energy consumption in kWh. Additionally, the optimization model improves other factors such as operational reliability through the reduction of maintenance activities.

The presented model, which includes the prediction of the best activation combination of pumping units for parallel pumping systems, takes advantage of online measurements and analysis of historical operation data and manufacturer information to achieve results of up to 25% savings in consumption. of input electrical energy.

The tests conducted using the new model show that a tuning process for the parameters estimated by the model, such as the inflow rate to the well, is required. It is also important to consider additional operational parameters such as the variation in the density of incoming wastewater, whose behavior is related to the time slot, affecting the estimation of pump speed.

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