Using Fractal Geometry in Studying Architectural and Urban Patterns

Aya A. Al-Dabbagh*, Khalid J. Aldeen Ismail

Architecture Department, University of Mosul, Mosul 41002, Iraq

Corresponding Author Email: aya.22enp51@student.uomosul.edu.iq

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ABSTRACT

The objective of this study is to delve into the correlation between fractal geometry and architectural formations across various historical periods. This will be achieved through two distinct dimensions. The first dimension will scrutinize the influence of fractal geometry in shaping the architectural and urban environment, while the second dimension will delve into the role of fractal geometry in analyzing architectural and urban patterns. The importance of the research within the practical part lies in the utilization of fractal geometry tools in studying the urban environment of the old city of Mosul with its original components. The methods and results of our study may prove valuable for future research, especially in the context of restoring the old city of Mosul, which suffered damage and distortion in its urban fabric due to conflicts involving ISIS. Our study aims to analyze the characteristics of different sectors within the old city, utilizing advanced fractal geometry tools. Specifically, we will compare the features of various sectors based on the box-counting method and fractal dimension, with the goal of estimating cohesion within the urban fabric.

1. INTRODUCTION

Fractal geometry is a useful technique in spatial analysis and can provide a fresh perspective on urban and regional systems. It is suggested that a city can be viewed as a fractal system with self-similar properties. This means that cities are suitable for fractal analysis, as they are formed by the gradual influence of local factors that create organized urban patterns [1]. Fractal patterns have a significant impact on human visual perception and processing, even at short time scales. Fractal patterns are so complex that they surpass the dimensions of the universe [2].

Urban patterns consist of many detached buildings or groups of buildings, and the distances between these buildings vary considerably [3]. The urban environment encompasses a vast array of human experiences, ranging from the design of buildings to the bustling activity of the city itself. Numerous studies have shown that this system is characterized by a fragmented and unpredictable nature [4]. The central rule or idea of the fractal architecture concept keeps details of different sizes together, thereby preventing monotony through variation [5].

The study of architectural and urban environments is crucial as it reflects the cultural level, economic and political power, and social system of a society. Fractal structures can be found in various architectural and urban formations. At first sight, these formations may seem to be in a state of transition from chaos to order. In this context, fractal geometry plays an essential role in showcasing the characteristics that allow these formations to grow and develop over time. Numerous studies have also indicated that urban patterns possess fractal characteristics. Fractal geometry provides more suitable tools to describe the properties of these formations in their structure. In contrast, Euclidean geometry interprets these complex systems as chaotic formations [6]. The architect can use the tools provided by fractal geometry to work with and make the most of its characteristics in various fields. These characteristics are inherent to fractal geometry [7]. Complex forms can be described effectively through the use of fractal geometry [8].

The importance of the study focuses on the role of fractal geometry tools in studying of morphological properties of old cities, especially focusing on the old city of Mosul with its original components, shown clearly in aerial photos taken by British pilots in the 1920s. The results of our study will provide a guideline for future studies, that are concerned with the restoration of the old city of Mosul, which suffered from the effects and damage as a result of the last ISIS war between 2014 and 2017.

The practical part of the study aims to analyze the characteristics of the old city of Mosul. We achieve this by dividing the urban site into several sectors based on their function and utilizing advanced fractal geometry tools. The fractal tool represented by the box-counting method, will reveal the morphological harmony between parts of the old city, which the research assumes has a direct relationship with the functions of those sectors.

To achieve the goal of studying, firstly we need a deeper understanding of the relationship between fractal geometry and architectural and urban patterns. This study tries to answer the following questions:

(1) How can fractal analysis be used to compare the
historical development of architectural styles across different cultures? (2) How can fractal analysis be used to determine the compositional features of the building and urban environment?

To answer these questions, this study will discuss two-axis: The first axis: The impact of fractal geometry on shaping urban architectural styles.

The second axis: The role of fractal geometry in analyzing urban architectural products.

In the practical part, the research will employ the box-counting method and fractal dimension as indicators to effectively compare the compositional characteristics of the different sectors in the old city of Mosul. Finally, the research results are presented and discussed.

2. THE IMPACT OF FRACTAL GEOMETRY IN THE FORMATION OF ARCHITECTURAL AND URBAN PATTERNS

This section examines the fractal concept as a design tool and a method that mimics natural shapes or uses fractal models with similar formal characteristics at various levels of designing an architectural product. The aim is to create a complex formation that forms a precise system in the dissimilar layers of the architectural structure, similar to the way natural forms are created. As a result, this section will cover many theories and studies that have examined the impact of fractal geometry in architecture, both at the architectural and urban levels, throughout the history of architectural development.

2.1 At the architectural level

Many studies indicate that there is a clear reflection of fractal formations on architectural products. Some of them dealt with the development of fractal structures across different architectural eras, while the other section dealt with specific architectural eras.

The study conducted by Shishin and Ismail [9] delved into the concept of organic unity that constitutes the structures of certain monumental buildings. The buildings are created using the basics of fractal geometry, which is considered geometric and natural shapes. The style employed in creating these buildings bears the hallmark traits of fractals, such as self-similarity, unity and diversity, fractal rhythm, hierarchy, capacity for growth and development, boundary uncertainty, and dynamic chaos. The study also discussed various approaches to creating these structures. In the early eras of historical architectural monuments, the elements of fractal geometry were regarded as intuition and the combination of three-dimensional and vital elements with geometric shapes found in nature. As architectural eras progressed, these aspects evolved into conscious, rational fractal patterns.

The study discusses four stages of architecture development, which reveal and use fractal structures. Each era has fractal properties that characterize those stages. The formations in these productions were sometimes conscious and unconscious, depending on the intellectual references that nourished those eras and the architectural era. In early eras, fractal geometric formations were formulated from natural shapes. Later, the use of natural and geometric forms and dynamic and anthropomorphic elements became more conscious, leading to later architectural trends. These trends are characterized by the conscious use of fractal and complex nature algorithms within parameters of non-linear architecture.

The study summarizes the difference in dealing with fractal geometry from unconscious formations in eras before modern architecture to conscious tools used by the architect in shaping his architectural products in later periods with Table 1. In trends of non-linear architecture in Table 1 and Table 2 [9].

Table 1. Discovery and application of fractal structures in various stages of architecture [9]

<table>
<thead>
<tr>
<th>Stages of Architecture Development</th>
<th>Primary Sources of Fractal Formations</th>
<th>The Nature (Method) of Using Fractals</th>
<th>Principles of Creating Fractal Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Historical Architecture (Pre-Modern)</td>
<td>Geometric shapes, natural shapes</td>
<td>Intuitive</td>
<td>Torsion, linear shapes, helicity, lattice</td>
</tr>
<tr>
<td>Modernist Architecture</td>
<td>Geometric shapes and natural organic shapes</td>
<td>Mostly intuitive, with elements of mathematical modeling</td>
<td>Limited number of iterations</td>
</tr>
<tr>
<td>Postmodern Architecture</td>
<td>Geometric and natural shapes and synthesis of natural and geometric shapes</td>
<td>Rational, based on new technologies and historical methods</td>
<td>Twisty, linear, spiral, metaphorical</td>
</tr>
</tbody>
</table>

Table 2. Analysis of the manifestation of fractal structures in famous architectural monuments [9]

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Architectural Products</th>
<th>Fractal Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sierpinski’s napkin</td>
<td>Paris Opera</td>
</tr>
<tr>
<td>2</td>
<td>Sierpinski’s napkin</td>
<td>St. Basil’s Church</td>
</tr>
<tr>
<td>3</td>
<td>A computer model of vegetation and logarithmic spiral</td>
<td>Hearst Tower-Norman Foster</td>
</tr>
</tbody>
</table>
According to a study conducted in 2009 by Larkham P.J., fractal geometry has the potential to be used as a design tool by incorporating the concept of "self-similarity." Bruce Goff's Price House is an example of a design that utilizes this concept, featuring similar triangles, hexagons, and triangular shapes to structure the house. The house incorporates sixty-degree angles that are multiplied, divided, and repeated using different shapes and materials. All the elements are interconnected, creating a holistic system with an advanced level of difficulty since the design varies somewhat throughout the pattern. While the different shapes are harmonious, interconnected, and self-similar, they are never identical, as seen in Figure 1 [10].

Modern architecture, particularly in the buildings designed by Frank Lloyd Wright, incorporates the relationship between scales and draws inspiration from natural forms. Additionally, intricate forms were created that deviated from basic rules, and certain structures retained similar characteristics from small to large (Figure 2) [11].

The design of the Grand Egyptian Museum is comprised of several large frames made up of smaller, decreasingly fractured components. To wrap the building, a 750m long and 46m high translucent wall was covered using the Sierpinski triangle. The architect drew inspiration for this design from the shape of the nearby pyramids and desired to present a modern version of it (Figure 3) [12].

Fractal geometry is a significant tool because it explains the complexity of nature and formations and characteristics that can grow and adapt continuously and interdependently. This helps designers achieve adaptation and growth in functionality and spatial requirements in the built environment through time, guided by sustainable natural systems [13].

We can observe a combination of lines and components in Zaha Hadid's works that display the properties of fractals through expansion and dynamic movement. These architectural structures seem to move in space, creating a perception of uncommon and exceptional structures, which results in a design style that is distinct in urban design concepts (Figure 4) [14].

Similarly, the architects employed the principles of fractal geometry in a Moscow complex consisting of a series of towers made of dynamic self-similar rectangular blocks, one section rotating around its axis with a slight sliding motion, and the other twisting in a spiral pattern, or parts creeping along. Its axis is to move and gives lightness to the structural structure (Figure 5) [14].

Peter Eisenman utilized Mandelbrot's book "Fractals: Form, Probability, and Dimension" as the basis for the House XI project. The design incorporated the principles of fractal geometry, including iterative and self-similarity concepts. Iteratively rotating and scaling the letter "L" form produced a composition with vertical symmetry at different sizes that is Eisenman's House XI. The efficient scalability of the letter "L" allowed for the creation of a fractal architecture (Figure 6) [15].
It is possible to consider any architectural product as having fractal properties if it possesses a fractal dimension. This assessment is based on certain structural mechanisms, such as integration and multiplicity, self-similarity, interconnected hierarchy, and growth across time and space. Research has shown that incorporating a fractal dimension into architectural products leads to greater harmony with nature, which exhibits similar characteristics in its complex system.

2.2 At the urban level

Architectural and urban formations have been studied extensively, and researchers have found that their irregular and organic shapes often reflect the principles of fractal geometry. This is seen in the patterns these forms make and in the way, they gradually occupy the space that becomes accessible to them. These processes of filling voids include the occurrence of new projects next to preexisting ones, the alteration of existing sites to allow for growth, and the fractal pattern of people moving via routes and open spaces as they go from their housing units to public facilities. The distribution of built-environment sizes, such as buildings, land parcels, road networks, and more, is likewise included in the patterns [7].

Urban environments possess certain characteristics, such as heterogeneity, self-similarity, and hierarchy, which are also fundamental properties of fractal structures. Additionally, fractals can efficiently represent the complexity, density, and heterogeneity of space by using a single value, the fractal dimension, which is scale-independent. This fractal nature has been an inherent feature of urban settings since ancient times, and much of the human mind's structure is a result of this relationship. In other words, the human brain is fractal and responds to the level of complexity in the environment perceived by the human eye [6].

Schumacher's works demonstrate integration through the models that were created in a dense system. The fractal structure of the relationships between the shapes characterizes these models. The shapes are free and self-organizing, and they repeat sequentially within a dynamic system, similar to nature (Figure 7) [2].

By combining serial repetition of modules with sharp lines that match mathematical data, Le Corbusier's urban designs showcase the concept of fractals (Figure 8) [2].

Federation Square in Melbourne is a prime example of a successfully implemented fractal architecture project. The building and surrounding square combine to form a distinctive urban and architectural structure, inspired by fractal geometry. The square has been designed to serve as a public space, providing both cultural and commercial functions. It plays a vital role in the overall concept, functioning as a hub that organizes the individual buildings surrounding it. Meanwhile, its precisely designed surface, made up of sandstone triangles of various sizes and colors arranged according to the Penrose tiling, creates a grand and impressive impression. The architects used non-periodic divisions in the project, which resulted in a chaotic yet periodically repeating pattern (Figure 9) [16].

Based on previous studies, we observed that some studies focused on the analytical aspect of architecture. They presented examples that incorporated fractal geometry, such as the study by Shishin and Ismail [9]. While other studies focused on the descriptive aspect of architecture and urban environments.

3. THE ROLE OF FRACTAL GEOMETRY IN ANALYZING ARCHITECTURAL AND URBAN PRODUCTS-PRACTICAL STUDY

The following axis describes the fractal concept as an analytical tool and scientific method used to measure complexity in architectural structures. This involves calculating the fractal dimension in order to characterize these products and integrate them into architectural curricula or evaluations. To identify the morphological characteristics of Old Mosul, fractal engineering tools will be used as a case study.

A 2009 study conducted by Larkham P.J. examined Tehran as a case study and utilized fractal dimension measurement to evaluate morphological change and classify patterns. This process requires two important factors: the presence of growth elements in the organic plan with diverse buildings and their
4. METHODOLOGY AND METHODS/MATERIALS

The method of fractal dimension analysis has been used by many researchers, they employed fractal analysis to characterize the complexity of urban and natural skylines, and have investigated fractal dimension in relation to urban design qualities and urban character [17], this method is built on the research of Lorenz [18] and Bovill [19], who suggested using the box-counting dimension method (DB) to determine how fractal an architectural design is, this fractal dimension is a numerical quantity that gives an indication of how completely a fractal seems to fill space, as one zooms down to finer scales [20].

The box-counting approach has been used extensively for architectural and urban analysis. For example, urban forms, street patterns, and skylines have been repeatedly described or analyzed using this method [21]. Here, the standard software application FracLac for image J has been used to calculate the fractal dimensions by the box-counting method, allowing one to make a fractal analysis of building projections [22]. When dealing with complicated images that other conventional approaches are unable to describe, DB makes it simple to find the fractal dimension of complex images. The idea behind the approach is that the degree of "roughness" and "irregularity" of the structure, which determines the object's degree of complexity [9].

The method is used as follows: a grid of a certain size (S1) is superimposed on the image and the number of cells including details of the image, which can then be calculated (N for s1). Then the size of the grid is reduced (S2) and again the number of cells is counted (N for s2).

The fractal dimension between two scales is then calculated by the connection between the difference in the number of boxes occupied and the variance of inverse mesh sizes. The calculation can be expressed mathematically by Eq. (1)

\[
DB(1 - 2) = \frac{\log N(s_2) - \log(s_1)/\log(1/s_2) - \log 1/s_1}{\log 1/s_2}
\] (1)

or

\[
DB(1 - 2) = \frac{\log N(s_2)/N(s_1))/\log 1/s_2}{\log 1/s_2}
\] (2)

S - size of the grid,
N - number of cells that cover the image details.
where, S is the size of the grid, and N is the number of cells that overlap with the image details [18, 19].

The procedure of the method used in this study will be as follows:

For a more objective analysis of the consistency between two levels of fractal graphs the correlation (or "CORREL" function in Microsoft Excel software) will be used.

The equation for the correlation coefficient is as follows:

\[
CORREL(X, Y) = \frac{\Sigma(x-x)(y-y)}{\sqrt{\Sigma(x-x)^2}\Sigma(y-y)^2}
\] (3)

where, \(x\) and \(y\) are example values of X (array1) and Y (array2) ("AVERAGE" (array1) and "AVERAGE" (array2) in the software of Microsoft Excel (2016).

This function displays the level of correlation between the graphs of the several data sets. Here, the character of the fractal dimension in different scales (which usually can be obviously...
The research methodology involves both quantitative and qualitative approaches. Quantitative values are represented by fractal dimension within the calculations of the box-counting method, and at the same time, these values represent qualitative characteristics of the urban environment.

The limits of the research in practical part, include the general urban level, leaving the field of study of the architectural level for future studies.

5. OLD CITY OF MOSUL-URBAN CHARACTERISTICS AND CASE STUDY

The city of Mosul fully represents those critical aspects of reconstruction listed above. Mosul is actually the expression of a very changeable urban whole, particularly in its residential areas, signifying a diffuse urban pattern that is difficult to identify because so few studies have been done on it. The city's sole monographic text was written in 1920, edited by the archaeologist Ernst Herzfeld. Mosul, which had to undergo reconstruction following the war events connected to the fight against ISIS, is a prime illustration of all those locations that run the risk of fading into obscurity in the lack of a strong typological consciousness [23]. The question that not opened, is the consistency of the city in the connections between sectors and collective space made up of cul-de-sac streets.

To study the area, it is divided into 10 sectors, based on the main paths that separate these sectors from each other, in order to compare them in terms of morphological characteristics in their original compositional structure through the measurement method described above. The study assumes that there is a similarity in the typological characteristics of these sectors, with a difference in the compositional characteristics according to the nature of the uses. For example, sector 1 is religious and includes the Grand Al-Nuri Mosque and also includes the Church of the Clock, while sector 5 includes the old market area of the city. Therefore, the study assumes the presence of diversity in some architectural characteristics of some of these sectors within a typological unit in its compositional urban content (Figure 11).

![Figure 11. (a) Existing plan of the old city of Mosul was photographed by British pilot in 1924 data, analysis, and results; (b) Division of the old city into 10 sectors](image)

6. DATA, ANALYSIS AND RESULTS

The study uses the box-counting method to calculate the fractal dimension for different sectors in Mosul. Sector 1 represents the religious part where the Grand Mosque is located, while sector 5 is the commercial market of the old city. The remaining sectors are residential. Table 3 shows the fractal measures for each sector on different scales, highlighting the differences in results. These results answer the second research question and demonstrate the potential of fractal geometry in defining the characteristics of urban environments, as presented in Table 4 and Figure 12.

![Figure 12. Represented results of fractal dimension of old city districts in Mosul](image)
7. DISCUSSION OF RESULTS AND CONCLUSION

During successive architectural eras, the studies showed a clear reflection of some fractal models and models on architectural products, consciously or unconsciously through a group of works by some architects that illustrate the application of fractal geometry in them to form patterns. Urban, thus clarifying the characteristics of these patterns represented by self-similarity, heterogeneity, and hierarchy, at the architectural and urban levels.

As for the results of the practical study, they indicate, based on the fractal scale values, that the morphology of the old city of Mosul is similar in terms of typological characteristics at the level of all its sectors. This is due to the unified natural, social, and technological factors that contributed to shaping the urban context and this is clearly indicated by Table 1 and Figure 12. We notice the harmony of fractal behavior between the different sectors of the ancient city.

The results also indicate that there is a difference in harmony in the fractal characteristics between the different sectors of the old city, especially between sector 1, which represents the religious center) and the neighboring residential sectors. We also notice the difference in structural characteristics between sector 5, which represents the commercial center) and the other residential sectors. This can be explained by the different functional type for each of sectors 1 and 5, which distinguishes them from the neighboring residential sectors. While we note the high harmony of the fractal characteristics between functionally similar sectors, such as sectors 6 and 7, among them, we also note that harmony between the two residential sectors 4 and 7. This supports the research hypothesis referred to above. The variation in harmony value between sectors, indicated by the results can be attributed to the different function of each sector. This observation highlights the diversity and distinctive urban characteristics within the old city of Mosul. Each sector’s unique purpose and role contribute to the overall complexity and richness of the urban fabric.

This is the answer to the second question in the research. Fractal geometry proved effective in identifying the morphological characteristics in the urban environment of the old city of Mosul by considering specific values. These values highlighted each sector individually and demonstrated the degree of cohesion or lack of cohesion between there and the adjacent sectors that differ in function.

It can be said that fractal geometry has provided analytical tools that can be used to study the morphological characteristics of the urban context in general and the traditional one in particular, through which conclusions can be reached that may be useful for understanding and comprehending the complex compositional structure of the traditional urban context. This is considered an important issue, especially within urban planning or urban development policies for traditional cities, in order to preserve the original morphological characteristics with an organic system and ensure that the urban fabric does not disintegrate. The results of the study can also be adopted in maintenance or rehabilitation work for the traditional urban fabric in the old city of Mosul, especially after it was subjected to comprehensive destruction after the last war, in order to preserve that architectural heritage.

A potential field of future studies could be a comparative analysis of urban characteristics (between the original fabric and present), that applies the same research tools to the old city of Mosul, but on an present map, might taken by newer tools like GPS. This would allow us to conduct a comparative study of the same areas in the city, taking into account the new reality of the situation, including the new sectors that were added after the changes brought about by the war. The aim would be to assess the cohesion of these new sectors and compare them to the original old ones.

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